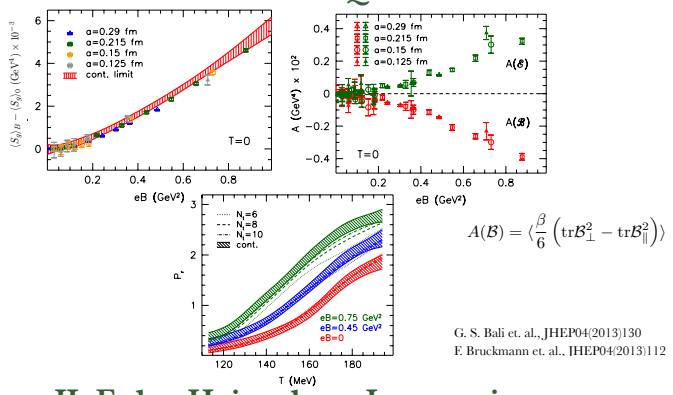
QCD effective potential with strong electromagnetic fields at zero and finite temperatures

Sho Ozaki[1], *Koichi Hattori*[2] *and Kazunori Itakura*[1]

1. *KEK Theory Center, IPNS, High Energy Accelerator Research Organization (KEK)*
2. *Theoretical Research Division, Nishina Center, RIKEN*

I. QCD vacuum in strong magnetic fields from lattice QCD



$$A(\mathcal{B}) = \langle \frac{\beta}{6}\left(\mathrm{tr}\mathcal{B}_\perp^2 - \mathrm{tr}\mathcal{B}_\parallel^2\right)\rangle$$

G. S. Bali et. al., JHEP04(2013)130
F. Bruckmann et. al., JHEP04(2013)112

II. Eular-Heisenberg Lagrangian for QCD+QED

We derive the Eular-Heisenberg Lagrangian for QCD + QED:

$$\mathcal{L}^{1+1T}_{QCD+QED} = \mathcal{L}^{1+1T}_{YM} + \mathcal{L}^{1+1T}_{q}$$

where $\mathcal{L}^{1+1T}_{YM}$ and $\mathcal{L}^{1+1T}_{q}$ are the YM part and quark part, respectively and expliciet forms are given as

$$\mathcal{L}^{1+1T}_{YM} = -\frac{i^\epsilon}{32\pi^2}\sum_{h=1}^{N_c^2-1}\int_0^\infty \frac{ds}{s^{3-\epsilon}}\left\{e^{-2igv_h as}e^{+2igv_h as} + e^{2gv_h bs} + e^{-2gv_h bs} - 2\right\}$$
$$\times \frac{gv_h as}{\sin(gv_h as)}\frac{gv_h bs}{\sinh(gv_h bs)}\left[1 + 2\sum_{n=1}^{\infty} e^{i\frac{h(s)}{4T^2}n^2}\cos\left(\frac{gv_h A_0}{T}n\right)\right]$$

$$\mathcal{L}^{1+1T}_{q} = \frac{i^\epsilon}{8\pi^2}\sum_{a=1}^{N_c}\sum_{i=1}^{N_f}\int_0^\infty \frac{ds}{s^{3-\epsilon}}e^{-im_{q_i}^2 s}(a_{a,i}s)(b_{a,i}s)\cot(a_{a,i}s)\coth(b_{a,i}s)$$
$$\times\left[1 + 2\sum_{n=1}^{\infty}(-1)^n e^{i\frac{h_{a,i}(s)}{4T^2}n^2}\cos\left(\frac{g\omega_a A_0}{T}n\right)\right]$$

w_a, v_h are color eigenvalues of quark and gluon.

Zero temperature part — Finite temperature part

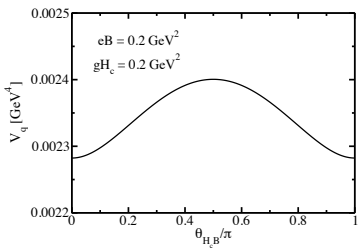
$$a_{a,i}^2 - b_{a,i}^2 = [(gw_a)^2(\vec{H}_c^2 - \vec{E}_c^2) + (eQ_{q_i})^2(\vec{B}^2 - \vec{E}^2) + 2gw_a eQ_{q_i}(\vec{H}_c\cdot\vec{B} - \vec{E}_c\cdot\vec{E})]$$
$$a_{a,i}b_{a,i} = -[(gw_a)^2\vec{E}_c\cdot\vec{H}_c + (eQ_{q_i})^2\vec{E}\cdot\vec{B} + gw_a eQ_{q_i}(\vec{E}_c\cdot\vec{B} + \vec{E}\cdot\vec{H}_c)]$$
$$h_{a,i}(s) = \frac{b_{a,i}^2 - \mathcal{E}_{a,i}}{a_{a,i}^2 + b_{a,i}^2}a_{a,i}\cot(a_{a,i}s) + \frac{a_{a,i}^2 + \mathcal{E}_{a,i}}{a_{a,i}^2 + b_{a,i}^2}b_{a,i}\coth(b_{a,i}s), \quad \mathcal{E}_{a,i} = (gw_a\vec{E}_c + eQ_{q_i}\vec{E})^2$$

