



Is Perfect Fluidity of the sQGP ***necessary*** in light of recent BES & D+Au & p+Au data from RHIC and LHC ?

Gyulassy Miklós
Wigner Research Center KFKI
(Columbia University)

Collaborators: Levai P, Vitev I, Biro T

Abstract:

Recent low $p_T < 2$ GeV azimuthal correlation data from the beam energy scan (BES) and D+Au at RHIC/BNL and the especially the surprising low p_T azimuthal $v_n(p_T)$ in p+Pb at LHC challenge long held assumptions about the necessity of perfect fluidity (minimal viscosity to entropy $\sim 1/4\pi$) to account for azimuthal asymmetric "flow" patterns in A+A.

Perfect fluidity is certainly sufficient to fit all A+A and even p+A, data,
but is it really necessary, i.e., is it a unique property of sQGP?

I discuss **basic** pQCD interference phenomena from beam jet color antenna arrays that may help unravel current BES+DA+pA vs AA puzzles without requiring perfect fluid hydrodynamic or CGC Glasma diagrams, but only LO Feynmann diagrams.

Slide 3-4: The Case for **Perfect Fluidity** and CGC/Glasma **Sufficiency** for Flow Harmonics in Non-central A+A as of QM12

Slides 5-14 : Trouble for Fluidity since 2012

- a) Beam Energy **independence**
- b) $p(D)+A = A+A$ v_2 and v_3 Beam size **independence**
- c) large **rapidity-even v1 Dipole**

Slides 15-27 : Is Perfect Fluidity **Necessary ??**

Are v_n in B+A mostly Nonabelian Wave interference?

(see also QM talks by Venugopalan , Dusling, Schenke... & poster of Biro, Schram)

Could **rapidity-even v odd Dipole, Triangular, etc** harmonics be just fingerprints of basic pQCD beam jet anisotropic bremsstrahlung?

Work in Progress, all order in opacity pQCD beam jets v_n
Bremsstrahlung in P+A via HIJING replacing ARIADNE with VGB

Non-Abelian Bremsstrahlung and Azimuthal Asymmetries in High Energy p+A

M. Gyulassy,^{1,2,*} P. Levai,¹ I. Vitev,³ and T. Biro¹

¹MTA WIGNER Research Centre for Physics, RMI, Budapest, Hungary

²Department of Physics, Columbia University, New York, NY 10027, USA

³Theoretical Division, Los Alamos National Laboratory, Los Alamos, NM 87545, USA

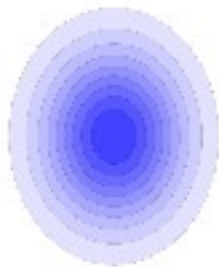
(Dated: May 13, 2014 v11)

Higher harmonic flow **Fourier starts at $n=1$**

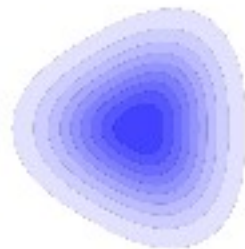


$$\frac{dN}{d\phi} = \frac{N}{2\pi} \left(1 + \sum_{n=2} (2v_n \cos[n(\phi - \psi_n)]) \right)$$

When including fluctuations, all moments appear:



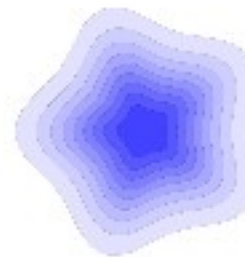
$n = 2$



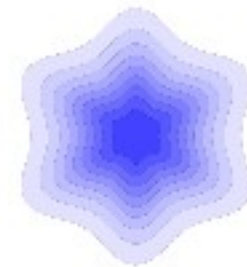
$n = 3$



$n = 4$



$n = 5$



$n = 6$

also v_1 and $n > 6$

Compute $v_n = \langle \cos[n(\phi - \psi_n)] \rangle$

with the event-plane angle $\psi_n = \frac{1}{n} \arctan \frac{\langle \sin(n\phi) \rangle}{\langle \cos(n\phi) \rangle}$

Part 1: case for perfect fluidity with CGC IS

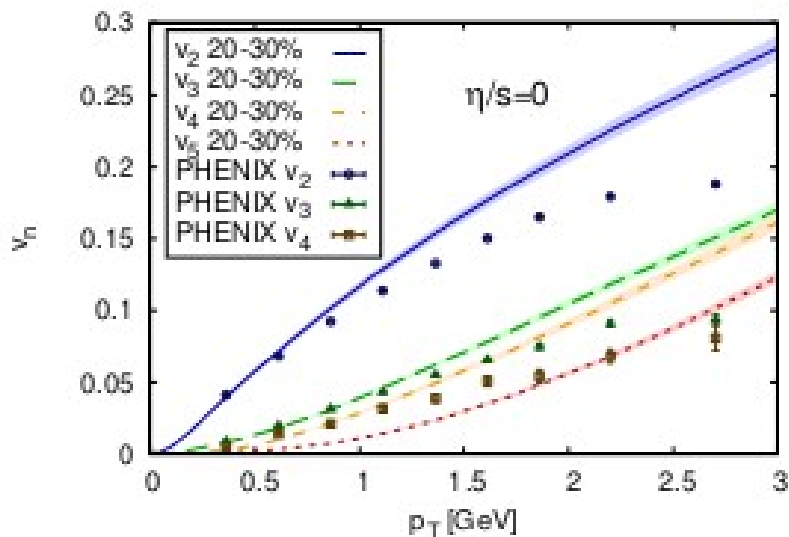
Using higher harmonics to determine η/s

Viscosity/entropy
Of perfect fluid QGP

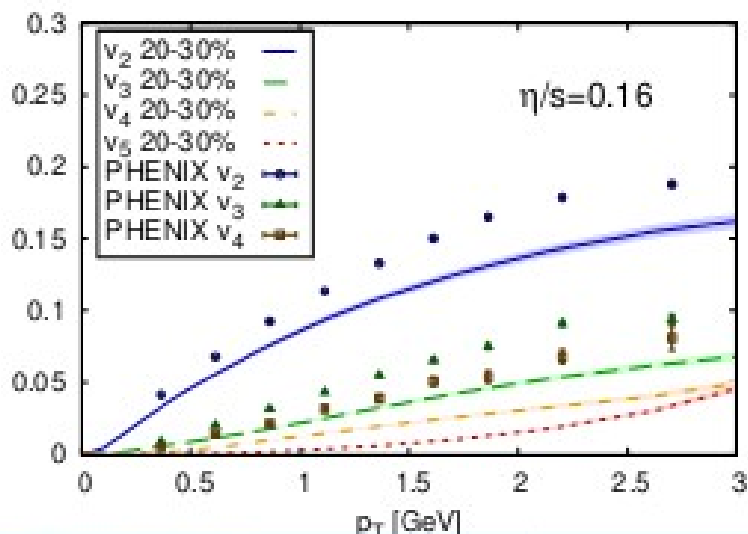
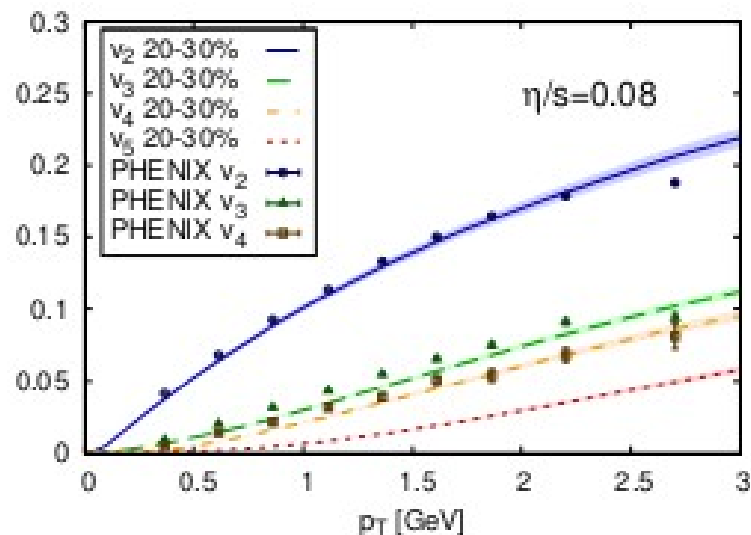


B. Schenke, S. Jeon, C. Gale, arXiv:1109.6289

Data is from event-plane method. Calculations are $\sqrt{\langle v_n^2 \rangle}$.



