## Correlations and fluctuations in high-energy nuclear collisions <br> -- a "flow" centric review

## Jiangyong Jia

- Ridge in small systems
- Collective phenomena in $\mathrm{A}+\mathrm{A}$


## The PHENIX d+Au ridge



Central



S. Huang


Clear excess at near-side, the "hidden ridge"

Central


Peripheral



## Be careful about Per-trigger yield...

Step structure here is simply a ZYAM anomaly


ZYAM can't see the ridge if the near-side shape is concave



## The tale of three ridges....



- Manifestation of QCD in different high density systems
- But is there an effective mechanism that rules them all? Is it initial state effect, final state effect or both?
- What is its detailed $\mathrm{p}_{\mathrm{T}}, \eta$, and centrality dependence? How these dependences compare between different systems?
pPb ridge properties summarized by harmonics

- $\mathrm{v}_{1}, \mathrm{v}_{2}, \mathrm{v}_{3}, \mathrm{~V}_{4}$ and $\mathrm{v}_{5}$, made possible with recoil subtraction
- $\mathrm{v}_{2}, \mathrm{v}_{3}$ out to 10 GeV , remain 3-5\%, small jet modifications?
- $\mathrm{v}_{\mathrm{n}}$ decrease with n for $\mathrm{n}=2-5$
- Significant $\mathrm{v}_{1}$ comparable with $\mathrm{v}_{3}$ at 4 GeV .
pPb ridge properties summarized by harmonics



Why ridge (and $\mathrm{v}_{2}, \mathrm{v}_{3}$ ) does not disappear at 10 GeV ?

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Is there global correlation in $\mathrm{p}+\mathrm{Pb}$ system?


Multi-particle and all particle correlation signal remain remarkably large in high-multiplicity events!!

## Comparison p+Pb with $\mathrm{Pb}+\mathrm{Pb}$

- Collectivity increase and decrease with system size.


Where and how the hydro-picture breaks down?
What is the correct effective theory? CGC+transport?

## Comparison of $\mathrm{p}+\mathrm{Pb}$ with $\mathrm{Pb}+\mathrm{Pb}$

- Why extrapolation of hydro prediction works so well? e.g. conformal scaling
G. Basar \& Teaney
- From the confomal analysis
- Find:

$\frac{L_{A A}}{L_{p A}}=\frac{\left\langle p_{T}\right\rangle_{p A}}{\left\langle p_{T}\right\rangle_{A A}}=1.3 \quad \frac{R_{A A}}{R_{p A}}=\frac{3.5}{2.6}=1.35$




## Comparison of $\mathrm{p}+\mathrm{Pb}$ with $\mathrm{Pb}+\mathrm{Pb}$

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- From the confomal analysis
$\frac{L_{A A}}{L_{p A}}=\frac{\left\langle p_{T}\right\rangle_{p A}}{\left\langle p_{T}\right\rangle_{A A}}=1.3$

D.Gangadharan

Detailed comparison between experiments are needed


A few observations/comments about flow in $A+A$ collisions

- Compare to RHIC results,
- Stronger radial flow and importance of hadronic rescattering.
- Poorer NCQ scaling.
- $\phi$ flow like a baryon (central) and meson (mid-central)
- Combination of mass and crosssection effects?

A. Dobrin \& Jan



- $\mathrm{Cu}+\mathrm{Au} \mathrm{v}_{1}$ from average dipolar geometry
- U+U: see some sensitivity to the initial state geometry.

Each collision system introduces its own uncertainty in geometry!

## Intra-event flow fluctuation and factorization

- Flow angle and amplitude fluctuates in $\mathrm{p}_{\mathrm{T}}($ and $\eta$ ) Ollitrault QM2012

$$
\tilde{r}_{n}\left(p_{T 1}, p_{T 2}\right):=\frac{\left\langle v_{n}\left(p_{T 1}\right) v_{n}\left(p_{T 2}\right) \cos \left[n\left(\Psi_{n}\left(p_{T 1}\right)-\Psi_{n}\left(p_{T 2}\right)\right)\right]\right\rangle}{\left\langle v_{n}\left(p_{T 1}\right) v_{n}\left(p_{T 2}\right)\right\rangle}
$$

- Breaking is largest for $\mathrm{v}_{2}$ in ultra-central $\mathrm{Pb}+\mathrm{Pb}$ collisions
- Much smaller for other harmonics and in other centralities (ALICE/ATLAS/CMS)
- Breaking of factorization $\mathrm{p}+\mathrm{Pb}$ at a few \% level D. Devetak also Y. Zhou



The strange $\mathrm{v}_{2}\left(\mathrm{p}_{\mathrm{T}}\right)$ shape!

