Energy loss and (de)coherence effects beyond the eikonal approximation

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Introduction

- Main Goal of Hard Probes: probe the hot and dense medium formed in heavy-ion collisions (QGP)
  - How? Indirect measurement through the modifications observed on jets (Jet Quenching)
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Jets in pp:

Parton branching described by pQCD:
- Vacuum splitting functions;
- Successive emissions follow angular ordering;

Universal hadronization prescription
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Modifications include:
- Energy loss by medium-induced gluon radiation;
- (De)coherence effects between successive emitters;
- Hadronization pattern due to colour flow;
- Elastic energy loss;
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In this talk!
Jet Quenching

Energy loss calculations:
- Soft gluon radiation limit;
- Extensions to account for hard limit;

[Ovanesyan et al 11, D'Eramo et al 11, LA et al 12]

LA, Armesto and Salgado [1204.2929]
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Broadening calculations:
- Eikonal approximation:
  - Only softest particle able to acquire transverse momentum
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\[
P_{2}(\mathbf{k}_{a}, \mathbf{k}_{b}, z; t_{L}, t_{0}) = 2g^{2}z(1-z)\int_{t_{0}}^{t_{L}} dt \int_{q, q', l} K(Q, l, z, p_{0}^{|l}; t) \\times P(k_{a} - p; t_{L}, t) P(k_{b} - (q + l - p); t_{L}, t) P(q; t, t_{0})
\]

Factorization of parton branching

LA, Armesto and Salgado [1204.2929]

[Ovanesyan et al 11, D’Eramo et al 11, LA et al 12]

[Idilbi et al 08, D’Eramo et al 10, Ovanesyan et al 11-12, Blaizot et al 13]

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- Coherent regime:
  - Subsequent emissions follow angular ordering
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P_2(k_a, k_b, z; t_L, t_0) = 2g^2 z(1 - z) \int_{t_0}^{t_L} dt \int_{q, Q, l} \mathcal{K}(Q, l, z, p_0^t; t) \times P(q - p; t_L, t) P(k_b - (q + l - p); t_L, t) P(q; t, t_0)
\]

Factorization of parton branching

[Blaizot, Dominguez, lancu and Mehtar-Tani [1311.5823]]

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Two different scales
\[ \Delta_{med} \approx 1 - e^{-\frac{1}{2} q \theta_{q}^{2} L^{2}} \]

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[Mehtar-Tani, Salgado and Tywoniuk [1112.5031]
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Kinematical Setup

- Extend previous works to account for:
  - Finite energy corrections to the energy loss;
  - Independent broadening of all propagating particles;
  - Colour correlation between different emitters.

beyond:

- Soft limit;
- Eikonal approximation;
- Small formation times (infinite medium).
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Ingredients:

- Kinematics:
  \[ p_0^+ \]
  \[ k_\perp = k \]
  \[ k_+ = z p_0^+ \]

- Medium:
  \( q_\perp = q \)
  \( q_+ = (1 - z)p_0^+ \)

- High-energy limit: \( p_0^+ >> |k|, |q| \)

Independent medium scatterings
Static scattering centers

\( \lambda << \mu^{-1} \)
Kinematical Setup

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For a frozen medium coloured configuration:

\[
G(x_0^+, x_0; L_+, x|p_+) = \int_{r(x_0^+)}^{r(L_+)} \mathcal{D}r(\xi) \exp \left\{ \frac{ip_+}{2} \int_{x_0^+}^{L_+} d\xi \left( \frac{dr}{d\xi} \right)^2 \right\} W(x_0^+, L_+; r(\xi))
\]

where: \( W(x_0^+, L_+; x) = \mathcal{P} \exp \left\{ ig \int_{x_0^+}^{L_+} dx_+ A_-(x_+, x) \right\} \)
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For a frozen medium coloured configuration:

$$G(x_{0+}, x_0; L_+, x | p_+) = \int_{r(x_{0+})=x_0}^{r(L_+)=x} Dr(\xi) \exp \left\{ \frac{i p_+}{2} \int_{x_{0+}}^{L_+} d\xi \left( \frac{dr}{d\xi} \right)^2 \right\} W(x_{0+}, L_+; r(\xi))$$

where: $W(x_{0+}, L_+; x) = \mathcal{P} \exp \left\{ ig \int_{x_{0+}}^{L_+} dx A_+(x, x) \right\}$
Kinematical Setup

