XXIV QUARK MATTER
DARMSTADT 2014

FEMTOSCOPIC CORRELATIONS OF TWO IDENTICAL PARTICLES WITH NONZERO SPIN IN THE MODEL OF ONE-PARTICLE MULTIPOLE SOURCES

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Abstract

The process of emission of two identical particles with nonzero spin S and different helicities is theoretically investigated within the model of one-particle multipole sources. Taking into account the unitarity of the finite rotation matrix and symmetry relations for d–functions, the general expression for probability of emission of two identical particles by two multipole sources with angular momentum J, averaged over the projections of angular momentum and over the space-time dimensions of the generation region, has been obtained. For the case of unpolarized particles, the additional averaging over helicities is performed and the formula for two-particle correlation function at sufficiently large 4-momentum difference q is derived. For particles with nonzero mass, this formula is simplified at the zero angle β between the particle momenta, and also at $J = S$.

The concrete cases of emission of two unpolarized photons by dipole and quadrupole sources, and emission of two "left" neutrinos ("right" antineutrinos) by sources with arbitrary J have been also considered, and the respective explicit expressions for the correlation function are obtained .

1 Probability of emission of two identical particles with nonzero spin by two multipole sources

In the framework of the model of independent sources [1] with the angular momentum J and the projections of angular momentum onto the coordinate axis z, equaling M and M', the amplitude of emission of two identical particles with the momentum $\mathbf{p}_1$, helicity λ_1 and momentum $\mathbf{p}_2$, helicity λ_2 has the following structure :

$$A_{M\,M'}(\mathbf{p}_1, \lambda_1; \mathbf{p}_2, \lambda_2) =$$

$$= D^{(J)}_{\lambda_1\,M}(\mathbf{n}_1) D^{(J)}_{\lambda_2\,M'}(\mathbf{n}_2)\, e^{i\,p_1 x_1}\, e^{i\,p_2 x_2} + D^{(J)}_{\lambda_2\,M}(\mathbf{n}_2) D^{(J)}_{\lambda_1\,M'}(\mathbf{n}_1)\, e^{i\,p_1 x_2}\, e^{i\,p_2 x_1}, \quad (1)$$

where x_1 and x_2 are the space-time coordinates of two multipole sources, $p_1 x_1 = E_1\,t_1 - \mathbf{p}_1 \mathbf{x}_1$, $p_2 x_2 = E_2\,t_2 - \mathbf{p}_2 \mathbf{x}_2$,

$$D^{(J)}_{\lambda_1 M}(\mathbf{n}_1) = D^{(J)}_{\lambda_1 M}(0, \theta_1, \phi_1) = \left(d_y(0, \theta_1, \phi_1)\, e^{i\,M \phi_1}\right)_{\lambda_1 M},$$

$$D^{(J)}_{\lambda_2 M'}(\mathbf{n}_2) = D^{(J)}_{\lambda_2 M'}(0, \theta_2, \phi_2) = \left(d_y(0, \theta_2, \phi_2)\, e^{i\,M' \phi_2}\right)_{\lambda_2 M'}, \quad (2)$$

are elements of the finite rotation matrix corresponding to the angular momentum J, $\mathbf{n}_1 = \mathbf{p}_1/|\mathbf{p}_1|$, $\mathbf{n}_2 = \mathbf{p}_2/|\mathbf{p}_2|$, θ_1, θ_2 and ϕ_1, ϕ_2 - polar and azimuthal angles of the momenta $\mathbf{p}_1$ and $\mathbf{p}_2$, respectively .

Thus, in accordance with Eq. (1), the probability of emission of two identical particles with spin S, respective 4-momenta p_1, p_2 and helicities λ_1, λ_2 by two multipole sources with the angular momentum J and projections M, M' of angular momentum onto the axis z amounts to :

$$W_{M M'}(p_1, \lambda_1; p_2, \lambda_2) = |\, D^{(J)}_{\lambda_1 M}(\mathbf{n}_1)|^2 |\, D^{(J)}_{\lambda_2 M'}(\mathbf{n}_2)|^2 + |\, D^{(J)}_{\lambda_1 M'}(\mathbf{n}_1)|^2 |\, D^{(J)}_{\lambda_2 M}(\mathbf{n}_2)|^2 +$$

$$+\, 2\, (-1)^{2S}\, \mathrm{Re}\left(\, D^{(J)}_{\lambda_1 M}(\mathbf{n}_1) D^{*(J)}_{\lambda_2 M}(\mathbf{n}_2) D^{*(J)}_{\lambda_1 M'}(\mathbf{n}_1) D^{(J)}_{\lambda_2 M'}(\mathbf{n}_2)\right) \cos(q\,x), \quad (3)$$

where $q = p_1 - p_2$ is the difference of 4-momenta of two identical particles and $x = x_1 - x_2$ is the difference of 4-coordinates of two one-particle multipole sources.