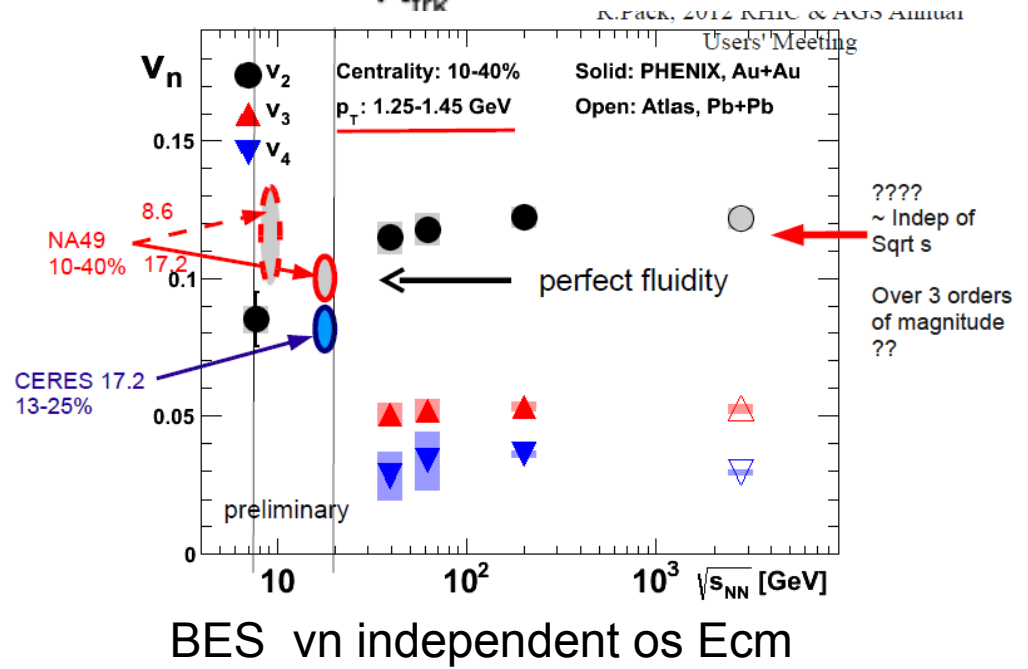
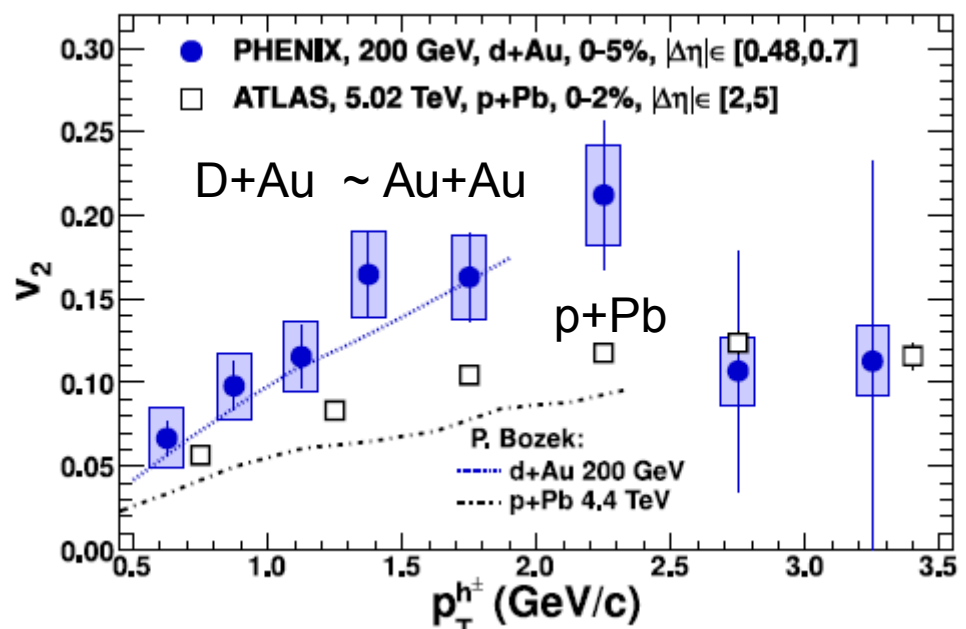
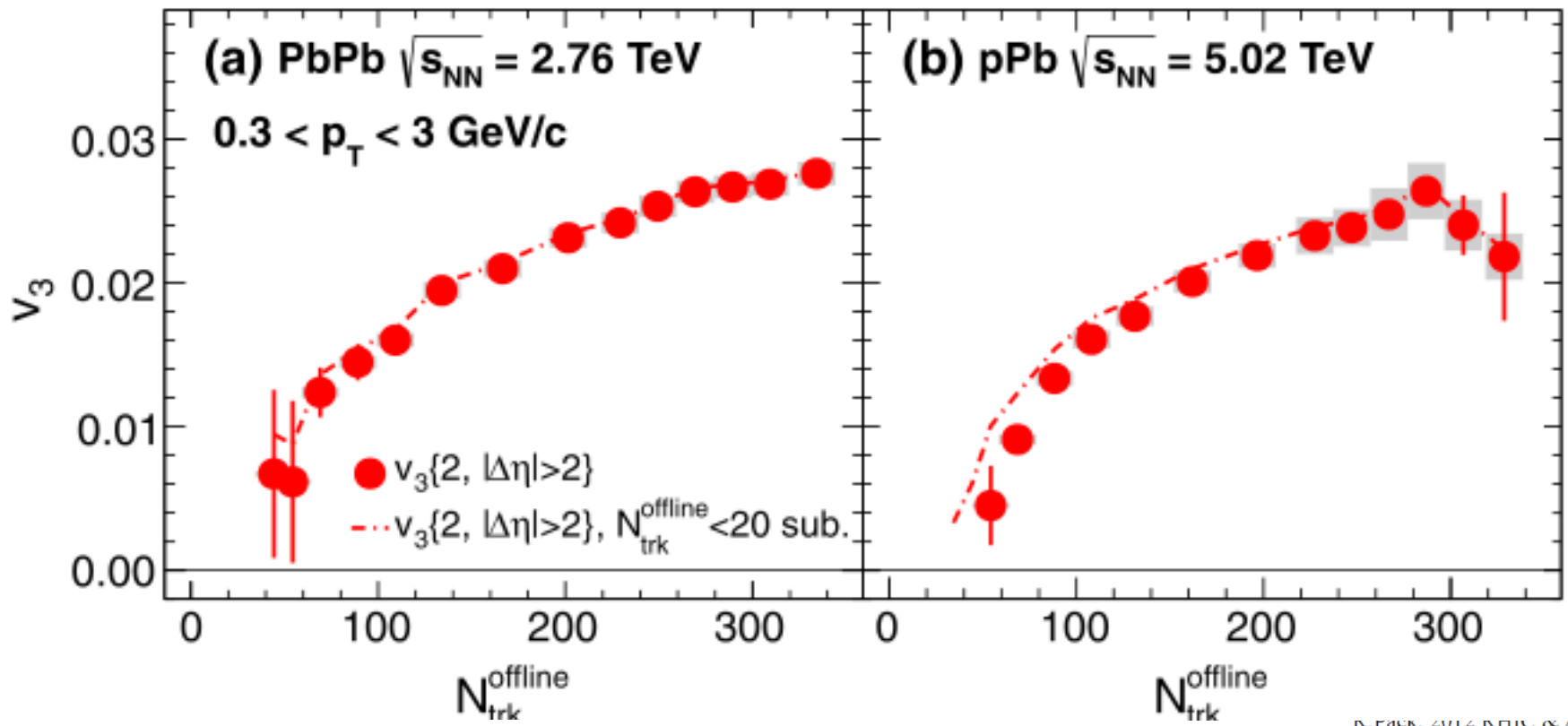
MC-Glauber initial conditions



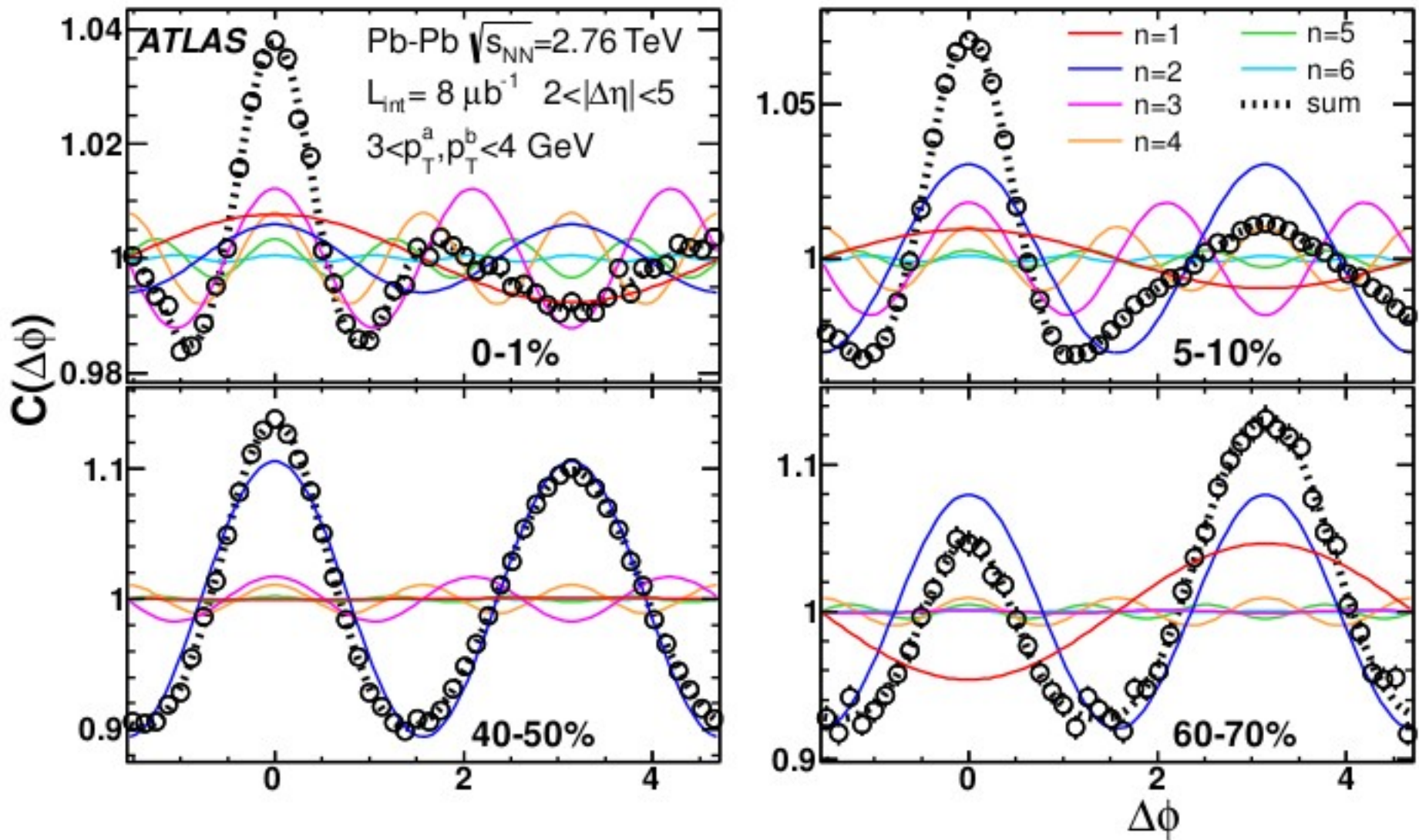
This is promising.
Need systematic study of all v_n as
function of initial conditions,
granularity, η/s , ...

Experimental data: PHENIX, arXiv:1105.3928

QM12 p(D) + A & BES surprises: $v_3(p+Pb) = v_3(Pb+Pb)$; $v_2(D+Au) = v_2(Au+Au)$; E_{cm} independent



PQCD Color Scintillations or Lumpy CGC perfect hydro response ?
or complex combo of many effects?

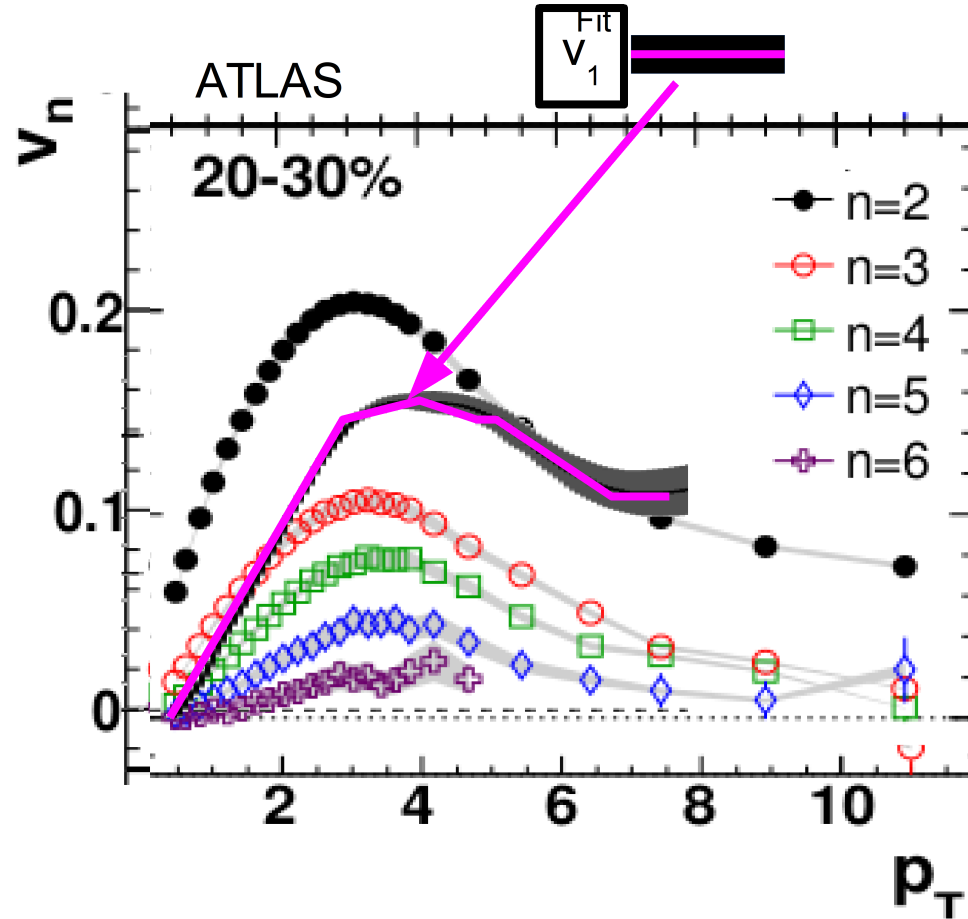


The magnitude of $v1,1 = \langle \cos[\phi_1 - \phi_2] \rangle$ **Red** is large for $p_1 \sim p_2 \sim 3-4$ GeV

After subtracting dijets and mom.consv a new rapidity even Dipole!

$$\chi^2 = \sum_{a,b} \frac{(v_{1,1}(p_T^a, p_T^b) - [v_1^{\text{Fit}}(p_T^a)v_1^{\text{Fit}}(p_T^b) - cp_T^a p_T^b])^2}{(\sigma_{a,b}^{\text{stat}})^2 + (\sigma_{a,b}^{\text{sys,p2p}})^2},$$

Rapidity-even, positive collective dipole $\langle \text{Cos}[\phi_1 - \phi_1] \rangle \sim -$ (negative mom.conserv. term)



Is v_1 an artifact of fit? See *

A hydro response? See Schenke

or is it a fingerprint of basic pQCD Color Scintillation?

