Collective dynamics in relativistic nuclear collisions

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Limits for the viscosity of strongly interacting matter:

- No direct measurements $\rightarrow$ extracting transport coefficients requires model for the spacetime evolution of the matter

**Fluid dynamics**

- Transport coefficients direct input to the model
- Easy to include transition from QGP to hadronic matter (EoS)
- Need: small gradients and close to local thermal equilibrium
Conservation laws

\[ \partial_\mu T^{\mu \nu} = 0 \]

\[ T^{\mu \nu} = e u^\mu u^\nu - (p + \Pi) \Delta^{\mu \nu} + \pi^{\mu \nu} \]

Israel-Stewart equations for dissipative parts of \( T^{\mu \nu} \)

shear-stress:

\[ \tau_\pi \frac{d}{d\tau} \pi^{\langle \mu \nu \rangle} + \pi^{\mu \nu} = 2\eta \nabla^{\langle \mu u^\nu \rangle} + \cdots \]

bulk pressure:

\[ \tau_\Pi \frac{d}{d\tau} \Pi + \Pi = -\zeta \nabla^{\mu} u^\mu + \cdots \]

Microscopic properties integrated into the coefficients:

\[ \eta(T, \{\mu_i\}) = \text{shear viscosity (resistance to deformations)} \]

\[ \zeta(T, \{\mu_i\}) = \text{bulk viscosity (resistance to volume changes)} \]
- Model for initial conditions (initial $T^\mu_\nu$)
- $\longrightarrow$ Spacetime evolution of $T^\mu_\nu$ from fluid dynamics
- $\longrightarrow$ Convert to observable particle spectra
Azimuthal deformations characterized by Fourier coefficients:

\[ v_1 = \text{directed flow}, \ v_2 = \text{elliptic flow}, \ v_3 = \text{triangular flow} \]

\[
\frac{dN}{dydp_T^2d\phi} = \frac{dN}{dydp_T^2} \left[ 1 + 2 \sum_{n=1}^{\infty} v_n(p_T) \cos(\phi - \psi_n) \right]
\]

Event-plane angle (direction of the deformation):

\[ \psi_n = \frac{1}{n} \arctan \left( \frac{\langle p_T \sin n\phi \rangle}{\langle p_T \cos n\phi \rangle} \right) \]

- \( v_n(p_T), \psi_n(p_T), dN/dy, \ldots \) characterize single event

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**Ensemble of events: Full characterization**

- Averages: \( \langle v_n \rangle, \langle \psi_n \rangle, \ldots \)
- Probability distributions: \( \mathcal{P}(v_n), \mathcal{P}(\psi_n), \ldots \)
- Correlations: \( \langle v_n, v_m \rangle, \langle \psi_n, \psi_m \rangle, \ldots \)
Azimuthal deform. of initial density characterized by eccentricities:

\[ \epsilon_{m,n} = -\frac{\int \mathrm{d}x\mathrm{d}y \ r^m \cos [n(\phi - \Psi_{m,n})] \varepsilon(x, y, \tau_0)}{\int \mathrm{d}x\mathrm{d}y \ r^m \varepsilon(x, y, \tau_0)} \]

Usually \( \epsilon_2 = \epsilon_{2,2}, \epsilon_3 = \epsilon_{3,2}, \ldots \)

\[ \Psi_{m,n} = \frac{1}{n} \arctan \frac{\int \mathrm{d}x\mathrm{d}y \ r^m \sin (n\phi) \varepsilon(x, y, \tau_0)}{\int \mathrm{d}x\mathrm{d}y \ r^m \cos (n\phi) \varepsilon(x, y, \tau_0)} + \frac{\pi}{n} \]

\( \Psi_{m,n} = \) participant plane angle (direction of deformation)

- \( \epsilon_{m,n}, \Psi_{m,n} \) characterize single event (initial energy density)

**Ensemble of events (initial conditions): Full characterization**

- Averages: \( \langle \epsilon_{m,n} \rangle, \langle \Psi_{m,n} \rangle, \ldots \)
- Probability distributions: \( \mathcal{P}(\epsilon_{m,n}), \mathcal{P}(\Psi_{m,n}) \)
- Correlations: \( \langle \epsilon_{m,n}, \epsilon_{m',n'} \rangle, \langle \Psi_{m,n}, \Psi_{m',n'} \rangle, \ldots \)
Fluid dynamics converts initial eccentricities to non-zero flow coefficients: $\varepsilon_n \rightarrow v_n$

**Ensemble of events (initial conditions): Full characterization**

- Averages: $\langle e_{m,n} \rangle, \langle \psi_{m,n} \rangle, \ldots$
- Probability distributions: $\mathcal{P}(e_{m,n}), \mathcal{P}(\psi_{m,n})$
- Correlations: $\langle e_{m,n}, e_{m',n'} \rangle, \langle \psi_{m,n}, \psi_{m',n'} \rangle, \ldots$

