Vector screening masses in the quark-gluon plasma and their physical significance

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Quark Matter 2014, Darmstadt, 20 May 2014

Support from PRISMA cluster of excellence, the Helmholtz-Institute Mainz and DFG grant ME 3622/2-1.

Introduction

Is there an indirect way of probing real-time physics in the Euclidean formulation of thermal field theory, not involving a numerically ill-posed analytic continuation?

A pole of the retarded correlator $G_R(\omega, k)$ in ω at fixed k can also be viewed as a pole in k at fixed ω; for $\omega = 2\pi nTi$, these are the screening masses computable on the lattice.

Outline:

- A concrete link between non-static correlation lengths and the photon production rate (via a certain potential V^+)
- Comparison between lattice calculation and EFT prediction for correlation lengths $\Longrightarrow$ test of the potential V^+.

Based on 1404.2404, JHEP (in press).

Computational setup for vector screening masses

$$
\begin{aligned}
G_{\mu\nu}^{(k_n)}(z) &\equiv \int_0^{1/T} \mathrm{d}\tau\, e^{ik_n\tau} \int_{\boldsymbol{x}} \Big\langle V_\mu(\tau,\boldsymbol{x},z)\; V_\nu(0)\Big\rangle_{\mathrm{c}} \\
&\stackrel{\mu=\nu}{=} \int_0^\infty \frac{\mathrm{d}\omega}{\pi}\, e^{-\omega|z|}\, \rho_{\mu\nu}^{(k_n)}(\omega)\;, \qquad k_n \equiv 2\pi n T.
\end{aligned}
$$

Dimensional reduction:
Keep only the Matsubara zero modes of the SU(3) gauge fields in the covariant derivatives $D_\mu = \partial_\mu - igA_\mu$.

With $\psi = \frac{1}{\sqrt{T}}\begin{pmatrix}\chi\\ \phi\end{pmatrix}$, leads to (at treelevel, in a certain repres. of the γ_μ's)

$$
\begin{aligned}
S_0 &= \sum_{\{p_n\}} \int_{\boldsymbol{x},z} \Bigg[i\chi^\dagger_{p_n}\left(p_n - gA_0 + D_3 - \frac{D_iD_i + i\sigma_3\epsilon_{ij}D_iD_j}{2p_n}\right)\chi_{p_n} \\
&\qquad + i\phi^\dagger_{p_n}\left(p_n - gA_0 - D_3 - \frac{D_iD_i + i\sigma_3\epsilon_{ij}D_iD_j}{2p_n}\right)\phi_{p_n} + \mathrm{O}\left(\frac{1}{p_n^2}\right)\Bigg]\;.
\end{aligned}
$$

△ Free propagators:

$$\begin{aligned} \langle \chi_{p_n}(z_1)\chi^\dagger_{p_n}(z_2)\rangle &\simeq \int_{p_3} e^{ip_3(z_1-z_2)}\frac{-i}{p_n+ip_3}\,, \\ \langle \phi_{p_n}(z_1)\phi^\dagger_{p_n}(z_2)\rangle &\simeq \int_{p_3} e^{ip_3(z_1-z_2)}\frac{-i}{p_n-ip_3}\,. \end{aligned}$$

Forward-propagating mesons are $\phi^\dagger_{p_n}\chi_{p'_n}$ and $\phi^\dagger_{p_n}\phi_{-p'_n}$ with $p_n, p'_n > 0$.