Now let us average this expression over the angular momentum projections M, M' and over the space-time dimensions of the emission region . In doing so, we take into account that, due to the unitarity of the finite rotation matrix, the following relations hold :

$$\sum_{M=-J}^{J} \mid D^{(J)}_{\lambda_1 M}(\mathbf{n}_1)|^2 = \sum_{M'=-J}^{J} \mid D^{(J)}_{\lambda_2 M'}(\mathbf{n}_2)|^2 =$$

$$= \sum_{M=-J}^{J} \mid D^{(J)}_{\lambda_2 M}(\mathbf{n}_2)|^2 = \sum_{M'=-J}^{J} \mid D^{(J)}_{\lambda_1 M'}(\mathbf{n}_1)|^2 = 1. \tag{4}$$

Let us remark that, without losing generality, we may choose the coordinate axis z as lying in the plane of the momenta $\mathbf{p}_1$ and $\mathbf{p}_2$, with the axis y being perpendicular to this plane. Then the azimuthal angles of the momenta $\mathbf{p}_1$ and $\mathbf{p}_2$ will be equal to zero: $\phi_1 = \phi_2 = 0$, and the angle $\beta = \theta_1 - \theta_2$ will have the meaning of angle between the momenta $\mathbf{p}_1$ and $\mathbf{p}_2$. In doing so, once again due to the unitarity of the finite rotation matrix, we obtain :

$$\sum_{M=-J}^{J} D^{(J)}_{\lambda_1 M}(\mathbf{n}_1) D^{*(J)}_{M \lambda_2}(\mathbf{n}_2) = \sum_{M=-J}^{J} \left(e^{-i\, J_y \theta_1}\right)_{\lambda_1 M} \left(e^{i\, J_y \theta_2}\right)_{M \lambda_2} =$$

$$= \left(e^{-i\, J_y (\theta_1 - \theta_2)}\right)_{\lambda_1 \lambda_2} = \left(d^{(J)}_y(\beta)\right)_{\lambda_1 \lambda_2} ; \tag{5}$$

$$\sum_{M'=-J}^{J} D^{(J)}_{\lambda_2 M'}(\mathbf{n}_2) D^{*(J)}_{M' \lambda_1}(\mathbf{n}_1) = \sum_{M'=-J}^{J} \left(e^{-i\, J_y \theta_2}\right)_{\lambda_2 M'} \left(e^{i\, J_y \theta_1}\right)_{M' \lambda_1} =$$

$$= \left(e^{i\, J_y (\theta_1 - \theta_2)}\right)_{\lambda_2 \lambda_1} = \left(d^{(J)}_y(-\beta)\right)_{\lambda_2 \lambda_1} . \tag{6}$$

Using the well-known symmetry relation $(d^{(J)}_y(\beta))_{\lambda_1 \lambda_2} = (d^{(J)}_y(-\beta))_{\lambda_2 \lambda_1}$ [2], we come to the result :

$$\overline{W_{MM'}}(p_1, \lambda_1; p_2, \lambda_2) = \frac{1}{(2J+1)^2} \left(2 + 2\, (d^{(J)}_{\lambda_1 \lambda_2}(\beta))^2 \, (-1)^{2S} \, \langle \cos(qx) \, \rangle\right) . \tag{7}$$

Let us emphasize that the quantity $r = (d^{(J)}_{\lambda_1\lambda_2}(\beta))^2$ has the meaning of the degree of non-orthogonality (non-distinguishability) of particle states with different helicities with respect to the momenta, the angle between which equals $\beta = \theta_1 - \theta_2$: $\langle\lambda_1|\lambda_2\rangle \neq 0$.

2 Correlation function for two unpolarized particles in the model of one-particle multipole sources

If the emitted identical particles with the momenta $\mathbf{p}_1, \mathbf{p}_2$ are unpolarized, then - after averaging over all the $(2S+1)$ values of helicity allowed at spin S - we obtain :

$$\overline{W}(q) = \Big(\, 2\,(2S+1)^2 +$$

$$+\,(-1)^{2S}\; 2 \sum_{\lambda_1=-S}^{S} \; \sum_{\lambda_2=-S}^{S} |\, d^{(J)}_{\lambda_1\lambda_2}(\beta)\,|^2 \,\langle\, \cos(qx)\,\rangle \Big) \, \frac{1}{(2J+1)^2} \; \frac{1}{(2S+1)^2} \; . \quad (8)$$

At sufficiently large momentum differences q the correlation function, normalized by unity, will take the form :

$$R(q) = 1 + \frac{(-1)^{2S}}{(2S+1)^2} \sum_{\lambda_1=-S}^{S} \; \sum_{\lambda_2=-S}^{S} |\, d^{(J)}_{\lambda_1\lambda_2}(\beta)\,|^2 \; \langle\, \cos(qx)\,\rangle \, . \quad (9)$$