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beyond:
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For a frozen medium coloured configuration:

\[
G(x_{0+}, x_0; L_+, x|p_+) = \int_{\mathbf{r}(x_{0+})=x_0}^{\mathbf{r}(L_+)=x} D\mathbf{r}(\xi) \exp \left\{ \frac{i}{2} \int_{x_{0+}}^{L_+} d\xi \left( \frac{d\mathbf{r}}{d\xi} \right)^2 \right\} W(x_{0+}, L_+; \mathbf{r}(\xi))
\]

where: \( W(x_{0+}, L_+; x) = \mathcal{P} \exp \left\{ ig \int_{x_{0+}}^{L_+} dx_+ A_- (x_+, x) \right\} \)
Contributions for a finite medium:

\[ S_{\text{out}} = -2\pi \delta (k_+ + q_+ - p_0^+) \frac{g}{4(k \cdot q)} T_{BA}^{a} \int_{x_0, x_1} e^{ix_0 \cdot p_0 - ix_1 \cdot (k+q)} \]
\[ \times G_{AA_1} (x_{0+}, x_0; L_+, x_1 | p_0^+) \vec{u}(q) \epsilon_\kappa^a (k + q) \gamma_+ \gamma_- M_h (p_0) \]

\[ S_{\text{in}} = 2\pi \delta (k_+ + q_+ - p_0^+) \frac{ig}{2} \int_{x_0, x_1, y, z} e^{ix_0 \cdot p_0 - iy \cdot q - iz \cdot k} \]
\[ \times G_{BB_1} (x_{1+}, x_1; L_+, y | q_+) T_{B_1 A}^{a_1} G_{AA_1} (x_{0+}, x_0; x_{1+}, y | p_0^+) \]
\[ \times G_{aa_1} (x_{1+}, x_1; L_+, z | k_+ \gamma_- M_h (p_0) \]
Contributions for a finite medium:

\[
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\]

\[
S_{\text{in}} = 2\pi\delta(k_+ + q_+ - p_{0+}) \frac{ig}{2} \int_{x_0, x_1, y, z} e^{i x_0 \cdot p_0 - i y \cdot q - i z \cdot k} \times G_{BB_1}(x_{1+}, x_1; L_+, y | q_+) T_{B_1A}^{a1} G_{AA_1}(x_{0+}, x_0; x_{1+}, y | p_{0+}) \times G_{aa_1}(x_{1+}, x_1; L_+, z | k_+) \tilde{u}(q) \epsilon_k^* \gamma_- M_h(p_0)
\]

Differential cross-section:

\[
\frac{d^2 I_{\text{tot}}}{d\Omega_q d\Omega_k} = \frac{1}{\sigma_{el}} |S_{\text{tot}}|^2 = \frac{1}{\sigma_{el}} \left( |S_{\text{out}}|^2 + |S_{\text{in}}|^2 + 2 \text{Re} |S_{\text{in}} S_{\text{out}}^\dagger| \right)
\]

with:

\[
d\Omega_p = \frac{dp_+ dp_-}{2p_+ (2\pi)^3}
\]

\[
\sigma_{el} = \sqrt{2} (2\pi)^3 |M_h(p_{0+})|^2
\]
Contributions for a finite medium:

\[
S_{out} = -2\pi\delta(k_+ + q_+ - p_0+) \frac{g}{4(k \cdot q)} T_{BA}^{a} \int_{x_0, x_1} e^{i x_0 \cdot p_0 - i x_1 \cdot (k + q)} \]
\[
\times G_{AA_1} (x_{0+}, x_0; L_+, x_1 | p_{0+}) \bar{u}(q) \xi_k (k + q) \gamma_+ \gamma_- M_h (p_0)
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\]

when \( \hat q_F L_+ \to \infty \)

vacuum spectrum is recovered

\[
S_{in} = 2\pi\delta(k_+ + q_+ - p_0+) \frac{ig}{2} \int_{x_0, x_1, y, z} e^{i x_0 \cdot p_0 - i y \cdot q - i z \cdot k} \]
\[
\times G_{BB_1} (x_{1+}, x_1; L_+, y | q_+) T_{B_1 A}^{a1} G_{AA_1} (x_{0+}, x_0; x_{1+}, y | p_{0+}) \]
\[
\times G_{aa_1} (x_{1+}, x_1; L_+, z | k_+) \bar{u}(q) \xi_k \gamma_- M_h (p_0)
\]