*Beware v_1 is notoriously tricky, see L. Csernai, H. Stoecker, JPG 2014

Below we show rapidity-even ATLAS v_1 Ridge component natural from pQCD Bremsstrahlung

Fuctuating Dipole can also appear

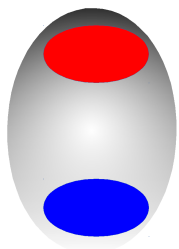


Higher harmonic flow

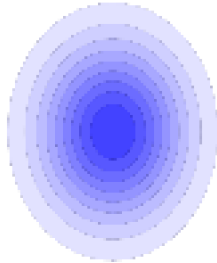
Fourier starts at $n=1$

$$\frac{dN}{d\phi} = \frac{N}{2\pi} \left(1 + \sum_{n=1} (2v_n \cos[n(\phi - \psi_n)]) \right)$$

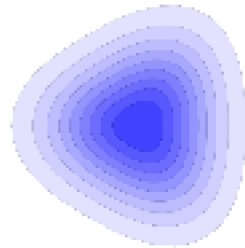
When including fluctuations, all moments appear:



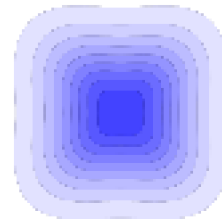
$n=1$



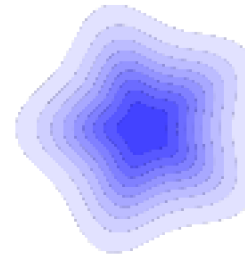
$n = 2$



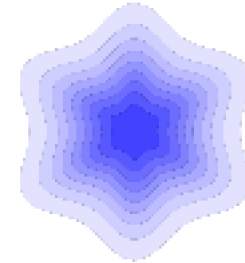
$n = 3$



$n = 4$



$n = 5$



$n = 6$

also v_1 and $n > 6$

Compute $v_n = \langle \cos[n(\phi - \psi_n)] \rangle$

with the event-plane angle $\psi_n = \frac{1}{n} \arctan \frac{\langle \sin(n\phi) \rangle}{\langle \cos(n\phi) \rangle}$

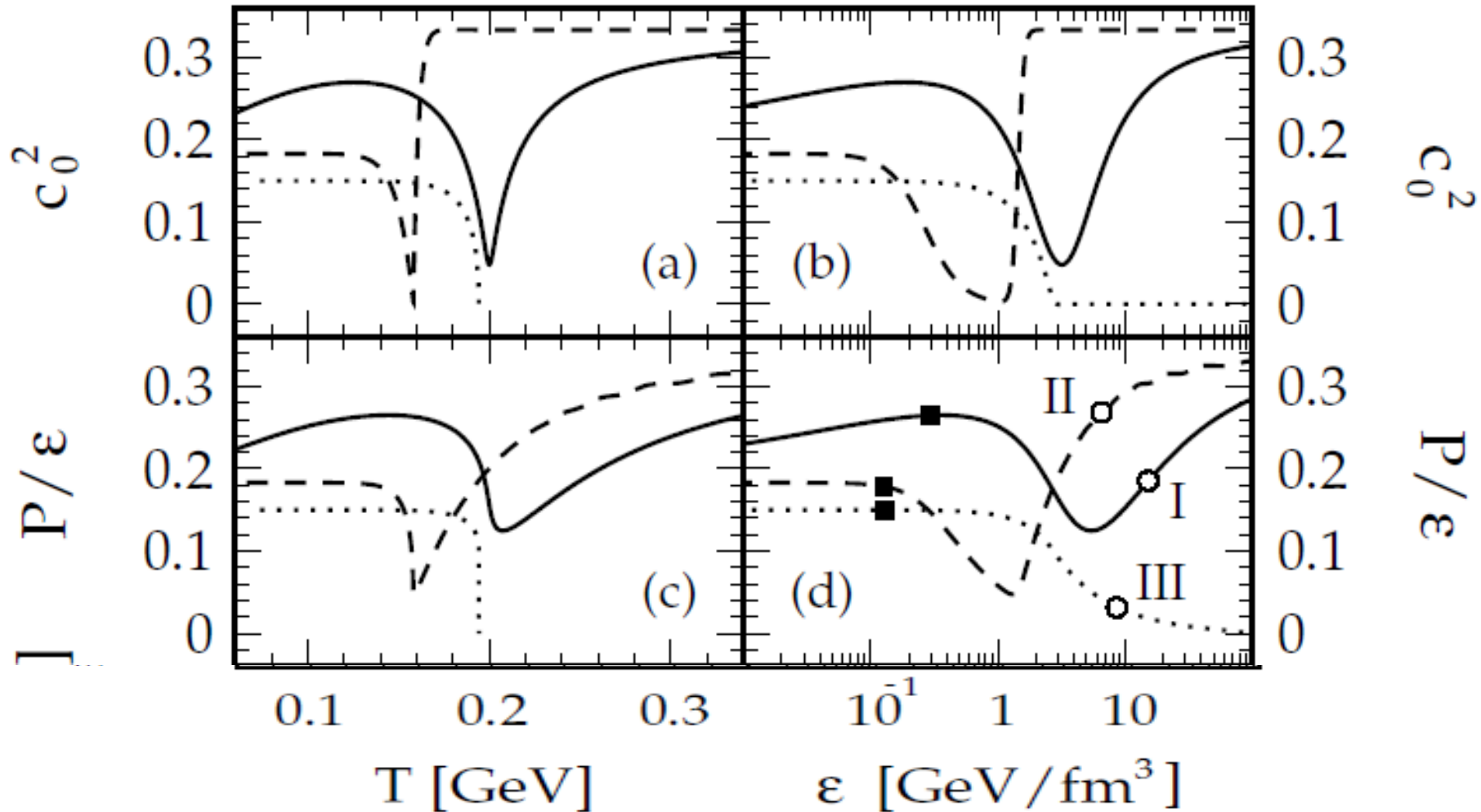
[! Danger ! Hydro can fit anything]

B.R. Schlei, D. Strottman, N. Xu

158-GeV/A Pb + Pb

[Nucl-th/9801045](#); [9710047](#), [9706037](#)

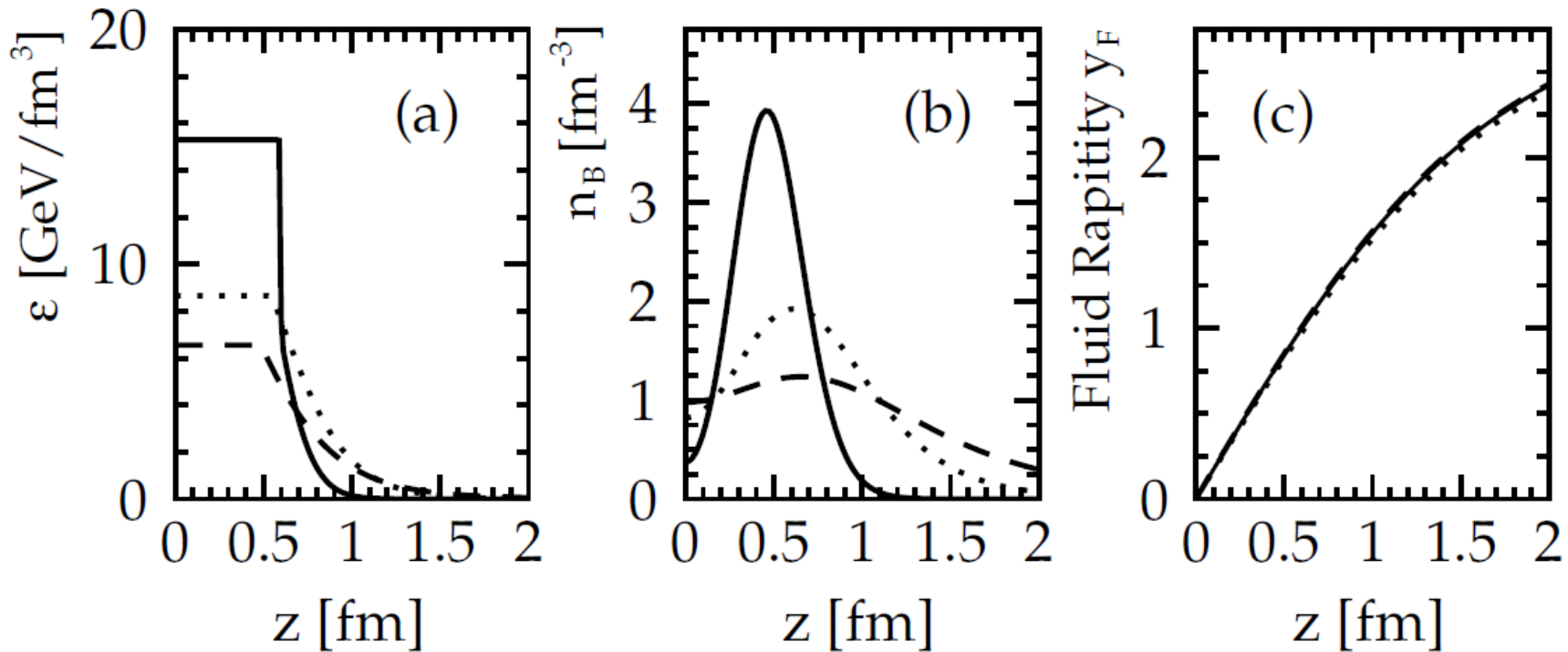
Ideal Landau 3D Hydro with different EOS I (solid), II (dashed), III(dotted)



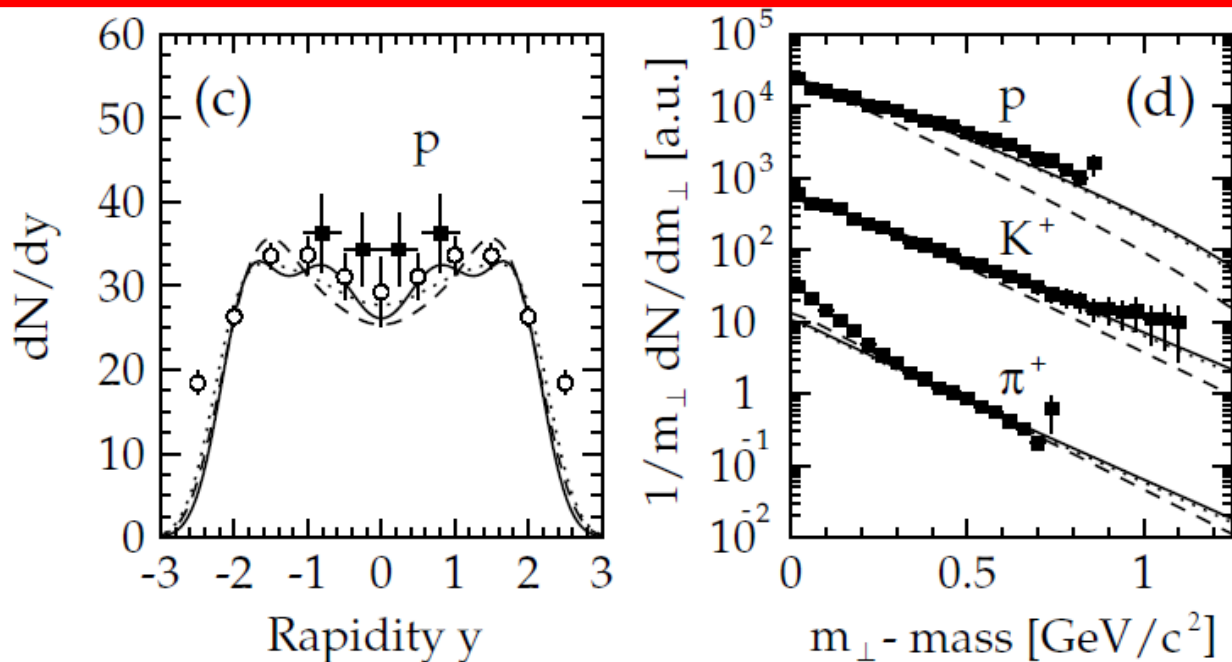
NA44 and NA49 data could be post-dicted with Ideal relativistic hydrodynamics with **ANY Equation of State**

Given freedom to adjust **Initial** and **Freeze-out** Conditions

Three different hydro evolutions that could fit SPS data



Adjusted initial/final conditions *sufficient* for even ideal hydro to fit the data



Mathematician's delight can be Physicist's Nightmare

K. Weierstrass (1885) Theorem:

“Über die analytische Darstellbarkeit sogenannter willkürlicher Functionen einer reellen Veränderlichen’

Sitzungsberichte der Königlich Preussischen Akademie der Wissenschaften zu Berlin, 1885 (II).

