Lattice QCD based equation of state at finite baryon density

Pasi Huovinen
J. W. Goethe Universität & Frankfurt Institute for Advanced Studies

Quark Matter 2014
May 20, 2014, Darmstadt, Germany

in collaboration with Peter Petreczky and Christian Schmidt

funded by BMBF under contract no. 06FY9092
Nuclear phase diagram

Temperature [MeV]

Quark–gluon plasma?

\( \epsilon \approx 0.16 \text{ GeV/fm}^3 \)

\( n_0 = 1.4 \cdot 10^{44}/\text{m}^3 \)

\( \epsilon \sim 0.7 \text{ GeV/fm}^3 \)

Tricritical point?

Hadrons:
- Baryons: \( p, n, ... \)
- Mesons: \( \pi, K, ... \)

 Everyday world

\( \equiv \frac{1}{3}(n_q - n_{\bar{q}}) \)

SC phases?

\(~5-10 \, n_0\)

Baryon density

~170

K

\( 2 \cdot 10^{12} \mathrm{K} \)

\( \downarrow \)

\( \epsilon \approx 0.7 \text{ GeV/fm}^3 \)
Taylor expansion for pressure

$$\frac{P}{T^4} = \sum_{i,j} c_{ij}(T) \left( \frac{\mu_B}{T} \right)^i \left( \frac{\mu_S}{T} \right)^j,$$

where

$$c_{ij}(T) = \frac{1}{i!j!} \frac{\partial^i}{\partial (\mu_B/T)^i} \frac{\partial^j}{\partial (\mu_S/T)^j} \frac{P}{T^4},$$

i.e. moments of baryon number and strangeness fluctuations and correlations

an EoS based on lattice calculations of these?
Continuum extrapolated second order coefficients (also $c_{11}$):

- $c_{20}$
- $c_{02}$

**HISQ**: hotQCD collaboration, Phys. Rev. D 86, 034509 (2012)

**stout**: Budapest-Wuppertal collaboration, JHEP 1201, 138 (2012)

- Are first coefficients enough?
Pressure in HRG at T = 150 MeV

full hadron resonance gas, or evaluate Taylor coefficients in HRG:

- Fourth and sixth order coefficients needed
- Evaluated using \textit{p4 action} with $N_\tau = 4$

$\Rightarrow$ large discretization effects?
Hadrons on lattice

• **16 pseudoscalar mesons** on lattice

• Hadron masses depend on lattice cutoff
  \[ \Rightarrow \text{ i.e. on temperature:} \]
  
  E.g. for pseudoscalar mesons on asqtad calculations

\[
\begin{align*}
  m^2_{ps_i} &= m^2_{ps0} + \frac{1}{r^2_1} \frac{a^i_{ps} x + b^i_{ps} x^2}{(1 + c^i_{ps} x)^{3_i}} \\
  x &= (a/r_1)^2 \\
  a &= \frac{1}{N \tau T}
\end{align*}
\]
30 MeV shift

cut=1.7GeV
cut=1.9GeV
p4, $N_\tau=4$

$N_\tau=8$, HISQ
Parametrization

\[ c_{ij}(T) = \frac{a_{ij1}}{\hat{T}^{n_{ij1}}} + \frac{a_{ij2}}{\hat{T}^{n_{ij2}}} + \frac{a_{ij3}}{\hat{T}^{n_{ij3}}} + \frac{a_{ij4}}{\hat{T}^{n_{ij4}}} + \frac{a_{ij5}}{\hat{T}^{n_{ij5}}} + \frac{a_{ij6}}{\hat{T}^{n_{ij6}}} + c_{ij}^{SB}, \]

where \( n_{kij} \) are integers with \( 1 < n_{kij} < 23 \),

and

\[ \hat{T} = \frac{T - T_s}{R}, \]

with \( T_s = 0.1 \) or 0 GeV, and \( R = 0.05 \) or 0.15 GeV.
Constraints:

\[
\begin{align*}
    c_{ij}(T_{sw}) &= c_{ij}^{HRG}(T_{sw}) \\
    \frac{d}{dT} c_{ij}(T_{sw}) &= \frac{d}{dT} c_{ij}^{HRG}(T_{sw}) \\
    \frac{d^2}{dT^2} c_{ij}(T_{sw}) &= \frac{d^2}{dT^2} c_{ij}^{HRG}(T_{sw}) \\
    \frac{d^3}{dT^3} c_{ij}(T_{sw}) &= \frac{d^3}{dT^3} c_{ij}^{HRG}(T_{sw})
\end{align*}
\]

at \( T_{sw} = 160 \text{ MeV} \) for second order coefficients
\( T_{sw} = 155 \text{ MeV} \) for fourth and sixth order coefficients

