Lattice QCD based equation of state at finite baryon density

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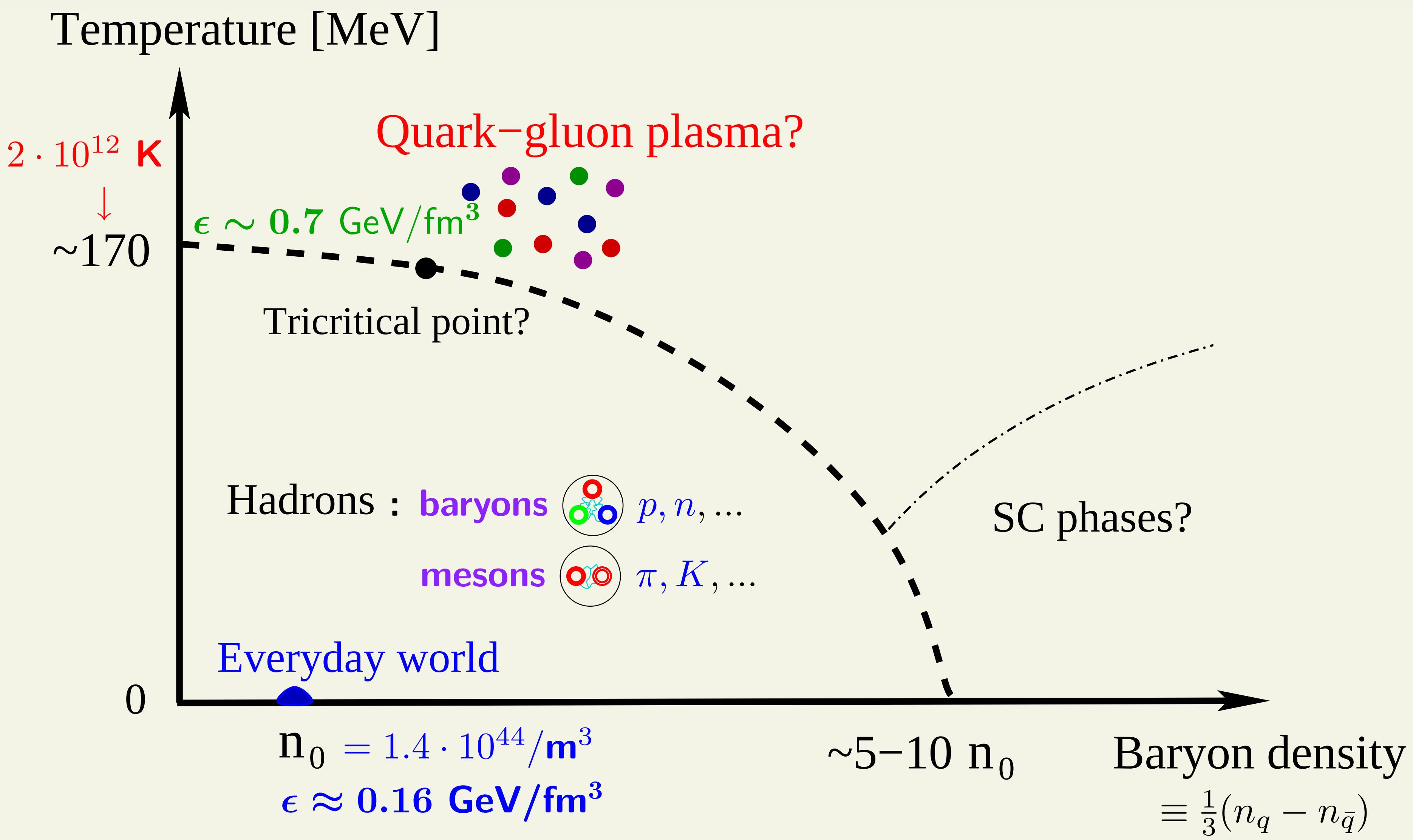
Nuclear phase diagram



Taylor expansion for pressure

$$\frac{P}{T^4} = \Sigma_{i,j}\, c_{ij}(T) \left(\frac{\mu_B}{T}\right)^i \left(\frac{\mu_S}{T}\right)^j,$$

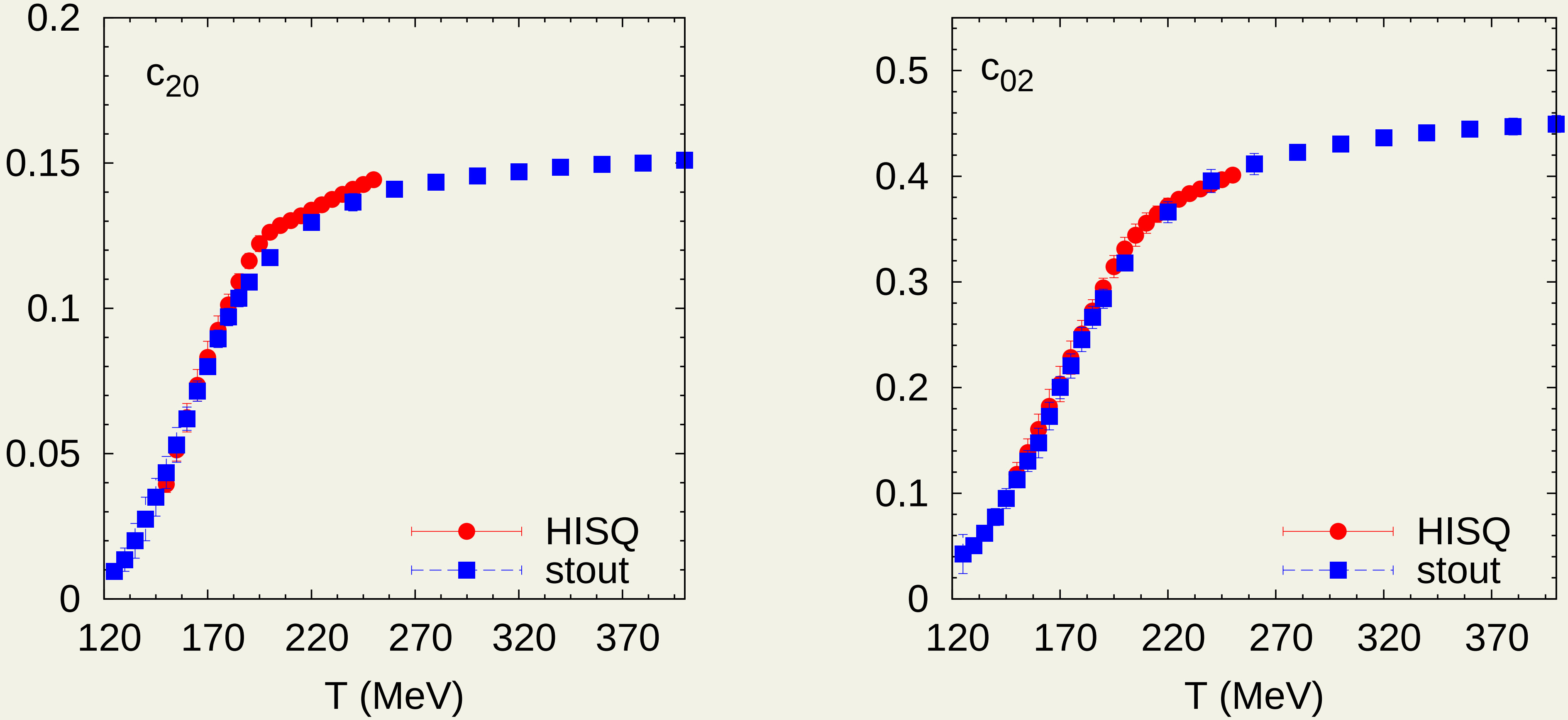
where

$$c_{ij}(T) = \frac{1}{i!j!}\frac{\partial^i}{\partial(\mu_B/T)^i}\frac{\partial^j}{\partial(\mu_S/T)^j}\frac{P}{T^4},$$

i.e. moments of baryon number and strangeness fluctuations and correlations

- an EoS based on lattice calculations of these?

Continuum extrapolated second order coefficients (also c_{11}):

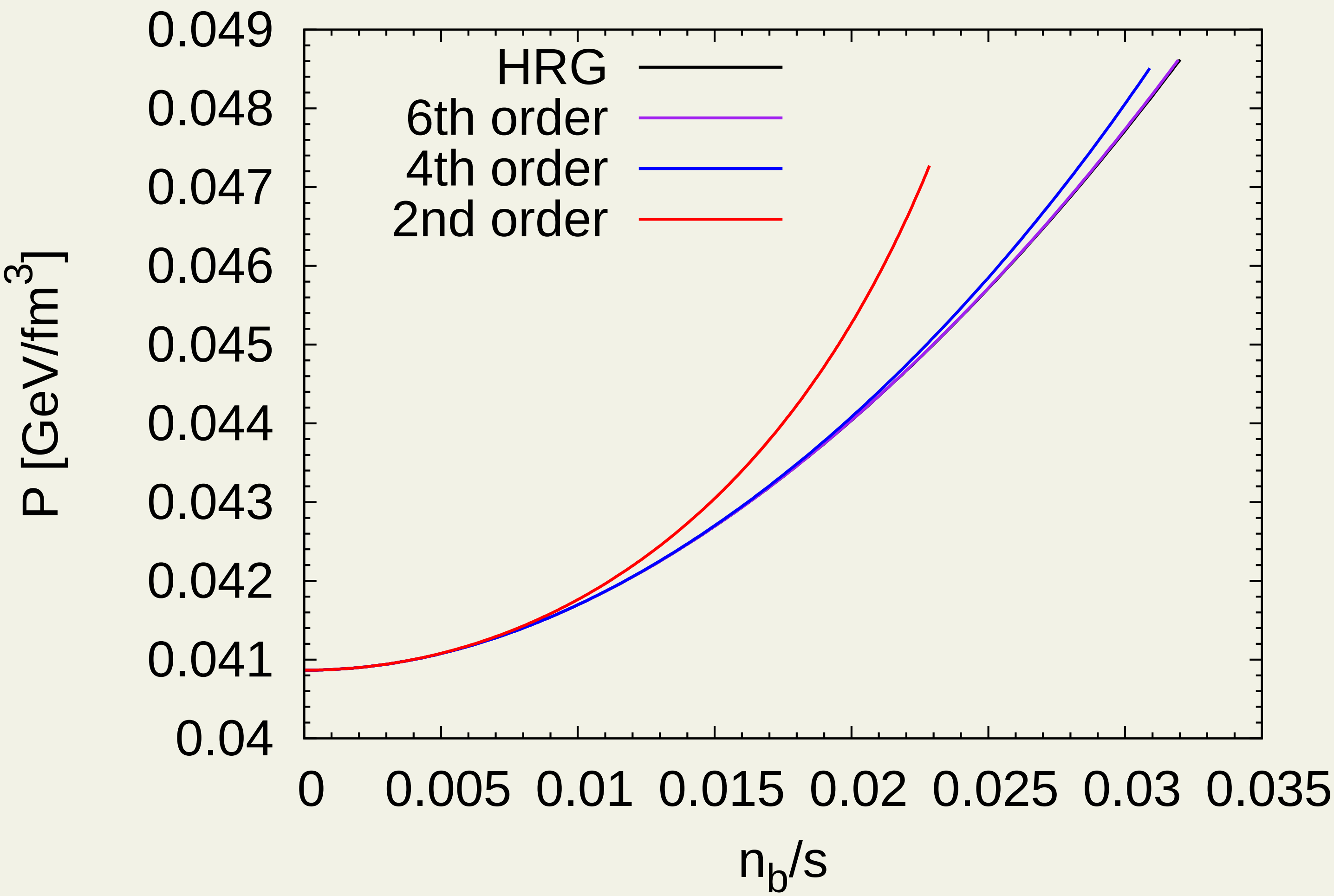


HISQ: hotQCD collaboration, Phys. Rev. D 86, 034509 (2012)
stout: Budapest-Wuppertal collaboration, JHEP 1201, 138 (2012)

- **Are first coefficients enough?**

Pressure in HRG at T = 150 MeV

full hadron resonance gas, or evaluate Taylor coefficients in HRG:



- **Fourth and sixth order coefficients needed**
- **Evaluated using p4 action with** $N_\tau = 4$