If f is a continuous real-valued function defined on the set $[a,b] \times [c,d]$ and $\varepsilon > 0$, then there exists a polynomial function in two variables such that

$$|f(x,y) - p(x,y)| < \varepsilon$$

for all x in $[a,b]$ and y in $[c,d]$.



Corrolary 1: Fourier transforms exists

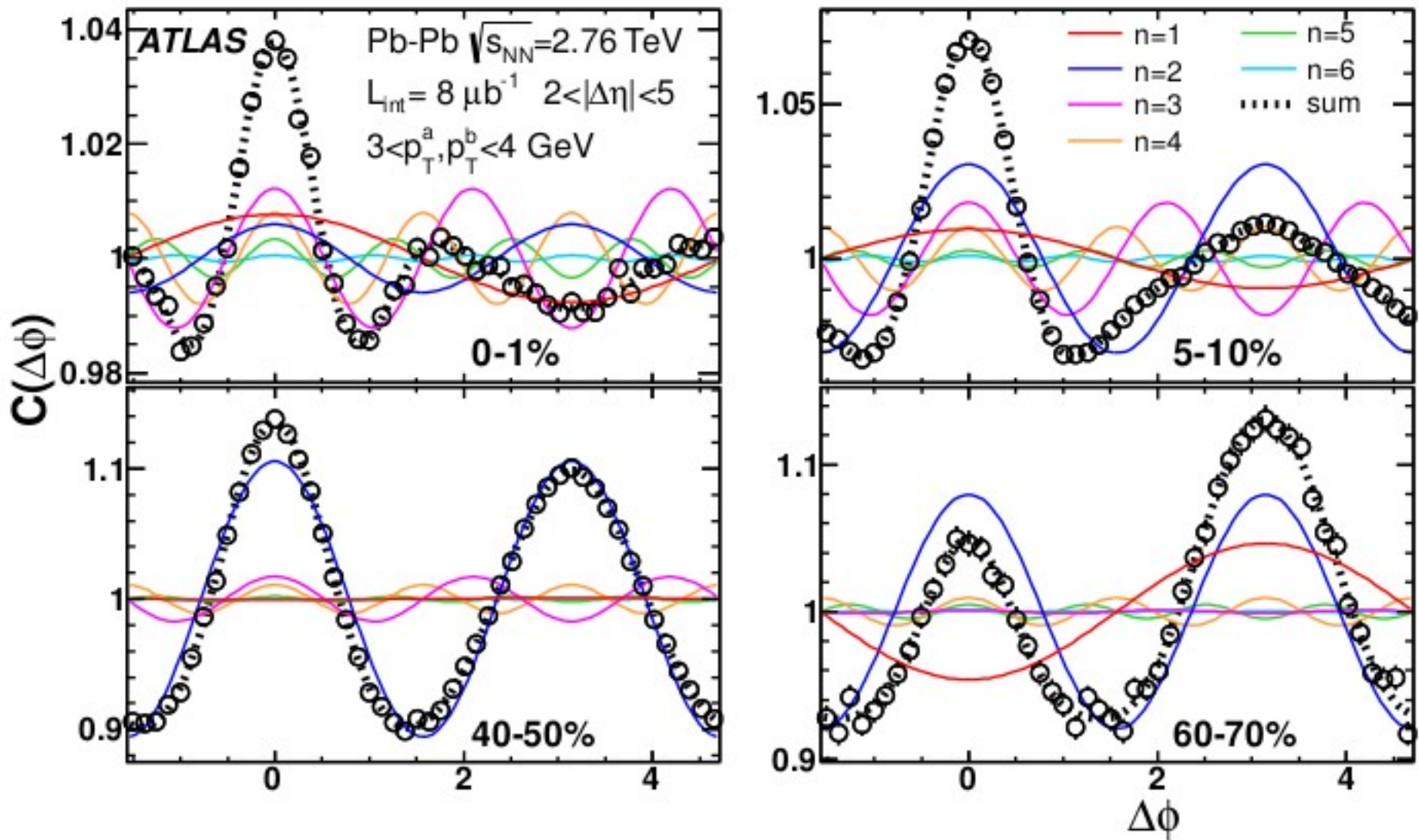


Corrolary 2: ideal hydro can fit anything



Weierstrass

pQCD Color Scintillations or Lumpy CGC+perfect hydro response ? or complex combo of many effects?



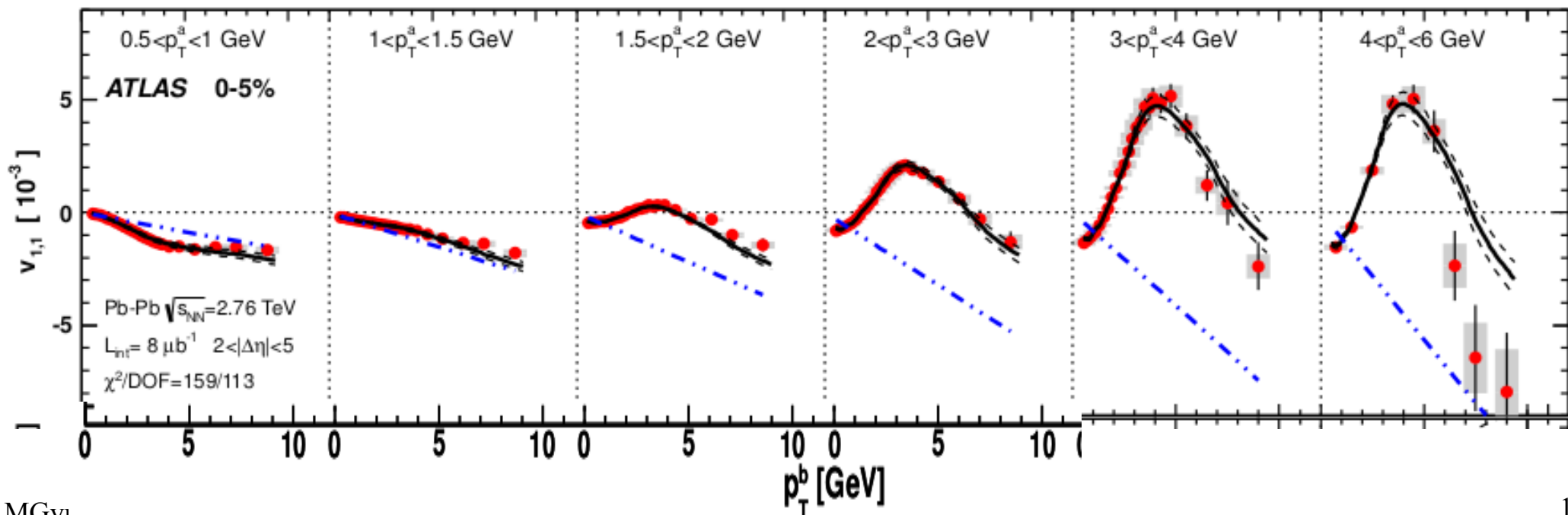
The magnitude of $v_{1,1} = \langle \cos[\phi_1 - \phi_1] \rangle$ **Red** is large for $p_1 \sim p_2 \sim 3-3$ GeV

General problem: Have we lost calibration control of v_n Barometer to verify lattice EOS?

The magnitude of $v_{1,1} = \langle \cos[\phi_1 - \phi_1] \rangle$ **Red** is large for $p_1 \sim p_2 \sim 3 \text{ GeV}$

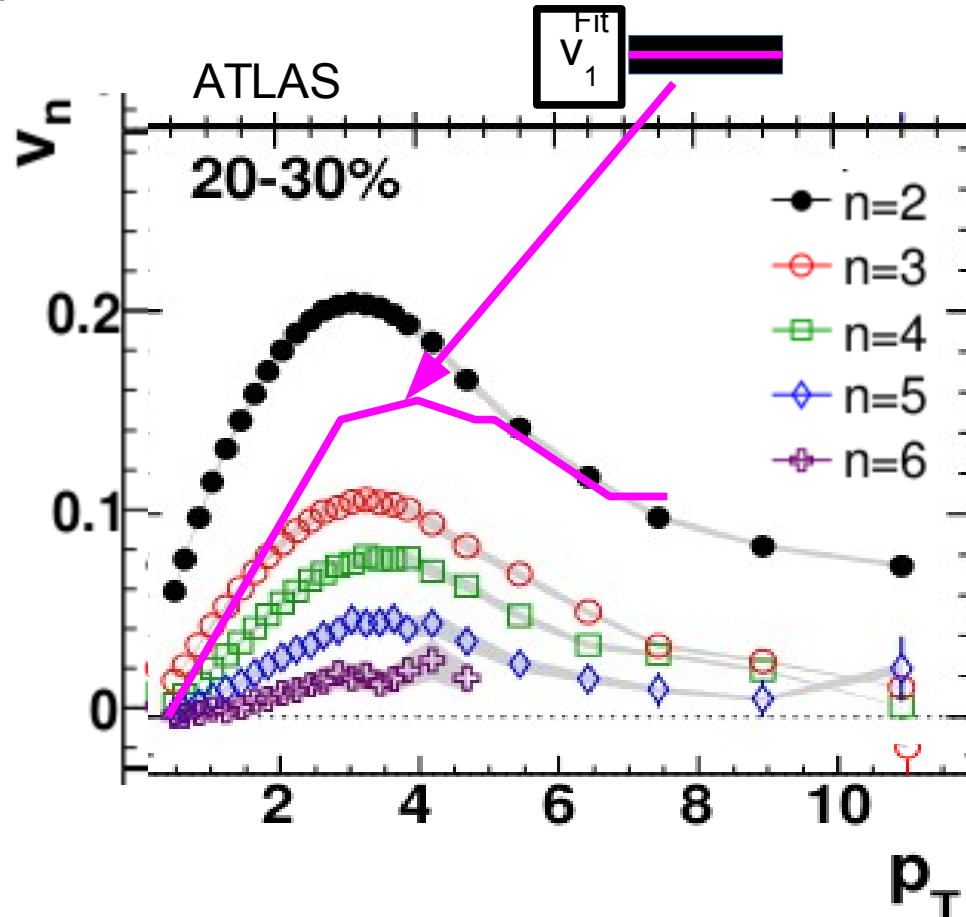
below). The rapidity-even v_1 signal is believed to arise from the dipole asymmetry of the nuclear overlap due to fluctuations in the initial geometry [11, 68]. This spatial asymmetry results in a dipole anisotropy of the pressure gradient, which drives the v_1 . If the rapidity-even v_1 depends weakly on η [68], similar to the higher-order v_n , its contribution to $v_{1,1}$ should be positive and vary weakly with $\Delta\eta$. In this case, Eq. 19 can be simplified to:

$$v_{1,1}(p_T^a, p_T^b) \approx v_1(p_T^a)v_1(p_T^b) - \frac{p_T^a p_T^b}{M \langle v_T^2 \rangle}. \quad (20)$$



$$\chi^2 = \sum_{a,b} \frac{(v_{1,1}(p_T^a, p_T^b) - [v_1^{\text{Fit}}(p_T^a)v_1^{\text{Fit}}(p_T^b) - cp_T^a p_T^b])^2}{(\sigma_{a,b}^{\text{stat}})^2 + (\sigma_{a,b}^{\text{sys,p2p}})^2},$$

Rapidity-even, positive collective dipole $\langle \text{Cos}[\phi_1 - \phi_1] \rangle \sim -$ (negative mom.conserv. term)



Is v_1 an artifact of fit? See *
 A hydro response ? See Schenke
 or is it a fingerprint of basic
 pQCD Color Scintillation?

