Structure of chromomagnetic fields in the glasma

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Outline

- CGC, Glasma, JIMWLK evolution as initial condition for CYM
- Spatial Wilson loop
- Magnetic field correlator

Talk based on: A. Dumitru, T.L., Y. Nara, `arXiv:1401.4124 [hep-ph]`; t.b.p. PLB

Comments:

- This talk is purely 2+1d boost-invariant.
- Only $Q_s\tau \lesssim 10$
- Starting point for isotropization

Gluon saturation, Glass and Glasma

Small x: the hadron/nucleus wavefunction is characterized by **saturation scale** $Q_s \gg \Lambda_{QCD}$.

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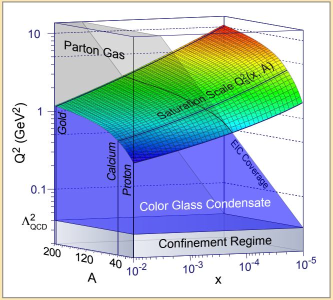
$$\Downarrow$$

$p_T \sim Q_s$: strong fields $A_\mu \sim 1/g$

- occupation numbers $\sim 1/\alpha_s$
- classical field approximation
- small α_s, but nonperturbative



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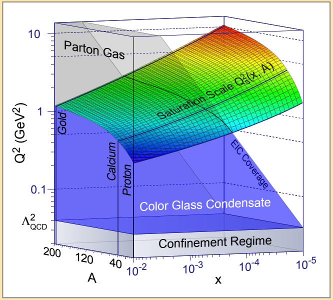


CGC: Effective theory for wavefunction of nucleus

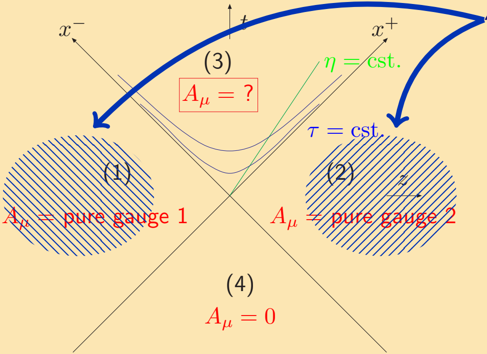
- Large x = source ρ, **probability** distribution $W_y[\rho]$
- Small x = classical gluon field A_μ + quantum flucts.

Glasma field configuration of two colliding sheets of CGC.

Distribution $W_y[\rho]$ from MV model or fixed/running coupling JIMWLK

Gluon fields in AA collision

Classical **Y**ang-**M**ills



2 pure gauges

$$A^i_{(1,2)} = \frac{i}{g} U_{(1,2)}(\mathbf{x}) \partial_i U^\dagger_{(1,2)}(\mathbf{x})$$

$$U_{(1,2)}(\mathbf{x}) = P e^{ig \int dx^- \frac{\rho(\mathbf{x}, x^-)}{\nabla^2}}$$

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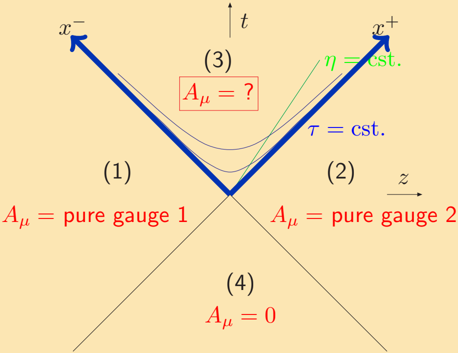

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$$A^i\Big|_{\tau=0} = A^i_{(1)} + A^i_{(2)}$$

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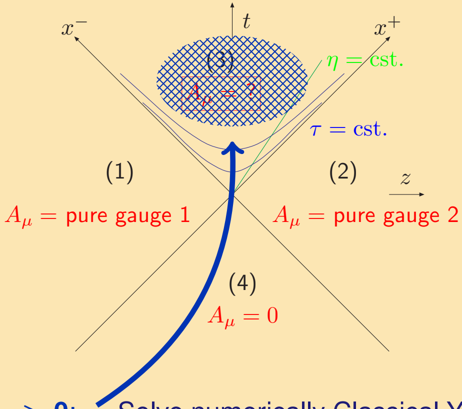