△ For instance, consider for $k_n > 0$

$$V_0^{(k_n)} = \sum_{0<p_n<k_n}\left(\chi^\dagger_{p_n}\chi_{p_n-k_n} + \phi^\dagger_{p_n}\phi_{p_n-k_n}\right)\,.$$

△ For given p_n, let $w(z,\boldsymbol{y})$ be the screening correlator of a local current and a point-split current with separation $\boldsymbol{y}$ at the sink in the EFT. For $z > 0$,

$$\begin{aligned} (\partial_z + \hat{H}^+)w(z,\boldsymbol{y}) &= 0\,, \qquad w(0,\boldsymbol{y}) = \delta^{(2)}(\boldsymbol{y}), \\ \hat{H}^+ &\equiv M_{\rm cm} - \frac{\nabla^2}{2M_{\rm r}} + V^+, \\ V^+_{\rm LO}(\boldsymbol{y}) &= \frac{g_E^2 C_F}{2\pi}\left[\ln\left(\frac{m_E y}{2}\right) + \gamma_{\rm E} + K_0(m_E y)\right]. \end{aligned}$$

△ The Fourier transform of $w(z, \boldsymbol{y})$ is closely related to the 'resolvent' g^+,

$$\left(\hat{H}^+ - \omega - i0^+\right) g^+(\omega, \boldsymbol{y}) = \delta^{(2)}(\boldsymbol{y}) \, .$$

△ Finally, we obtain for the screening spectral function

$$\rho_{00}^{(k_n)}(\omega) = - \sum_{0<p_n<k_n} 2N_c T \lim_{\boldsymbol{y}\to\boldsymbol{0}} \mathrm{Im}\, g^+(\omega, \boldsymbol{y}) \, .$$

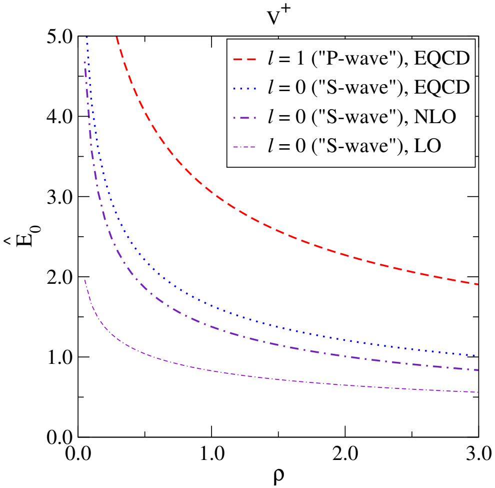
△ Close resemblance with the corresponding equations appearing in the LPM resummation of longitudinal modes for photon or dilepton production (same potential V^+, looking for an s-wave bound state) [Aurenche, Gelis, Moore, Zakaret hep-ph/0211036].

△ Potential V^+ can be defined non-perturbatively using a (modified) Wilson loop and has been computed in 3d lattice simulations [Caron-Huot 0811.1603; Panero, Rummukainen, Schäfer 1307.5850]

△ Idea: test the predictions for non-static screening masses resulting from solving the Schrödinger equation for the 2+1d potential V^+.

$\bar{Q}Q$ Binding energy in the EFT

$$g^+(\omega, \boldsymbol{y}) = \sum_{i=0}^{\infty} \frac{\psi_i(\boldsymbol{y})\psi_i^*(\boldsymbol{0})}{E_i - \omega - i0^+}\,, \quad \rho_{00}^{(k_n)}(\omega) = -2\pi N_c T \sum_{0<p_n<k_n} \sum_{i=0}^{\infty} \delta(E_i - \omega)|\psi_i(\boldsymbol{0})|^2$$

