Possible resolution of the early thermalization problem in relativistic heavy-ion collisions: decoherence entropy from the Glasma

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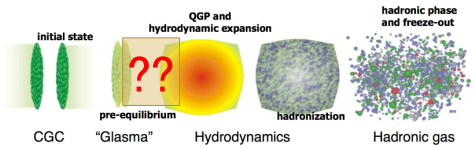
Introduction: entropy production in HIC

Possible entropy production in initial stage of Heavy-Ion Collisions (HIC) is studied in classical Yang-Mills (CYM) dynamics.



(from B.Muller, Phys. Scripta T158, 014004 (2013))

Entropy: $S = -\mathrm{Tr}(\rho \log \rho)$ ρ : density matrix

Pure state : $S = 0$ → (decoherence) Mixed state : $S \neq 0$

Here, the entropy (decoherence entropy) is calculated under the assumptions:

1) The gluons in the initial stage of HIC can be described by **coherent states**
   ⇒ enables us to calculate the matrix elements of ρ from CYM dynamics.
2) Complete decoherence takes place and the density matrix in the Fock state basis becomes diagonal at a certain time.

Entropy from CYM

(Kunihiro, Muller, Ohnishi, Schafer, Takahashi and Yamamoto '10; Iida, Kunihiro, Muller, Ohnishi, Schafer and Takahashi '13)

CYM The Hamiltonian for CYM in non-expanding geometry is written as

$$H = \frac{1}{2}\sum_{x,a,i} E_i^a(x)^2 + \frac{1}{4}\sum_{x,a,i,j} F_{ij}^a(x)^2, \quad F_{ij}^a(x) = \partial_i A_j^a(x) - \partial_j A_i^a(x) + \sum_{b,c} f^{abc} A_i^b(x) A_j^c(x)$$

(in temporal gauge $A_0^a = 0$)

The canonical equation of motion:

$$\dot{A}_i^a(x) = E_i^a(x), \dot{E}_i^a(x) = \sum_j \partial_j F_{ji}^a(x) + \sum_{b,c,j} f^{abc} A_j^b(x) F_{ji}^c(x)$$

… solved numerically from the glasma-like initial condition.

※We check that the Gauss' law is fairly kept during the calculation.

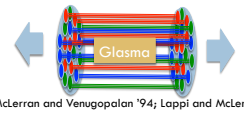
(Iida, Kunihiro, Muller, Ohnishi, Schafer and Takahashi '13)

Entropy Replacement: $\langle\alpha|q|\alpha\rangle \to A_i^a(t,\mathbf{k}), \langle\alpha|p|\alpha\rangle \to E_i^a(t,\mathbf{k})$

$$S(t) = -\sum_{\mathbf{k},a,i,n} P_n(\alpha_{i\mathbf{k}}^a(t)) \log P_n(\alpha_{i\mathbf{k}}^a(t))$$

$$P_n(\alpha_{i\mathbf{k}}^a) \equiv \exp(-|\alpha_{i\mathbf{k}}^a|^2)|\alpha_{i\mathbf{k}}^a|^{2n}/n! \qquad \alpha_{i\mathbf{k}}^a(t) = \frac{1}{\sqrt{2\omega_{\mathbf{k}}}}(\omega_{\mathbf{k}} A_i^a(t,\mathbf{k}) + iE_i^a(t,\mathbf{k}))$$

Glasma & Coherent state



(McLerran and Venugopalan '94; Lappi and McLerran '06 …)

In rest frame, nucleons are saturated by gluons
⇒ High energy gluons are behaved as strong color source for low energy gluons
classical treatment is good!

Coherent state: $|\alpha\rangle = \exp(i\alpha a^\dagger - i\alpha a)|0\rangle \quad a|\alpha\rangle = \alpha|\alpha\rangle$
is a best quantum analogue of classical states.

Assumption: **gluon states at initial stage ~ coherent state**

(Muller and Schafer ('03, '06) ; Fries, Muller and Schafer '09)

Important formulae of a coherent state

- Eigenvalue of a coherent state: $\alpha = (m\omega\langle\alpha|q|\alpha\rangle + i\langle\alpha|p|\alpha\rangle)/\sqrt{2m\omega}$
  for the Hamiltonian $H = p^2/2m + m\omega^2 q^2/2$
- Probability of finding n-particle state $|n\rangle$ in a coherent state:
  $P_n(\alpha) \equiv |\langle n|\alpha\rangle|^2 = \exp(-|\alpha|^2)|\alpha|^{2n}/n!$ … Poisson distribution

Expression of entropy

We assume (complete) decoherence … density matrix is diagonal:

$$\rho_{mn} = \langle m|\alpha\rangle\langle\alpha|n\rangle \qquad \rho_{mn} \to |\langle n|\alpha\rangle|^2\delta_{mn} = P(n)\delta_{mn}$$

Entropy: $S = -\mathrm{Tr}(\rho\log\rho) = -\sum_n P_n(\alpha)\log P_n(\alpha)$

※ We do not consider the process of decoherence in the study.