$\Rightarrow$ **large discretization effects?**

Hadrons on lattice

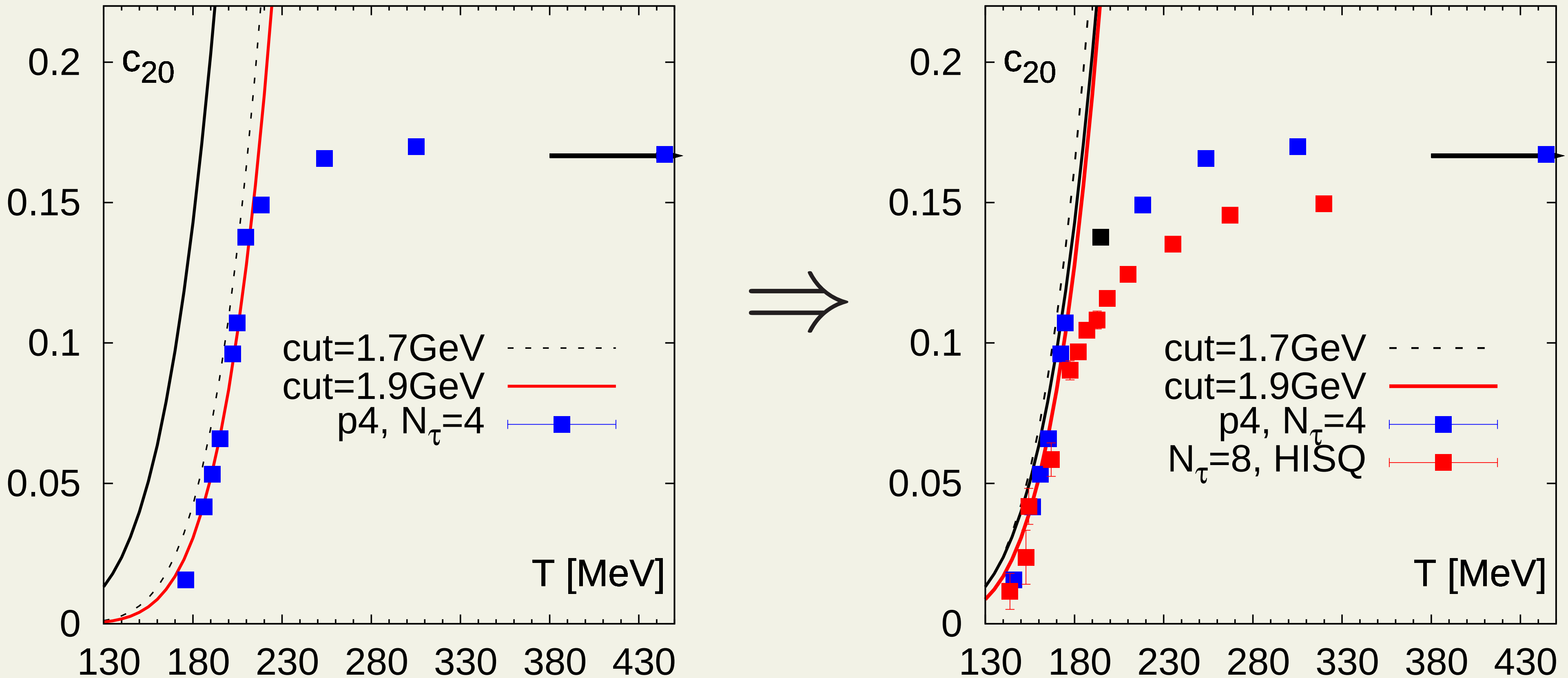
- **16 pseudoscalar mesons on lattice**

- **Hadron masses depend on lattice cutoff**

⇒ i.e. on temperature:
E.g. for pseudoscalar mesons on asqtad calculations

$$
\begin{aligned}
m^2_{\mathrm{ps}_i} &= m^2_{\mathrm{ps}_0} + \frac{1}{r_1^2}\frac{a^i_{\mathrm{ps}}x + b^i_{\mathrm{ps}}x^2}{(1+c^i_{\mathrm{ps}}x)^{\beta_i}} \\
x &= (a/r_1)^2 \\
a &= \frac{1}{N_\tau T}
\end{aligned}
$$

30 MeV shift



c_{20}
cut=1.7GeV
cut=1.9GeV
p4, N_τ=4
T [MeV]
0.2
0.15
0.1
0.05
0
130 180 230 280 330 380 430
$\Rightarrow$
c_{20}
cut=1.7GeV
cut=1.9GeV
p4, N_τ=4
N_τ=8, HISQ
T [MeV]
0.2
0.15
0.1
0.05
0
130 180 230 280 330 380 430

Parametrization

$$c_{ij}(T) = \frac{a_{ij1}}{\hat{T}^{n_{ij1}}} + \frac{a_{ij2}}{\hat{T}^{n_{ij2}}} + \frac{a_{ij3}}{\hat{T}^{n_{ij3}}} + \frac{a_{ij4}}{\hat{T}^{n_{ij4}}} + \frac{a_{ij5}}{\hat{T}^{n_{ij5}}} + \frac{a_{ij6}}{\hat{T}^{n_{ij6}}} + c_{ij}^{SB},$$

where n_{kij} are integers with $1 < n_{kij} < 23$,
and

$$\hat{T} = \frac{T - T_s}{R},$$

with $T_s = 0.1$ or 0 GeV, and $R = 0.05$ or 0.15 GeV.

Constraints:

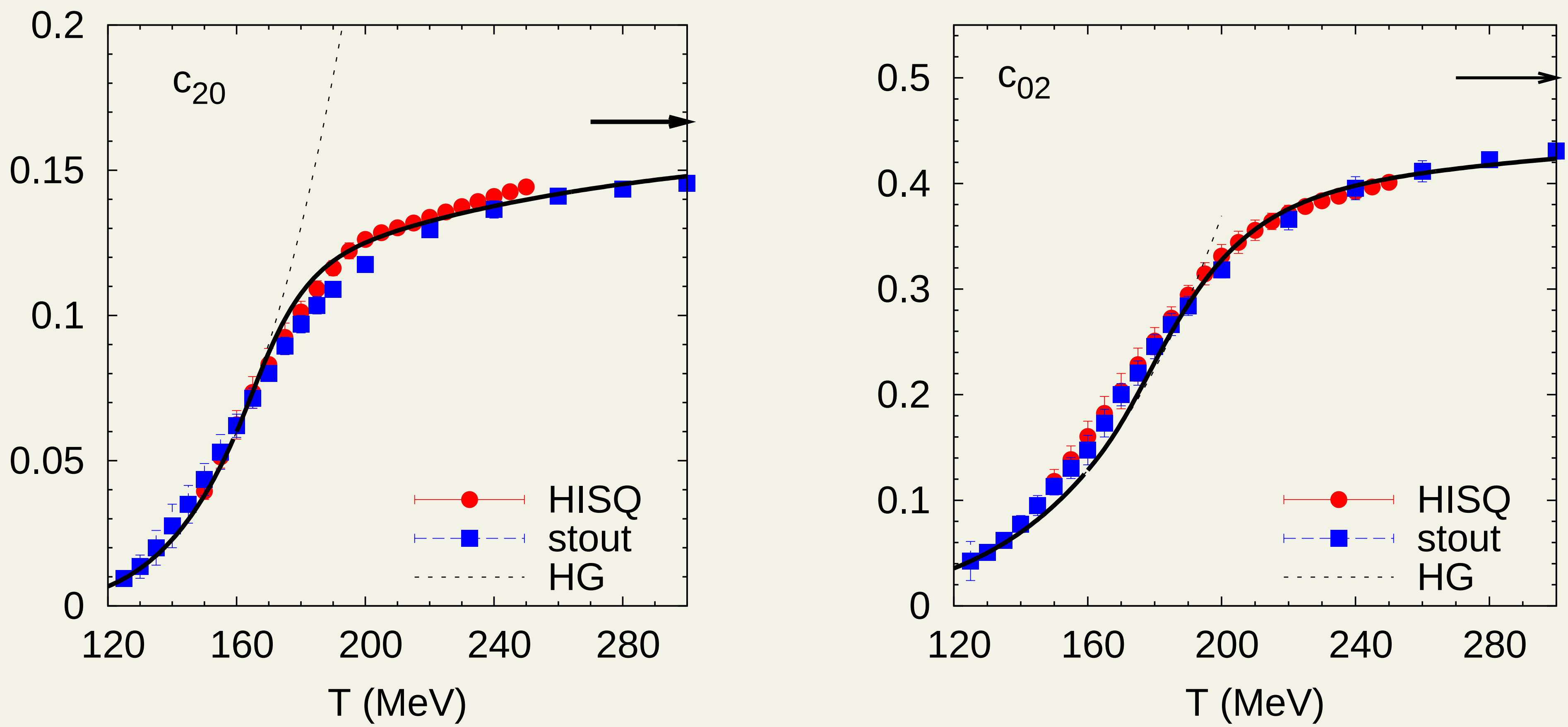
$$
\begin{aligned}
c_{ij}(T_{\rm sw}) &= c_{ij}^{\rm HRG}(T_{\rm sw}) \\
\frac{\rm d}{{\rm d}T}\, c_{ij}(T_{\rm sw}) &= \frac{\rm d}{{\rm d}T}\, c_{ij}^{\rm HRG}(T_{\rm sw}) \\
\frac{{\rm d}^2}{{\rm d}T^2}\, c_{ij}(T_{\rm sw}) &= \frac{{\rm d}^2}{{\rm d}T^2}\, c_{ij}^{\rm HRG}(T_{\rm sw}) \\
\frac{{\rm d}^3}{{\rm d}T^3}\, c_{ij}(T_{\rm sw}) &= \frac{{\rm d}^3}{{\rm d}T^3}\, c_{ij}^{\rm HRG}(T_{\rm sw})
\end{aligned}
$$

at $T_{\rm sw} = 160$ MeV for second order coefficients
$T_{\rm sw} = 155$ MeV for fourth and sixth order coefficients

3rd derivative to quarantee smooth behaviour of speed of sound:

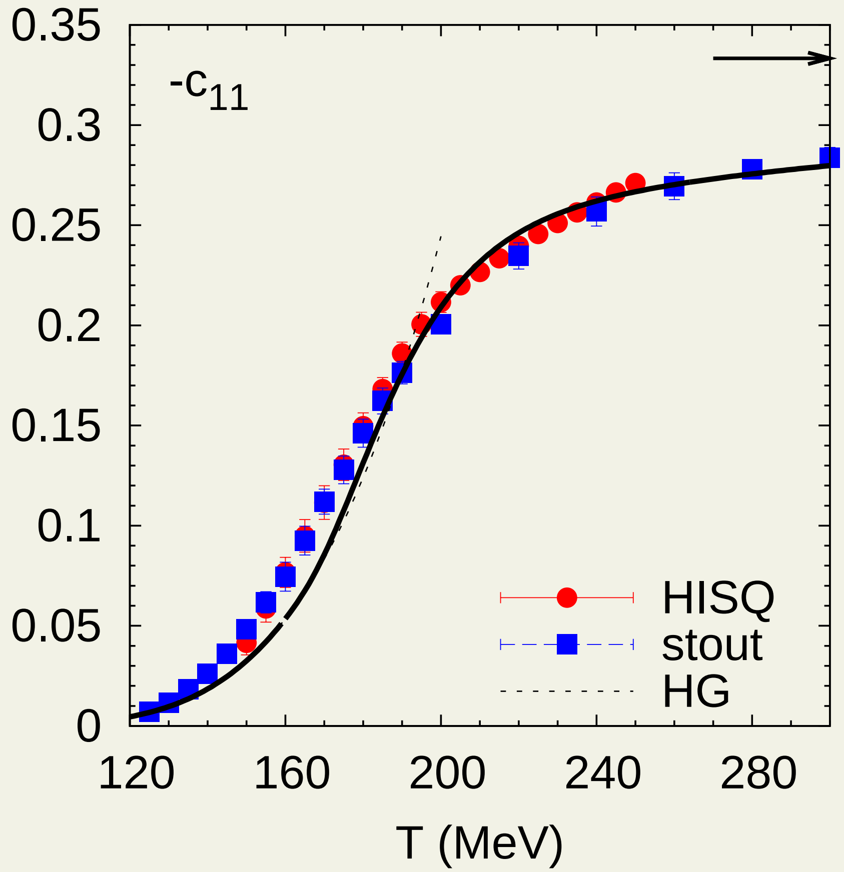
$$
c_s^2 \propto \frac{{\rm d}^2}{{\rm d}T^2}\, c_{ij}
$$

c_{20} and c_{02}

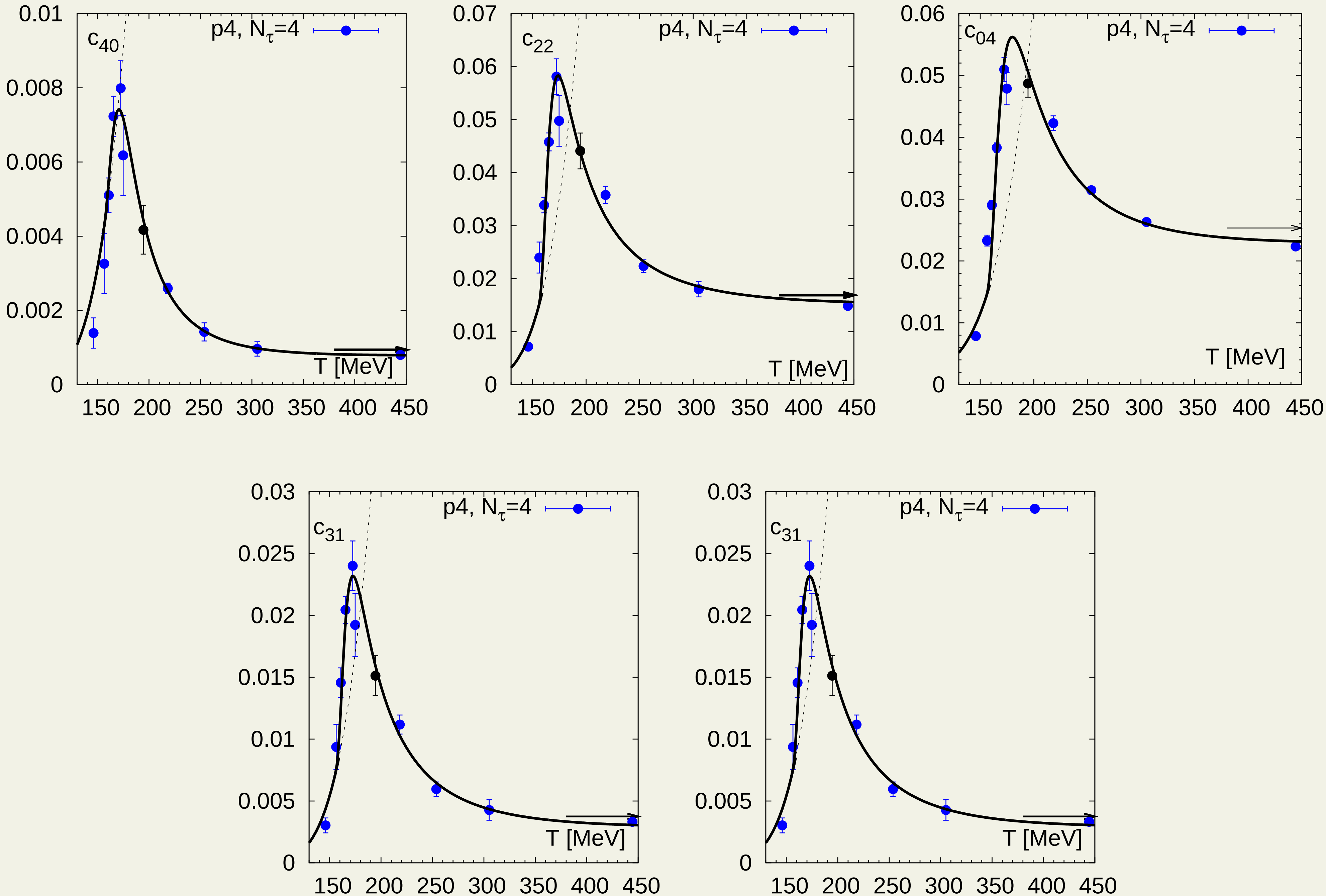


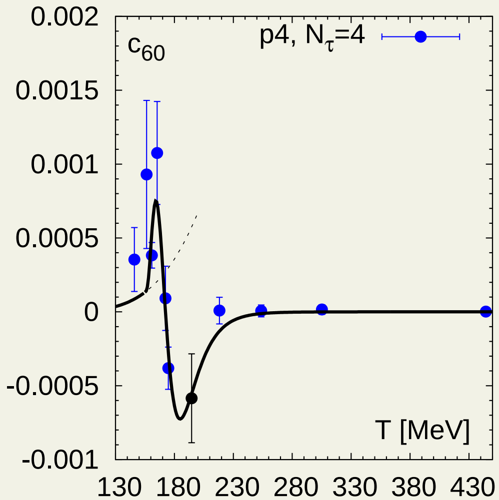
c20
0.2
0.15
0.1
0.05
0
120
160
200
240
280
T (MeV)
HISQ
stout
HG
c02
0.5
0.4
0.3
0.2
0.1
0
120
160
200
240
280
T (MeV)
HISQ
stout
HG

c_{11}

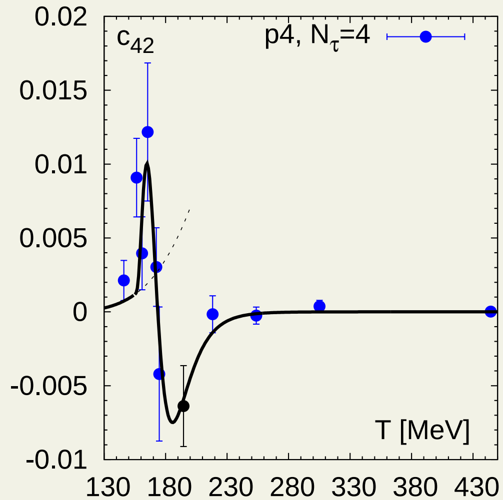


-c11
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
120
160
200
240
280
T (MeV)
HISQ
stout
HG