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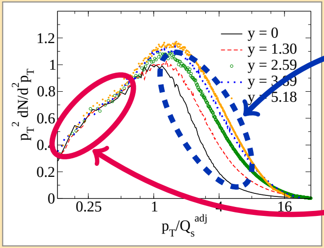
$\tau > 0$: Solve numerically Classical Yang-Mills **CYM** equations.
This is the **glasma** field $\Longrightarrow$ Then average over ρ.

Interpretation: Gluons with $p_T \sim Q_s$ — strings of size $R \sim 1/Q_s$

Universality in the IR spectrum?



(Here $y = \ln \sqrt{s}$:

$y = 0$ is MV, larger y JIMWLK)

- Scaled gluon spectrum in the UV depends on anomalous dimension $\Longrightarrow$ different for MV, JIMWLK
- IR seems to **scale**

But this requires gauge fixing

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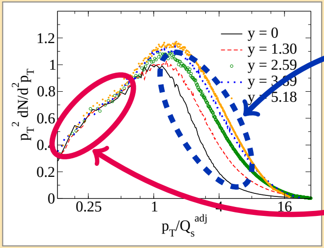

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Gauge inv. probe for $p_T \lesssim Q_s$?

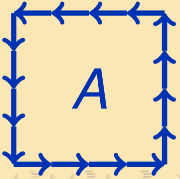
Spatial Wilson loop

$$W(A) = \frac{1}{N_c} \operatorname{Tr} \mathbb{P} \exp\left\{ ig \oint_A d\mathbf{x} \cdot \mathbf{A} \right\}$$

2d lattice: transverse links:

$$\uparrow = U_i(\mathbf{x}) = \exp\{igaA_i\}$$

$$W(A) = \frac{1}{N_c} \operatorname{Tr}$$

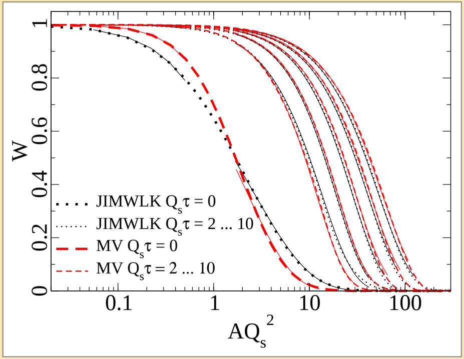


Measure Wilson loops

$N_c = 2$, only MV in Dumitru, Nara, Petreska [arXiv:1302.2064], PRD 2013. Here $N_c = 3$.

Calculation is simple:

- Construct initial glasma fields at $\tau = 0$ using e.g.
  - MV model
  - rcJIMWLK
  - fcJIMWLK
- Evolve forward in τ
- Measure $W(A)$



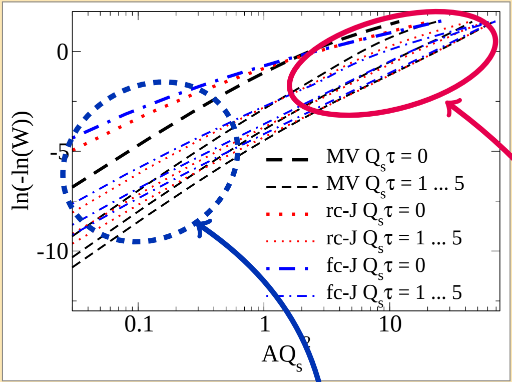
Both UV ($AQ_s^2 \lesssim 1$) and IR ($AQ_s^2 \gtrsim 1$) fit with parametrization

$$W = \exp\left\{-(\sigma A)^{\gamma}\right\}$$

(Fits are solid lines in figure)