- Linear response dominates: $v_{n} \propto \varepsilon_{n}$ for all n
- Models have difficulty explain $\mathrm{v}_{2} \approx \mathrm{v}_{3}$
- Importance of nucleon-nucleon correlation and bulk viscosity? G.Denicol


## Ultra-central collisions




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## Event-by-Event fluctuations

## Geometry and harmonic flow



$$
\vec{\epsilon}_{n} \equiv \epsilon_{n} e^{i n \Phi_{n}^{*}} \equiv-\frac{\left\langle r^{n} e^{i n \phi}\right\rangle}{\left\langle r^{n}\right\rangle}
$$

$$
\frac{d N}{d \phi} \propto 1+2 \sum_{n} \mathrm{v}_{\mathrm{n}} \cos n\left(\phi-\Phi_{n}\right)
$$

$$
\vec{v}_{n} \equiv v_{n} e^{i n \Phi_{n}}
$$

- How $\left(\varepsilon_{\mathrm{n}}, \Phi_{\mathrm{n}}{ }^{*}\right)$ are transferred to $\left(\mathrm{v}_{\mathrm{n}}, \Phi_{\mathrm{n}}\right)$ ?
- What is the nature of final state (non-linear) dynamics?


## Experimental observables

Many little bangs


$$
p\left(v_{n}, v_{m}, \ldots, \Phi_{n}, \Phi_{m}, \ldots\right)=\frac{1}{N_{\mathrm{evts}}} \frac{d N_{\mathrm{evts}}}{d v_{n} d v_{m} \ldots d \Phi_{n} d \Phi_{m} \cdots}
$$

Angular component captured by cosines

$$
\begin{aligned}
& \frac{d N_{\text {evts }}}{d \Phi_{1} d \Phi_{2} \ldots d \Phi_{l}} \propto \sum_{c_{n}=-\infty}^{\infty} a_{c_{1}, c_{2}, \ldots, c_{l}} \cos \left(c_{1} \Phi_{1}+c_{2} \Phi_{2} \ldots+c_{l} \Phi_{l}\right) \\
& a_{c_{1}, c_{2}, \ldots, c_{l}}=\left\langle\cos \left(c_{1} \Phi_{1}+c_{2} \Phi_{2}+\ldots+c_{l} \Phi_{l}\right)\right\rangle \\
&\left\langle\cos \left(c_{1} \Phi_{1}+2 c_{2} \Phi_{2} \ldots+l c_{l} \Phi_{l}\right)\right\rangle, c_{1}+2 c_{2} \ldots+l c_{l}=0
\end{aligned}
$$

1104.4740, 1209.2323, 1203.5095 ,1312.3572

|  | Probability distribution | Cumulants |
| :---: | :---: | :---: |
| Flow amplitudes | $p\left(v_{n}\right), p\left(v_{n}, v_{m}\right)$ | $v_{n}\{2 k\},\left\langle v_{n}^{2} v_{m}^{2}\right\rangle-\left\langle v_{n}^{2}\right\rangle\left\langle v_{m}^{2}\right\rangle$ |
| Event-plane correlation | $p\left(\Phi_{n}, \Phi_{m}, \ldots\right)$ | $\left\langle\vec{v}_{n} \vec{v}_{m} \ldots\right\rangle$ or |
|  |  | $\left\langle\cos \left(c_{1} \Phi_{1}+\ldots+l c_{l} \Phi_{l}\right)\right\rangle$ |

## $\mathrm{v}_{\mathrm{n}}\{2 \mathrm{k}\}$ in $\mathrm{Pb}+\mathrm{Pb}$ collisions




4t-birt-3278

- Provide information about the underlying $\mathrm{p}\left(\mathrm{v}_{\mathrm{n}}\right)$ distribution
- $\mathrm{v}_{2}\{4\} \sim \mathrm{v}_{2}\{6\} \sim \mathrm{v}_{2}\{8\} \rightarrow$ Gaussian fluctuation around mean $\mathrm{v}_{2}{ }^{\mathrm{RP}}$ :

$$
p\left(\vec{v}_{n}\right)=\frac{1}{2 \pi \delta_{v_{n}}^{2}} e^{-\left(\vec{v}_{n}-\vec{v}_{n}^{\mathrm{RP}}\right)^{2} /\left(2 \delta_{v_{n}}^{2}\right)}
$$