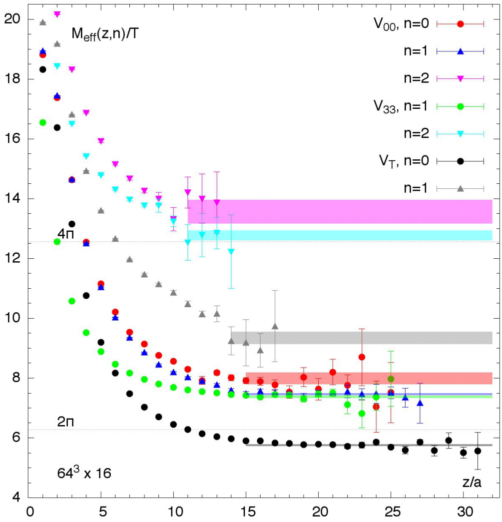


$$\rho = \frac{g_E^2 C_F M_\mathrm{r}}{\pi m_\mathrm{E}^2}$$

$$E_\mathrm{full} = M_\mathrm{cm} + \frac{g_E^2 C_F}{2\pi}\hat{E}$$

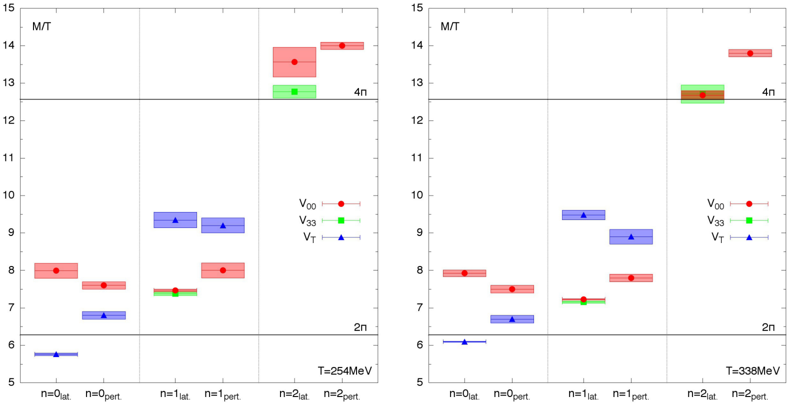
$$(m_\mathrm{E} = \mathrm{O}(gT), \quad g_E^2 = g^2 T + \dots)$$

Effective mass plot of screening correlators at $T = 250$ MeV



Two-flavor QCD $\quad m_\pi(T=0) = 270$ MeV $\quad 12 \times 64^3$ and 16×64^3.

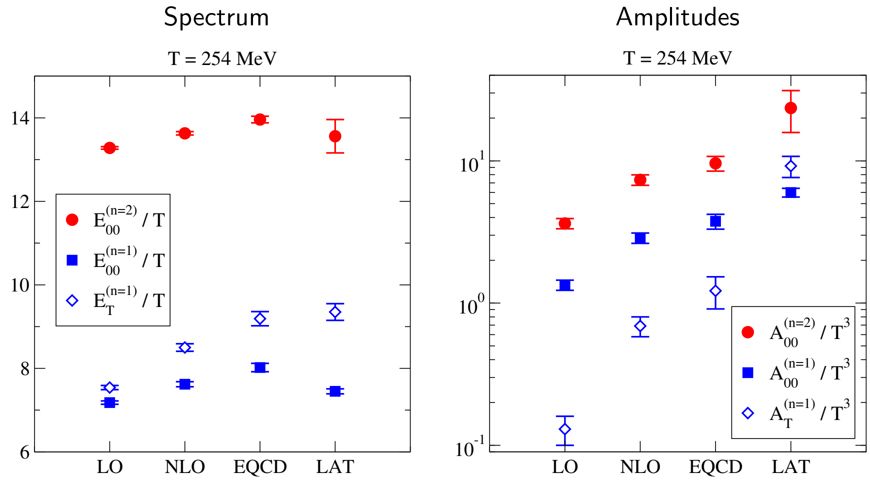
Vector screening masses: lattice vs. EFT



$T = 250$ MeV $\qquad$ $T = 340$ MeV

Satisfactory agreement between lattice QCD and the EFT predictions.

Perturbative vs. non-perturbative potential



Indication that the non-perturbative potential leads to better agreement with the results of full QCD simulations.

Outlook

- 'Integrating out' the non-static gauge modes perturbatively seems to be a decent approximation even at $T = 250$ MeV.
- Using a non-perturbative potential V^+ improves the predictions for the non-static screening masses.
- This study adds confidence to the applicability of EFT methods for the study of phenomenologically interesting observables at temperatures relevant to heavy ion collision experiments.
- Extend relation of non-static screening masses and real-time rates to a fully non-perturbative level?