FEMTOSCOPIC CORRELATIONS OF TWO IDENTICAL PARTICLES WITH NONZERO SPIN IN THE MODEL OF ONE-PARTICLE MULTIPOLe SOURCES

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Abstract

The process of emission of two identical particles with nonzero spin $S$ and different helicities is theoretically investigated within the model of one-particle multipole sources. Taking into account the unitarity of the finite rotation matrix and symmetry relations for $d$-functions, the general expression for probability of emission of two identical particles by two multipole sources with angular momentum $J$, averaged over the projections of angular momentum and over the space-time dimensions of the generation region, has been obtained. For the case of unpolarized particles, the additional averaging over helicities is performed and the formula for two-particle correlation function at sufficiently large 4-momentum difference $q$ is derived. For particles with nonzero mass, this formula is simplified at the zero angle $\beta$ between the particle momenta, and also at $J = S$.

The concrete cases of emission of two unpolarized photons by dipole and quadrupole sources, and emission of two “left” neutrinos (“right” antineutrinos) by sources with arbitrary $J$ have been also considered, and the respective explicit expressions for the correlation function are obtained.
1 Probability of emission of two identical particles with nonzero spin by two multipole sources

In the framework of the model of independent sources [1] with the angular momentum $J$ and the projections of angular momentum onto the coordinate axis $z$, equaling $M$ and $M'$, the amplitude of emission of two identical particles with the momentum $\mathbf{p}_1$, helicity $\lambda_1$ and momentum $\mathbf{p}_2$, helicity $\lambda_2$ has the following structure:

$$A_{M,M'}(\mathbf{p}_1, \lambda_1; \mathbf{p}_2, \lambda_2) =$$

$$= D^{(J)}_{\lambda_1 M}(\mathbf{n}_1)D^{(J)}_{\lambda_2 M'}(\mathbf{n}_2) e^{ip_1 x_1} e^{ip_2 x_2} + D^{(J)}_{\lambda_2 M}(\mathbf{n}_2)D^{(J)}_{\lambda_1 M'}(\mathbf{n}_1) e^{ip_1 x_2} e^{ip_2 x_1}, \quad (1)$$

where $x_1$ and $x_2$ are the space-time coordinates of two multipole sources, $p_1 x_1 = E_1 t_1 - \mathbf{p}_1 \mathbf{x}_1$, $p_2 x_2 = E_2 t_2 - \mathbf{p}_2 \mathbf{x}_2$,

$$D^{(J)}_{\lambda_1 M}(\mathbf{n}_1) = D^{(J)}_{\lambda_1 M}(0, \theta_1, \phi_1) = \left( d_y(0, \theta_1, \phi_1) e^{i M \phi_1} \right)_{\lambda_1 M},$$

$$D^{(J)}_{\lambda_2 M'}(\mathbf{n}_2) = D^{(J)}_{\lambda_2 M'}(0, \theta_2, \phi_2) = \left( d_y(0, \theta_2, \phi_2) e^{i M' \phi_2} \right)_{\lambda_2 M'}, \quad (2)$$

are elements of the finite rotation matrix corresponding to the angular momentum $J$, $\mathbf{n}_1 = \mathbf{p}_1/|\mathbf{p}_1|$, $\mathbf{n}_2 = \mathbf{p}_2/|\mathbf{p}_2|$, $\theta_1, \theta_2$ and $\phi_1, \phi_2$ - polar and azimuthal angles of the momenta $\mathbf{p}_1$ and $\mathbf{p}_2$, respectively.

Thus, in accordance with Eq. (1), the probability of emission of two identical particles with spin $S$, respective 4-momenta $p_1, p_2$ and helicities $\lambda_1, \lambda_2$ by two multipole sources with the angular momentum $J$ and projections $M, M'$ of angular momentum onto the axis $z$ amounts to:

$$W_{M,M'}(p_1, \lambda_1; p_2, \lambda_2) = | D^{(J)}_{\lambda_1 M}(\mathbf{n}_1)|^2 | D^{(J)}_{\lambda_2 M'}(\mathbf{n}_2)|^2 + | D^{(J)}_{\lambda_2 M}(\mathbf{n}_2)|^2 | D^{(J)}_{\lambda_1 M'}(\mathbf{n}_1)|^2 +$$

