Fluctuations in Relativistic Causal Hydrodynamics

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Abstract

We calculate hydrodynamic-fluctuations within the framework of the second order hydrodynamics of Müller, Israel and Stewart and its generalization to the third order. We have also calculated the fluctuations for several other causal hydrodynamical equations. We show that the form for the Onsager-coefficients and form of the correlation-functions remains same as those obtained by the relativistic Navier-Stokes equation. Further we numerically investigate evolution of the correlation function using the one dimensional boost-invariant (Bjorken) flow. We compare the correlation functions obtained using the causal hydrodynamics with the correlation-function for the relativistic Navier-Stokes equation.

Introduction

A macroscopic theory such as hydrodynamics provides a simplest possible description of a complicated many-body system in terms of space-time evolution of the mean or averaged quantities like energy density, pressure, flow velocity etc. The fluctuation theory studies small deviations from the mean behavior and it can help in calculating correlation functions for the macroscopic variables[1]. It is well-known that relativistic Navier-Stokes theory exhibit acausal behavior and it can give rise to unphysical instabilities. Indeed these issues do not arise in the second-order causal hydrodynamics theory developed by Müller, Israel and Stewart (MIS)[3]. In this work we apply Onsager theory to MIS equations and also to the hydrodynamics models developed by Denicol, Koide and Rischke (DKR) [6], Jaiswal, Bhalerao and Pal (JBP)[5] and other models based on MIS approach [4]. Further, we apply these results to study the hydrodynamical evolution of correlation functions using 1+0 dimensional Bjorken flow.

Fluctuations and correlations in Hydrodynamics

In thermodynamic equilibrium entropy of the system S is a function of the additive quantities x_k. In equilibrium, S satisfies the condition $X_k = -\frac{\partial S}{\partial x_k} = 0$.

However, when the system is slightly away from the equilibrium the generalized forces $X_k \neq 0$ and $\frac{dx_i}{dt} = -\gamma_{ik} X_k + \xi_i$, describes the flux associated with the quantity x_i, where ξ_i are the random forces and γ_{ik} are the Onsager coefficients.

- In a phenomenological theory time rate of change of the total entropy $\frac{dS}{dt}$ is given by,

$$\frac{dS}{dt} = -\frac{dx_i}{dt} X_i \,, \tag{1}$$

,

- Correlation between ξ_i is given by the formula,

$$\langle \xi_i(t_1)\xi_k(t_2)\rangle = (\gamma_{ik} + \gamma_{ki})\delta(t_1 - t_2). \tag{2}$$

- The expressions for the energy momentum tensor $T^{\mu\nu}$ and the current-density J^μ_B for a viscous fluid are,

$$T^{\mu\nu} = T^{\mu\nu}_{id} + \Delta T^{\mu\nu} + S^{\mu\nu}, \tag{3}$$

$$J^\mu_B = n_B u^\mu + \nu^\mu + I^\mu, \tag{4}$$

where, $T^{\mu\nu}_{id} = \epsilon u^\mu u^\nu - p\Delta^{\mu\nu}$ →energy momentum tensor of ideal fluid.
$\Delta T^{\mu\nu} = \Delta T^{\mu\nu}_{vis} + \Delta T^{\mu\nu}_{heat}$[2].

- The equations of hydrodynamics are the conservation equations, $\partial_\mu J^\mu_B = 0$, $\partial_\mu T^{\mu\nu} = 0$.

Equations for dissipative fluxes and fluctuation correlations in MIS

- In Landau-Lifshitz frame the expression for the non-equilibrium entropy four-current is,

$$S^\mu = su^\mu - \frac{\mu_B}{T}\nu^\mu - \left(\beta_0\Pi^2 - \beta_1 q_\nu q^\nu + \beta_2 \pi_{\nu\lambda}\pi^{\nu\lambda}\right)\frac{u^\mu}{2T} - \frac{\alpha_0 \Pi q^\mu}{T} + \frac{\alpha_1 \pi^{\mu\nu} q_\nu}{T}. \tag{5}$$