Medium Component

\[\ |...| ^2 = \text{average over spins, colour and medium profile} \]
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\times G_{aa_1}(x_{1+}, x_1; L_+, z | k_+ \tilde{u}(q) \gamma_+ \gamma_- M_h(p_0)
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with:

\[
\sigma_{el} = \sqrt{2} (2\pi)^3 |M_h(p_{0+})|^2
\]

\[
d\Omega_p = \frac{dp_+ dp}{2p_+(2\pi)^3}
\]
Medium averages

- Schematic representation of the in-in term of the spectrum ($|S_{\text{in}}|^2$):

```
S_{\text{in}} \rightarrow g \rightarrow q \leftarrow q \leftarrow S_{\text{in}}^\dagger
```

$S_{\text{in}}$
Medium averages

- Schematic representation of the in-in term of the spectrum ($|S_{\text{in}}|^2$):

\[ S_{\text{in}} \]

\[ S_{\text{in}}^\dagger \]

\[ x_0^+ \]

\[ x_1^+ \]

\[ x_2^+ \]

\[ L^+ \]

I  II  III

High energy approximation:

⇒ Decomposition with a fixed number of propagators:

⇒ 3 different regions

\[ t_{\text{form}} = x_2^+ x_1^+ \]
Medium averages

Schematic representation of the in-in term of the spectrum ($|S_{\text{in}}|^2$):

High energy approximation:

$\Rightarrow$ Decomposition with a fixed number of propagators:

$\Rightarrow$ 3 different regions

$t_{\text{form}} = x_2, x_1$

Factorisation of the colour structure and transverse momentum dynamics:

$$G_{AB}(x_+, x; y_+, y) = \int_{r(x_+)=x}^{r(y_+)=y} D\mathbf{r}(\xi) \exp \left\{ \frac{ip_+}{2} \int_{x_+}^{y_+} d\xi \left( \frac{d\mathbf{r}}{d\xi} \right)^2 \right\} W_{AB}(x_+, y_+; \mathbf{r}(\xi))$$

Kinetic part (Broadening)

Colour part
Medium averages

○ Schematic representation of the in-in term of the spectrum ($|S_{in}|^2$):

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\]

Kinetic part (Broadening)

Implies path integrals evaluation

Colour part

Implies computation of n-field correlators
Medium averages

- Path integral resolution:
  - Dipole approximation: $C_F n(\xi) \sigma(r) \approx \frac{1}{2} q_F^2 r^2 + O(r^2 \ln r^2)$
  - Semi-classical method:

$$G_0(x_+, x; y_+, y) = \int_{r(x_+) = x}^{r(y_+) = y} \mathcal{D}r(\xi) \exp \left\{ \frac{i p_+}{2} \int_{x_+}^{y_+} d\xi \left( \frac{dr}{d\xi} \right)^2 \right\}$$

$$= \frac{1}{(2\pi i)^{D/2}} \left| \det \left( - \frac{\partial^2 R_{cl}}{\partial y_i \partial x_i} \right) \right|^{1/2} e^{i R_{cl}(x_+, x; y_+, y)}$$

D = n\text{\textsuperscript{o} of dimensions}

Classical action: $R_{cl} = \int d\xi \mathcal{L}(\xi)$

EOM: $\frac{d}{d\xi} \frac{\partial \mathcal{L}}{\partial \dot{r}} - \frac{\partial \mathcal{L}}{\partial r} = 0 \Rightarrow r = r_{cl}(\xi)$

Dominant contribution for the average trajectory given by the classical path

+ Fluctuations of the classical action
Medium averages

- Calculation of n-field correlators (at large $N_c$):