- Non-zero $\mathrm{v}_{3}\{4\}$ (ALICE) and also $\mathrm{v}_{4}\{4\}$ (ATLAS)


## Cumulants from traditional method and from $\mathrm{p}\left(\mathrm{v}_{2}\right)$



## Cumulants from traditional method and from $\mathrm{p}\left(\mathrm{v}_{2}\right)$



- Measuring $\mathrm{p}\left(\mathrm{v}_{2}\right)$ is equivalent to cumulants, more intuitive and simpler systematics
- Non-Bessel Gaussian is reflected by a $2 \%$ change beyond $4^{\text {th }}$ order cumulants
- Flow response is linear for $\mathrm{v}_{2}$ and $\mathrm{v}_{3}: v_{n} \propto \varepsilon_{n}$ and $\Phi_{n} \approx \Phi_{n}^{*}$ i.e.

$$
v_{2} e^{-i 2 \Phi_{2}} \propto \epsilon_{2} e^{-i 2 \Phi_{2}^{*}}, \quad v_{3} e^{-i 3 \Phi_{3}} \propto \epsilon_{3} e^{-i 3 \Phi_{3}^{*}}
$$

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$$

- Higher-order flow arises from EP correlations., e.g. :


Ollitrault, Luzum,
Teaney, Li,
Heinz,Chun....


$$
v_{4} e^{-i 4 \Phi_{4}} \propto \varepsilon_{4} e^{-i 5 \Phi_{4}^{*}}+c v_{2}^{2} e^{-i 4 \Phi_{2}}+\ldots \quad \quad v_{5} e^{-i 5 \Phi_{5}} \propto \varepsilon_{5} e^{-i 5 \Phi_{5}^{*}}+c v_{2} v_{3} e^{-i\left(2 \Phi_{2}+3 \Phi_{3}\right)}+\ldots
$$

More info by selecting on event-shape

arXiv:1208.4563 arxiv:1311.7091

- Select events with certain $\mathrm{v}_{2}{ }^{\text {obs }}$ in Forward Rapidity:

More info by selecting on event-shape

$\mathrm{FCal} \mathrm{v}_{2}^{\mathrm{obs}}$
arXiv:1208.4563 arxiv:1311.7091

- Fix centrality, then select events with certain $\mathrm{v}_{2}{ }^{\text {obs }}$ in Forward rapidity:
$\rightarrow$ ATLAS: measure $\mathrm{v}_{\mathrm{n}}$ via two-particle correlations in $|\boldsymbol{n}|<2.5$
Fix system size and change ellipticity!!

More info by selecting on event-shape

S. Mohapatra

# $\mathrm{v}_{\mathrm{n}}-\mathrm{v}_{2}$ correlations: centrality dependence 

- First correlation without event $\mathrm{v}_{2}$-selection, $5 \%$ steps

S. Mohapatra


## $\mathrm{v}_{\mathrm{n}}-\mathrm{v}_{2}$ correlations: within fixed centrality

- Fix system size and vary the ellipticitiy!


## Probe $\mathrm{p}\left(\mathrm{v}_{\mathrm{n}}, \mathrm{v}_{2}\right)$





Linear correlation for forward
$\mathrm{v}_{2}$-selected bin $\rightarrow$ viscous
damping controlled by
system size, not shape

## $\mathrm{v}_{\mathrm{n}}-\mathrm{v}_{2}$ correlations: within fixed centrality

- Fix system size and vary the ellipticitiy!
- Overlay $\varepsilon_{3}-\varepsilon_{2}$ and $\varepsilon_{4}-\varepsilon_{2}$ correlations, rescaled


## Probe $p\left(v_{n}, v_{2}\right)$


quadratic rise from nonlinear coupling to $v_{2}{ }^{2}$

## $\mathrm{v}_{\mathrm{n}}-\mathrm{v}_{2}$ correlations: within fixed centrality

- Fix system size and vary the ellipticitiy!

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- Overlay $\varepsilon_{3}-\varepsilon_{2}$ and $\varepsilon_{4}-\varepsilon_{2}$ correlations, rescaled


Linear correlation for forward $\mathrm{v}_{2}$-selected bin $\rightarrow$ viscous damping controlled by system size, not shape


Clear anti-correlation, mostly initial geometry effect!!

quadratic rise from nonlinear coupling to $v_{2}{ }^{2}$ initial geometry do not work!!