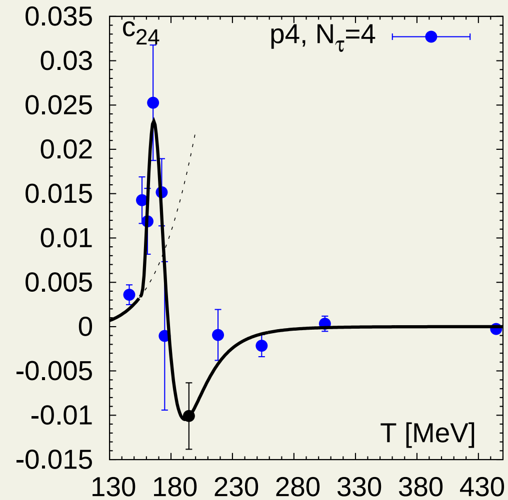

c40
p4, Nτ=4
T [MeV]
c22
p4, Nτ=4
T [MeV]
c04
p4, Nτ=4
T [MeV]
c31
p4, Nτ=4
T [MeV]
c31
p4, Nτ=4
T [MeV]



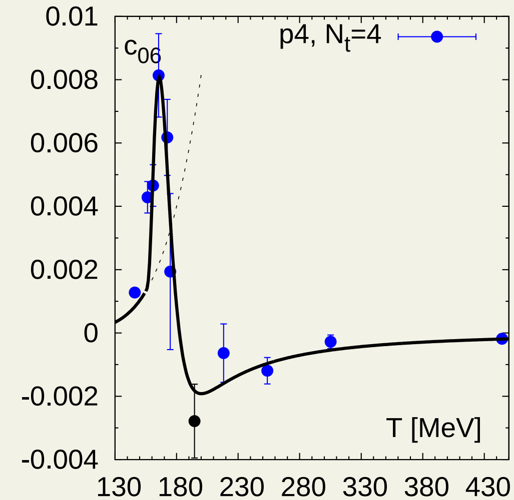
c_{60}
p4, N_τ=4
T [MeV]
0.002
0.0015
0.001
0.0005
0
-0.0005
-0.001
130 180 230 280 330 380 430



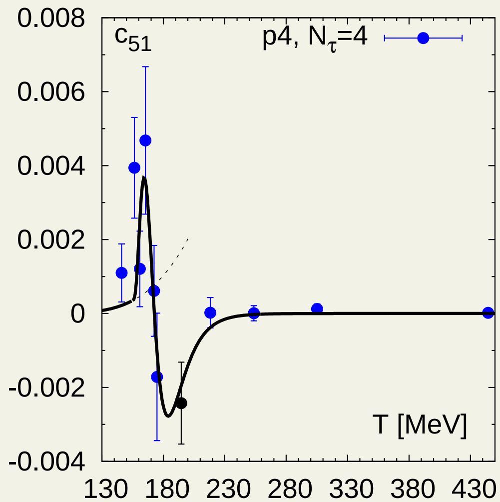
c_{42}
p4, N_τ=4
T [MeV]
0.02
0.015
0.01
0.005
0
-0.005
-0.01
130 180 230 280 330 380 430



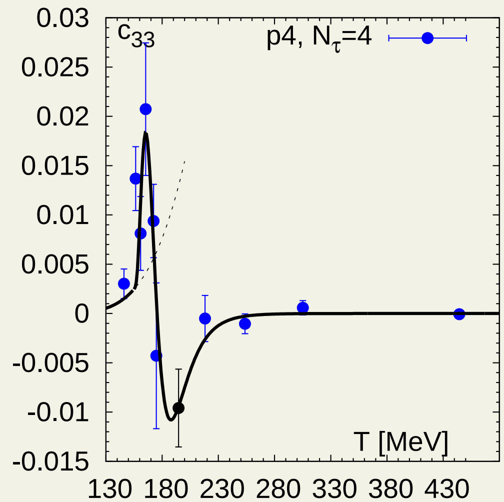
c_{24}
p4, N_τ=4
T [MeV]
0.035
0.03
0.025
0.02
0.015
0.01
0.005
0
-0.005
-0.01
-0.015
130 180 230 280 330 380 430



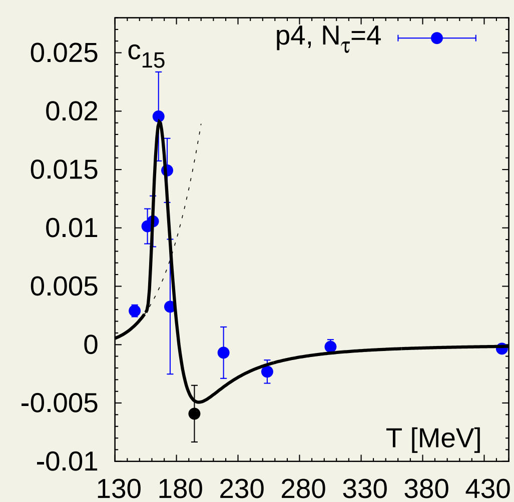
c_{06}
p4, N_t=4
T [MeV]
0.01
0.008
0.006
0.004
0.002
0
-0.002
-0.004
130 180 230 280 330 380 430



c_{51}
p4, N_τ=4
T [MeV]
0.008
0.006
0.004
0.002
0
-0.002
-0.004
130 180 230 280 330 380 430

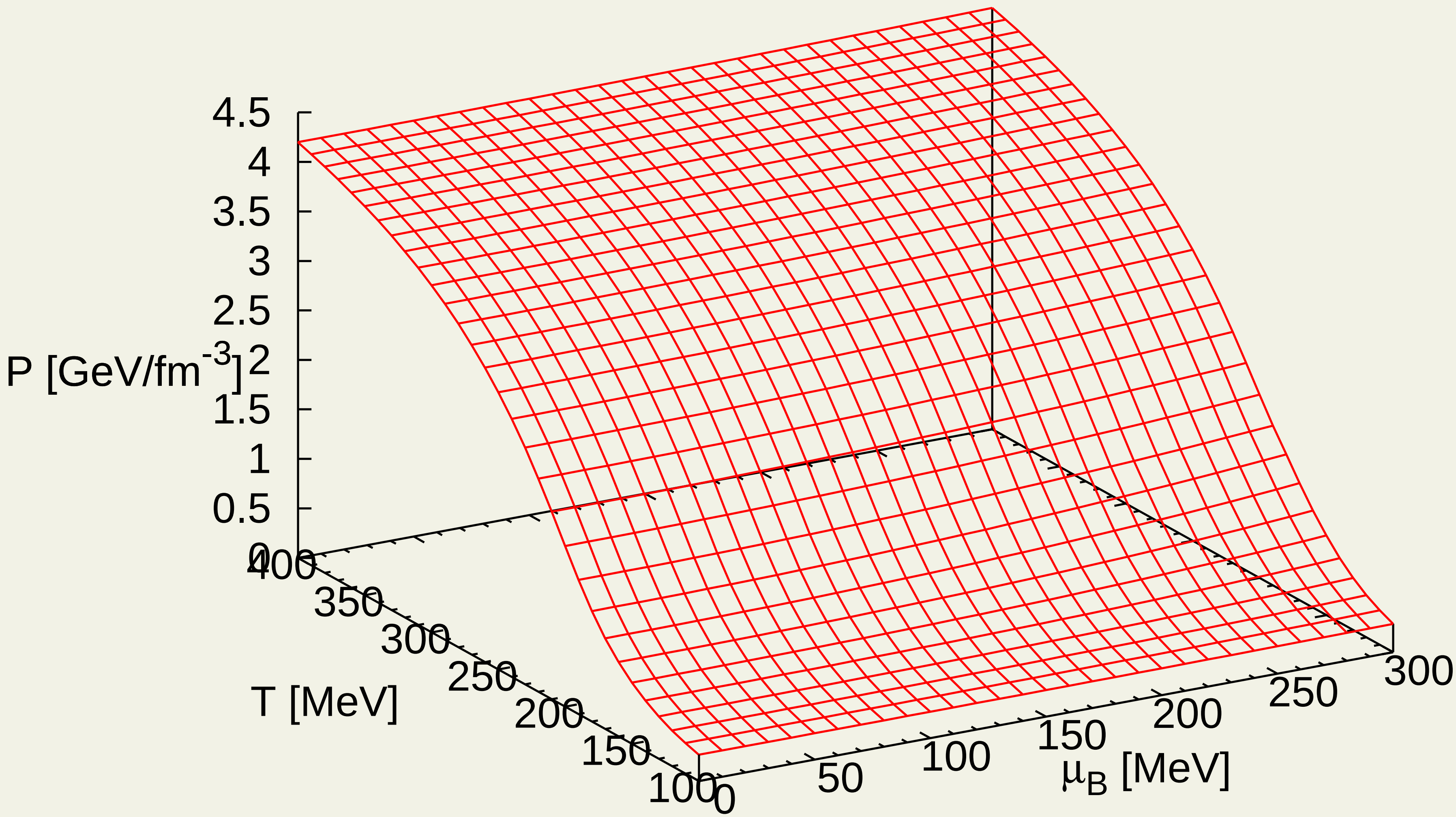


c_{33}
p4, N_τ=4
T [MeV]
0.03
0.025
0.02
0.015
0.01
0.005
0
-0.005
-0.01
-0.015
130 180 230 280 330 380 430



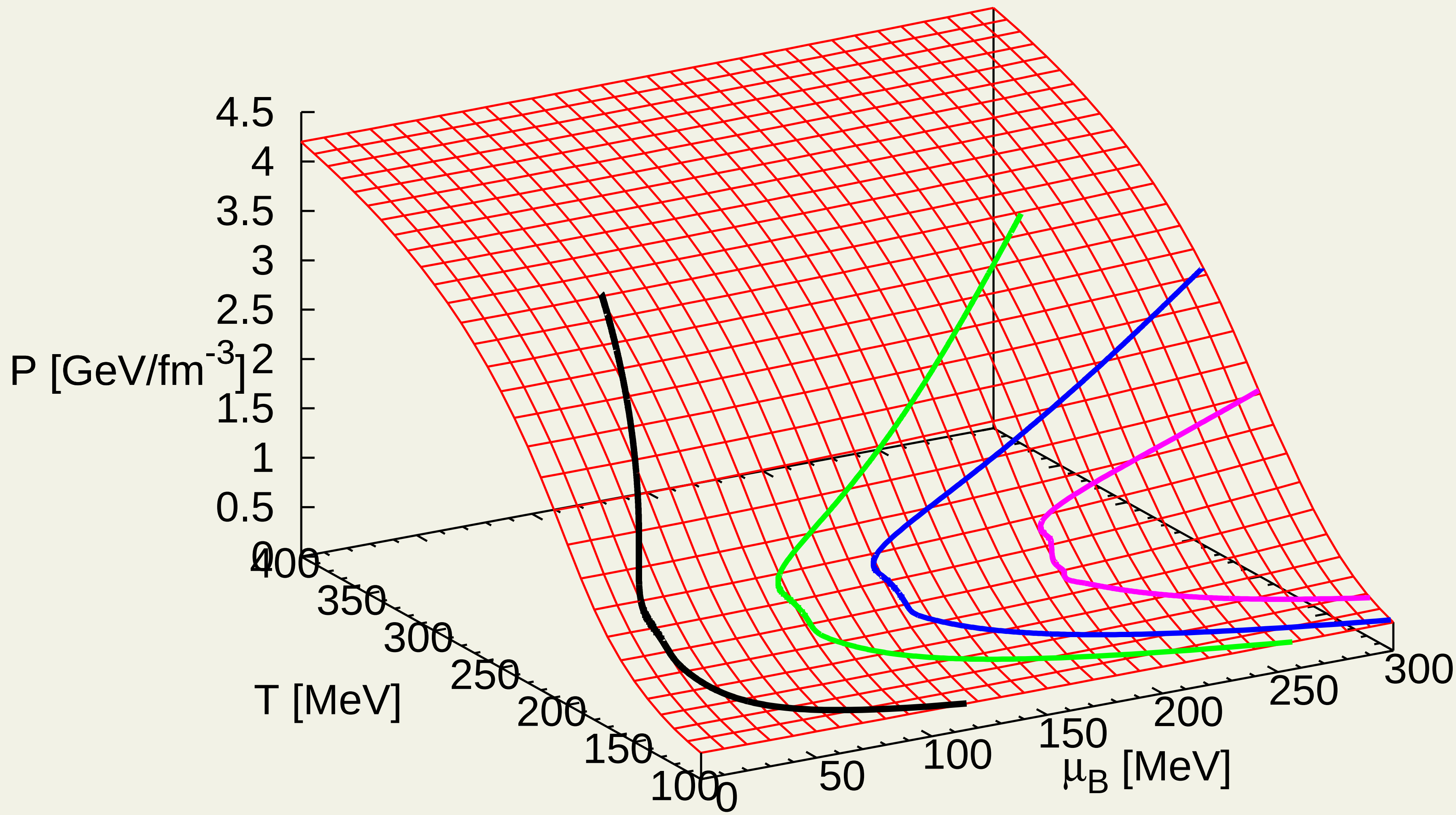
c_{15}
p4, N_τ=4
T [MeV]
0.025
0.02
0.015
0.01
0.005
0
-0.005
-0.01
130 180 230 280 330 380 430