Infinitesimal expansion of the Wilson line:

$$W_{ij}(x_{0+}, L_+; x) = \left[ \delta_{i\alpha} \left( 1 - \frac{C_F}{2} B(\xi, L_+; 0) \right) + ig \int_{x_{0+}}^{\xi} dx_+ A_- (x_+, x) T_{i\alpha} \right] V_{\alpha j} (\xi, L_+; x)$$

where: $\delta^{ab} B(x_{i+}, x_{f+}; x - y) = g^2 \int_{x_{i+}}^{x_{f+}} dx_+ dy_+ \langle A_-^a (x_+, x) A_-^b (y_+, y) \rangle$
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\]

where: $\delta^{ab} B(x_{i+}, x_{f+}; x - y) = g^2 \int_{x_{i+}}^{x_{f+}} dx_+ dy_+ \langle A_+(x_+, x) A_-(y_+, y) \rangle$
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Where: $\delta^{ab} B(x_{i+}, x_{f+}; x - y) = g^2 \int_{x_{i+}}^{x_{f+}} dx_+ dy_+ \langle A^a_-(x_+, x) A^b_-(y_+, y) \rangle$
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Applying to a 2-field correlator:

$$\frac{1}{N} \text{Tr} \langle W(x) W^\dagger(x) \rangle = e^{-C_F v(x-y)}$$

where:

$$v(x - y) = B(0) - B(x - y) = \frac{1}{2} \int dx_+ \sigma(x - y) n(x_+)$$

Dipole cross-section:

$$\sigma(x - y) = 2g^2 \int \frac{dq}{(2\pi)^2} |a(q)|^2 \left( 1 - e^{-i\cdot(q)(x-y)} \right)$$
Medium averages

Schematic representation of the in-in term of the spectrum $|\text{Sin}|^2$ at large $N_c$:
Medium averages

- Schematic representation of the in-in term of the spectrum ($|S_{\text{in}}|^2$) at large $N_c$:

\[
\langle \text{Tr} \left[ W(x_g)W^\dagger(x_{g'}) \right] \text{Tr} \left[ W^\dagger(x_g)W(x_g)W^\dagger(x_{g'})W(x_{g'}) \right] \rangle = \text{Tr} \langle W(x_g)W^\dagger(x_{g'}) \rangle \text{Tr} \langle W^\dagger(x_g)W(x_g)W^\dagger(x_{g'})W(x_{g'}) \rangle
\]

At large $N_c$: factorization into a dipole $\times$ quadrupole

2-point function  4-point function  6-point function
Medium averages

- Schematic representation of the in-in term of the spectrum ($|\Sigma|^2$) at large $N_c$:

\[
\langle \text{Tr} \left[ W(x_{q_g})W^\dagger(x_{\bar{g}_{q}}) \right] \text{Tr} \left[ W^\dagger(x_{q_g})W(x_{\bar{g}_{q}})W^\dagger(x_{\bar{q}_g})W(x_{q_g}) \right] \rangle = \\
= \text{Tr} \langle W(x_{q_g})W^\dagger(x_{\bar{g}_{q}}) \rangle \text{Tr} \langle W^\dagger(x_{q_g})W(x_{\bar{g}_{q}})W^\dagger(x_{\bar{q}_g})W(x_{q_g}) \rangle
\]

At large $N_c$: factorization into a dipole x quadrupole

- $q$ and $g$ colour connected
- $\bar{q}$ and $\bar{g}$ colour connected
- $g$ and $\bar{g}$ colour connected
Medium averages

- Schematic representation of the in-in term of the spectrum ($|\text{Sin}|^2$) at large $N_c$:

\[
\langle \text{Tr} \left[ W(x_g)W^\dagger(x_{\bar{g}}) \right] \text{Tr} \left[ W^\dagger(x_g)W(x_{\bar{g}})W^\dagger(x_{\bar{q}})W(x_q) \right] \rangle = \text{Tr} \langle W(x_g)W^\dagger(x_{\bar{g}}) \rangle \text{Tr} \langle W^\dagger(x_g)W(x_{\bar{g}})W^\dagger(x_{\bar{q}})W(x_q) \rangle
\]

At large $N_c$: factorization into a dipole $\times$ quadrupole

- $q$ and $g$ colour connected
- $\bar{q}$ and $\bar{g}$ colour connected
- $q$ and $\bar{q}$ colour connected
- $g$ and $\bar{g}$ colour connected

Coherent state

Decoherent state
Medium averages

- Schematic representation of the in-in term of the spectrum ($|\text{Sin}|^2$) at large $N_c$:

\[
\langle \text{Tr} \left[ W(x_q)W^\dagger(x_{\bar{q}}) \right] \text{Tr} \left[ W^\dagger(x_q)W(x_{\bar{q}})W^\dagger(x_{\bar{q}})W(x_q) \right] \rangle = \text{Tr} \langle W(x_q)W^\dagger(x_{\bar{q}}) \rangle \text{Tr} \langle W^\dagger(x_q)W(x_{\bar{q}})W^\dagger(x_{\bar{q}})W(x_q) \rangle
\]