In particular, if $\beta = 0$, then we have $d^{(J)}_{\lambda_1\lambda_2}(0) = \delta_{\lambda_1\lambda_2}$, and formula (9) is simplified:

$$R(q) = 1 + (-1)^{2S} \, \frac{1}{2S+1} \, \langle\, \cos(qx)\,\rangle \, . \quad (10)$$

Besides, taking into account the unitarity of the matrix $d^{(J)}_{\lambda_1\lambda_2}(\beta)$, it is easy to see from Eq. (9) that at $J = S$ formula (10) is valid at any angles between the momenta $\mathbf{p}_1$ and $\mathbf{p}_2$. Let us stress that Eq.(10) is related to particles with nonzero mass .

3 Special cases of pair correlations of two unpolarized photons and two neutrinos

In the case of emission of two unpolarized photons, when the mass equals zero, spin $S = 1$ and each of the helicities λ_1, λ_2 takes only two $(2S)$ values: -1 and 1, irrespective of the momentum direction, the correlation function for dipole sources has the form [3] :

$$R(q) =$$

$$= 1 + \frac{1}{4} \left[(d^{(1)}_{11}(\beta))^2 + (d^{(1)}_{-1,1}(\beta))^2 + (d^{(1)}_{-1,-1}(\beta))^2 + (d^{(1)}_{1,-1}(\beta))^2 \right] \langle \cos(qx) \rangle . \tag{11}$$

Taking into account the equalities :

$$d^{(1)}_{11}(\beta) = d^{(1)}_{-1,-1}(\beta) = \frac{1 + \cos\beta}{2}, \quad d^{(1)}_{1,-1}(\beta) = d^{(1)}_{-1,1}(\beta) = \frac{1 - \cos\beta}{2}, \tag{12}$$

we find :

$$R(q) = 1 + \frac{1}{4}(1 + \cos^2\beta) \langle \cos(qx) \rangle. \tag{13}$$

At very small angles between the photon momenta ($\beta \ll 1$) we obtain:

$$R(q) = 1 + \frac{1}{2} \langle \cos(qx) \rangle. \tag{14}$$

For the case of quadrupole sources , the correlation function is as follows:

$$R(q) =$$

$$= 1 + \frac{1}{4} \left[(d^{(2)}_{11}(\beta))^2 + (d^{(2)}_{-1,1}(\beta))^2 + (d^{(2)}_{-1,-1}(\beta))^2 + (d^{(2)}_{1,-1}(\beta))^2 \right] \langle \cos(qx) \rangle . \tag{15}$$

Using the equalities :

$$d^{(2)}_{11}(\beta) = d^{(2)}_{-1,-1}(\beta) = \frac{1 + \cos\beta}{2} (2\cos\beta - 1), \tag{16}$$

$$d^{(2)}_{1,-1}(\beta) = d^{(2)}_{-1,1}(\beta) = \frac{1-\cos\beta}{2}\,(2\cos\beta + 1), \tag{17}$$

we find the correlation function of two unpolarized photons emitted by the quadrupole sources :

$$R(q) = 1 + \frac{1}{4}\,\left(4\cos^4\beta - 3\cos^2\beta + 1\right)\,\langle\cos(qx)\rangle\,. \tag{18}$$

At $\beta \approx 0$ we have : $R(q) = 1 + \frac{1}{2}\langle\cos(qx)\rangle$, i.e. here we also obtain the standard formula (see Eq. (14)), corresponding to two directions of polarization for each of the photons [3] .

Let us consider also the case of emission of two "left" neutrinos (two "right" antineutrinos), with helicity taking only one value $\lambda_1 = \lambda_2 = +\frac{1}{2}$. For this case, the correlation function in the model of multipole sources is as follows :

$$R(q) = 1 - (d^{(J)}_{\frac{1}{2}\frac{1}{2}}(\beta))^2\,\langle\cos(qx)\rangle\,. \tag{19}$$

In particular, at $J = S = \frac{1}{2}$ we obtain :

$$R(q) = 1 - \cos^2\frac{\beta}{2}\,\langle\cos(qx)\rangle\,. \tag{20}$$

In the limit $\beta \to 0$ Eq. (20) gives:

$$R(q) = 1 - \langle\cos(qx)\rangle\,. \tag{21}$$

References

[1] M.I. Podgoretsky, EChAYa, **20**, 266 (1989) .

[2] L.D. Landau and E.M. Lifshitz, *Quantum Mechanics. Nonrelativistic theory* (Nauka, Moscow, 1989), § 58 .

[3] V.L. Lyuboshitz and M.I. Podgoretsky, Yad. Fiz. **58**, 33 (1995) [Phys. At. Nucl. **58**, 30 (1995)] .