P/T^4



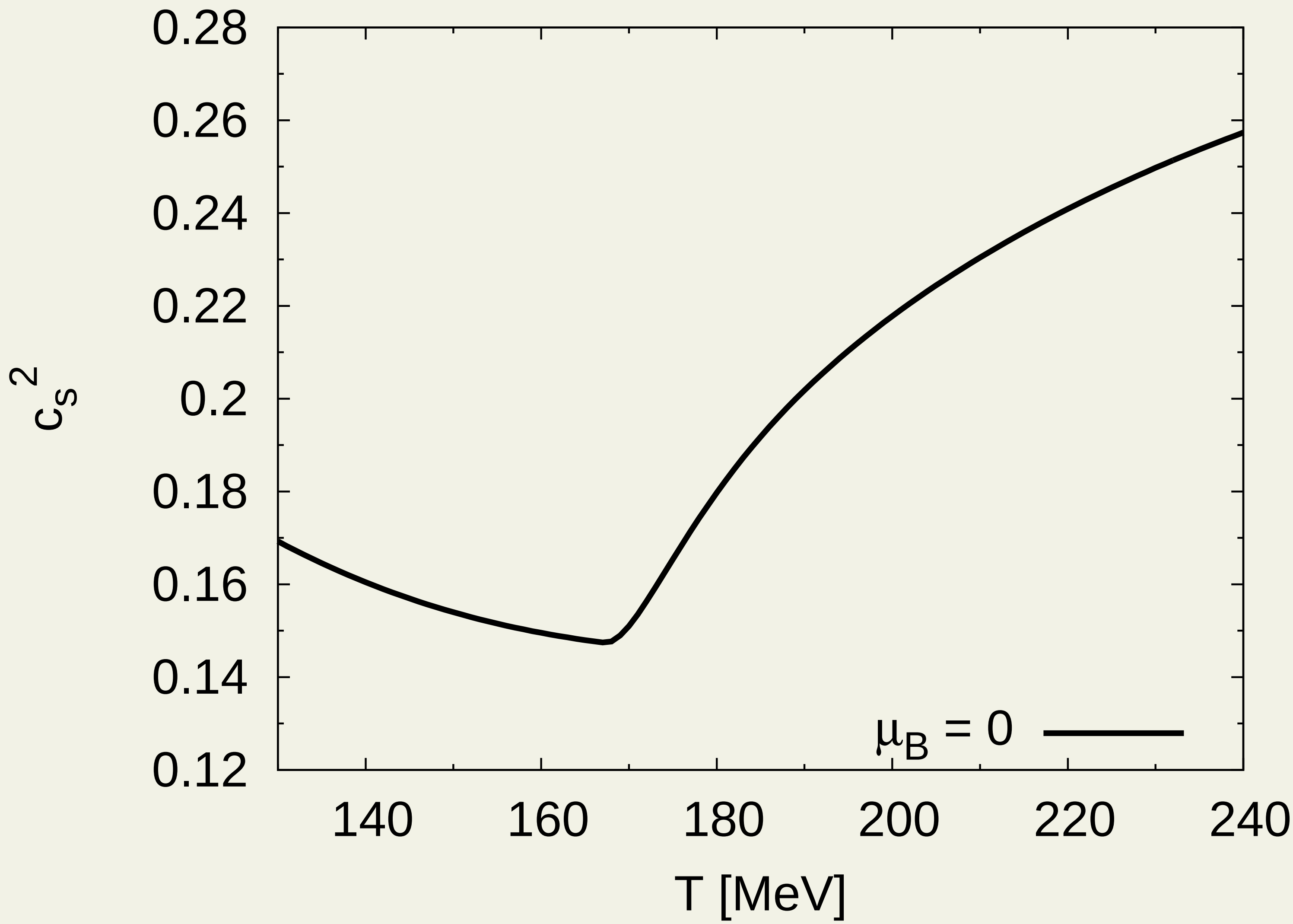
P [GeV/fm-3]
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
T [MeV]
400
350
300
250
200
150
100
μB [MeV]
0
50
100
150
200
250
300

P/T^4



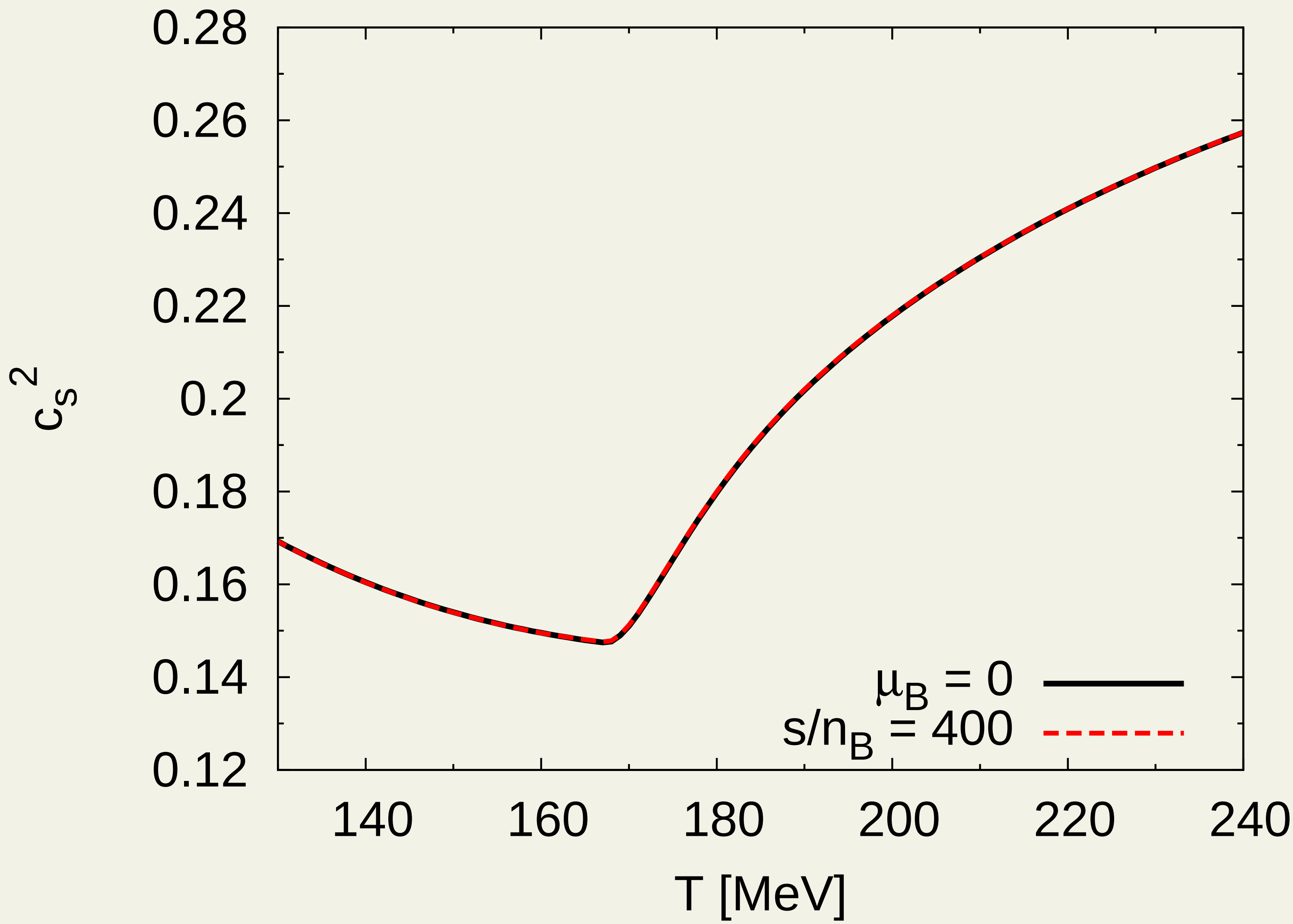
P [GeV/fm-3]
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
T [MeV]
400
350
300
250
200
150
100
μB [MeV]
0
50
100
150
200
250
300

Speed of sound along $s/n_b = \mathrm{const.}$ curve



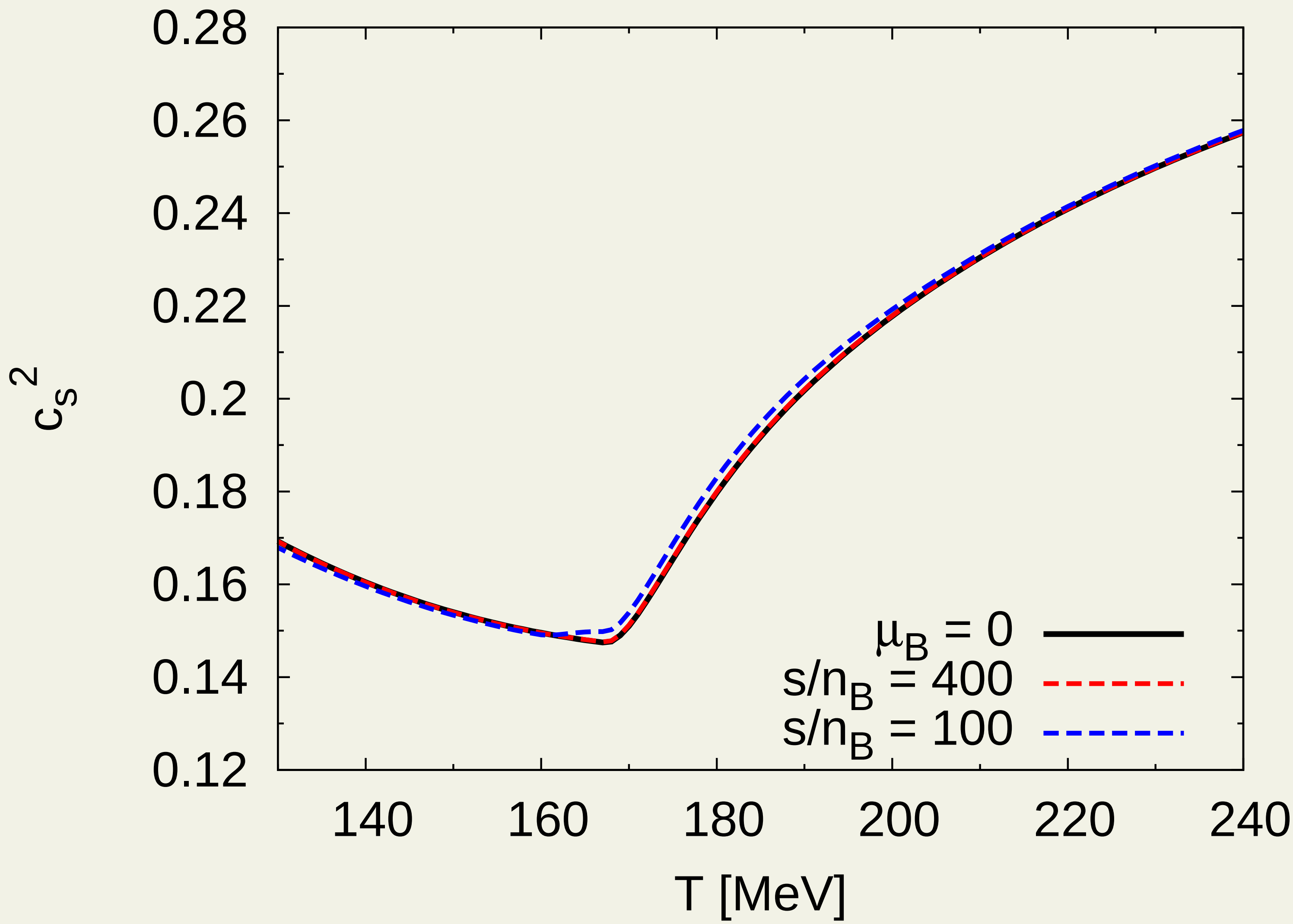
- $n_b = 0$ **parametrized Budapest-Wuppertal trace anomaly, arXiv:1309.5258**