At large $N_c$: factorization into a dipole $\times$ quadrupole

(larger number of swaps suppressed by $O(N^{-2})$)

Coherent state

somewhere, in between, there should be a swap between the colour configurations

Decoherent state
Medium averages

- Result of the quadrupole:
  \[
  \text{Tr} \left\langle W^\dagger(x_g)W(x_{\bar{g}})W^\dagger(x_{\bar{q}})W(x_{q}) \right\rangle_{(x_{2+},L_+)} = e^{Nm_{22}} + \int_{x_{2+}}^{L_+} d\tau e^{Nm_{11}(x_{2+},\tau)} m_{12}(\tau) e^{Nm_{22}(\tau,L_+)}
  \]

where:

- \( m_{11} = -\frac{1}{2} [v(x_g - x_{\bar{q}}) + v(x_q - x_{\bar{g}})] \) (coherent prop.):
- \( m_{22} = -\frac{1}{2} [v(x_{\bar{g}} - x_g) + v(x_{\bar{q}} - x_q)] \) (independent prop.):
- \( m_{12} = -\frac{1}{2} [v(x_{\bar{g}} - x_q) - v(x_q - x_g) - v(x_{\bar{q}} - x_{\bar{g}}) - v(x_g - x_{\bar{q}})] \)
Medium averages

- Result of the quadrupole:
  \[ \text{Tr} \langle W^\dagger(x_g)W(x_{\bar{g}})W^\dagger(x_{\bar{q}})W(x_q) \rangle_{(x_{2+},L_+)} = e^{Nm_{22}} + \int_{x_{2+}}^{L_+} d\tau e^{Nm_{11}(x_{2+},\tau)} m_{12}(\tau) e^{Nm_{22}(\tau,L_+)} \]

  complete
  independent piece

where:

- \( m_{11} = -\frac{1}{2} [v(x_g - x_{\bar{g}}) + v(x_q - x_{\bar{q}})] \) (coherent prop.):
- \( m_{22} = -\frac{1}{2} [v(x_{\bar{g}} - x_g) + v(x_{\bar{q}} - x_q)] \) (independent prop.):
- \( m_{12} = -\frac{1}{2} [v(x_{\bar{g}} - x_q) - v(x_{\bar{q}} - x_g) - v(x_{\bar{g}} - x_q) - v(x_{\bar{q}} - x_g)] \)
Medium averages

- Result of the quadrupole:
  
  \[
  \text{Tr} \left< W^\dagger(x_g)W(x_{\bar{g}})W^\dagger(x_{\bar{q}})W(x_q) \right>_{(x_{2+},L_+)} = e^{N m_{22}} + \int_{x_{2+}}^{L_+} d\tau e^{N m_{11}(x_{2+}, \tau)} m_{12}(\tau) e^{N m_{22}(\tau, L_+)}
  \]

  complete
  independent piece
  coherent propagation up to \( \tau \)

  where:
  
  \[ m_{11} = -\frac{1}{2} \left[ v(x_g - x_{\bar{g}}) + v(x_q - x_{\bar{q}}) \right] \]  (coherent prop.):

  \[ m_{22} = -\frac{1}{2} \left[ v(x_{\bar{g}} - x_g) + v(x_{\bar{q}} - x_q) \right] \]  (independent prop.):

  \[ m_{12} = -\frac{1}{2} \left[ v(x_{\bar{g}} - x_q) - v(x_q - x_g) - v(x_{\bar{g}} - x_q) - v(x_q - x_g) \right] \]
Medium averages

- Result of the quadrupole:
  \[
  \text{Tr} \left\langle W^\dagger(x_g)W(x_\bar{g})W^\dagger(x_q)W(x_{\bar{q}}) \right\rangle_{(x_{2+}, L_+)} = e^{Nm_{22}} + \int_{x_{2+}}^{L_+} d\tau e^{Nm_{11}(x_{2+}, \tau)} m_{12}(\tau) e^{Nm_{22}(\tau, L_+)}
  \]

  complete independent piece coherent propagation up to \(\tau\)
  independent piece Local swap at \(\tau\)

where: \[m_{11} = -\frac{1}{2} [v(x_\bar{g} - x_q) + v(x_q - x_\bar{g})]\] (coherent prop.):

\[m_{22} = -\frac{1}{2} [v(x_\bar{g} - x_q) + v(x_q - x_\bar{g})]\] (independent prop.):

\[m_{12} = -\frac{1}{2} [v(x_\bar{g} - x_q) - v(x_q - x_\bar{g}) - v(x_\bar{g} - x_q) - v(x_q - x_\bar{g})]\]
Medium averages