II. Zero temperature part

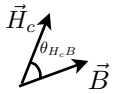
Here, we concentrate on the pure chromomangnetic fields with U(1)$_{em}$ magnetic fields. In this case, we can perform the proper time integral analytically, and the effective potential at zero temperature reads

$$V^1_{YM} = \frac{11N_c}{96\pi^2}(gH_c^2)\left\{\log\left(\frac{gH_c}{\mu^2}\right) - c_g + \frac{1}{N_c}\sum_{h=1}^{N_c^2-1}v_h^2\log|v_h|\right\}$$
$$V^1_q = \sum_{a=1}^{N_c}\sum_{i=1}^{N_f}\left\{-\frac{a_{a,i}^2}{24\pi^2}\left[\log\left(\frac{2a_{a,i}}{\mu^2}\right) + 12\zeta'\left(-1, \frac{m_{q_i}^2}{2a_{a,i}}\right) - 1\right]\right.$$
$$\left. + \frac{a_{a,i}m_{q_i}^2}{8\pi^2}\log\left(\frac{2a_{a,i}}{m_{q_i}^2}\right) - \frac{m_{q_i}^4}{16\pi^2}\left[\log\left(\frac{2a_{a,i}}{m_{q_i}^2}\right) + \frac{1}{2}\right]\right\}$$

♠ Anisotropy of the QCD vacuum in magnetic field



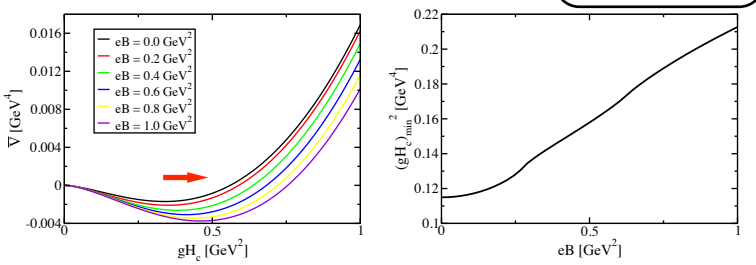
$$a_{a,i} = \sqrt{(gw_a\vec{H}_c + eQ_{q_i}\vec{B})^2} = \sqrt{(gw_a H_c)^2 + (eQ_{q_i}B)^2 + 2gw_a eQ_{q_i}H_c B\cos\theta_{H_cB}}$$



The chromomagnetic field prefers to be parallel (anti-parallel) to the external B-field, which is consistent with lattice QCD.

♠ Gluonic magnetic catalysis in chromomagnetic condensate

$\alpha_s = 1$ at $\mu = 1$ GeV^2



- The chromomagnetic condensate increases with an increasing B-field.
- This result supports the recent observed gluonic magnetic catalysis at zero temperature in lattice QCD.

III. finite temperature part (preliminary)

We restrict ourselves to color SU(2) case for simplicity. The Polyakov loop is defined as

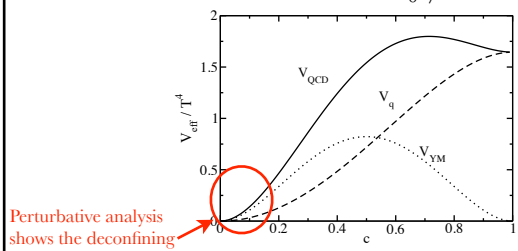
$$P(\vec{x}) = \langle L(\vec{x})\rangle = \langle \frac{1}{N_c}\mathrm{tr}\mathcal{T}\exp\left\{ig\int_0^\beta d\tau A_0(\tau,\vec{x})\right\}\rangle$$

In the Polyakov gauge, $\partial_0 A_0 = 0$, $A_0 = a_0\frac{\tau_3}{2}$,

$$L = \cos(\pi c)\,, \quad c = \frac{gA_0}{2\pi T}$$

♠ Weiss potential of SU(2) QCD with vanishing fields

$$E_c = H_c = E = B = 0$$
$$A_0 \neq 0$$



Center symmetry
$c \leftrightarrow 1 - c$

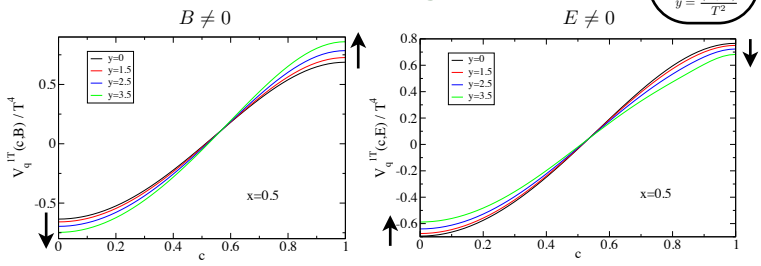
Quark loop explicitly breaks the center symmetry.

Perturbative analysis shows the deconfining phase.

♠ Explicite center symmetry breaking in electric and magnetic fields

$x = \frac{m_q^2}{T^2}$, $y = \frac{(E,B)}{T^2}$

$B \neq 0$ $E \neq 0$



- The magnetic field enhances the explicite breaking of the center symmetry.
- On the other hand, the electric field suppresses the explicite symmetry breaking.

IV. Summary and outlook

- We analyze the QCD vacuum with strong electromagnetic fields at zero and finite temperatures in one-loop level.
- Non-perturbative analysis with FRG is ongoing.

References

[1] S. Ozaki, Phys. Rev. D 89, 054022 (2014)
[2] S. Ozaki, K. Hattori and K. Itakura, in progress