Initial geometry describe $\mathrm{v}_{3}-\mathrm{v}_{2}$ but fails $\mathrm{v}_{4}-\mathrm{v}_{2}$ correlation s . Mohapatra




- Fit $v_{4}=\sqrt{c_{0}^{2}+c_{1}^{2} v_{2}^{4}}$ to separate linear $\left(\varepsilon_{4}\right)$ and non-linear $\left(\mathrm{v}_{2}{ }^{2}\right)$ component





Linear-component provide independent constraints on viscosity

- Fit $v_{4}=\sqrt{ } c_{0}^{2}+c_{1}^{2} v_{2}^{4}$ to separate linear $\left(\varepsilon_{4}\right)$ and non-linear $\left(\mathrm{v}_{2}{ }^{2}\right)$ component




See details at https://atlas.web.cern.ch/Atlas/GROUPS/PHYSICS/CONFNOTES/ATLAS-CONF-2014-022/


## Future prospects: my humble opinion

## (I) : Precision event-shape selection

- Different collision system e.g. $\mathrm{He}^{3}+\mathrm{Au}$, June $16^{\text {th }}$ ! P. ROMATSCHKE Nagle, et al (MM), arXiv: I 3 | 2.4565

Intrinsic trangularity


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- Different collision system e.g. $\mathrm{He}^{3}+\mathrm{Au}$, June $16^{\text {th }}$ !
P. ROMATSCHKE

Nagle, et al (MM), arXiv: $13 \mid 2.4565$

Intrinsic trangularity


- Event-shape selections on $v_{2}$ and/or $v_{3} \rightarrow$ Fix size, change $\varepsilon_{2}$ and $\varepsilon_{3}$
- $\mathrm{v}_{\mathrm{n}}, \mathrm{HBT}, \mathrm{R}_{\mathrm{AA}}, \mathrm{CME}$ etc..

Schukraft, Timmins, and Voloshin, arXiv:1208.4563 Huo, Mohapatra, JJ arxiv:1311.7091 |ncreasing $\varepsilon_{2}$


Increasing $\varepsilon_{3}$


## (II) : understand jet-medium interaction

- How (mini)-jet are thermalized in medium?
- Difficult due to dominance of collective flow
- Until 2010, triangular flow was interpreted as "Mach-cone"
- Event-shape selection technique can help!
- Require events to have small $\mathrm{v}_{\mathrm{n}}$, less flow subtraction.



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- $\eta \times \phi$ space are dominated by fake-jets or "hydro-jets"


No boost invariance!!


Curtsey of L.Pang and X.N Wang, EbyE 3D hydro+AMPT condition

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- Require events to have small $\mathrm{v}_{\mathrm{n}}$, less flow subtraction.
- $\eta \times \phi$ space are dominated by fake-jets or "hydro-jets"
- They can be found by jet-reco algorithm (vetoing good jets) Then analysis spectrum or study substructure?


(III) : flow longitudinal dynamics

- Shape of participants in two nuclei not the same due to fluctuation $\varepsilon_{n}^{\mathrm{F}}, \boldsymbol{\Phi}_{n}^{\text {FF }} \neq \varepsilon_{n}^{\mathrm{B}}, \boldsymbol{\Phi}_{n}^{\text {AB }^{\text {B }}}$
- Particles are produced by independent fragmentation of wounded nucleons, emission function $f(\eta)$ not symmetric in $\eta \rightarrow$ Wounded nucleon model


## (III) : flow longitudinal dynamics



- Eccentricity vector interpolates between $\vec{\epsilon}_{n}^{\mathrm{F}}$ and $\bar{\epsilon}_{n}^{\mathrm{B}}$

$$
\begin{array}{cl}
\vec{\epsilon}_{n}^{\mathrm{tot}}(\eta) \approx \alpha(\eta) \vec{\epsilon}_{n}^{\mathrm{F}}+(1-\alpha(\eta)) \vec{\epsilon}_{n}^{\mathrm{B}} \equiv \epsilon_{n}^{\mathrm{tot}}(\eta) e^{i n \Phi_{n}^{* \mathrm{tot}}(\eta)} \begin{array}{ll}
\text { Asymmetry: } & \varepsilon_{n}^{\mathrm{F}} \neq \varepsilon_{n}^{\mathrm{B}} \\
\alpha(\eta) \text { determined by } \mathrm{f}(\mathrm{\eta}) & \text { Twist: } \\
\Phi_{n}^{* \mathrm{~F}} \neq \Phi_{n}^{* \mathrm{~B}}
\end{array}
\end{array}
$$