$\downarrow$

**Hydrodynamic response (EoS, $\eta/s, \zeta/s, \ldots$)**

$\downarrow$

**Ensemble of events (spectra): Full characterization**

- Averages: $\langle v_n \rangle, \langle \psi_n \rangle, \ldots$
- Probability distributions: $\mathcal{P}(v_n), \mathcal{P}(\psi_n), \ldots$
- Correlations: $\langle v_n, v_m \rangle, \langle \psi_n, \psi_m \rangle$

- Determine matter properties: $\eta/s(T, \mu_i), \zeta/s(T, \mu_i), \ldots$
- Initial state must be determined simultaneously
limits for $\eta/s$

(assume $\eta/s = \text{constant}$)
Uncertainty from initial conditions


- **UrQMD + (2+1)D viscous fluid dynamics (VISHNU)**
- \( \eta/s \sim 0.08 - 0.24 \) (RHIC)
- Large uncertainty (factor 2-3) from the initial conditions (MC-KLN \( \eta/s \sim 0.20 \) vs. MC-Glauber \( \eta/s \sim 0.08 \))
Identified hadrons $\pi$, K, p: elliptic flow


- UrQMD + (2+1)-D viscous hydro (VISHNU)
- Mass ordering of elliptic flow (prediction of hydrodynamics)
- Good agreement with the data
Identified hadrons $\pi$, $K$, $p$: $p_T$-spectra


- UrQMD + (2+1)-D viscous hydro (hybrid)
- Mass dependence of $p_T$ slopes & multiplicities
Higher harmonics: $v_2$, $v_3$, $v_4$, ... 

Gale, Jeon, Schenke, Tribedy and Venugopalan, PRL 110, 012302 (2013)

Constraints to initial conditions: $v_2/v_n$ ratio depends on IC
- IP-Glasma initial state + viscous hydrodynamics (MUSIC) $\rightarrow v_n$'s with $\eta/s = 0.20$ (LHC) or $\eta/s = 0.12$ (RHIC)
- $\eta/s$ (RHIC) $\neq \eta/s$ (LHC): indication of $T$-dependence of $\eta/s$?
Fluid dynamical behaviour: all the data explained by functions:

\[ p(T, \{\mu_i\}), \eta/s(T, \{\mu_i\}), \zeta/s(T, \{\mu_i\}), \ldots \]

properties of matter: should not change with \( \sqrt{s} \) or initial state
\[ \eta/s = \text{constant} \rightarrow \eta/s(T) \]
$\eta/s(T)$ from LHC $v_n$'s


+ fluctuations (preliminary results) poster by R. Paatelainen

- initial state: pQCD + local saturation (EKRT revisited)
- $v_n$'s do not give unique constraints to $\eta/s(T)$
- If we require minimum near $T_c \longrightarrow$ Constant $\eta/s$ gives upper limit for $\eta/s(T \sim T_c)$
$\eta/s(T)$ from LHC and RHIC $v_n$’s

+ fluctuations (preliminary results) poster by R. Paatelainen

LHC $\rightarrow$ RHIC: more sensitivity to hadronic viscosity
Beam energy scan

- More sensitivity to the properties of hadronic matter

Karpenko, Bleicher, Huovinen and Petersen, arXiv:1310.0702 [nucl-th]

- Same model that works at LHC and top RHIC energy works at lower $\sqrt{s}$ as well
- J. Auvinen (talk): $\sqrt{s}$ dependence of $v_3$ more sensitive probe of hadronic viscosity
Similar mass ordering of $v_n$ (also $\langle p_T \rangle$) as in AA collisions.

Here $\eta/s = 0.08$ (consistent with AA collisions with Glauber $N_{bin} + N_{wn}$ mixture, but not with MC-KLN/IP-Glasma/pQCD + saturation that require larger $\eta/s$).

$\eta/s = 0.08$ also in other models that describe the flow data.

Talks: P. Romatschke, V. Kozlov, K. Werner.
Ultracentral AA: Bulk viscosity and NN-correlations

- CMS ultracentral AA collisions $v_2 \sim v_3$
- Hard to reproduce with $\eta/s$ alone
- Add bulk viscosity + NN-correlations (talk by G. Denicol)

Bulk viscosity + correlations - IPGlasma

$$\frac{\zeta}{s} = b \times \frac{\eta}{s} \left( \frac{1}{3} - c_s^2 \right)^2$$

MUSIC 2.0

0-1% - LHC
Flow fluctuations: constraints to initial conditions
Strong correlation between $v_{2/3}$ and $\varepsilon_{2/3}$, i.e. $v_n \sim C\varepsilon_n$

At least within sufficiently narrow centrality bin:
$v_n/\varepsilon_n \sim \text{constant} \ (n = 2, 3)$

Relative fluctuations of $\varepsilon_n \rightarrow$ relative fluctuations of $v_n$

Probability distributions $P(\delta v_n) = P(\delta \varepsilon_n)$, $\delta v_n = (v_n - \langle v_n \rangle)/\langle v_n \rangle$
Flow fluctuations