Speed of sound along $s/n_b = \text{const.}$ curve



$s/n_b = 400 \leftrightarrow \sqrt{s_{\rm NN}} = 200\,\text{GeV}$

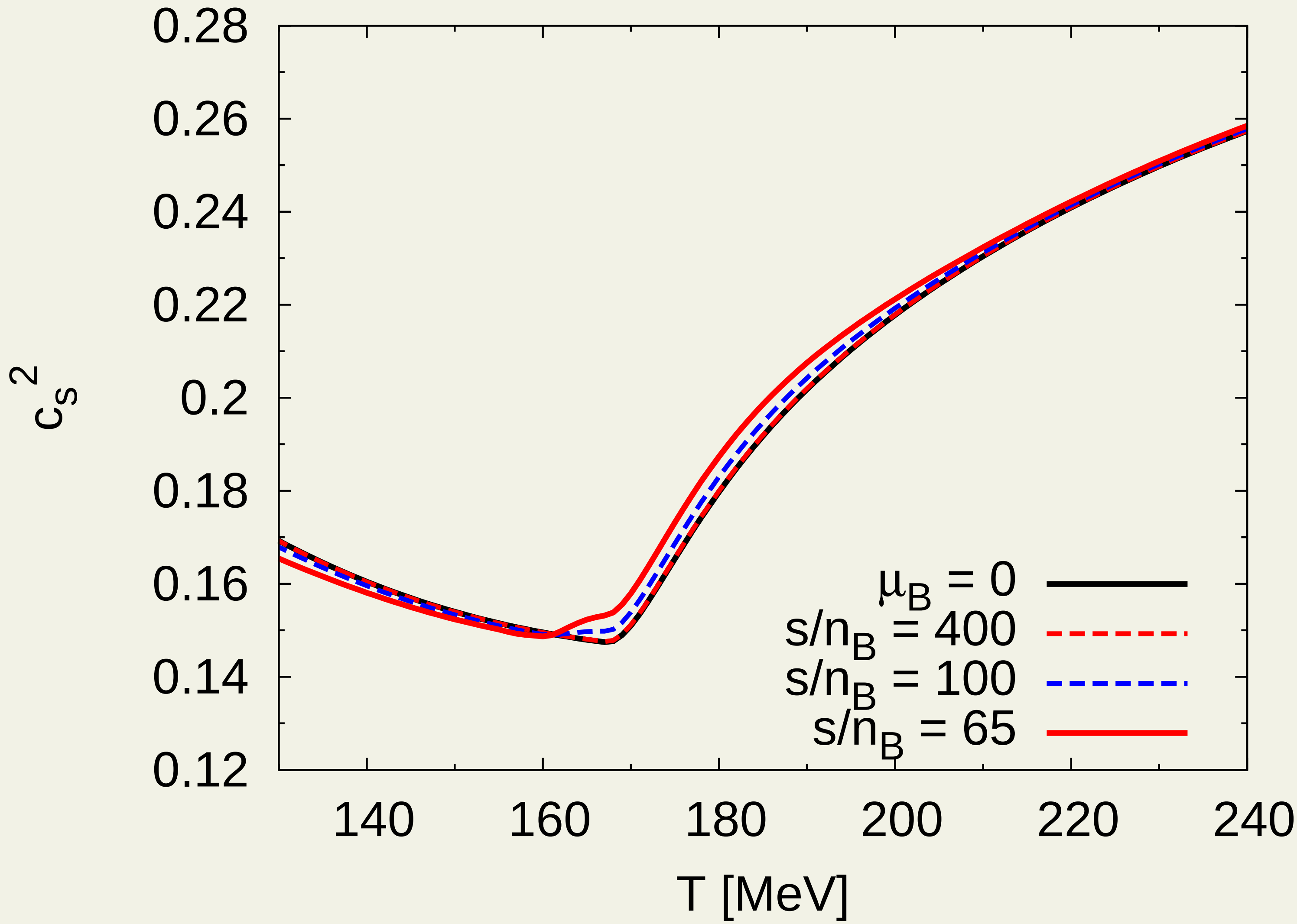
Speed of sound along $s/n_b = \text{const.}$ curve



$s/n_b = 400 \leftrightarrow \sqrt{s_{\rm NN}} \sim 200\,\text{GeV}$

$s/n_b = 100 \leftrightarrow \sqrt{s_{\rm NN}} \sim 64\,\text{GeV}$

Speed of sound along $s/n_b = \mathrm{const.}$ curve

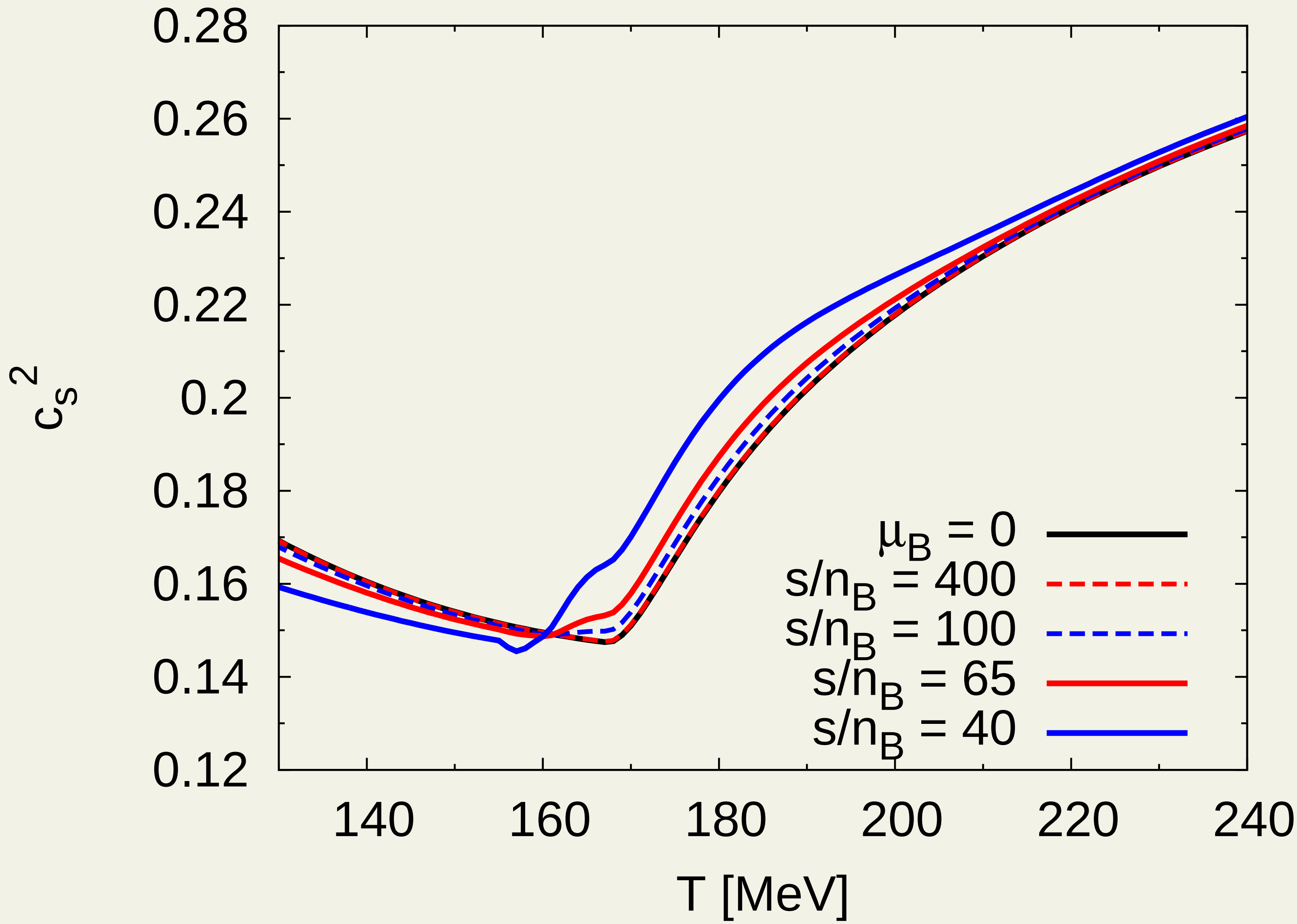


$s/n_b = 400 \leftrightarrow \sqrt{s_{\mathrm{NN}}} \sim 200\,\mathrm{GeV}$

$s/n_b = 65 \leftrightarrow \sqrt{s_{\mathrm{NN}}} \sim 39\,\mathrm{GeV}$

$s/n_b = 100 \leftrightarrow \sqrt{s_{\mathrm{NN}}} \sim 64\,\mathrm{GeV}$

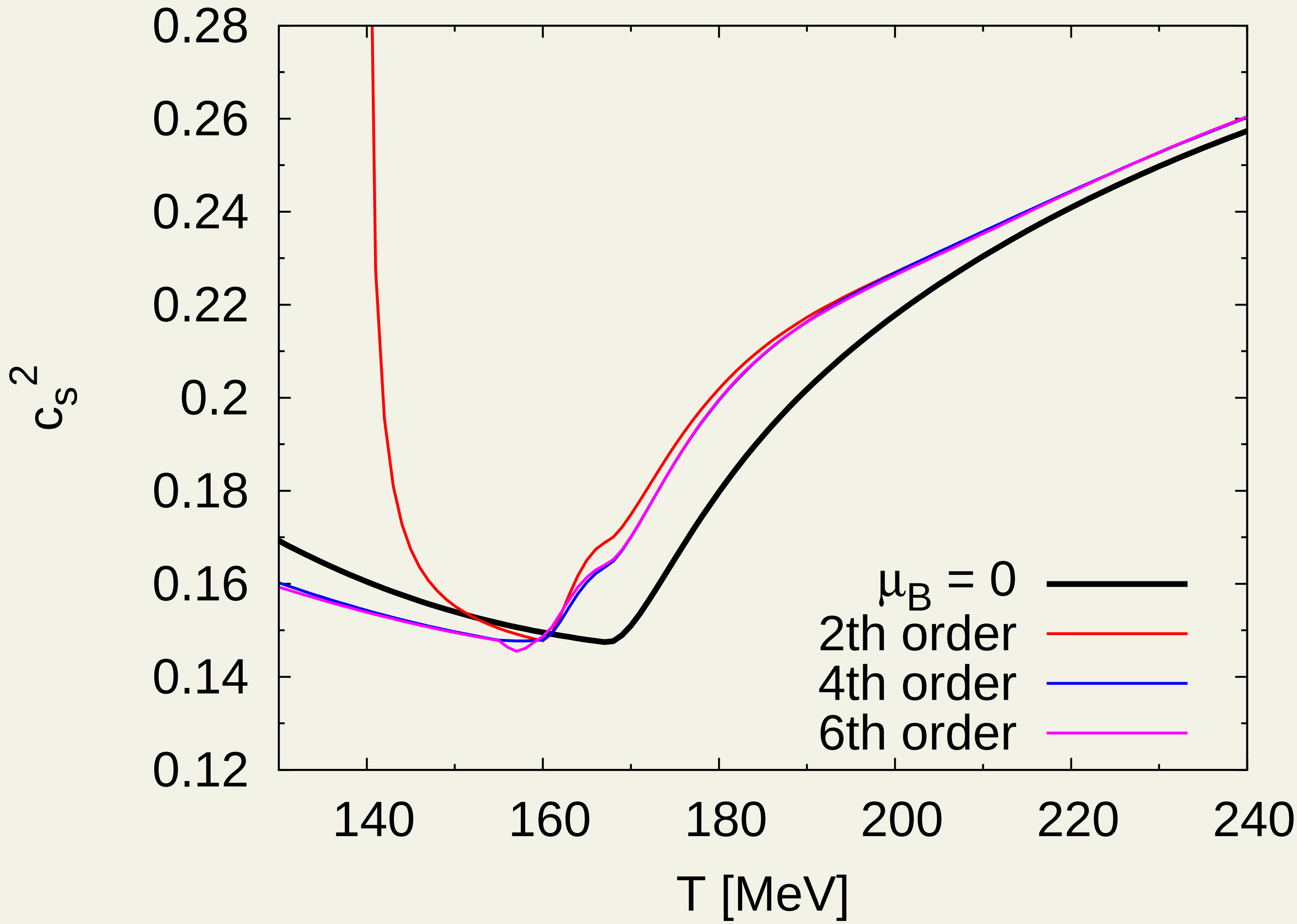
Speed of sound along $s/n_b = \text{const.}$ curve



$s/n_b = 400 \leftrightarrow \sqrt{s_{\rm NN}} \sim 200\,\text{GeV}$
$s/n_b = 65 \leftrightarrow \sqrt{s_{\rm NN}} \sim 39\,\text{GeV}$

$s/n_b = 100 \leftrightarrow \sqrt{s_{\rm NN}} \sim 64\,\text{GeV}$
$s/n_b = 40 \leftrightarrow \sqrt{s_{\rm NN}} \sim 17\,\text{GeV}$