*Beware v_1 is notoriously tricky, see L. Csernai, H. Stoecker, JPG 2014

Here we take rapidity-even ATLAS v_1 Ridge component as physics beyond naïve v_2, v_3 flow

II. FIRST ORDER IN OPACITY (GB) BREMSSTRAHLUNG AND AZIMUTHAL ASYMMETRIES v_n

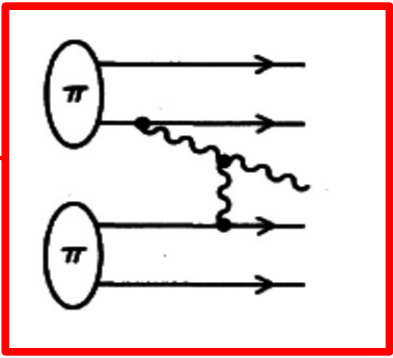
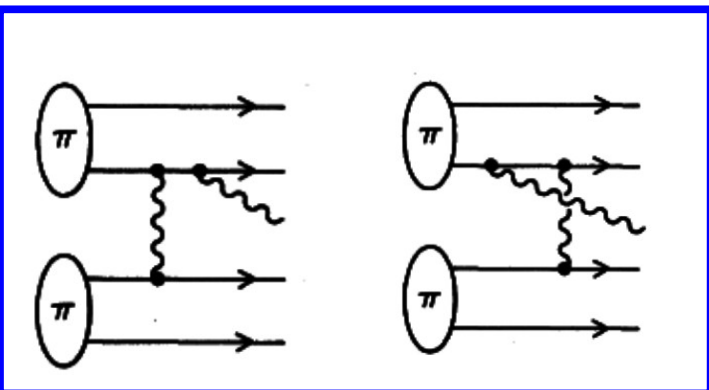
The above puzzles with BES and $D + Au$ at RHIC and with $p + Pb$ at LHC and models proposed so far motivate us to consider simpler more basic perturbative QCD sources of azimuthal asymmetries. The well known non-abelian bremsstrahlung Gunion-Bertsch (GB) formula[29] for the soft gluon radiation single inclusive distribution is

$$\frac{dN_g^{(1)}}{d\eta d^2\mathbf{k} d^2\mathbf{q}} \equiv f(\eta, \mathbf{k}, \mathbf{q})$$

$$= \frac{C_R \alpha_s}{\pi^2 k^2} \frac{\mu^2}{(\mathbf{q}^2 + \mu^2)} \frac{P_\eta}{(\mathbf{k} - \mathbf{q})^2 + \mu^2}$$

$$\equiv \frac{F \cdot P}{A_{kq} - \cos(\phi - \psi)}$$

Color Dipole
Form factor

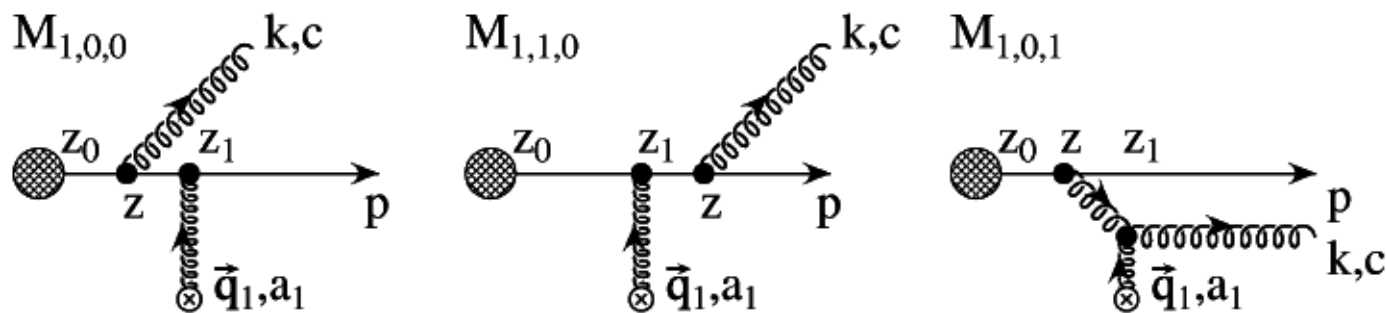


Gluon Bremsstrahlung peaks in transverse direction near net momentum transfer $\vec{Q} = (Q, \Psi)$ that also defined reaction Event Plane (EP)

Basic Non-Abelian feature: uniform *rapidity-even* distributed (unlike QED)

Of course also peaks in beam direction $1/k^2$ (as in QED)

Basic lowest order pQCD Bremsstrahlung



$$M_{1,0,1} = J(p)e^{ipx_0}(-i) \int \frac{d^2\mathbf{q}_1}{(2\pi)^2} v(0, \mathbf{q}_1) e^{-i\mathbf{q}_1 \cdot \mathbf{b}_1} \times 2i g_s \frac{\boldsymbol{\epsilon} \cdot (\mathbf{k} - \mathbf{q}_1)}{(\mathbf{k} - \mathbf{q}_1)^2} e^{i(\omega_0 - \omega_1)z_1} (e^{i\omega_1 z_1} - e^{i\omega_1 z_0}) [c, a_1] T_{a_1}$$

$$M_{1,0,0} + M_{1,1,0} = J(p)e^{ipx_0}(-i) \int \frac{d^2\mathbf{q}_1}{(2\pi)^2} v(0, \mathbf{q}_1) e^{-i\mathbf{q}_1 \cdot \mathbf{b}_1} \times (-2i g_s) \frac{\boldsymbol{\epsilon} \cdot \mathbf{k}}{\mathbf{k}^2} e^{i\omega_0 z} (c a_1 T_{a_1} - a_1 c T_{a_1})$$

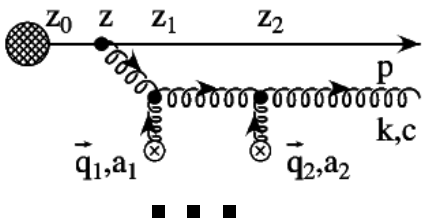
$$\frac{dE_{\text{ind}}^{(1)}}{dx} = \frac{C_R \alpha_S}{\pi} \frac{L}{\lambda} E \int \frac{d\mathbf{k}^2}{\mathbf{k}^2 + m_g^2 + M^2 x^2} \int \frac{d^2 \mathbf{q}_1}{\pi} \frac{\mu^2}{(\mathbf{q}_1^2 + \mu^2)^2} \times 2 \frac{\mathbf{k} \cdot \mathbf{q}_1 (\mathbf{k} - \mathbf{q}_1)^2 + (m_g^2 + M^2 x^2) \mathbf{q}_1 \cdot (\mathbf{q}_1 - \mathbf{k})}{\left(\frac{4Ex}{L}\right)^2 + ((\mathbf{k} - \mathbf{q}_1)^2 + M^2 x^2 + m_g^2)^2},$$

LPM
Formation
Coherence

Triple gluon vertex
Radiation Dipole focuser
Into reaction plane

“Dead Cone”
Massive quarks
HTL gluons

A higher order opacity amplitude



$L/\lambda=1$

$L/\lambda=2$

$L/\lambda=M$

$$k - q_1 \longrightarrow k - (q_1 + q_2) \longrightarrow k - (q_1 + \dots + q_M)$$

e-b-e In Plane focusing gets stronger with opacity

ϕ is the azimuthal angle of \mathbf{k} and ψ is the azimuthal angle of \mathbf{q} and abbreviations

$$A \equiv A_{kq} \equiv (k^2 + q^2 + \mu^2)/(2kq) \geq 1$$

$$F \equiv F_{kq} \equiv \frac{C_R \alpha_s}{\pi^2 k^2} \frac{\mu^2}{(\mathbf{q}^2 + \mu^2)} \frac{1}{2kq} P_\eta$$

Kinematic rapidity envelope $P_\eta \equiv (1 - e^{Y_T - \eta})^{n_f} (1 - e^{\eta - Y_P})^{n_f}$,

$$\begin{aligned} v_n(k, q, \psi) f_0(k, q) &\equiv \int \frac{d\phi}{2\pi} \cos(n\phi) f(\eta, k, \phi, q, \psi) \\ &= F \int \frac{d\phi}{2\pi} \frac{\cos(n\phi)}{A - \cos(\phi - \psi)} \\ &= \cos(n\psi) F \int \frac{d\phi}{2\pi} \frac{\cos(n\phi)}{A - \cos(\phi)} . \end{aligned}$$

$f_0 \equiv \int d\phi f = \int d\phi d^7 N / d\eta dk^2 d\phi dq^2 d\psi$ is the ϕ integrated single gluon inclusive

$$dN / d\eta dk^2 = F_{kq} P_\eta / (A_{kq}^2 - 1)^{1/2}$$

A single GB color antennas has analytic vn:

$$A_{kq} \equiv (k^2 + q^2 + \mu^2)/(2k q) \geq 1$$

$$v_1^{GB}(k, q, \psi) = \cos[\psi](A_{kq} - \sqrt{A_{kq}^2 - 1})$$

1/ Dipole size

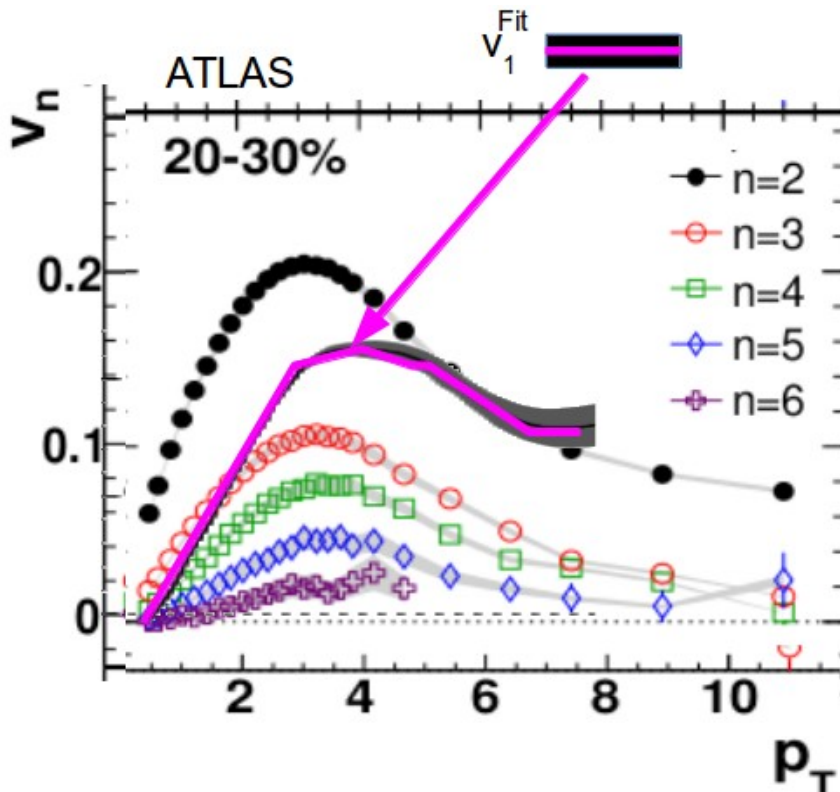
$$\lim_{\mu \rightarrow 0} v_1^{GB}(k, q, 0) = (k/q) \theta(q - k)$$

