New Theoretical Developments for the Physics of Strongly Coupled Systems

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Outline

Quantum Simulations in Atomic and Condensed Matter Physics

Wilson's Lattice Gauge Theory versus Quantum Link Models

Quantum Simulators for $U(1)$ Quantum Link Models

Atomic Quantum Simulators for Non-Abelian Gauge Theories

Conclusions

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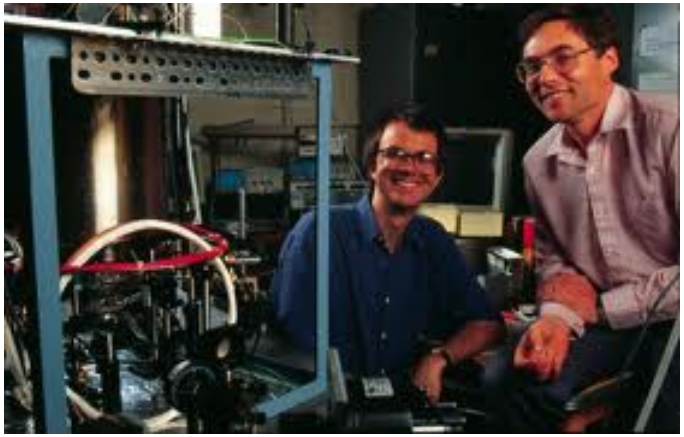
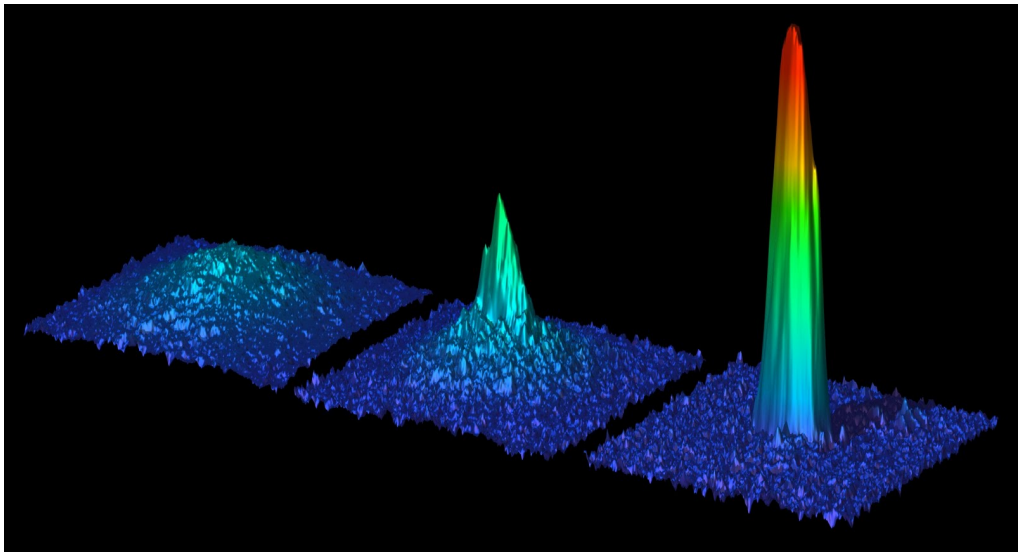
Feynman's Vision of 1982

"I'm not happy with all the analyses that go with just the classical theory, because nature isn't classical, dammit, and if you want to make a simulation of nature, you'd better make it quantum mechanical, and by golly it's a wonderful problem, because it doesn't look so easy."

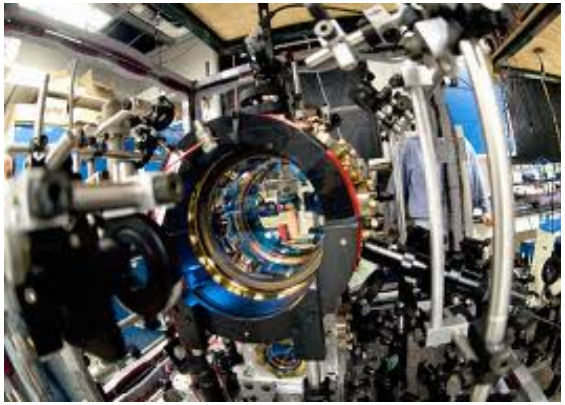
Bose-Einstein condensation in ultracold atomic gases