- Now identification between the phenomenological variables $(\dot{x}_1, \dot{x}_2)$ and the hydrodynamical variables are, $\dot{x}_1 \to \Delta T^{\mu\nu}_{vis}$, $\dot{x}_2 \to q^\mu$.
So the Onsager coefficients are,

$$\gamma_{11} = 2T\left[\left(\eta\Delta^{\mu\alpha}\Delta^{\nu\beta} - \frac{1}{3}\eta\Delta^{\mu\nu}\Delta^{\alpha\beta}\right) + \frac{1}{2}\zeta\Delta^{\mu\nu}\Delta^{\alpha\beta}\right]\frac{1}{\Delta V}, \quad \gamma_{22} = -\frac{\lambda T^2 \Delta^{\mu\nu}}{\Delta V}. \tag{6}$$

Following the phenomenological theory, the correlation functions are,

$$\langle S^{\mu\nu}_{vis}(x_1) S^{\alpha\beta}_{vis}(x_2)\rangle = 2T\left[\eta(\Delta^{\mu\alpha}\Delta^{\nu\beta} + \Delta^{\mu\beta}\Delta^{\nu\alpha}) + (\zeta - \frac{2}{3}\eta)\Delta^{\mu\nu}\Delta^{\alpha\beta}\right]\delta(x_1 - x_2), \tag{7}$$

$$\langle I^\mu(x_1) I^\nu(x_2)\rangle = -2\lambda T^2 \Delta^{\mu\nu}\delta(x_1 - x_2), \tag{8}$$

$$\langle S^{\mu\nu}_{vis}(x_1) I^\alpha(x_2)\rangle = 0. \tag{9}$$

Equations for dissipative fluxes and correlations for other Hydrodynamic models

Third order hydrodynamics

- From the third order hydrodynamics[?], we obtain the viscous correlation functions here,

$$\langle S^{\mu\nu}_{vis}(x_1) S^{\alpha\beta}_{vis}(x_2)\rangle = 2T\left[\eta(\Delta^{\mu\alpha}\Delta^{\nu\beta} + \Delta^{\mu\beta}\Delta^{\nu\alpha}) - \frac{2}{3}\eta\Delta^{\mu\nu}\Delta^{\alpha\beta}\right]\delta(x_1 - x_2). \tag{10}$$

JBP hydrodynamics

Bhalerao et. al. have constructed the expression for the entropy four-current S^μ generalized from the Boltzmann's H-function. We calculate the divergence of entropy four-current and find the correlation functions as,

$$\langle S^{\mu\nu}_{vis}(x_1) S^{\alpha\beta}_{vis}(x_2)\rangle = 2T\left[\eta(\Delta^{\mu\alpha}\Delta^{\nu\beta} + \Delta^{\mu\beta}\Delta^{\nu\alpha}) + (\zeta - \frac{2}{3}\eta)\Delta^{\mu\nu}\Delta^{\alpha\beta}\right]\delta(x_1 - x_2), \tag{11}$$

$$\langle I^\mu(x_1) I^\nu(x_2)\rangle = -2\lambda T \Delta^{\mu\nu}\delta(x_1 - x_2), \tag{12}$$

$$\langle S^{\mu\nu}_{vis}(x_1) I^\alpha(x_2)\rangle = 0. \tag{13}$$

DKR hydrodynamics

Following the relativistic hydrodynamics by Denicol et. al. we find the viscous correlation functions as,

$$\langle S^{\mu\nu}_{vis}(x_1) S^{\alpha\beta}_{vis}(x_2)\rangle = 2T\left[\beta_\pi\tau_\pi(\Delta^{\mu\alpha}\Delta^{\nu\beta} + \Delta^{\mu\beta}\Delta^{\nu\alpha}) + (\beta_\Pi\tau_\Pi - \frac{2}{3}\beta_\pi\tau_\pi)\Delta^{\mu\nu}\Delta^{\alpha\beta}\right]\delta(x_1 - x_2). \tag{14}$$

Calculation of correlation functions for boost-invariant Bjorken flow

- We calculated the spatio-temporal evolution of the correlation functions in different models of hydrodynamics considered above for the Bjorken flow in heavy-ion collisions.
- The viscous-correlations in different forms of hydrodynamics, namely MIS, JBP and third order(TO) hydrodynamics are,

$$\langle f(y_1,\tau_1) f(y_2,\tau_2)\rangle = \frac{2T(\tau_1)}{A\tau_1 w^2(\tau_1)}\left[\frac{4}{3}\eta(\tau_1) + \zeta(\tau_1)\right]\delta(\tau_1 - \tau_2)\delta(y_1 - y_2), \tag{15}$$

- The correlation functions are normalized to the initial value of the correlation obtained using the Navier-Stokes theory.