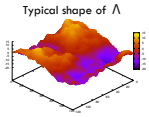
Initial condition: Glasma-like

(Romatschke and Venugopalan '06; Fukushima '11; Fukushima and Gelis '12)

In this study, we construct glasma-like initial condition ※non-expanding geometry

1) Solving the Poisson equations:
   $-\partial_\perp^2 \Lambda^{(t)}(\mathbf{x}_\perp) = \rho^{(t)}(\mathbf{x}_\perp), -\partial_\perp^2 \Lambda^{(p)}(\mathbf{x}_\perp) = \rho^{(p)}(\mathbf{x}_\perp)$
   ($\rho^{(p)}, \rho^{(t)}$: Gaussian random color sources for projectile and target nucleons)

Typical shape of Λ



2) $\alpha_i^{(t)}, \alpha_i^{(p)}$ are constructed from Wilson lines
   $\alpha_i^{(t)} = -\frac{1}{ig}V\partial_i V^\dagger, \alpha_i^{(p)} = -\frac{1}{ig}W\partial_i W^\dagger \qquad V^\dagger(\mathbf{x}_\perp) = e^{ig\Lambda^{(t)}(\mathbf{x}_\perp)}, W^\dagger(\mathbf{x}_\perp) = e^{ig\Lambda^{(p)}(\mathbf{x}_\perp)}$
3) Gauge fields, electric fields and magnetic fields are obtained by
   $A_i = \alpha_i^{(t)} + \alpha_i^{(p)}, A_z = 0 \quad E_i = 0, E_z = ig\sum_i\left(\left[\alpha_i^{(t)}, \alpha_i^{(p)}\right]\right), B_i = 0, B_z = ig\left(\left[\alpha_1^{(p)}, \alpha_2^{(t)}\right] + \left[\alpha_1^{(t)}, \alpha_2^{(p)}\right]\right).$
4) Fluctuations in z direction are added to the above electric fields
   $\delta E_i(\mathbf{x}_\perp, z) = \partial_z F(z)\delta\tilde{E}_i(\mathbf{x}_\perp), \delta E_z(\mathbf{x}_\perp, z) = -F(z)D_i\delta\tilde{E}_i(\mathbf{x}_\perp).$
   $\langle\delta\tilde{E}_i(\mathbf{x}_\perp)\delta\tilde{E}_j(\mathbf{y}_\perp)\rangle = \delta_{ij}\delta(\mathbf{x}_\perp - \mathbf{y}_\perp) \qquad \langle F(z)F(z')\rangle = \Delta^2\delta(z - z')$

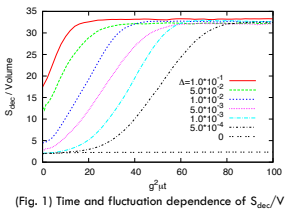
Numerical results

Time dependence of S_{dec}/V (Fig.1)

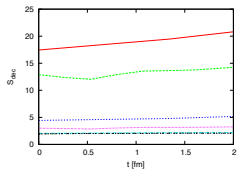
- **No long. fluctuation, Δ=0:**
  S_{dec}/V ~ 2.5 and not enhanced
  … "2D entropy" (p_z=0 mode)
- **With long. fluctuations, Δ≠0:**
  Δ≦1.0^{-3}
  - S_{dec}/V ~ 2.5 at t=0
  - S_{dec}/V increases after a certain time dependeing on Δ
    … realization of "3D entropy"

  Δ>1.0^{-3}
  - S_{dec}/V > 2.5 at t=0 depending on Δ
    … "intrinsic entropy" in Glasma
  - S_{dec}/V increases after a certain time dependeing on Δ … "3D entropy"

Unstable against fluctuations



(Fig. 1) Time and fluctuation dependence of S_{dec}/V



(Fig. 2) S_{dec}/V plotted against time with physical unit

•S_{dec} plotted against physical time scale

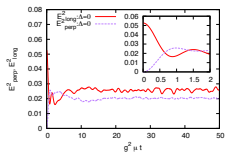
[Rough estimation of the scale]

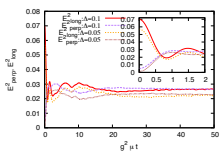
$$L^2 \sim \pi R_{\mathrm{Au}}^2 \to 1/a = g^2\mu \sim 0.32\mathrm{GeV}(a \sim 0.63\mathrm{fm})$$

(Fig.2) : S_{dec} does not increase largely for 0<t<2fm.
… "intrinsic entropy" is important?
Not conclusive: **large uncertainty of the scale determination**

Parameters for the calculation

volume	Δ	$\epsilon(g^2\mu)^4$	ϵ(GeV/fm^4)	ratio of E^2
20^3	0	0.1711	0.3476	0
20^3	1.0×10^{-3}	0.1711	0.3476	3.278×10^{-5}
20^3	5.0×10^{-3}	0.1712	0.3478	8.194×10^{-4}
20^3	1.0×10^{-2}	0.1714	0.3482	3.278×10^{-3}
20^3	5.0×10^{-2}	0.1814	0.3685	8.194×10^{-2}
20^3	1.0×10^{-1}	0.21	0.4266	3.278×10^{-1}





- **Isotropization:** E_{perp}^2 & E_{long}^2 with and without fluctuations
  - without fluctuations (left figure) … E_{perp}^2 & E_{long}^2 become close, but never coincide.
  - with fluctuations (right figure) … E_{perp}^2 & E_{long}^2 are almost coincide after t=45 for Δ=0.05 and after t=40 for Δ=0.1.

… Fluctuations make the isotropization earlier.

Summary & Conclusion

- Possible entropy production in initial stage of Heavy-Ion Collisions (HIC) is studied in classical Yang-Mills (CYM) dynamics in non-expanding geometry.
- We have proposed a new prescription for evaluating an (decoherence) entropy in the initial stage of HIC using CYM.

Results:

- Longitudinal fluctuations Δ largely enhance the entropy production:
  For small Δ, the entropy is small at t=0, and increases after a certain time.
  (realization of "3D entropy")
  For large Δ, the entropy is already large at t=0 ("intrinsic entropy")
  compared to that with no fluctuation, and increases for a certain time.
- Taking into physical scale: "intrinsic entropy" is important? ← more study is needed
- Isotropization: Fluctuations make the isotropization earlier.