- Result of the quadrupole:
  \[
  \text{Tr} \left\langle W^\dagger(x_g) W(x_\bar{g}) W^\dagger(x_\bar{q}) W(x_q) \right\rangle_{(x_{2+}, L_+)} = e^{Nm_{22}} + \int_{x_{2+}}^{L_+} d\tau e^{Nm_{11}(x_{2+}, \tau)} m_{12}(\tau) e^{Nm_{22}(\tau, L_+)}
  \]
  
  where:
  \[
  m_{11} = -\frac{1}{2} \left[ v(x_\bar{g} - x_\bar{q}) + v(x_q - x_g) \right] \quad \text{(coherent prop.)}
  \]
  \[
  m_{22} = -\frac{1}{2} \left[ v(x_\bar{g} - x_g) + v(x_q - x_\bar{q}) \right] \quad \text{(independent prop.)}
  \]
  \[
  m_{12} = -\frac{1}{2} \left[ v(x_\bar{g} - x_\bar{q}) - v(x_q - x_g) - v(x_\bar{g} - x_q) - v(x_q - x_\bar{g}) \right]
  \]
  
  complete
  independent piece
  coherent propagation up to \( \tau \)
  Local swap
  at \( \tau \)
  independent propagation from \( \tau \)
  to \( L \)
Medium averages

- Result of the quadrupole:
  \[
  \text{Tr} \left\langle W^\dagger(x_g)W(x_{\bar{g}})W^\dagger(x_q)W(x_q) \right\rangle_{(x_{2+}, L_+)} = e^{Nm_{22}} + \int_{x_{2+}}^{L_+} d\tau \, e^{Nm_{11}(x_{2+}, \tau)} \, m_{12}(\tau) \, e^{Nm_{22}(\tau, L_+)}
  \]
  complete piece
  independent piece
  coherent propagation up to \( \tau \)
  Local swap at \( \tau \)
  independent propagation from \( \tau \) to \( L \)

  \[
  \text{where: } m_{11} = -\frac{1}{2} \left[ v(x_g - x_{\bar{g}}) + v(x_q - x_q) \right] \quad (\text{coherent prop.}): \\
  m_{22} = -\frac{1}{2} \left[ v(x_g - x_g) + v(x_q - x_{\bar{q}}) \right] \quad (\text{independent prop.}): \\
  m_{12} = -\frac{1}{2} \left[ v(x_g - x_{\bar{q}}) - v(x_q - x_g) - v(x_{\bar{g}} - x_q) - v(x_{\bar{q}} - x_g) \right]
  \]

  Factorising the independent propagation:
  \[
  \text{Tr} \left\langle W^\dagger(x_g)W(x_{\bar{g}})W^\dagger(x_{\bar{q}})W(x_q) \right\rangle_{(x_{2+}, L_+)} = e^{Nm_{22}} \left\{ 1 + \int_{x_{2+}}^{L_+} d\tau \, e^{N(m_{11} - m_{22})(x_{2+}, \tau)} \, m_{12}(\tau) \right\}
  \]
Medium averages

- Result of the quadrupole:

$$\text{Tr} \left\langle W^\dagger(x_g)W(x_{\bar{g}})W^\dagger(x_{\bar{q}})W(x_q) \right\rangle_{(x_{2+},L_+)} = e^{Nm_{22}} + \int_{x_{2+}}^{L_+} d\tau e^{N\left(m_{11}(x_{2+},\tau) + m_{12}(\tau) \right)} e^{Nm_{22}(\tau,L_+)}$$

where:

$$m_{11} = -\frac{1}{2} [v(x_{\bar{g}} - x_{\bar{q}}) + v(x_q - x_g)] \quad \text{(coherent prop.)}$$

$$m_{22} = -\frac{1}{2} [v(x_{\bar{g}} - x_{\bar{g}}) + v(x_q - x_{\bar{q}})] \quad \text{(independent prop.)}$$

$$m_{12} = -\frac{1}{2} [v(x_{\bar{g}} - x_{\bar{q}}) - v(x_q - x_g) - v(x_{\bar{g}} - x_q) - v(x_q - x_{\bar{g}})]$$