$$+ 2 \left( -1 \right)^{2S} \text{Re} \left( D^{(J)}_{\lambda_1 M}(\mathbf{n}_1)D^{*(J)}_{\lambda_2 M'}(\mathbf{n}_2)D^{*(J)}_{\lambda_1 M'}(\mathbf{n}_1)D^{(J)}_{\lambda_2 M'}(\mathbf{n}_2) \right) \cos(q x), \quad (3)$$

where $q = p_1 - p_2$ is the difference of 4-momenta of two identical particles and $x = x_1 - x_2$ is the difference of 4-coordinates of two one-particle multipole sources.
Now let us average this expression over the angular momentum projections $M, M'$ and over the space-time dimensions of the emission region. In doing so, we take into account that, due to the unitarity of the finite rotation matrix, the following relations hold:

$$
\sum_{M=-J}^{J} |D_{\lambda_1 M}^{(J)}(n_1)|^2 = \sum_{M'=-J}^{J} |D_{\lambda_2 M'}^{(J)}(n_2)|^2 =
$$

$$
\sum_{M=-J}^{J} |D_{\lambda_2 M}^{(J)}(n_2)|^2 = \sum_{M'=-J}^{J} |D_{\lambda_1 M'}^{(J)}(n_1)|^2 = 1. \quad (4)
$$

Let us remark that, without losing generality, we may choose the coordinate axis $z$ as lying in the plane of the momenta $p_1$ and $p_2$, with the axis $y$ being perpendicular to this plane. Then the azimuthal angles of the momenta $p_1$ and $p_2$ will be equal to zero: $\phi_1 = \phi_2 = 0$, and the angle $\beta = \theta_1 - \theta_2$ will have the meaning of angle between the momenta $p_1$ and $p_2$. In doing so, once again due to the unitarity of the finite rotation matrix, we obtain:

$$
\sum_{M=-J}^{J} D_{\lambda_1 M}^{(J)}(n_1)D^{*\, (J)}_{M\lambda_2}(n_2) = \sum_{M=-J}^{J} (e^{-iJy\theta_1})_{\lambda_1 M} (e^{iJy\theta_2})_{M\lambda_2} =
$$

$$
= \left(e^{-iJy(\theta_1-\theta_2)}\right)_{\lambda_1 \lambda_2} = \left(d_{y}^{(J)}(\beta)\right)_{\lambda_1 \lambda_2}; \quad (5)
$$

$$
\sum_{M'=-J}^{J} D_{\lambda_2 M'}^{(J)}(n_2)D^{*\, (J)}_{M'\lambda_1}(n_1) = \sum_{M'=-J}^{J} (e^{-iJy\theta_2})_{\lambda_2 M'} (e^{iJy\theta_1})_{M'\lambda_1} =
$$

$$
= \left(e^{iJy(\theta_1-\theta_2)}\right)_{\lambda_2 \lambda_1} = \left(d_{y}^{(J)}(-\beta)\right)_{\lambda_2 \lambda_1}. \quad (6)
$$

Using the well-known symmetry relation $(d_{y}^{(J)}(\beta))_{\lambda_1 \lambda_2} = (d_{y}^{(J)}(-\beta))_{\lambda_2 \lambda_1}$ [2], we come to the result:

$$
W_{M, M'}(p_1, \lambda_1; p_2, \lambda_2) = \frac{1}{(2J+1)^2} \left( 2 + 2 \left(d_{\lambda_1 \lambda_2}^{(J)}(\beta)\right)^2 (-1)^{2S} \cos(qx) \right). \quad (7)
$$
Let us emphasize that the quantity $r = (d_{\lambda_1\lambda_2}^{(J)}(\beta))^2$ has the meaning of the degree of non-orthogonality (non-distinguishability) of particle states with different helicities with respect to the momenta, the angle between which equals $\beta = \theta_1 - \theta_2 : \langle \lambda_1 | \lambda_2 \rangle \neq 0$.

2 Correlation function for two unpolarized particles in the model of one-particle multipole sources

If the emitted identical particles with the momenta $p_1, p_2$ are unpolarized, then after averaging over all the $(2S + 1)$ values of helicity allowed at spin $S$ - we obtain:

$$\overline{W}(q) = \left( 2 (2S + 1)^2 + \right)$$

$$+ (-1)^{2S} \frac{1}{(2S + 1)^2} \sum_{\lambda_1 = -S}^{S} \sum_{\lambda_2 = -S}^{S} |d_{\lambda_1\lambda_2}^{(J)}(\beta)|^2 \{ \cos(qx) \} \frac{1}{(2J + 1)^2} \frac{1}{(2S + 1)^2}. \quad (8)$$