Speed of sound along $s/n_b = 40$ curve

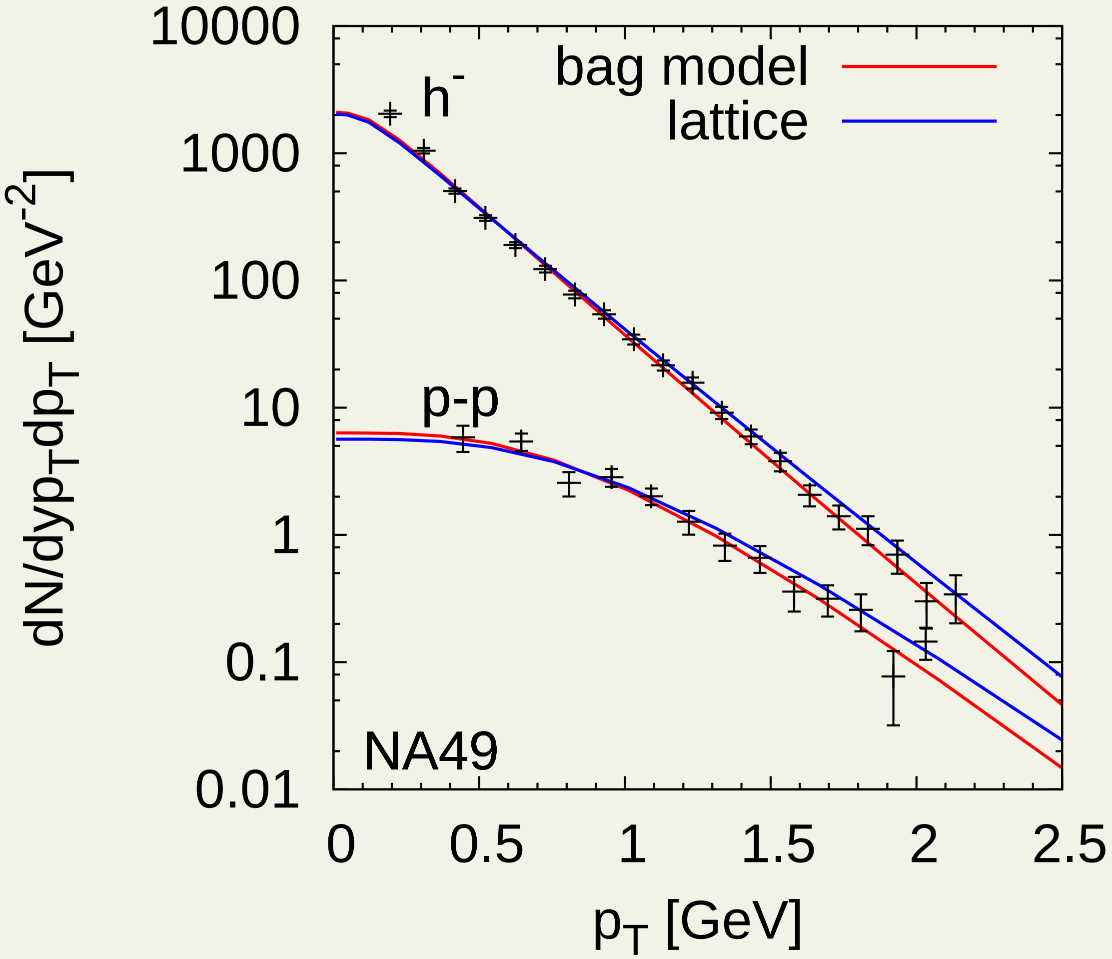


- each correction smaller than previous

⇒ expansion under control

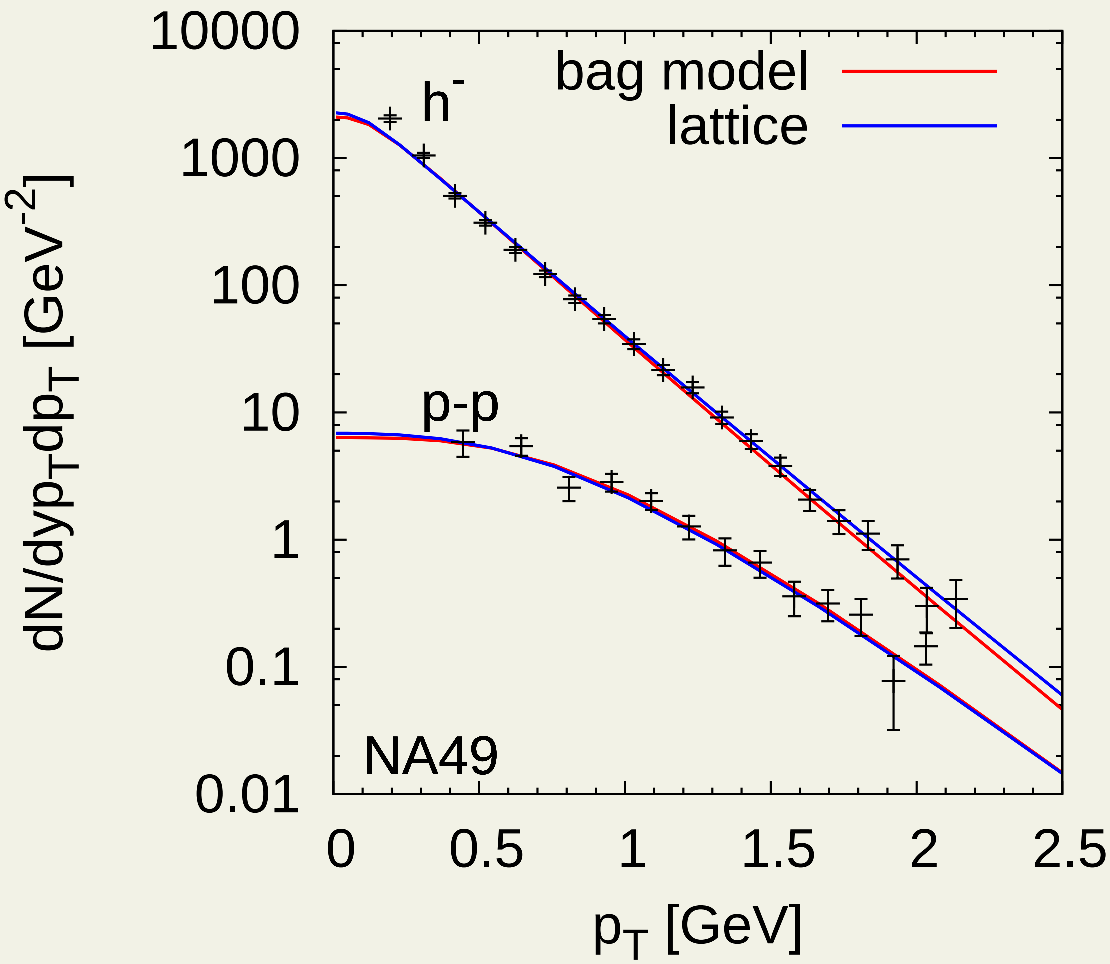
- 4th order essential at low temperatures!

p_T-spectra at SPS



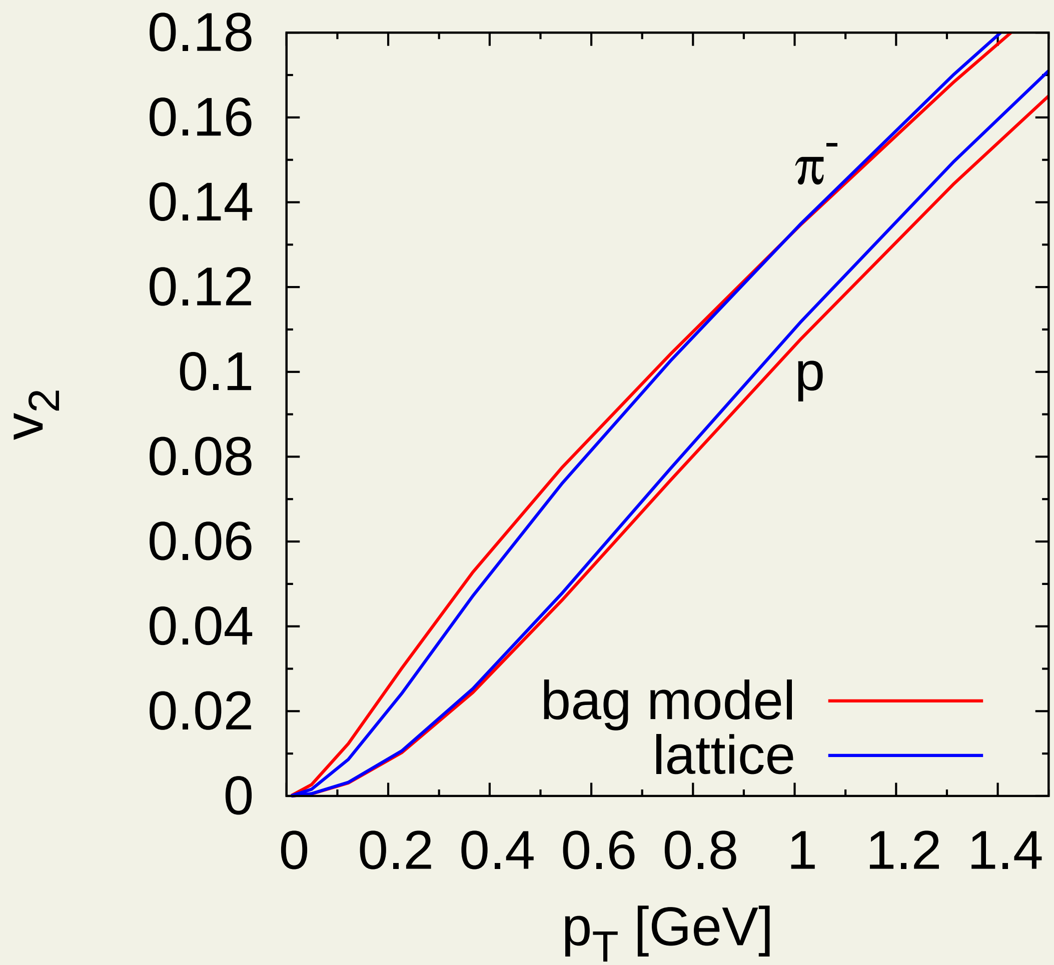
- **harder EoS, more transverse flow, flatter spectra**

p_T-spectra at SPS



- $T_{\rm fo} \approx 120$ **MeV (bag)** $\Rightarrow$ **130 MeV (lattice)**

v_2 at SPS ($b = 7$ fm)



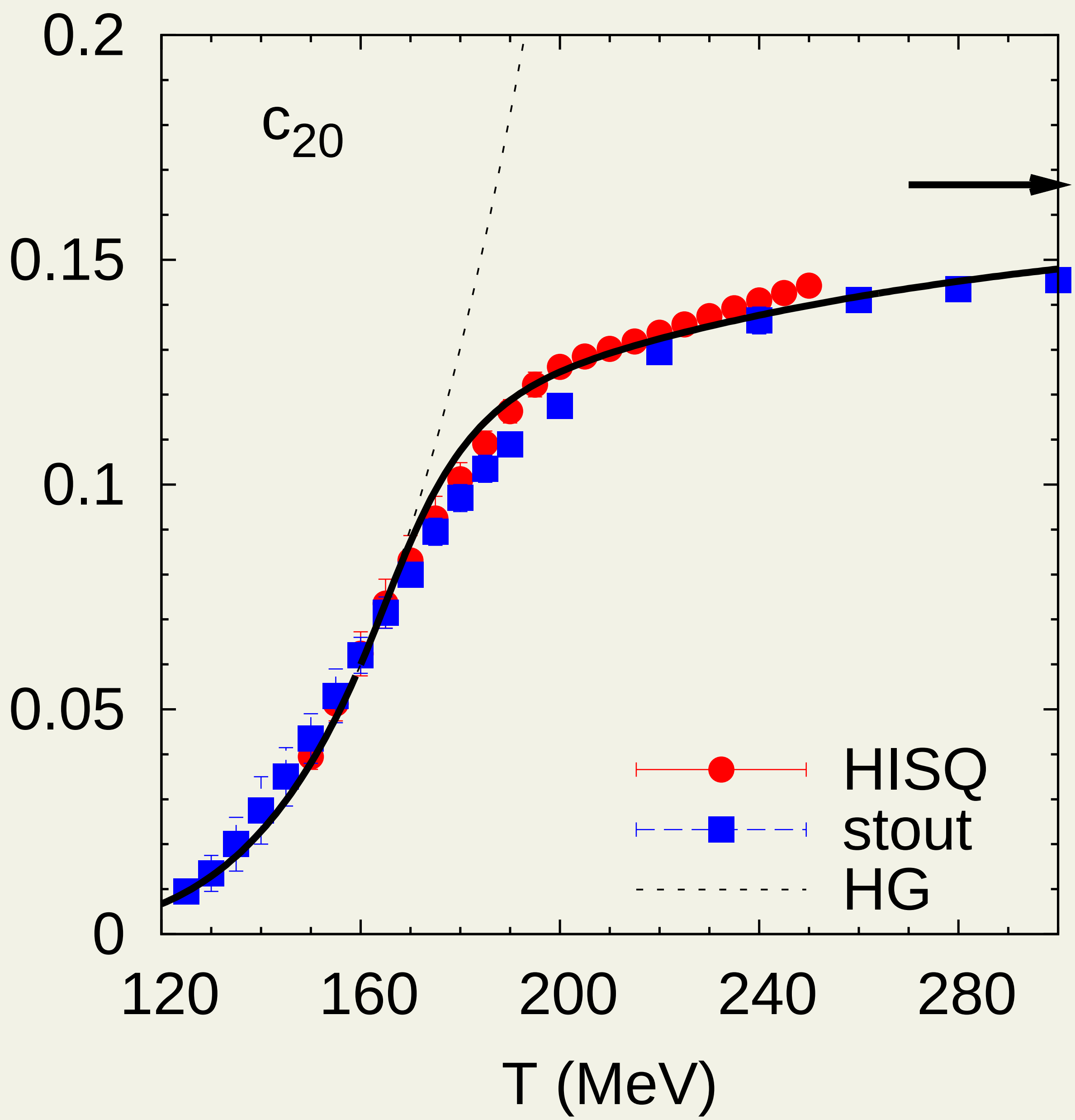
- $T_{\rm fo} \approx 120$ **MeV (bag)** $\Rightarrow$ **130 MeV (lattice)**

Conclusions

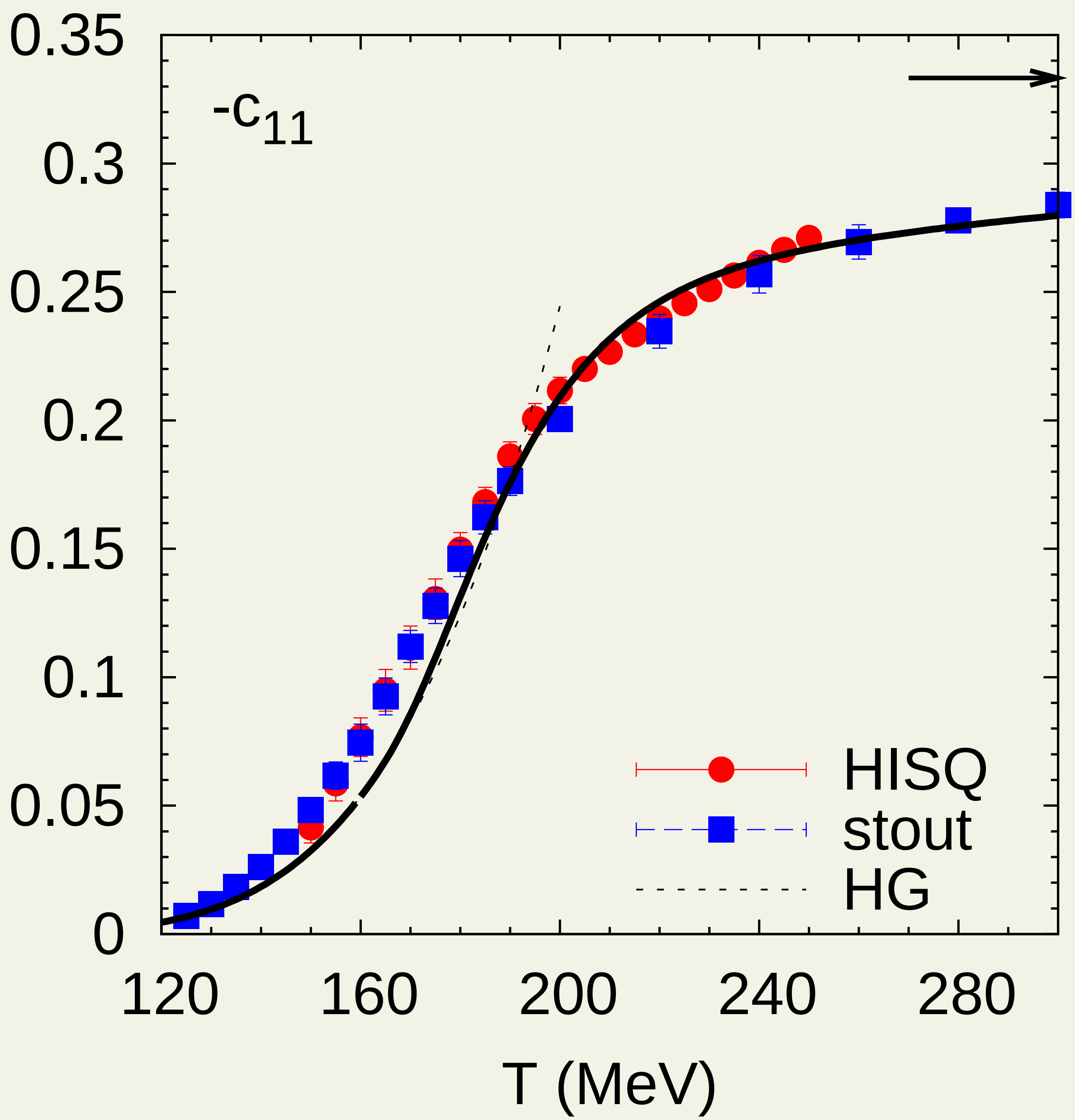
- EoS at finite baryon densities based on lattice QCD calculations of baryon number and strangeness fluctuations and correlations
  - extension to baryon densities at SPS energies requires 4th and 6th order coefficients
- lattice spacing dependence of hadron masses explains the difference between HRG and lattice QCD
  - 30 MeV shift in temperature
- effect on flow when compared to bag model EoS tiny at SPS and (some?) RHIC low energy scan energies