$$v_n^{GB}(k, q, \psi) = \cos[n\psi] (v_1^{GB}(k, q, 0))^n$$

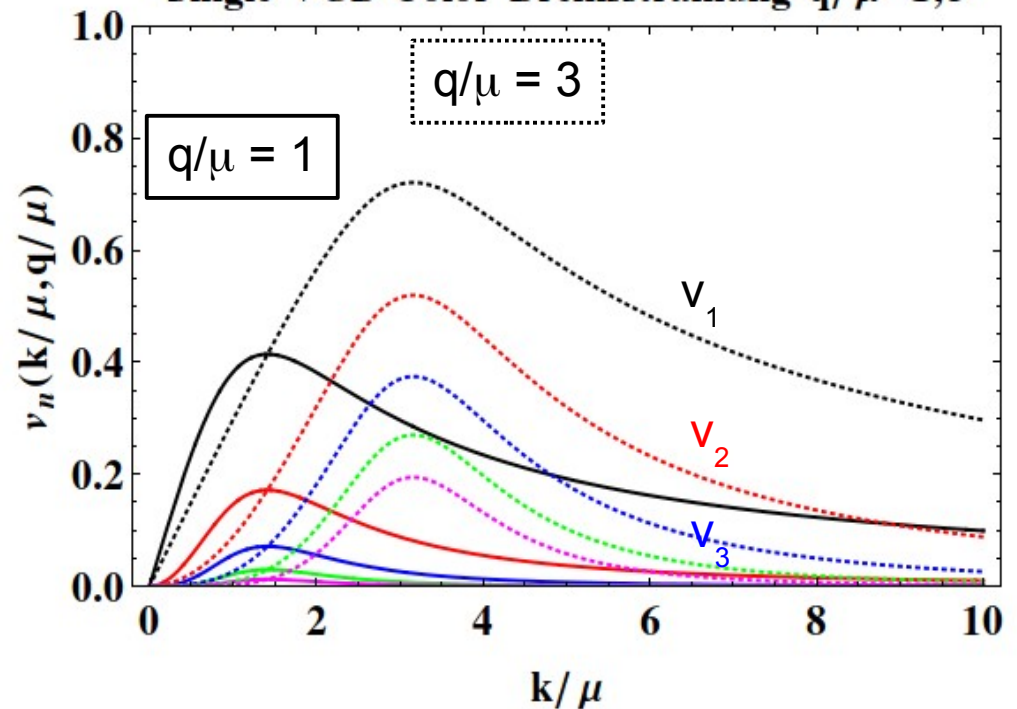
$$\lim_{\mu \rightarrow 0} v_n^{GB}(k, q, 0) = (k/q)^n \theta(q - k) .$$

Perfect $v_n^{1/n} = v_1$ Scaling

Two particle vn from ATLAS



Single VGB Color Bremsstrahlung $q/\mu=1,3$

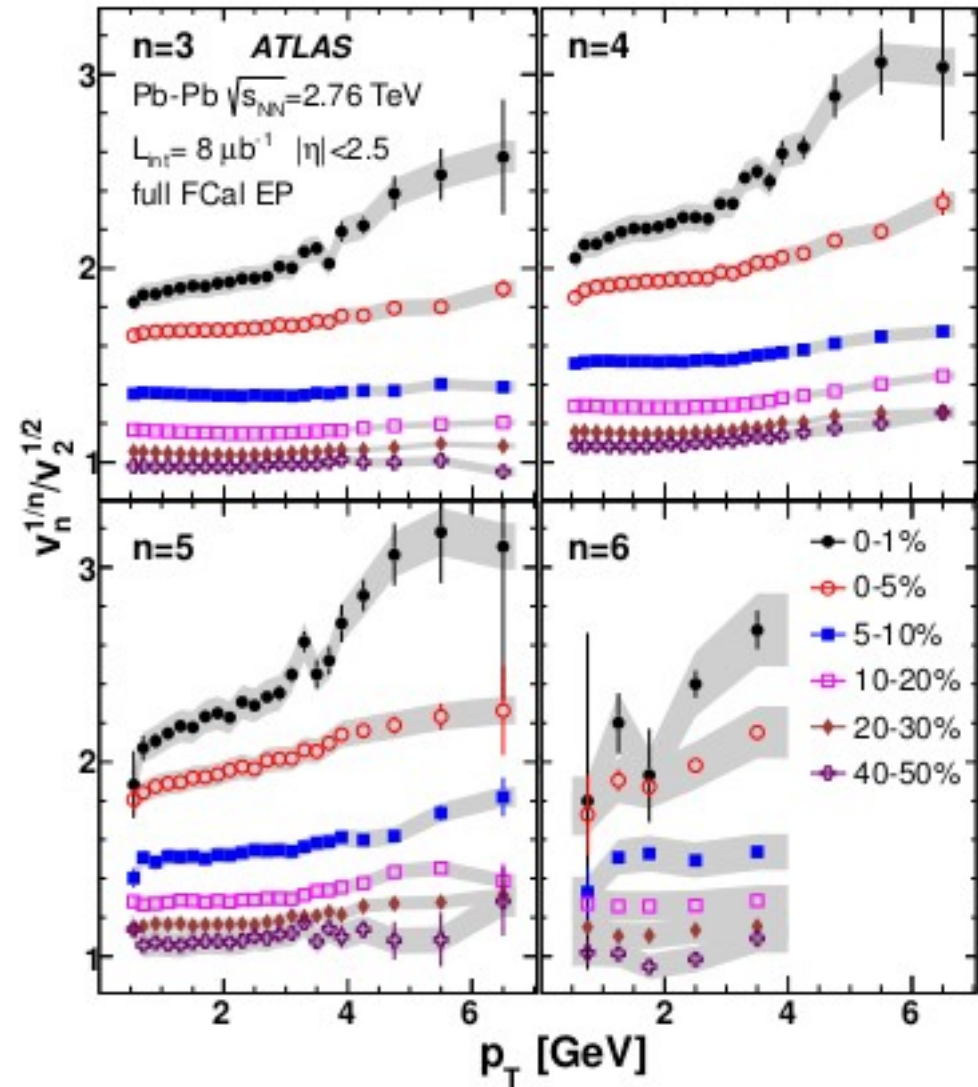
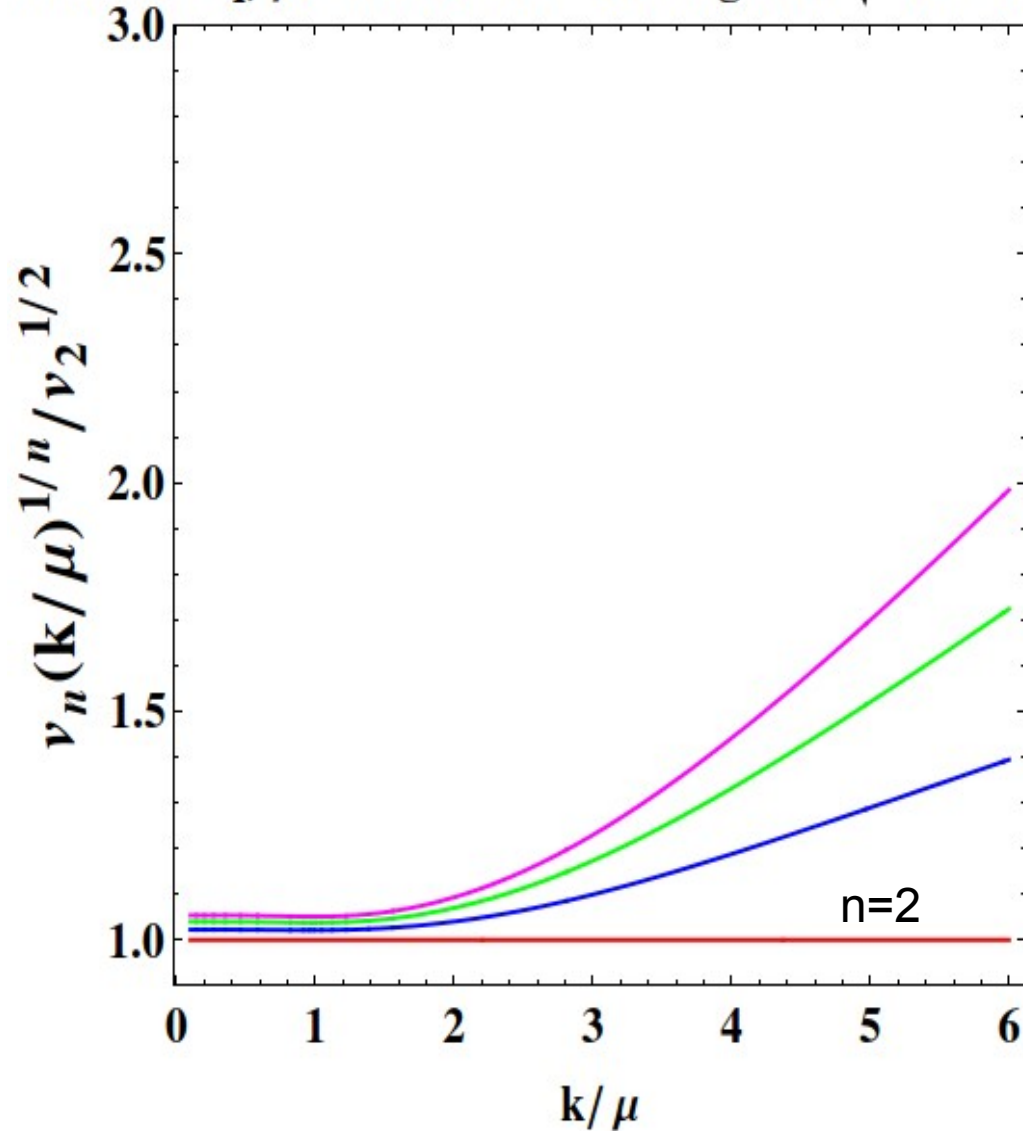


Fixed q GB pQCD Bremsstrahlung harmonics scale perfectly via 1/n power law

$$[v_n^{GB}(k, q, 0)]^{1/n} = [v_m^{GB}(k, q, 0)]^{1/m}$$

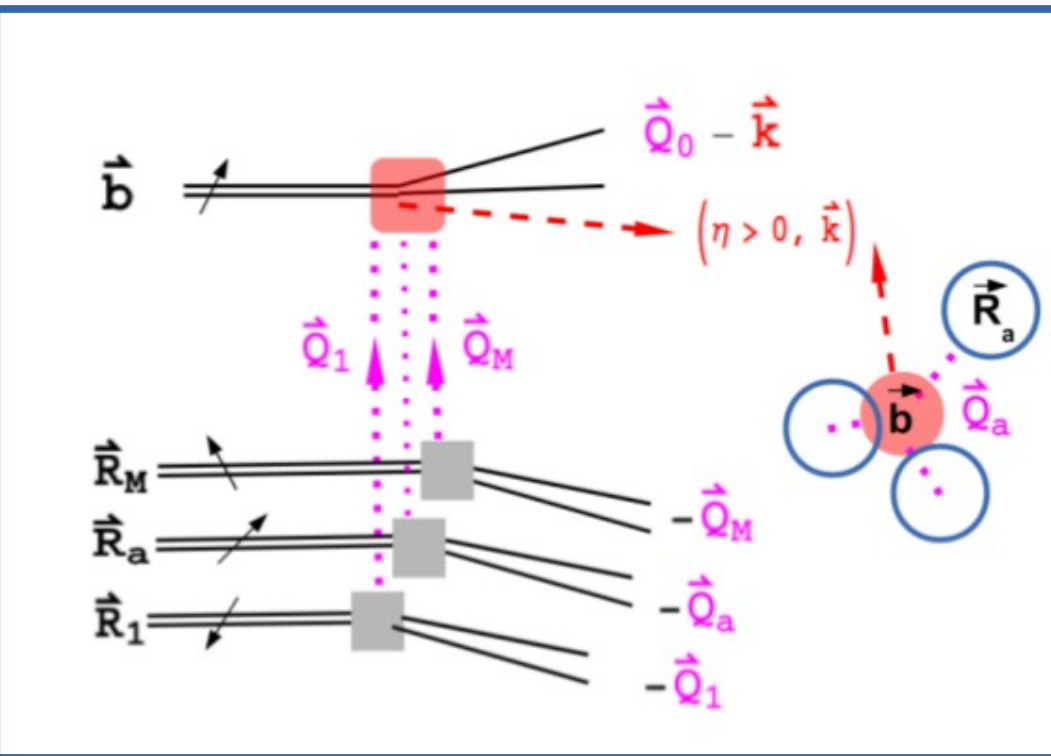
For Yukawa averaged $\langle q/\mu \rangle = \sqrt{M}$, GB 1/n scaling hold for $k < \sqrt{M}$ and breaks down for $k > \sqrt{M}$