Factoring the independent propagation:

$$\text{Tr} \left\langle W^\dagger(x_g)W(x_{\bar{g}})W^\dagger(x_{\bar{q}})W(x_q) \right\rangle_{(x_{2+},L_+)} = e^{Nm_{22}} \left[ 1 + \int_{x_{2+}}^{L_+} d\tau e^{N(m_{11} - m_{22}) (x_{2+},\tau)} m_{12}(\tau) \right]$$

$$\Delta_{med} \text{ by definition}$$
Generalized $\Delta_{med}$

- Generalized $\Delta_{med}$ accounts for the broadening of the particles:

$$\Delta_{med} = 1 + \int_{x_{2+}}^{L+} d\tau \ e^{N(m_{11}-m_{22})(x_{0+},\tau)} N_{m_{12}}(\tau)$$

$$= 1 + \int_{x_{2+}}^{L+} d\tau \ \hat{q}_F \ (x_q - x_{\bar{q}}) \cdot (x_g - x_{\bar{g}}) \ e^{-\hat{q}_F \int_{x_{2+}}^{\tau} d\xi (x_q - x_{\bar{q}}) \cdot (x_g - x_{\bar{g}})}$$

, in the dipole approximation
Generalized $\Delta_{med}$

- Generalized $\Delta_{med}$ accounts for the broadening of the particles:

$$
\Delta_{med} = 1 + \int_{x_{2+}}^{L+} d\tau e^{N(m_{11}-m_{22})(x_{0+},\tau)} Nm_{12}(\tau)
$$

$$
= 1 + \int_{x_{2+}}^{L+} d\tau \hat{q}_F (x_q - x_{\hat{q}}) \cdot (x_g - x_{\bar{q}}) \left| \frac{\hat{q}_F}{N} \right| e^{-\hat{q}_F \int_{x_{2+}}^{L+} d\xi (x_q - x_{\hat{q}}) \cdot (x_q - x_{\bar{q}})}\ , \text{in the dipole approximation}
$$

- Able to recover previous results?
Generalized $\Delta_{med}$

- Generalized $\Delta_{med}$ accounts for the broadening of the particles:

\[
\Delta_{med} = 1 + \int_{x_{2+}}^{L_+} d\tau \, e^{N(m_{11} - m_{22})(x_{0+}, \tau)} N m_{12}(\tau)
\]

\[
= 1 + \int_{x_{2+}}^{L_+} d\tau \, \hat{q}_F \cdot (x_q - x_{\bar{q}}) \cdot (x_g - x_{\bar{g}}) \left| e^{-\hat{q}_F \int_{x_{2+}}^\tau d\xi (x_q - x_{\bar{q}}) \cdot (x_q - x_{\bar{q}})} \right| , \text{in the dipole approximation}
\]

- Able to recover previous results?
  - Soft (hard) limit: $x_{q(g)} = x_{\bar{q}(\bar{g})} \Rightarrow \Delta_{med} = 1$
Generalized $\Delta_{\text{med}}$

- Generalized $\Delta_{\text{med}}$ accounts for the broadening of the particles:

$$\Delta_{\text{med}} = 1 + \int_{x_2^+}^{L+} d\tau \, e^{N(m_{11}-m_{22})(x_{0+},\tau)} Nm_{12}(\tau)$$

$$= 1 + \int_{x_2^+}^{L+} d\tau \, \hat{q}_F (x_q - x_{\bar{q}}) \cdot (x_g - x_{\bar{g}})|_\tau e^{-\hat{q}_F \int_{x_2^+}^{\tau} d\xi (x_q - x_{\bar{q}}) \cdot (x_q - x_{\bar{g}})} , \text{ in the dipole approximation}$$