Fit to Wilson loop area dependence

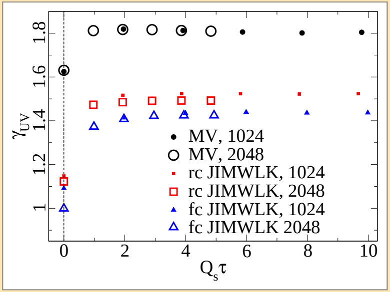
$$W = \exp\{-(\sigma A)^\gamma\} \Longleftrightarrow \ln(-\ln W) = \gamma \ln(AQ_s^2) + \gamma \ln(\sigma/Q_s^2)$$

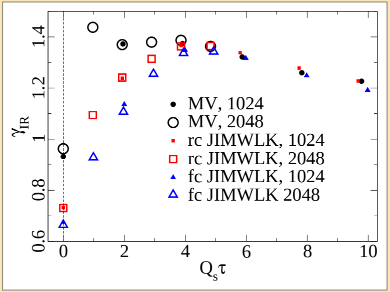


Main observations

- UV (small loop): initial slope γ stays
- IR (big loop): all init. conditions collapse to universal behavior

Wilson loop scaling exponents





UV ($e^{-3.5} < AQ_s^2 < e^{-0.5}$)

IR ($e^{0.5} < AQ_s^2 < e^{5}$)

Remembers initial condition

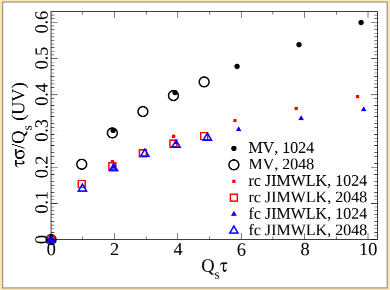
Initial conditions collapse to $\gamma_{IR} \approx 1.2$ decreasing slowly with τ

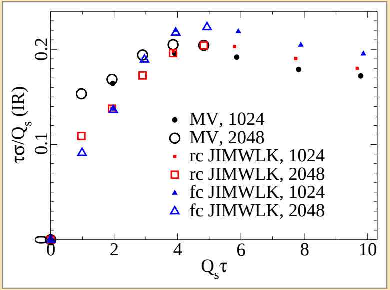
"String tension" coefficients σ

In expanding system fields naturally decrease as

$$\tau \gg 1/Q_s \implies A_\mu \sim 1/\sqrt{\tau} \implies \sigma/Q_s^2 \sim 1/(Q_s\tau)$$

Plot "string tension" σ as scaling variable $\sigma\tau/Q_s$





UV: initial conditions differ

IR: even σ universal within $\sim$ 10%

(Note: the numerical value of σ depends on the convention used to define Q_s)

At $\tau = 0$: $\sigma/Q_s^2 \approx 0.55 \ldots 0.6$ (UV) and $\sigma/Q_s^2 \approx 0.35 \ldots 0.45$ (IR)

Wilson loop and magnetic field correlator

Wilson loop measures magnetic flux:

$$W(A) = \frac{1}{N_c} \mathrm{Tr}\, \mathbb{P} \exp\left\{ ig \oint_A \mathrm{d}\mathbf{x} \cdot \mathbf{A} \right\} \quad \text{“}=\text{”} \quad \frac{1}{N_c} \mathrm{Tr} \exp\left\{ ig \int \mathrm{d}^2\mathbf{x} B_z(\mathbf{x}) \right\}$$

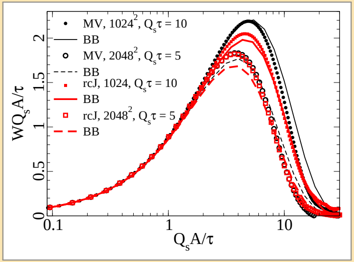
If magnetic field consists of **uncorrelated Gaussian domains**:

$$\langle W(A) \rangle = \exp\left\{ -\frac{1}{2}\frac{1}{N_c} \mathrm{Tr} \left\langle \left[\int \mathrm{d}^2\mathbf{x} g B_z(\mathbf{x}) \right]^2 \right\rangle \right\}$$