Yukawa $\langle q/\mu \rangle = 3$ ave $v_n^{1/n}$ scaling wrt $\sqrt{v_2}$ GB

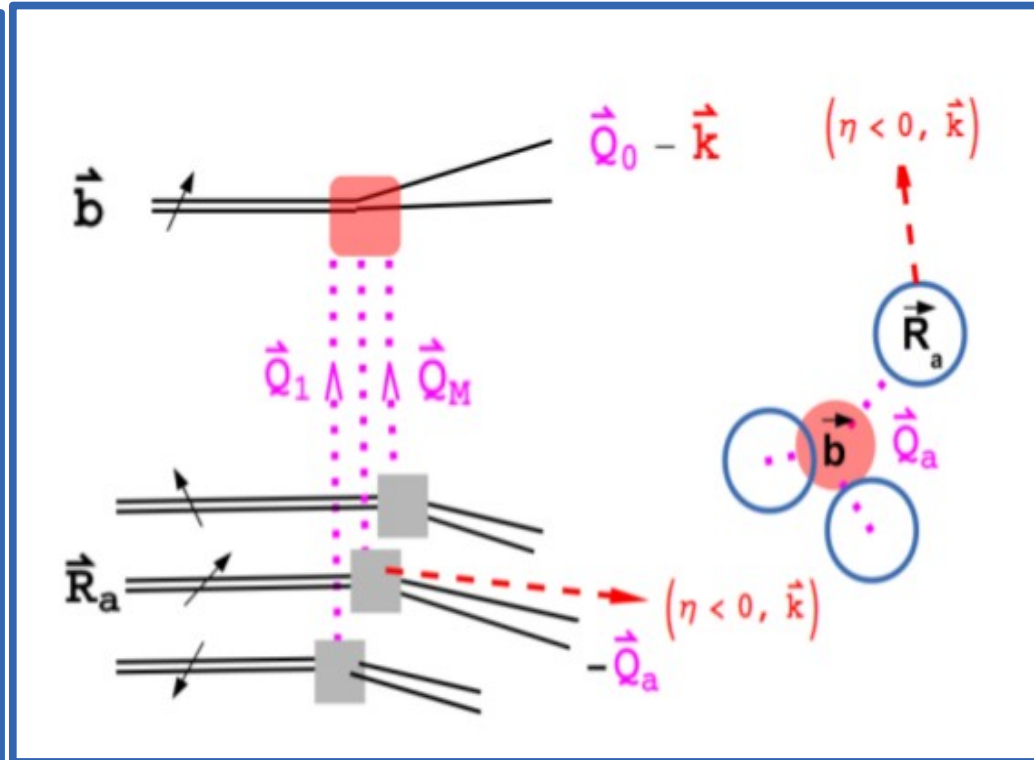


Combined projectile and target participants soft recoil Bemsstrahlung

Projectile Beam Jet Brems



Participant Recoil Target Beam Jets Brems



Target dipoles act
Coherently if transverse
separation cannot be resolved

$$R_{ij} \lesssim d(k) = \frac{c}{k}$$

$1 < M < N$ coherent target clusters defined by Resolution scale $1/k$

If $i \in I_a$ and $j \in I_a$ as well as $j \in I_b$, then j is added to I_a if its $\langle d_{ij} \rangle_{i \in I_a} < \langle d_{ij} \rangle_{i \in I_b}$

Vitev all order in opacity multiple scattering generalization of GB Brems

$$dN_{coh}^{VGB}(\mathbf{k}) = \sum_{n=1}^{\infty} \int d^2\mathbf{Q} P_n^{el}(\mathbf{Q}) dN^{GB}(\mathbf{k}, \mathbf{Q})$$

$$P_n^{el}(\mathbf{Q}) = \exp[-\chi] \frac{\chi^n}{n!} \int \left\{ \prod_{j=1}^n \frac{d^2\mathbf{q}_j}{\sigma_{el}} \frac{d\sigma_{el}}{d^2\mathbf{q}_j} \right\} \\ \times \delta^2(\mathbf{Q} - (\mathbf{q}_1 + \dots + \mathbf{q}_n))$$

Cumulative momentum transfer from n coherent scatterings

At n=N th order in opacity with M coherent target clusters that can resolved by k Projectile plus Target bremsstrahlung sums to

$$dN^{M,N} = dN_P^N(\eta, \mathbf{k}_1; \mathbf{Q}_P) + dN_T^{M,N}(\eta, \mathbf{k}_1; \{\mathbf{Q}_a\}) \\ = \sum_{a=0}^M \frac{B_{1a}}{(\mathbf{k}_1 + \mathbf{Q}_a)^2 + \mu_a^2},$$

$$B_{ia} \equiv F_{k_i, Q_a} P_a(\eta_i) \\ \mathbf{Q}_0 \equiv -\mathbf{Q}_P = -\sum_a \mathbf{Q}_a$$

2 glue Brems in independent emission approx

$$dN_2^{N,M}(\mathbf{k}_1, \mathbf{k}_2) = \sum_{a=0}^M \sum_{b=0}^M \frac{B_{1a}}{A_{1a} - \cos(\phi_1 + \psi_a)} \frac{B_{2b}}{A_{2b} - \cos(\phi_2 + \psi_b)}$$

Two gluon relative $\text{Cos}(n(\phi_1 - \phi_2))$ analytic azimuthal harmonics VGA color antennas

$$\begin{aligned}
 f_n^{N,M}(k_1, k_2) &\equiv \int_{-\pi}^{\pi} d\Phi \int_{-\pi}^{\pi} d\Delta\phi \cos(n\Delta\phi) dN_2^{N,M}(k_1, \Phi + \Delta\phi/2, k_2, \Phi - \Delta\phi/2) \\
 &= \sum_{a,b=0}^M B_{1a} B_{2b} \int_{-\pi}^{\pi} d\Phi' \frac{1}{A_{1a} - \cos(\Phi')} \int_{-\pi}^{\pi} d\Delta\phi \frac{\cos(n\Delta\phi)}{A_{2b} - \cos((\Phi' + \psi_b - \psi_a) - \Delta\phi)} \\
 &= \sum_{a,b=0}^M B_{1a} B_{2b} f_{0,1,a} f_{0,2,b} \boxed{(v_1^{GB}(k_1, Q_a) v_1^{GB}(k_2, Q_b))^n \cos(n(\psi_b - \psi_a))}
 \end{aligned}$$

$$f_{n,1,a} = \int_{-\pi}^{\pi} d\Phi \frac{\cos(n\Phi)}{A_{1a} - \cos(\Phi)} = (v_1^{GB}(k_1, Q_a))^n f_{0,1,a} = \frac{(A_{k_1, Q_a} - \sqrt{A_{k_1, Q_a}^2 - 1})^n}{\sqrt{A_{k_1, Q_a}^2 - 1}}$$

$$v_n^{M,N}\{2\}[k_1, k_2] \equiv \langle \cos(n(\phi_1 - \phi_2)) \rangle_{k_1, k_2} = \frac{\langle f_n^{M,N}(k_1, k_2) \rangle}{\langle f_0^{M,N}(k_1, k_2) \rangle}$$

$$\langle \dots \rangle = \int \left\{ \prod_{a=0}^M d\mathbf{Q}_a \right\} \delta\left(\sum_{a=0}^M \mathbf{Q}_a\right) \sum_{m_1, \dots, m_M} \delta\left(N - \sum_{a=1}^M m_a\right) p_{\{m_j\}}^{M,N} P_{m_1}^{el}(\mathbf{Q}_1) \dots P_{m_1}^{el}(\mathbf{Q}_M)$$

A. Special Z_n color antenna arrays

From Eq.(36,42) it is clear that a particularly simple special case arises if $M = n - 1$ target clusters have similar number of members $m_a = N/M = N/(n - 1)$ and transfer similar $Q_a^2 = N/M\mu^2$ to the projectile but only at special angles , $\{\psi_a\} = 2\pi a/n$, corresponding to the discrete unitary group of $n = M + 1$ roots of unity,

$$Z_n = \{z_{a,n} = e^{i2\pi a/n} | a = 0, \dots, n - 1; \sum_{a=0}^{n-1} z_{a,n} = 0\} . \quad (43)$$

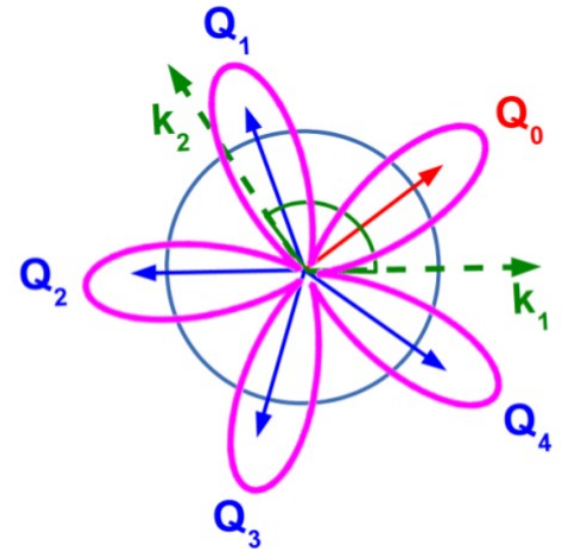
For this special geometry of projectile and target color dipole antennas the double sum over a and b decouple because

$$\cos(n(\psi_a - \psi_b)) = \cos(2\pi(a - b)) = 1 , \quad (44)$$

$$v_n^{M,N} \{2\}(k_1, k_2) \xrightarrow{Z_n} \delta_{n,M+1} v_{M+1}^{GB}(k_1, Q_0) v_{M+1}^{GB}(k_2, Q_0)$$

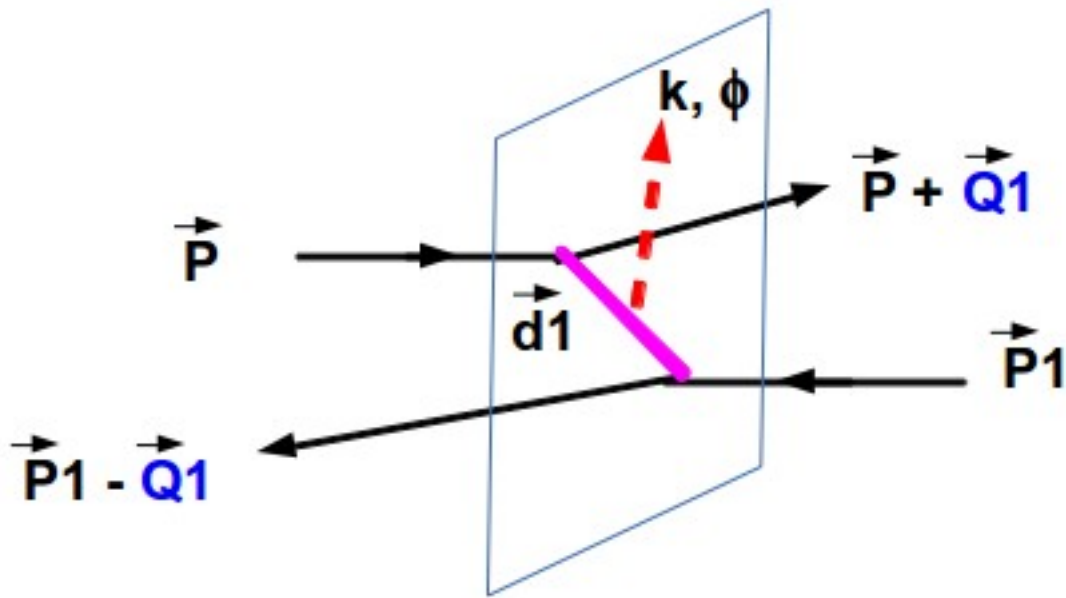
$$\frac{v_n^{M,N} \{2\}(k_1, k_2)}{v_{M+1}^{GB}(k_2, Q_0)} \xrightarrow{Z_n} \delta_{n,M+1} (v_1^{GB}(k_1, Q_0))^{M+1}$$