- Able to recover previous results?
  - Soft (hard) limit: $x_q(g) = x_{\bar{q}(\bar{g})} \Rightarrow \Delta_{\text{med}} = 1 \quad \text{OK!}$
  - Infinite medium (small $t_{\text{form}}$):

$$\exp \left\{ -\hat{q}_F \int_{x_2^+}^{\tau} d\xi (x_q - x_{\bar{q}}) \cdot (x_{\bar{g}} - x_g) \right\} \leq \exp \left\{ -\hat{q}_F (x_q - x_{\bar{q}}) \cdot (x_{\bar{g}} - x_g)|_{x_2^+} (\tau - x_{2+}) \right\}$$

$$\Delta \tau \sim \frac{1}{\Delta p} \sim \frac{1}{\sqrt{\hat{q}_F L_+}} \Rightarrow \Delta_{\text{med}} \approx 1 + \int_{x_2^+}^{L+} d\tau \frac{1}{L} \exp \left\{ -\frac{\tau - x_{2+}}{x_{2+}} \right\}$$
Generalized $\Delta_{med}$

- Generalized $\Delta_{med}$ accounts for the broadening of the particles:

$$\Delta_{med} = 1 + \int_{x_{2+}}^{L_+} d\tau \, e^{N(m_{11} - m_{22})(x_{0+}, \tau)} \, Nm_{12}(\tau)$$

$$= 1 + \int_{x_{2+}}^{L_+} d\tau \, \hat{q}_F (x_q - x_{\bar{q}}) \cdot (x_g - x_{\bar{g}})|_{\tau} \, e^{-\hat{q}_F \int_{x_{2+}}^{\tau} d\xi (x_q - x_{\bar{q}}) \cdot (x_q - x_g)} , \text{ in the dipole approximation}$$

- Able to recover previous results?
  - Soft (hard) limit: $x_{q(g)} = x_{\bar{q}(g)} \Rightarrow \Delta_{med} = 1$ \textbf{Ok!}
  - Infinite medium (small $t_{form}$):

$$\exp \left\{ -\hat{q}_F \int_{x_{2+}}^{\tau} d\xi (x_q - x_{\bar{q}}) \cdot (x_g - x_{\bar{g}}) \right\} \leq \exp \left\{ -\hat{q}_F (x_q - x_{\bar{q}}) \cdot (x_q - x_{\bar{g}}) \bigg|_{x_{2+}} (\tau - x_{2+}) \right\}$$

$$\Delta \tau \sim \frac{1}{\Delta p} \sim \frac{1}{\sqrt{\hat{q}_F L_+}} \Rightarrow \Delta_{med} \approx 1 + \int_{x_{2+}}^{L_+} d\tau \, \frac{1}{L} \exp \left\{ -\frac{\tau - x_{2+}}{x_{2+}} \right\}$$

Suppressed by $L_+$

When $L_+ \to \infty \Rightarrow \Delta_{med} = 1$ \textbf{Ok!}
Two different regimes:
Two different regimes:

Random walk of initial quark

Random walk of initial quark
Two different regimes:

Random walk of initial quark

Quark and gluon are correlated (gluon formation time)
Two different regimes:

- Random walk of initial quark
- Quark and gluon are correlated (gluon formation time)
- Quark and gluon act coherently (coherent propagation controlled by dipole distance)
- Independent broadening of final partons
Two different regimes:

Random walk of initial quark

Quark and gluon are correlated (gluon formation time)

Quark and gluon act coherently (coherent propagation controlled by dipole distance)

Medium-induced gluon radiation is suppressed

Random walk of initial quark

Quark and gluon are correlated (gluon formation time)

Independent broadening of final partons

Loss of energy is more efficient
Conclusions

- Able to unify in a single expression:

Broadening:
- Eikonal approximation:
- Beyond eikonal approximation:

Energy loss:
- Soft gluon radiation limit;
- Extensions to account for hard limit;

\[
\int \mathcal{D}x_q(\xi) \mathcal{D}x_{\bar{q}}(\xi) \mathcal{D}x_g(\xi) \mathcal{D}x_{\bar{g}}(\xi) \exp \left\{ \frac{ip_0+}{2} \int_{x_2^+}^{L_+} d\xi \left[ (1-z) (\dot{x}_q^2 - \dot{x}_{\bar{q}}^2) + z (\dot{x}_g^2 - \dot{x}_{\bar{g}}^2) \right] \right\}
\times e^{-\frac{N}{2} v(x_g-x_{\bar{g}})} \left\{ e^{Nm_{22}} + \int_{x_2^+}^{L_+} d\tau e^{Nm_{11}(x_{0+},\tau)} Nm_{12} e^{Nm_{22}(\tau,L_+)} \right\}
\]

(Anti)angular ordering:
- Coherent regime:
- Decoherent regime: