Eigenmode Analysis of Anisotropic Flow

Rajeev S. Bhalerao[1], Jean-Yves Ollitrault[2], Subrata Pal[1] and Derek Teaney[3]

[1]Tata Institute of Fundamental Research, Homi Bhabha Road, Mumbai 400 005, India
[2]CNRS, URA2306, Inst. de physique theorique de Saclay, F-91191 Gif-sur-Yvette, France
[3] Dept. of Physics & Astronomy, State University of New York, Stony Brook, NY 11794, USA

1 Motivation

Methods currently used to analyze anisotropic flow, v_n, (event-plane, cumulants, etc.), were devised before the importance of event-to-event flow fluctuations was recognized [1].

We argue that flow fluctuations can be obtained directly from data by fully exploiting the information contained in the two-particle correlation matrix [2] $\langle \cos n\Delta\phi \rangle$, where $\Delta\phi$ is the azimuthal separation between two particles (in general from different pseudorapidity bins), and $\langle \cdots \rangle$ denotes an average over pairs of particles in an event and then over events in a centrality class.

Our new method uses the eigenmodes and eigenvalues of the two-particle correlation matrix. It can be used to extract information on the pseudorapidity- and transverse-momentum-dependence of flow fluctuations. We test the applicability of this method with Monte-Carlo simulations using the transport model AMPT [3].

2 Method

Divide the detector acceptance into N bins in pseudorapidity (η). For each event and each η bin, define the flow vector $Q_n(\eta)$ in harmonic n as

$$Q_n(\eta) \equiv \sum_{j=1}^{M(\eta)} \exp(in\phi_j),$$

where $M(\eta)$ is the number of particles in the bin and ϕ_j is the azimuthal angle of a particle. Define the two-point correlator or the correlation matrix $V_{n\Delta}(\eta,\eta')$ as [4]

$$V_{n\Delta}(\eta,\eta') \equiv \left\langle Re\, Q_n(\eta) Q_n^*(\eta') \right\rangle .$$

where $\langle \cdots \rangle$ denotes average over events in a centrality class. The quantity inside the brackets is the sum of $\cos n\Delta\phi$ over all pairs in the same event.

If there are no flow fluctuations or nonflow correlations, the correlation matrix factorizes

$$V_{n\Delta}(\eta,\eta') = \langle M(\eta)M(\eta')\rangle v_n(\eta) v_n(\eta').$$

Experimentally, factorization is known to be only approximate [2].
Theoretically, flow fluctuations also break factorization [5].

In general, $V_{n\Delta}(\eta,\eta')$ is symmetric and can be diagonalized. If there are no flow fluctuations or nonflow correlations, one eigenvalue is positive and all others are 0. If there are flow fluctuations but no nonflow correlations, there is more then one positive eigenvalue, but no negative eigenvalue. Short-range, nonflow correlations generally yield both positive and negative eigenvalues. Thus the results can be used to probe the validity of the flow hypothesis.

The correlation matrix $V_{n\Delta}(\eta,\eta')$ can be written as a sum of eigenmodes

$$V_{n\Delta}(\eta,\eta') = \sum_{\alpha=1}^{N} \langle M(\eta)M(\eta')\rangle v_n^{(\alpha)}(\eta) v_n^{(\alpha)}(\eta').$$

(We have assumed for simplicity that all eigenvalues are positive). Each term in the sum corresponds to a different component of flow fluctuations.

The leading mode (i.e., one with the largest eigenvalue) corresponds to the usual anisotropic flow (Figure 1). It depends little on the pseudorapidity η. $v_2(\eta)$ increases strongly with centrality, whereas the higher harmonics increase mildly with centrality.

Other eigenmodes correspond to flow fluctuations (Figure 2). They typically oscillate as a function of η. Target-projectile symmetry implies that the eigenmodes are either even or odd in η, up to statistical and systematic errors. The leading mode is even and the sub-leading modes alternate in parity. As expected from flow fluctuations, they increase mildly with centrality.

The above analysis can be easily extended to $v_n(p_T)$ and $v_n(\eta, p_T)$. In Fig. 3, we show the p_T dependence of the leading and the first subleading modes for n=2,3.

3 Results

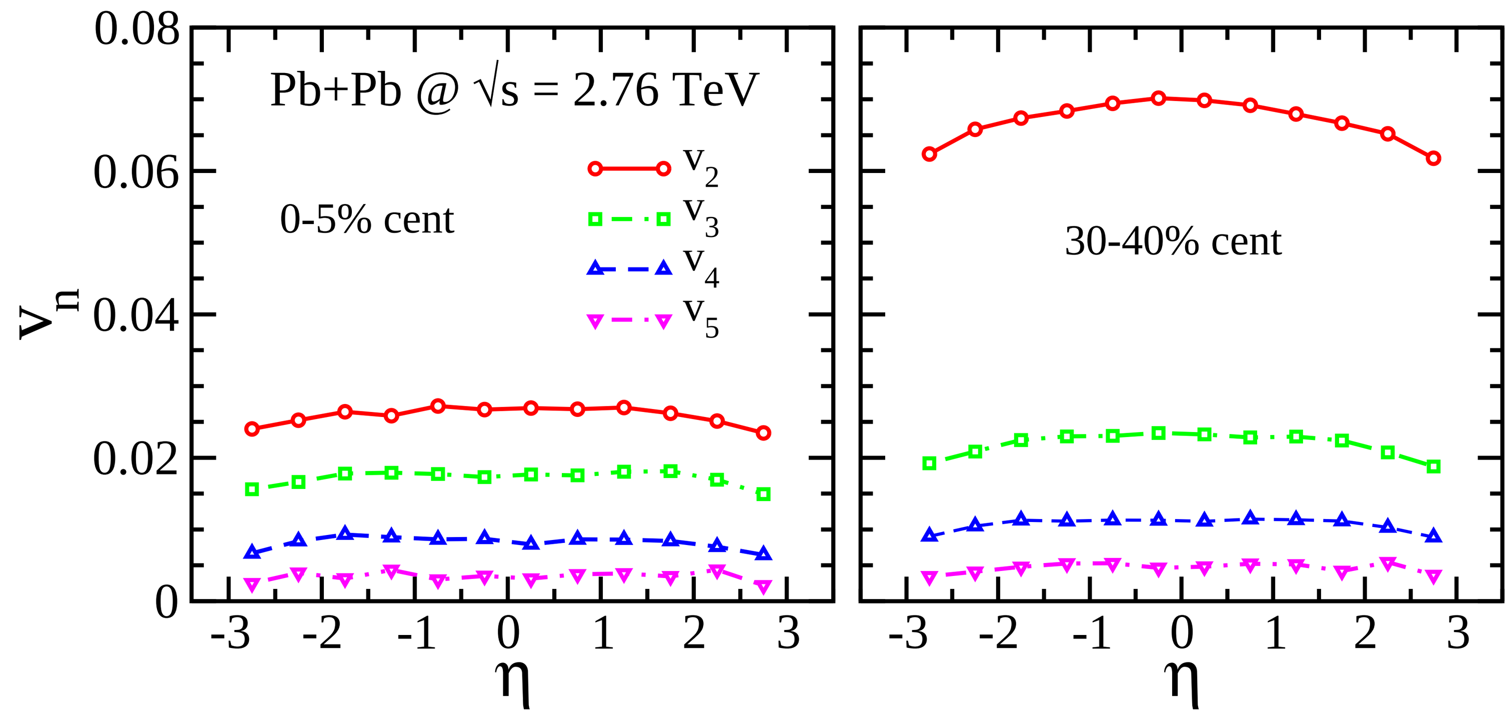

Figure 1: Simulations using the transport code AMPT [3]: results for the largest eigenmode $v_n(\eta)$ — corresponding to the usual anisotropic flow — for $n = 2, 3, 4, 5$ in Pb-Pb collisions at 2.76 TeV and two centralities.

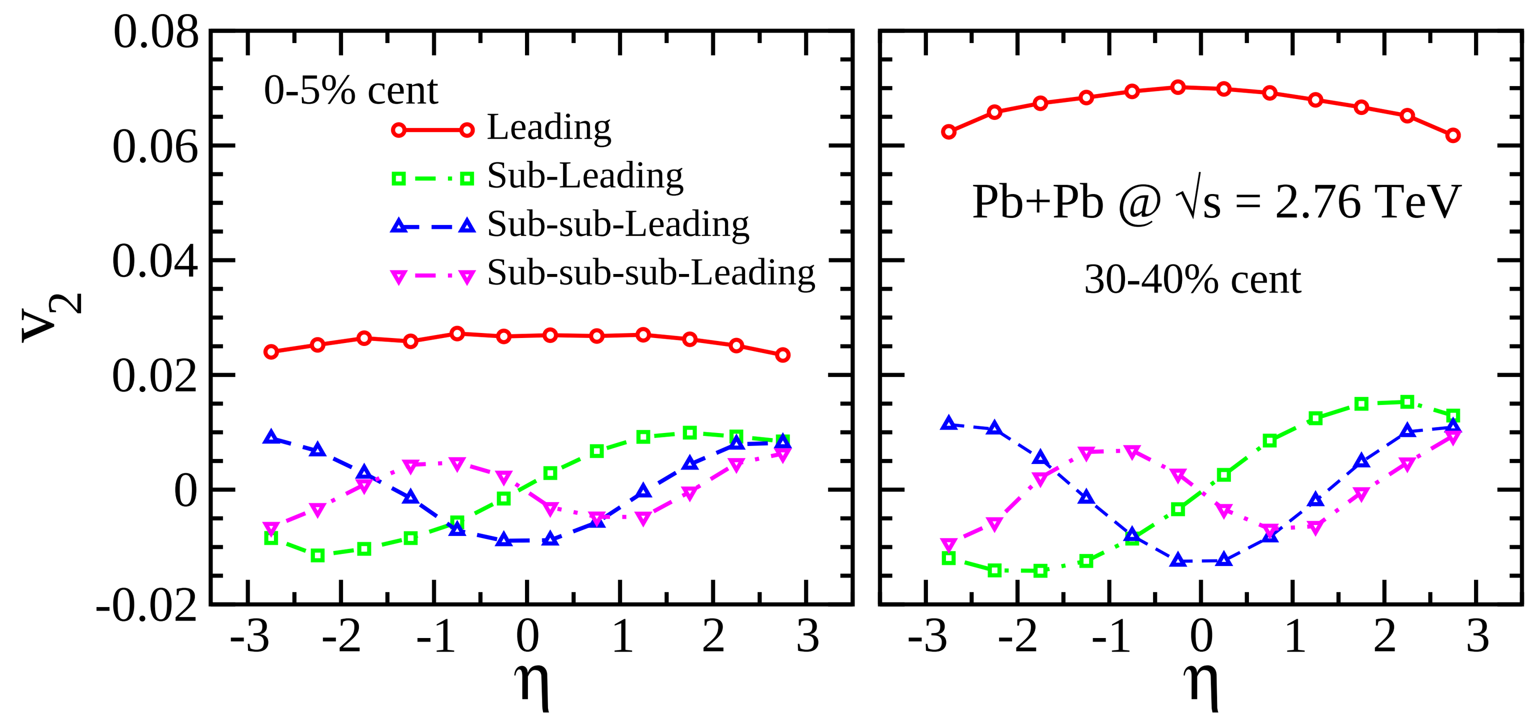

Figure 2: First four eigenmodes of elliptic flow $v_2(\eta)$. Subleading eigenmodes correspond to flow fluctuations.

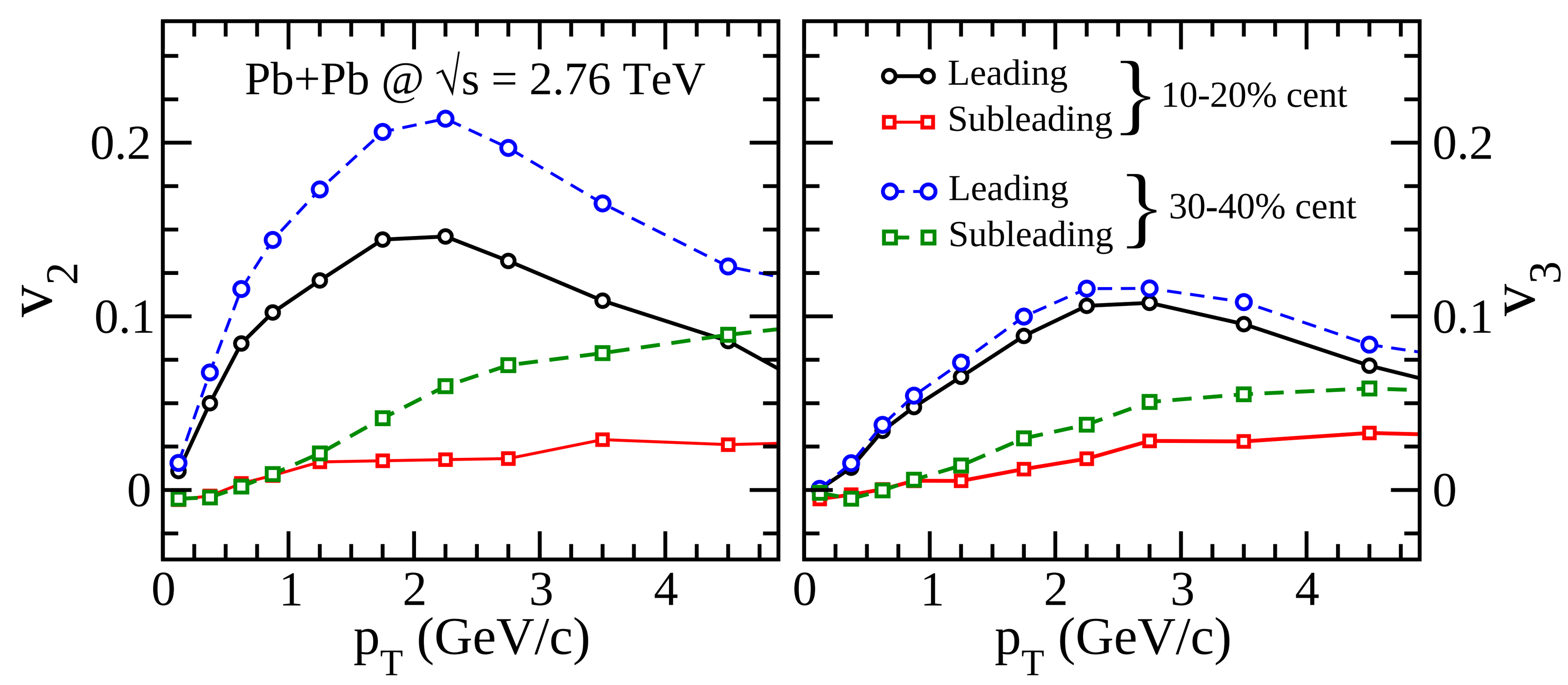

Figure 3: Leading and the first subleading eigenmodes of the elliptic and triangular flows as a function of the transverse momentum.

4 Discussion

This new method, unlike traditional analysis methods, makes use of *all* the information contained in two-particle azimuthal correlations. Specifically, it uses the detailed information on how they depend on the pseudorapidity (and/or transverse momentum) of both particles. This information is expressed in a handy way in terms of eigenvalues and eigenvectors, which can be directly compared with a model calculation.

The condition that the largest eigenvalues should be positive is a new, nontrivial test of the "flow hypothesis" that long-range correlations are due to collective flow.

References

[1] PHOBOS Collaboration, Phys. Rev. Lett. **98**, 242302 (2007).

[2] ALICE Collaboration, Phys. Lett. B **708**, 249 (2012).

[3] Z. -W. Lin *et al.*, Phys. Rev. C **72**, 064901 (2005).

[4] B. Alver and G. Roland, Phys. Rev. C **81**, 054905 (2010).

[5] F. G. Gardim *et al.*, Phys. Rev. C **87**, 031901 (2013).