Zn color dipole antennas produce perfect pitch
Unique multipole v_n azimuthal harmonics



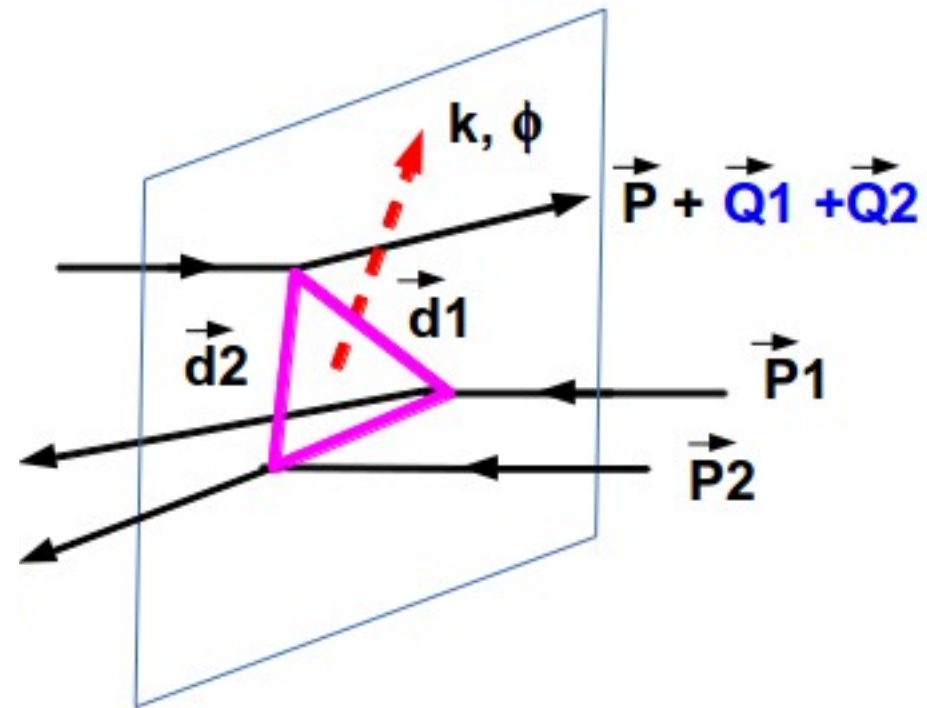
Z_5 “star fish” antenna array with $M = 4$ target clusters recoiling off projectile with $Q_0 = -\sum_{a=1}^M Q_a$

Classical Color Field Produced by Interfering 3 dipole currents



Two BG dipole antenna array

Produce only $n=2,4,6, \dots$



Three BG dipole antenna array

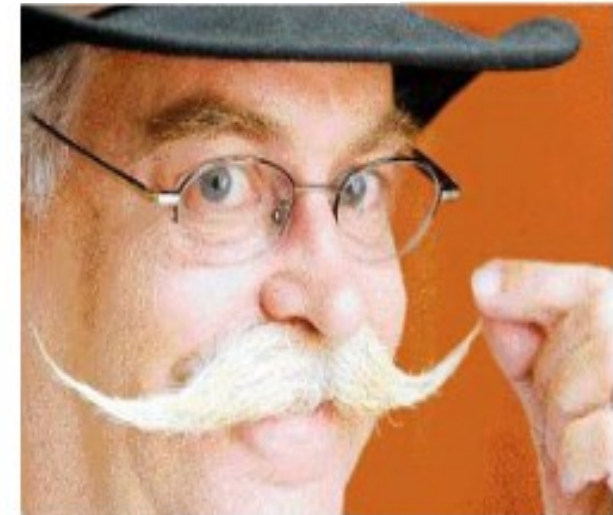
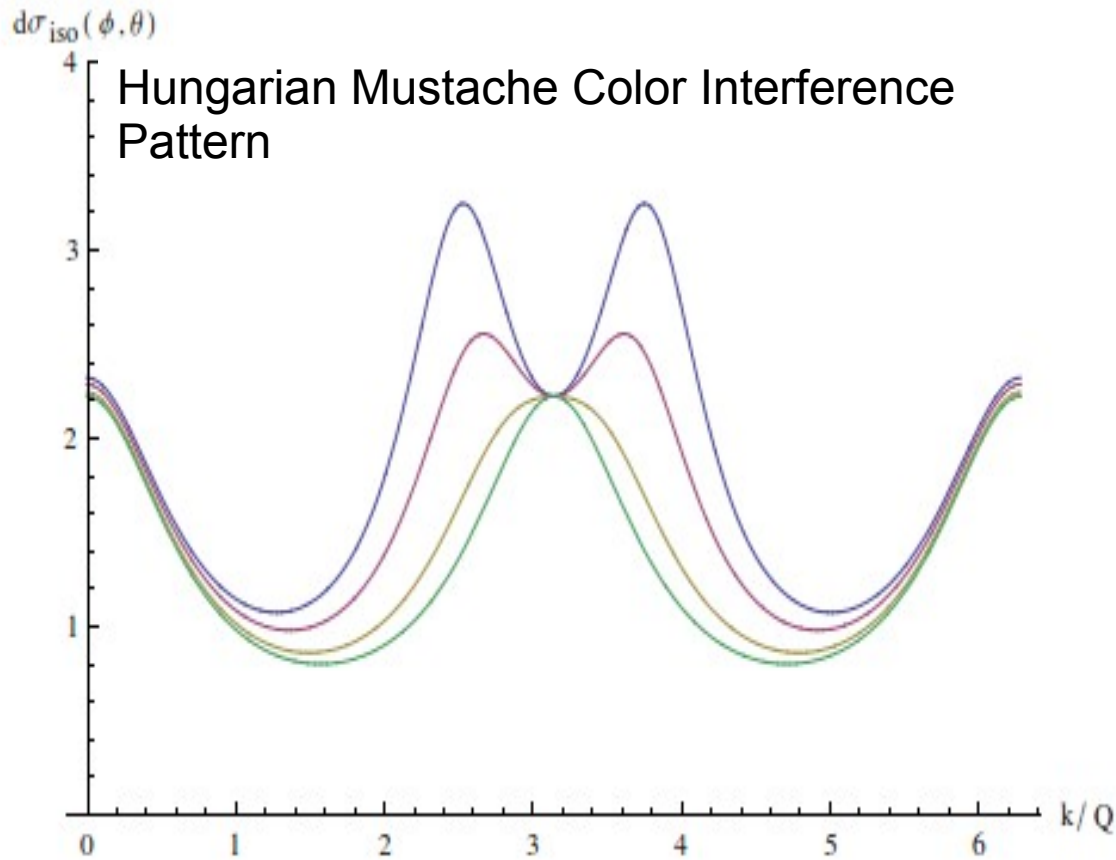
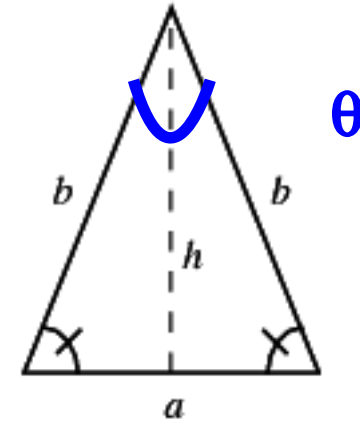
Produce all $n=1, 2, 3, 4, \dots$

Example: Color Scintillating Arrays can easily produce large v2, v3 Interferences

Isosceles Triangle GB antenna (Magyar Bajusz) array

$$\frac{d\sigma_{\text{iso}}}{d\eta d\mathbf{k} d\mathbf{k} d\phi} [\{\vec{Q}_a\}] = \frac{C_A \alpha}{k^2} \sum_{a=1}^3 \frac{Q_a^2}{k^2 + Q_a^2 - 2k Q_a \cos[\phi - \Theta_a]}$$

$$v_n(k/Q_1, \theta) = \int d\phi \cos[n\phi] d\sigma_{\text{iso}} / \left(\int d\phi d\sigma_{\text{iso}} \right)$$



MG, P. Levai, I. Vitev, and T. Biro work in progress

B. Random recoil color antenna arrays

Another simple limit is when the recoil azimuthal angles ψ_a are in random $[0, 2\pi]$ and the \mathbf{Q}_a are distributed with a Gaussian of same width squared $\langle Q_a^2 \rangle = Q_T^2 = (N/M)\mu^2$ for $a \in [1, \dots, M]$ In this antenna array t, the projectile \mathbf{Q}_0 is also Gaussian distributed with zero mean but with an enhanced second moment,

$$\langle Q_0^2 \rangle = MQ_T^2 = N\mu^2 . \quad (48)$$

Unlike for perfect n^{th} harmonic antenna arrays with Eq.(49), in the random Gaussian distributed case

$$\cos(n(\psi_a - \psi_b)) = \delta_{a,b} , \quad (49)$$

$$f_n^{N,M}(k, k) \xrightarrow{\text{Gauss}} \int d^2\mathbf{Q} \left\{ \frac{\exp[-Q^2/(2N\mu^2)]}{2\pi N\mu^2} + M \frac{\exp[-Q^2/(2(N/M)\mu^2)]}{2\pi(N/M)\mu^2} \right\} \{B_{kQ} f_{0,k,Q} v_n^{GB}(k, Q)\}^2$$

$$\sqrt{f_n^{N,M}(k, k)} \approx \left(\frac{C_R \alpha_s \mu^2}{\pi^2 k^2} \right) \left\{ \frac{P_P(\eta)}{(N+1)\mu^2} \frac{(v_1^{GB}(k, \sqrt{N}\mu))^n}{((k^2 + (N+1)\mu^2)^2 - 4Nk^2\mu^2)^{1/2}} + \frac{M(\eta)}{(N/M(\eta)+1)\mu^2} \frac{(v_1^{GB}(k, \sqrt{N/M(\eta)}\mu))^n}{((k^2 + (N/M(\eta)+1)\mu^2)^2 - 4(N/M(\eta))k^2\mu^2)^{1/2}} \right\}$$

Mixture of two scale GB harmonics with $Q^2 \sim N\mu^2$ from projectile beam jet
And $M Q^2 \sim (N/M)\mu^2$ recoil target beam jet clusters that depends of eta

**C: Work in progress implement correlated VGA brems into HIJING Monte Carlo generator
Replace phi ave ARIADNE in JETSET with VGA recoil correlated gluon Bremsstrahlung**

Slide 3-4: The Case for **Perfect Fluidity** and CGC/Glasma **Sufficiency** for Flow Harmonics in Non-central A+A as of QM12

Slides 5-14 : Trouble for Fluidity since 2012

- a) Beam Energy **independence**
- b) $p(D)+A = A+A$ v_2 and v_3 Beam size **independence**
- c) large **rapidity-even v1 Dipole**

Slides 15-27 : Is Perfect Fluidity **Necessary ??**

Are v_n in B+A mostly Nonabelian Wave interference?

(see also QM talks by Venugopalan , Dusling, Schenke... & poster of Biro, Schram)

Could **rapidity-even v odd Dipole, Triangular, etc harmonics be just fingerprints of basic pQCD beam jet anisotropic bremsstrahlung?**

(My current working hypothesis is Yes, and that it will increase η/s fits closer to 1985 pQCD)

**Work in Progress: all order in opacity pQCD beam jets v_n
Bremsstrahlung in P+A via HIJING replacing ARIADNE with VGB**

Non-Abelian Bremsstrahlung and Azimuthal Asymmetries in High Energy p+A

M. Gyulassy,^{1,2,*} P. Levai,¹ I. Vitev,³ and T. Biro¹

¹MTA WIGNER Research Centre for Physics, RMI, Budapest, Hungary

²Department of Physics, Columbia University, New York, NY 10027, USA

³Theoretical Division, Los Alamos National Laboratory, Los Alamos, NM 87545, USA

(Dated: May 13, 2014 v11)