Backups

c_{20}

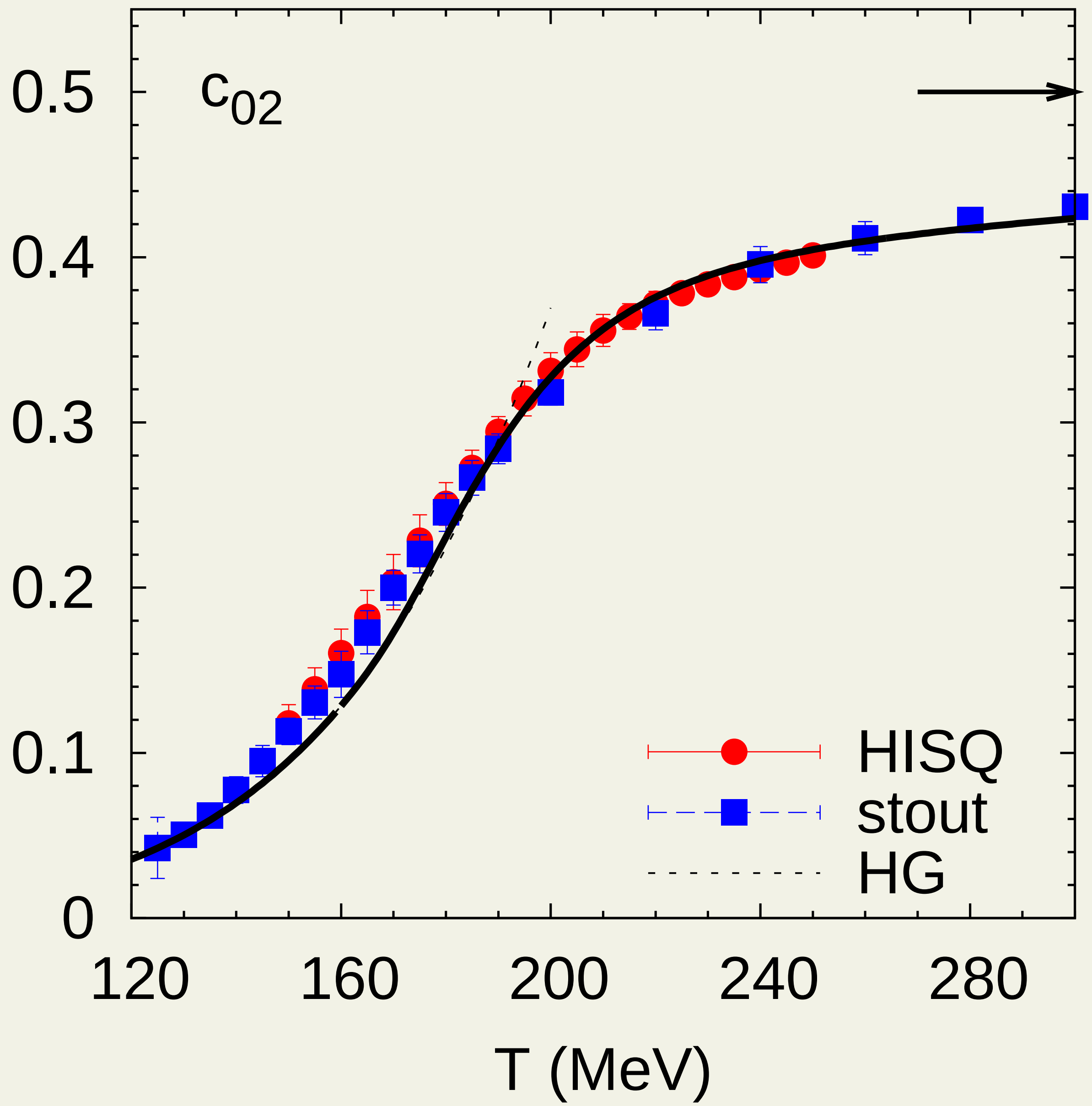


c_{11}

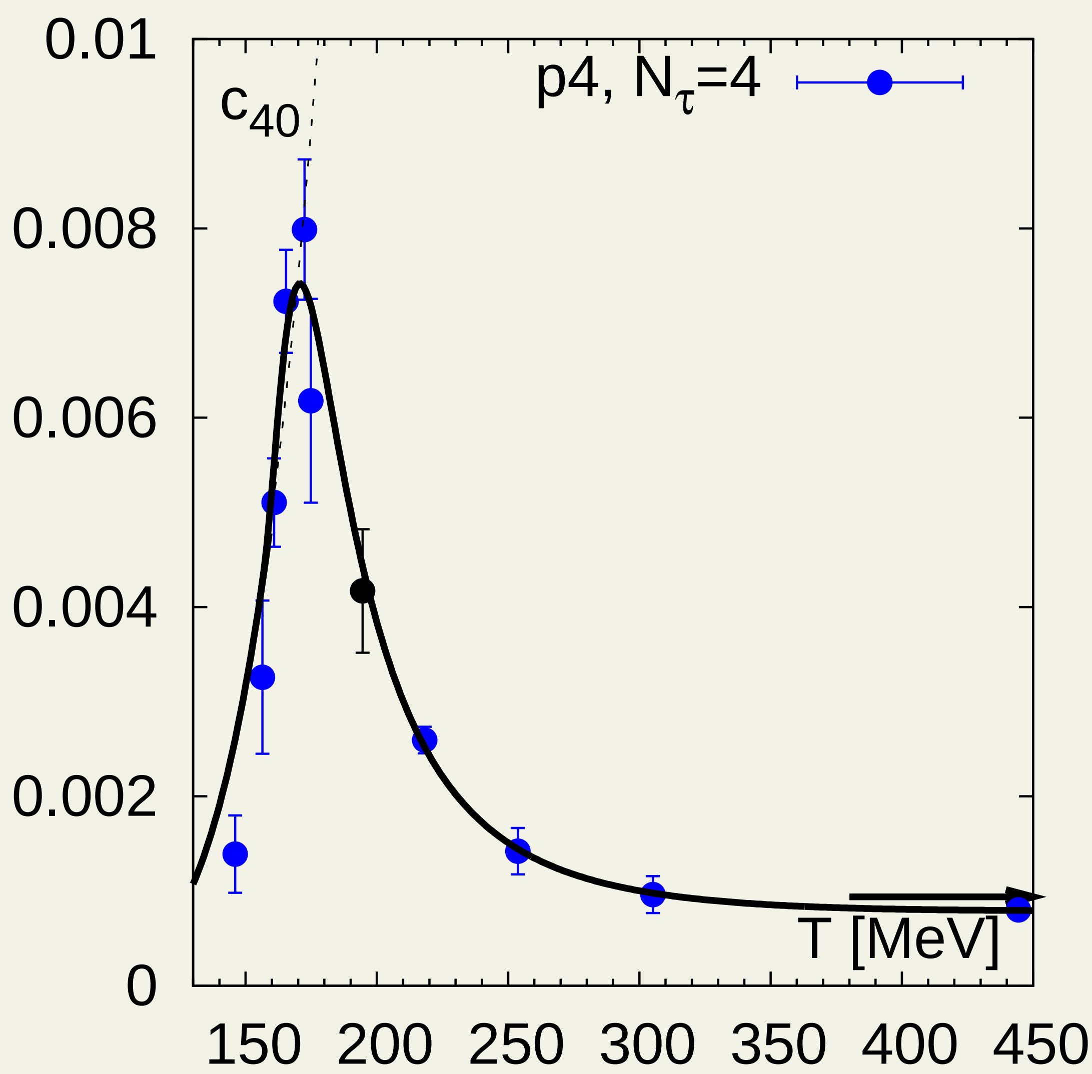


0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
-c11
HISQ
stout
HG
120
160
200
240
280
T (MeV)