3rd derivative to guarantee smooth behaviour of speed of sound:

\[
c_s^2 \propto \frac{d^2}{dT^2} c_{ij}
\]
$c_{20}$ and $c_{02}$

![Graph showing $c_{20}$ and $c_{02}$ as functions of T (MeV)](image_url)
$c_{11}$
$P/T^4$

![Graph showing $P$ vs $T$ with $P$ in GeV/fm$^{-3}$ and $T$ in MeV, and contours labeled with $\mu_B$ in MeV. The graph includes labeled axes and a color scale.](image)
Speed of sound along $s/n_b = \text{const.}$ curve

- $n_b = 0$ parametrized Budapest-Wuppertal trace anomaly, arXiv:1309.5258
Speed of sound along $s/n_b = \text{const.}$ curve

\[ s/n_b = 400 \leftrightarrow \sqrt{s_{NN}} = 200 \text{ GeV} \]
Speed of sound along $s/n_b = \text{const.}$ curve

$s/n_b = 400 \leftrightarrow \sqrt{s_{NN}} \sim 200 \text{ GeV}$

$s/n_b = 100 \leftrightarrow \sqrt{s_{NN}} \sim 64 \text{ GeV}$
Speed of sound along $s/n_b = \text{const.}$ curve

$s/n_b = 400 \leftrightarrow \sqrt{s_{NN}} \sim 200 \text{ GeV}$
$s/n_b = 65 \leftrightarrow \sqrt{s_{NN}} \sim 39 \text{ GeV}$

$s/n_b = 100 \leftrightarrow \sqrt{s_{NN}} \sim 64 \text{ GeV}$
Speed of sound along $s/n_b = \text{const.}$ curve

\[c_s^2 \begin{array}{c}
\mu_B = 0 \\
\text{s/n}_B = 400 \\
\text{s/n}_B = 100 \\
\text{s/n}_B = 65 \\
\text{s/n}_B = 40
\end{array}\]

$s/n_b = 400 \leftrightarrow \sqrt{s_{\text{NN}}} \sim 200 \text{ GeV}$
$s/n_b = 65 \leftrightarrow \sqrt{s_{\text{NN}}} \sim 39 \text{ GeV}$
$s/n_b = 100 \leftrightarrow \sqrt{s_{\text{NN}}} \sim 64 \text{ GeV}$
$s/n_b = 40 \leftrightarrow \sqrt{s_{\text{NN}}} \sim 17 \text{ GeV}$
Speed of sound along $s/n_b = 40$ curve

- each correction smaller than previous
  ⇒ expansion under control
- 4th order essential at low temperatures!
• harder EoS, more transverse flow, flatter spectra
$p_T$-spectra at SPS

$\frac{dN}{d\eta dp_T} \ [GeV^{-2}]$

- $h^-$
- $pp$

**NA49**

$T_{fo} \approx 120 \ \text{MeV (bag)} \Rightarrow 130 \ \text{MeV (lattice)}$
\( v_2 \) at SPS \( (b = 7 \text{ fm}) \)

- \( T_{fo} \approx 120 \text{ MeV (bag)} \Rightarrow 130 \text{ MeV (lattice)} \)
Conclusions

- EoS at finite baryon densities based on lattice QCD calculations of baryon number and strangeness fluctuations and correlations
- Extension to baryon densities at SPS energies requires 4th and 6th order coefficients
- Lattice spacing dependence of hadron masses explains the difference between HRG and lattice QCD
- 30 MeV shift in temperature
- Effect on flow when compared to bag model EoS tiny at SPS and (some?) RHIC low energy scan energies
$c_{02}$

![Graph showing $c_{02}$ vs. $T$ (MeV)]

- HISQ
- stout
- HG
$C_{40}$

$p_4, N_\tau=4$

$T [\text{MeV}]$
\( C_{31} \)

\[ p4, N_\tau = 4 \]

\[ T \ [	ext{MeV}] \]
$C_{22}$

\[c_{22} \quad p4, N_\tau=4\]
$c_{04}$

$p4, N_\tau=4$

$T \text{ [MeV]}$

$T$ values range from 150 to 450 MeV.

The graph shows the function $c_{04}$ as a function of temperature $T$. The data points are marked with blue circles, and error bars are indicated. The curve fitted to the data suggests a peak around $T = 200$ MeV.
$v_2$ at SPS ($b = 7$ fm)

\begin{itemize}
  \item $T_{fo} \approx 120$ MeV (both)
\end{itemize}