At sufficiently large momentum differences $q$ the correlation function, normalized by unity, will take the form:

$$R(q) = 1 + \frac{(-1)^{2S}}{(2S + 1)^2} \sum_{\lambda_1 = -S}^{S} \sum_{\lambda_2 = -S}^{S} |d_{\lambda_1\lambda_2}^{(J)}(\beta)|^2 \{ \cos(qx) \}. \quad (9)$$

In particular, if $\beta = 0$, then we have $d_{\lambda_1\lambda_2}^{(J)}(0) = \delta_{\lambda_1\lambda_2}$, and formula (9) is simplified:

$$R(q) = 1 + (-1)^{2S} \frac{1}{2S + 1} \{ \cos(qx) \}. \quad (10)$$

Besides, taking into account the unitarity of the matrix $d_{\lambda_1\lambda_2}^{(J)}(\beta)$, it is easy to see from Eq. (9) that at $J = S$ formula (10) is valid at any angles between the momenta $p_1$ and $p_2$. Let us stress that Eq.(10) is related to particles with nonzero mass.
3 Special cases of pair correlations of two unpolarized photons and two neutrinos

In the case of emission of two unpolarized photons, when the mass equals zero, spin $S = 1$ and each of the helicities $\lambda_1, \lambda_2$ takes only two ($2S$) values: $-1$ and $1$, irrespective of the momentum direction, the correlation function for dipole sources has the form [3]:

$$ R(q) = 1 + \frac{1}{4} \left[ (d_{11}^{(1)}(\beta))^2 + (d_{-1,1}^{(1)}(\beta))^2 + (d_{-1,-1}^{(1)}(\beta))^2 + (d_{1,-1}^{(1)}(\beta))^2 \right] \langle \cos(qx) \rangle . $$

(11)

Taking into account the equalities:

$$ d_{11}^{(1)}(\beta) = d_{-1,-1}^{(1)}(\beta) = \frac{1 + \cos \beta}{2}, \quad d_{1,-1}^{(1)}(\beta) = d_{-1,1}^{(1)}(\beta) = \frac{1 - \cos \beta}{2}, $$

(12)

we find:

$$ R(q) = 1 + \frac{1}{4} (1 + \cos^2 \beta) \langle \cos(qx) \rangle . $$

(13)

At very small angles between the photon momenta ($\beta \ll 1$) we obtain:

$$ R(q) = 1 + \frac{1}{2} \langle \cos(qx) \rangle . $$

(14)

For the case of quadrupole sources, the correlation function is as follows:

$$ R(q) = 1 + \frac{1}{4} \left[ (d_{11}^{(2)}(\beta))^2 + (d_{-1,1}^{(2)}(\beta))^2 + (d_{-1,-1}^{(2)}(\beta))^2 + (d_{1,-1}^{(2)}(\beta))^2 \right] \langle \cos(qx) \rangle . $$

(15)

Using the equalities:

$$ d_{11}^{(2)}(\beta) = d_{-1,-1}^{(2)}(\beta) = \frac{1 + \cos \beta}{2} (2 \cos \beta - 1), $$

(16)
we find the correlation function of two unpolarized photons emitted by the quadrupole sources:

\[
R(q) = 1 + \frac{1}{4} \left( 4 \cos^4 \beta - 3 \cos^2 \beta + 1 \right) \langle \cos(qx) \rangle. \tag{18}
\]

At \( \beta \approx 0 \) we have: \( R(q) = 1 + \frac{1}{2} \langle \cos(qx) \rangle \), i.e. here we also obtain the standard formula (see Eq. (14)), corresponding to two directions of polarization for each of the photons [3].

Let us consider also the case of emission of two “left” neutrinos (two “right” antineutrinos), with helicity taking only one value \( \lambda_1 = \lambda_2 = + \frac{1}{2} \). For this case, the correlation function in the model of multipole sources is as follows:

\[
R(q) = 1 - (d_{\frac{3}{2}, \frac{3}{2}}^{(J)})^2 \langle \cos(qx) \rangle. \tag{19}
\]

In particular, at \( J = S = \frac{1}{2} \) we obtain:

\[
R(q) = 1 - \cos^2 \frac{\beta}{2} \langle \cos(qx) \rangle. \tag{20}
\]

In the limit \( \beta \to 0 \) Eq. (20) gives:

\[
R(q) = 1 - \langle \cos(qx) \rangle. \tag{21}
\]

References