Results and Discussion

- In these plots, the ideal equation of state, $\epsilon = 3p$ with the pressure given by the bag model, $p = \frac{\pi^2}{30}T^4$ is used. The initial temperature is $T_i = 0.310$ GeV and initial viscous stress $\pi = \eta\frac{4}{3\tau}$.
- We find that the form of the correlation functions given by all different models of hydrodynamics are strikingly similar to the correlation functions obtained using relativistic Navies-Stokes theory
- For the one-dimensional boost-invariant (Bjorken) flow for which the temporal evolution of the correlation functions are calculated, the correlations are proportional to $(\epsilon + p)/\tau$, whose evolution varies with different hydrodynamical models.

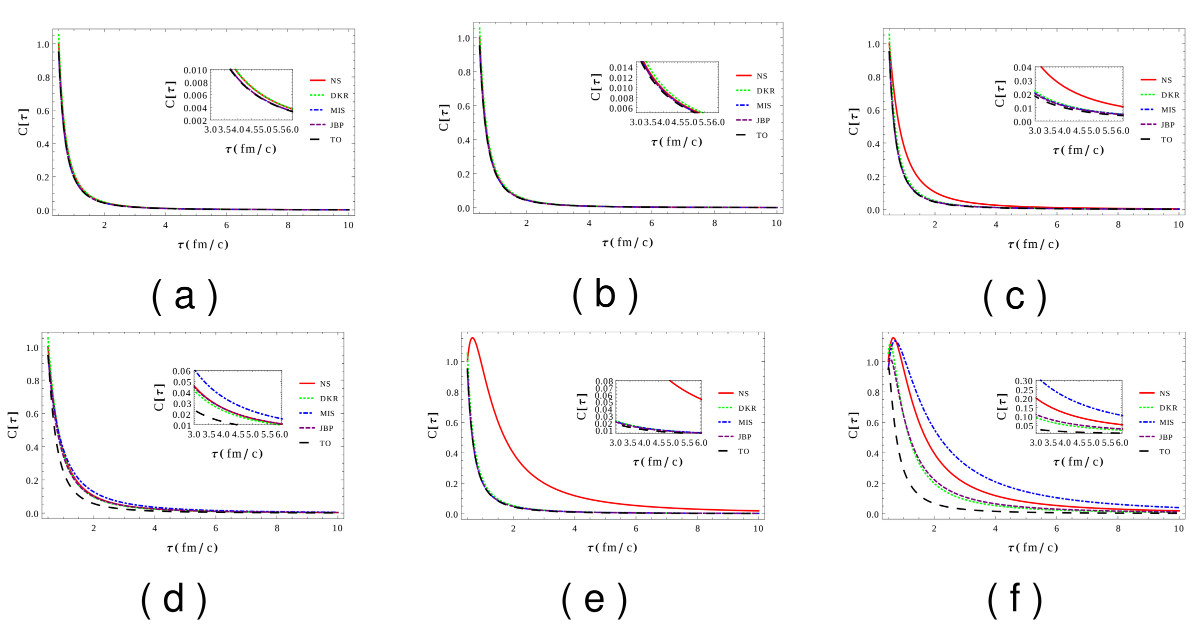

Figure 1: *Time evolution of the function* $C(\tau)_{[E]}$ *with same initial temperature* $T_i = 310\ MeV$. $\eta_{DKR} = \frac{4T}{3\sigma}$
.

- In figures 1(a-b) when η_{DKR}/s is close to $(1/4\pi)$, all the correlations overlap with each other. This is expected as all the viscous hydrodynamics models should approach the ideal hydrodynamics limit when $\eta/s \approx 1/4\pi$, the minimum value.
- The rise of correlations in NS theory compared to other causal versions of hydrodynamics may be attributed to the unphysical behavior of NS theory. These results indicate that it may be possible to distinguish between the correlation function from the Navier-Stoke theory from all other causal hydrodynamics models.
- Figure (2) corresponds to the case when $\frac{\eta}{s}$ equals to $0.08, 0.56$ and 1.60(kept same for all models). The correlations obtained using relativistic Navier-Stokes theory has unphysical behavior for higher values of η/s which differs from the correlations obtained from all other causal hydrodynamics models.
- In conclusion, from the numerical example(Bjorken flow) we considered to calculate the correlation functions, this numerical results can not be compared with the experimental data. However, the results give us some idea about how the correlations in different hydrodynamic models compare with each other.

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