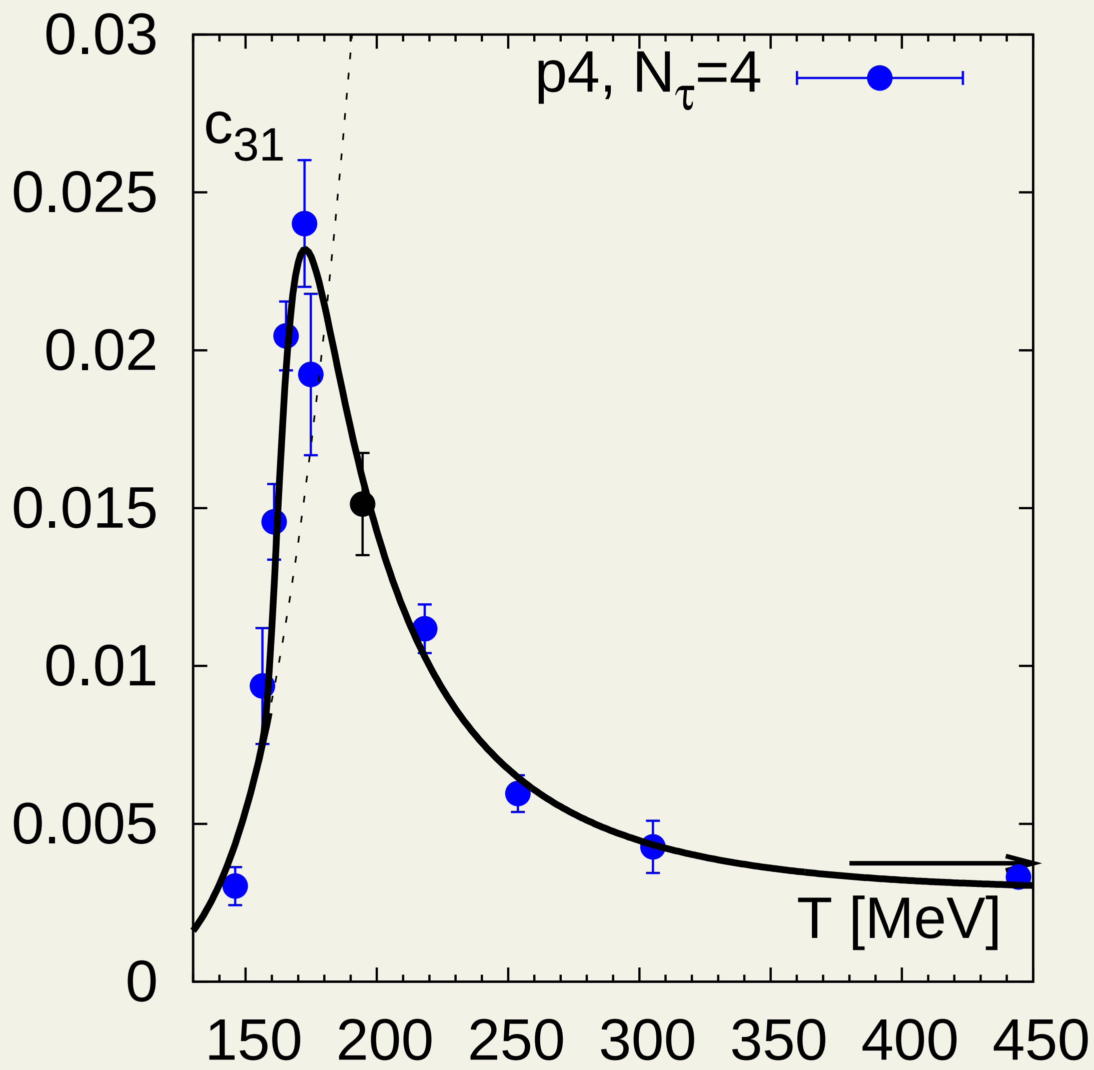
c_{02}



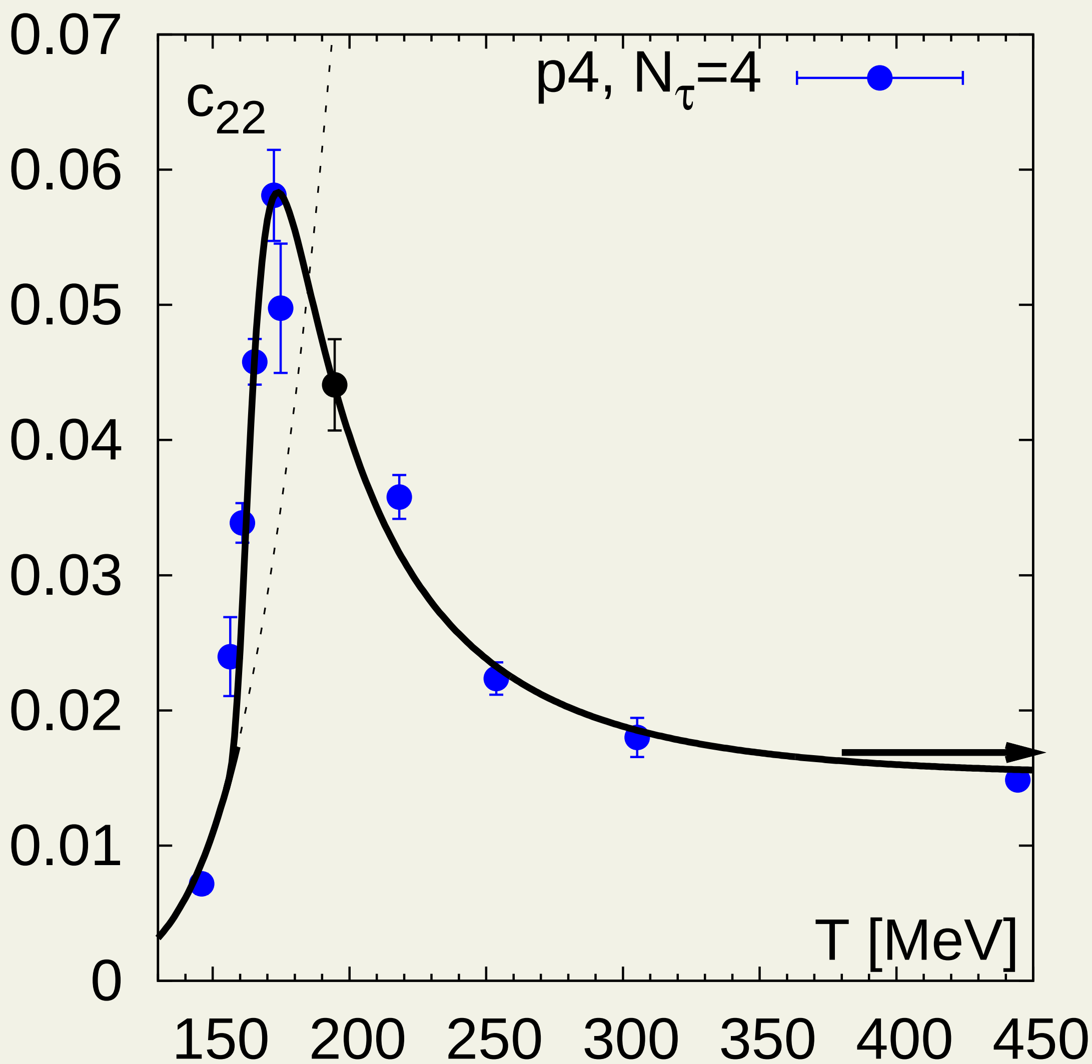
c_{40}


0.01
0.008
0.006
0.004
0.002
0
150 200 250 300 350 400 450
c40
p4, Nτ=4
T [MeV]

c_{31}

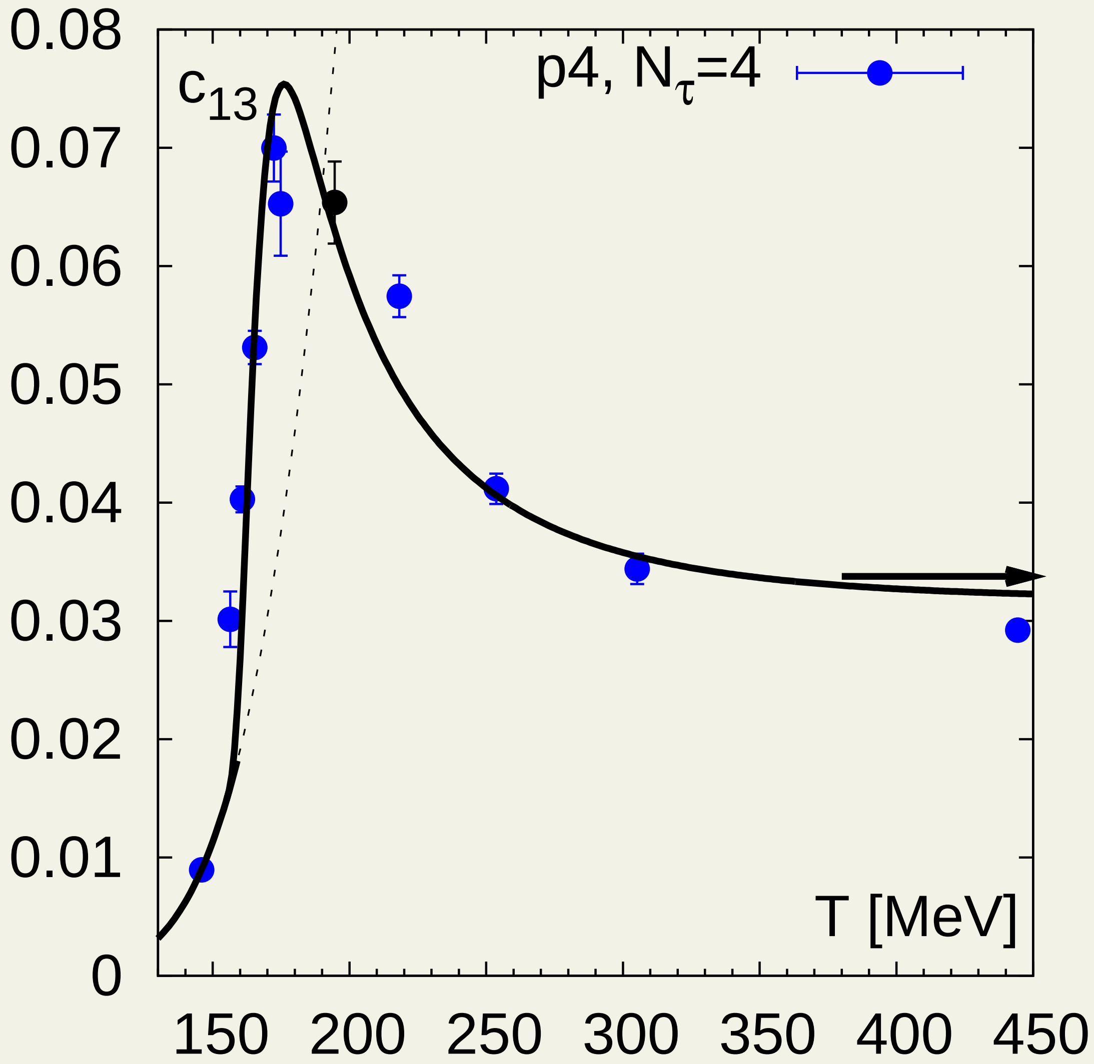

0.03
0.025
0.02
0.015
0.01
0.005
0
150
200
250
300
350
400
450
c31
p4, Nτ=4
T [MeV]

c_{22}



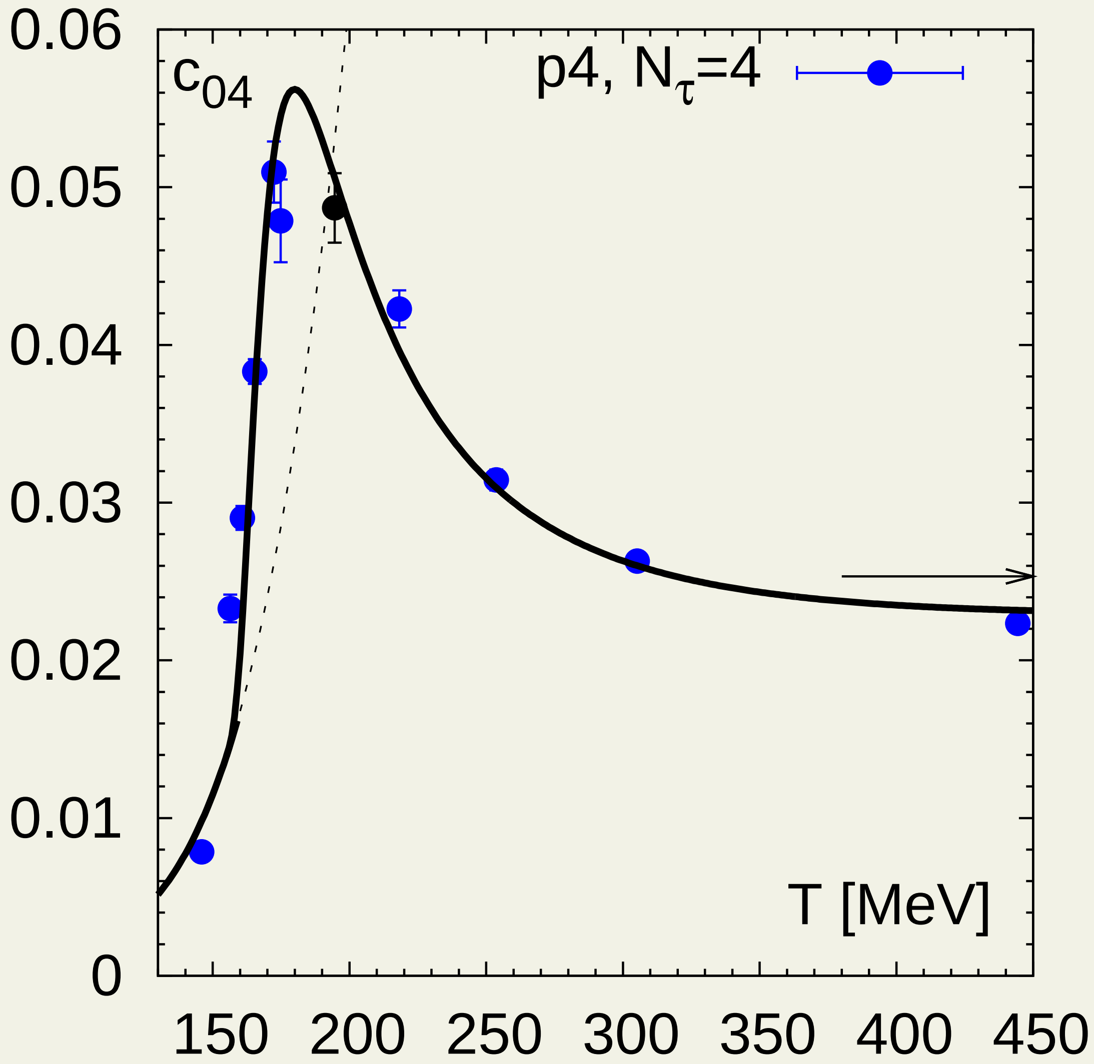
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
150
200
250
300
350
400
450
c22
p4, Nτ=4
T [MeV]

c_{13}



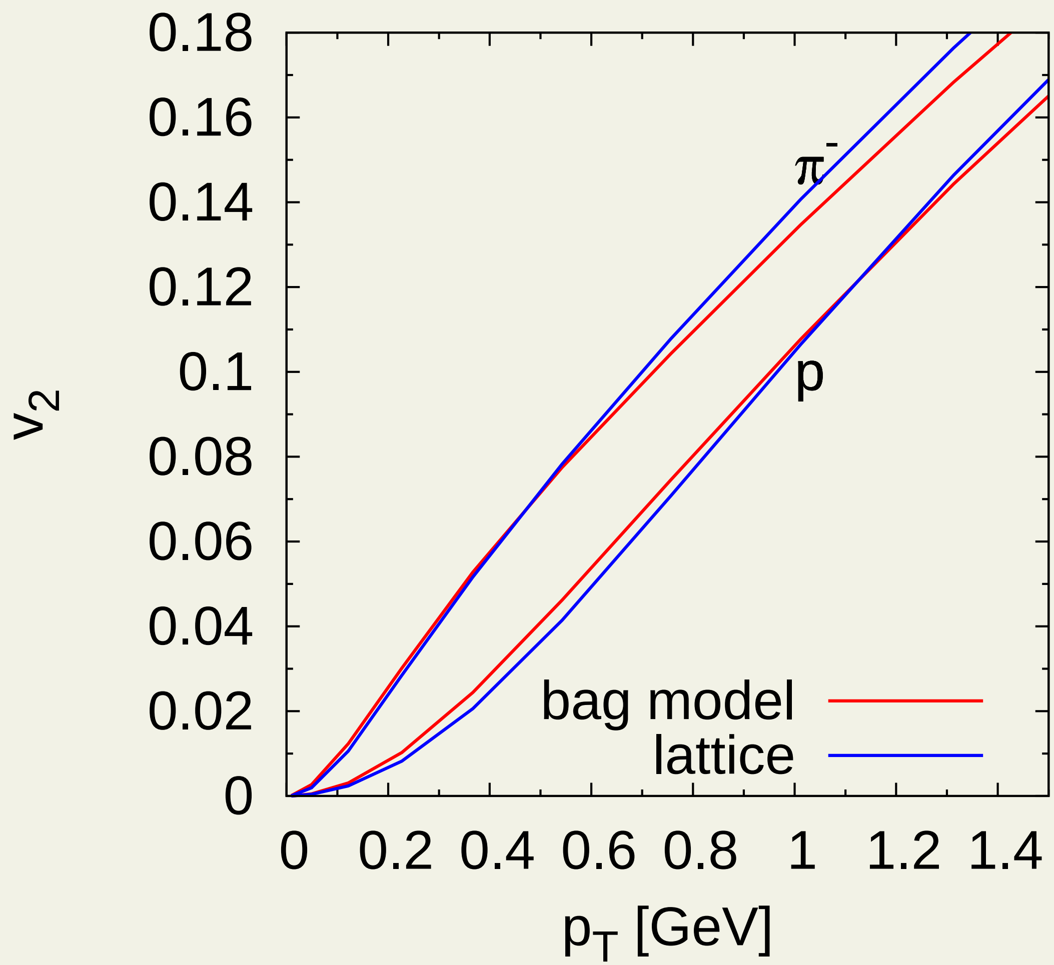
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
c13
p4, Nτ=4
T [MeV]
150
200
250
300
350
400
450

c_{04}



0.06
0.05
0.04
0.03
0.02
0.01
0
150
200
250
300
350
400
450
c04
p4, Nτ=4
T [MeV]

v_2 at SPS ($b = 7$ fm)



- $T_{\text{fo}} \approx 120$ **MeV (both)**