G. Aad et al. [ATLAS Collaboration], JHEP 1311, 183 (2013)

- $P(v_2)$ compared to MC-Glauber and MC-KLN $P(\varepsilon_2)$
- MC-KLN: too narrow
- MC-Glauber: too wide
Variations of MC Glauber model


\[ s \propto \rho_{bc}^\alpha \]
\[ s \propto \rho_{wn}^\beta \]
\[ s \propto f \rho_{bc} + (1-f) \rho_{wn} \]

\[ \rho_{wn}/bc = \sum_{i=1}^{N_{wn,bc}} C_i e^{-\frac{(r-r_i)^2}{2\sigma^2}} \]

\( r_i \): positions of wounded nucleons or binary collisions from MC Glauber

Fluctuations in \( C_i \) or \( \sigma \) do not matter:
P(\( \delta v_n \)) measures geometry fluctuations, not fluctuations in particle/entropy production (Talk by T. Renk)
$\nu_3$ fluctuations

$\nu_3$ fluctuations identical for each case/centrality ($\nu_3$ is from random geometry fluctuations, not from underlying average nuclear overlap geometry)

Universal fluctuation-driven eccentricities

(talk by L. Yan)

Universal fluctuation spectrum, \( P(\varepsilon) = 2\alpha \varepsilon (1 - \varepsilon^2)^{\alpha-1} \)

Assume linear response \( v_n = C_n \varepsilon_n \)

→ predictions for \( v_2\{2\}/v_2\{4\}, v_2\{4\}/v_2\{6\}, v_2\{6\}/v_2\{8\} \)

Confirmed by CMS: talk by Quan Wang
IP-Glasma initial conditions

- Classical Yang-Mills
- Pre-thermal evolution
- Viscous fluid dynamics (MUSIC)
- Good agreement with the data over several centrality classes (talk by B. Schenke)
non-linear $v_2$, $\varepsilon_2$ correlation

+ fluctuations (preliminary results) poster by R. Paatelainen

- Near-central collisions (small $\varepsilon_2$): $v_2 \propto \varepsilon_2$
- Peripheral collisions (large $\varepsilon_2$): $v_2$ still strongly correlated to $\varepsilon_2$ (but not linearly)
effect on distributions

+ fluctuations (preliminary results) poster by R. Paatelainen

- Near-central collisions: follows from eccentricity
- Peripheral collisions: wider distributions after hydro
- Same observation with IP-Glasma IC (talk by B. Schenke)
- pQCD + saturation IC: works (blue)
- MC Glauber $N_{\text{bin}} + N_{\text{wn}}$: too wide (red)
Initial state constraints from $\varepsilon_2/\varepsilon_3$ ratio


- Estimate allowed range of $\varepsilon_2/\varepsilon_3$ ratio
- Constraints from $v_2/v_3$ data + fluid dynamics
Event-plane correlations $\langle \cos(N(\Psi_n - \Psi_m)) \rangle$


- Constraints to $\eta/s$ and initial state.
- MC-Glauber ($\eta/s = 0.08$) vs MC-KLN ($\eta/s = 0.20$): clearly different correlators
  Behaviour of correlators: non-linear response to initial state geometry $v_4$ generated by $\epsilon_4$ and $(\epsilon_2)^2$

$\langle \cos(N(\Psi_n - \Psi_m)) \rangle$ from the event-plane method strongly dependent on the event-plane resolution

- Large number of hydro runs randomly in the parameter space
  \[\rightarrow\] avoid huge number of runs
- Interpolate \[\rightarrow\] global fits
- Error estimates for the parameters (e.g. \(\eta/s\)) (diagonals)
- Identify correlators between the parameters (off diagonals)
- Here: example in 6–dimensional parameter space
Collective flow without fluid dynamics


BAMPS:

Boltzmann equation with pQCD $2 \leftrightarrow 2$ and $2 \leftrightarrow 3$ cross-sections

Talk by F. Senzel

Elliptic flow:

$\eta/s(T)$:

Pb+Pb, $\sqrt{s}=2.76$ TeV
improved GB, $X=0.3$
running coupling $\varepsilon=0.6$ GeV/fm$^3$

CMS, charged particles, $v_2(4)$

averaged running coupling $<\alpha_s>$, $n_f=0$
averaged running coupling $<\alpha_s>$, $n_f=3$
shear viscosity / entropy density $\eta/s$, $n_f=0$
shear viscosity / entropy density $\eta/s$, $n_f=3$
More interesting talks not covered here:

- Including thermal fluctuations to fluid dynamics (noise):
  - T. Hirano
  - J. Kapusta
- Anisotropic hydrodynamics: change the expansion basis
  - U. Heinz
- Mode-by-mode fluid dynamics: perturbative approach to fluctuations
  - S. Flörchinger
- New hydro codes
  - C. Nonaka
  - V. Rolando (ECHO-QGP)
- $\eta/s$ from acoustic scaling
  - R. Lacey
- And many more . . .
Summary

- Impressive agreement with several low-$p_T$ observables
- $\eta/s = 0.08 - 0.24$ (assuming constant $\eta/s$)
- $\eta/s(T)$ still not well constrained by $v_n$ data
- Fluid dynamical behaviour: all systems described by same $\eta/s(T, \{\mu_i\})$ (not yet clear)
- More constraints available: Fluctuation spectra, correlations
- Call for global analysis. Practical way: emulators