(Connect B:s at different locations with gauge link for gauge invariance)

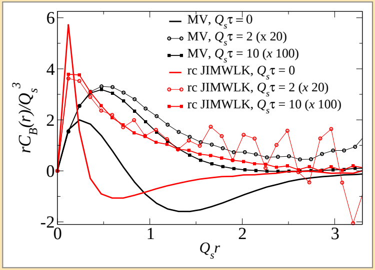
$\Longrightarrow$ $W(A)$ related to $\langle B(\mathbf{x}) B(\mathbf{y}) \rangle$, which we also measure

Check: compare $W(A)$ with reconstruction from BB-correlator



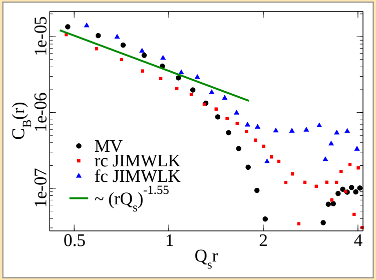
Magnetic field correlator

However: no obvious scaling seen in *BB*-correlator



Same on log plot

$$C(|\mathbf{x}-\mathbf{y}|) \equiv \mathrm{Tr}\left\langle [B(\mathbf{x})B(\mathbf{y})]_{\text{gauge link}}\right\rangle$$



Straight line: $\sim (rQ_s)^{-1.55}$.

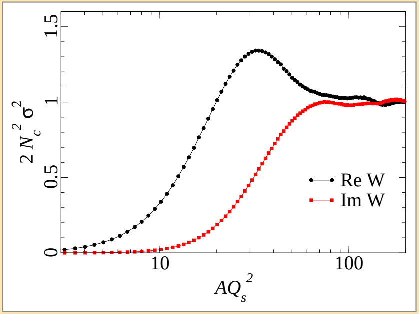
(For $C(r) \sim (rQ_s)^{-\alpha}$ one would get $\gamma = 2 - \alpha/2 \iff \alpha = 4 - 2\gamma$;
from $W(A)$ measured $\gamma = 1.22$, which gives $\alpha = 1.55$)

Wilson loop fluctuations and eigenvalue distributions

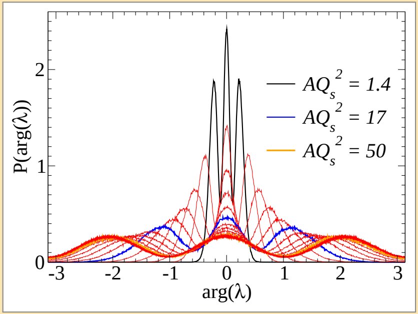
Work in progress, Dumitru, T.L., Nara

How are the Wilson lines distributed in SU(3)?

Fluctuations of Re W and Im W



Eigenvalue λ phase distribution:



For large areas A both look like random SU(3) matrices:

$$\sigma^2(\mathrm{Re}\,W) = \sigma^2(\mathrm{Im}\,W) = \frac{1}{2N_c^2}$$

$$P(\varphi \equiv \arg(\lambda)) = \frac{1}{2\pi}\left(1 + \frac{2}{3}\cos 3\varphi\right)$$

Conclusions

- CYM initial state for AA collision can be studied nonperturbatively by lattice methods
- Universal behavior for $p_T \ll Q_s$ seen in gluon spectrum,
  - I.e. produced gluon spectrum is independent of the unintegrated gluon distribution of the colliding nuclei.
- New observation: similar universality seen in spatial Wilson loop, for areas $A \gtrsim 1/Q_s^2$
  - Nontrivial area dependence of the loop $W \sim \exp\{A^{-1.2}\}$