- Hence $\vec{v}_{n}(\eta) \approx c_{n}(\eta)\left[\alpha(\eta) \vec{\epsilon}_{n}^{\mathrm{F}}+(1-\alpha(\eta)) \vec{\epsilon}_{n}^{\mathrm{B}}\right]$ for $\mathrm{n}=2,3$
- Picture verified in AMPT simulations, magnitude estimated 1403.6077

Require $\varepsilon_{2}^{\mathrm{F}}>\varepsilon_{2}^{\mathrm{B}}$ see $v_{2}(+\eta)>v_{2}(-\eta)$


## Initial state twist and asymmetry survives collective expansion

Play a bigger role for $\mathrm{Cu}+\mathrm{Au}, \mathrm{U}+\mathrm{U}$ and $\mathrm{p}+\mathrm{A}$ system

CMS Preliminary $\mathrm{pPb} \sqrt{\mathrm{s}_{\mathrm{NN}}}=5.02 \mathrm{TeV}$
Require $\Phi_{n}^{* \mathrm{~F}}>\Phi_{n}^{* \mathrm{~B}}$ see $\Phi_{2}(+\eta)>\Phi_{2}(-\eta)$


Backup

## Elliptic flow of identified particles

Identified $\mathrm{K}_{\mathrm{S}}$ and $\wedge$ \& charged hadrons
$v_{2}$ (and $v_{3}$ ) from
2-particle correlations
show mass ordering In pPb and PbPb (stronger in pPb ) and $\approx$ quark scaling (better in pPb )


## Vs dependence of final spatial eccentricity



- Gradual decrease of $\varepsilon_{\mathrm{f}}$ as function of $\sqrt{S}$.
- Hydro predicts stronger decrease,
- UrQMD works but it probably under-predicts the flow.


## Intra-event flow fluctuation and factorization

- Flow angle and amplitude fluctuates in $\mathrm{p}_{\mathrm{T}}($ and $\eta$ ) Ollitrault QM2012

$$
\tilde{r}_{n}\left(p_{T 1}, p_{T 2}\right):=\frac{\left\langle v_{n}\left(p_{T 1}\right) v_{n}\left(p_{T 2}\right) \cos \left[n\left(\Psi_{n}\left(p_{T 1}\right)-\Psi_{n}\left(p_{T 2}\right)\right)\right]\right\rangle}{\left\langle v_{n}\left(p_{T 1}\right) v_{n}\left(p_{T 2}\right)\right\rangle}
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- Much smaller for other harmonics and in other centralities
- Very small (2-3\%) breaking for high-multiplicity pPb collisions
- Be aware of non-flow bias from di-jets, recoil subtraction is necessary in

$$
1.0 \mathrm{GeV} / \mathrm{c}<\mathrm{p}_{\mathrm{T}}^{\text {trig }}<1.5 \mathrm{GeV} / \mathrm{c} 1.5 \mathrm{GeV} / \mathrm{c}<\mathrm{p}_{\mathrm{T}}^{\text {trig }}<2.0 \mathrm{GeV} / \mathrm{c} 2.0 \mathrm{GeV} / \mathrm{c}<\mathrm{p}_{\mathrm{T}}^{\text {trig }}<2.5 \mathrm{GeV} / \mathrm{c} 2.5 \mathrm{GeV} / \mathrm{c}<\mathrm{p}_{\mathrm{T}}^{\text {trig }}<3.0 \mathrm{GeV} / \mathrm{c}
$$



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- Very small (2-3\%) breaking for high-multiplicity pPb collisions
- Be aware of non-flow bias from di-jets, recoil subtraction is necessary in order to compare with theory

Kozlov et.al.:arXiv:1405.3976


## Beam Energy scan: search for CEP



- Looking for non-monotonic change with $\sqrt{ }$ s

Looking for non-monotonic change with $\sqrt{ }$ s
 G. Wang \& B. Mohanty

## Looking for non-monotonic change with $\sqrt{ }$ s

- Shallow dips observed at $\sim 10-20 \mathrm{GeV}$ for several observables HBT analysis


More refined measurements with BES II and theory input!!