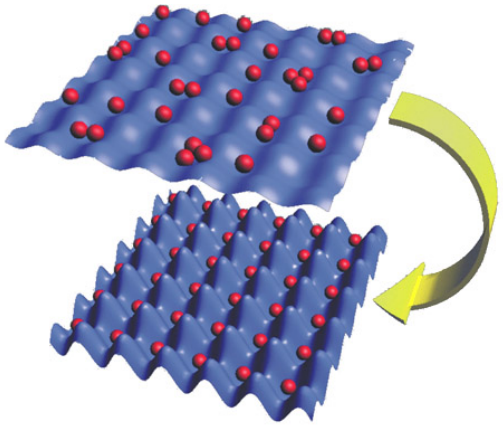


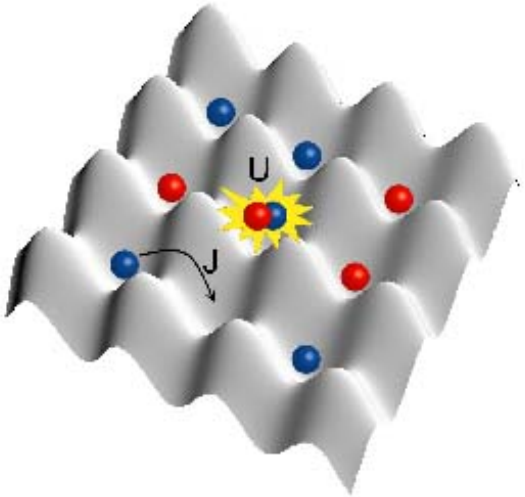
Eric Cornell, Carl Wieman, Wolfgang Ketterle, 1995



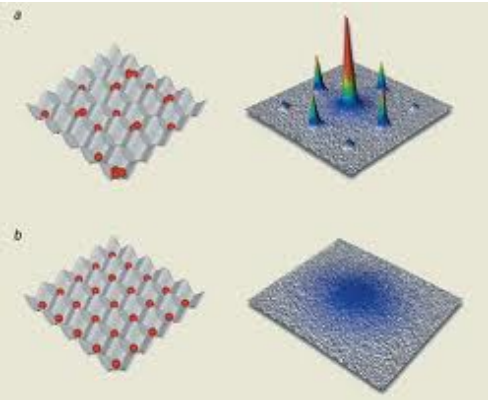
Ultra-cold atoms in optical lattices as analog quantum simulators







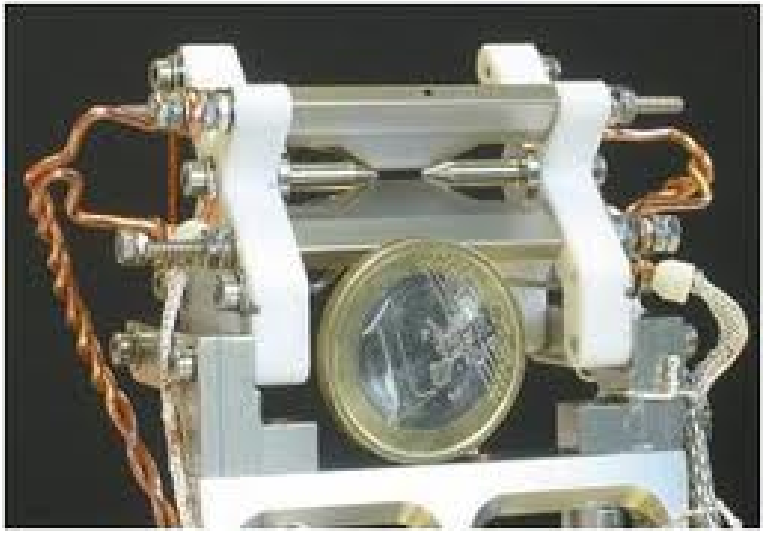
Superfluid-Mott insulator transition in the bosonic Hubbard model



Theodor Hänsch Immanuel Bloch

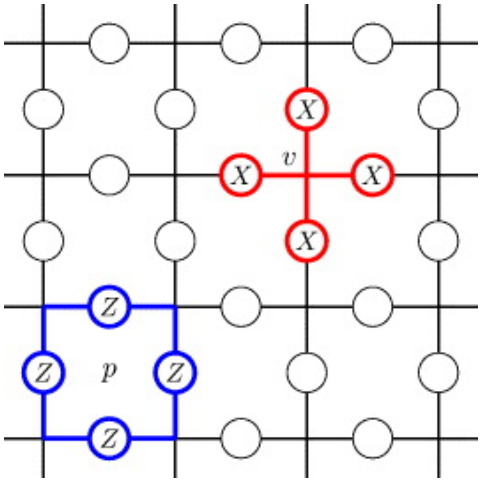
M. Greiner, O. Mandel, T. Esslinger, T. Hänsch, I. Bloch, Nature 415 (2002) 39.

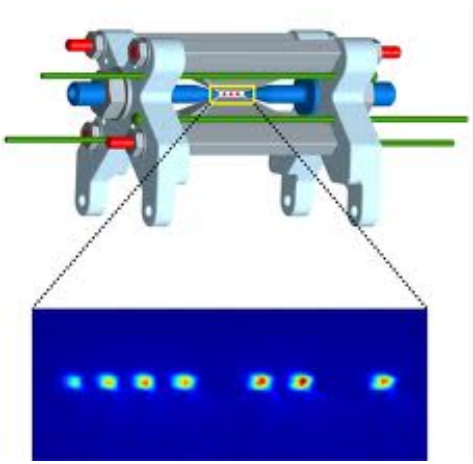
Ion trap as a quantum computer?



J. I. Cirac, P. Zoller, Phys. Rev. Lett. 74 (1995) 4091.

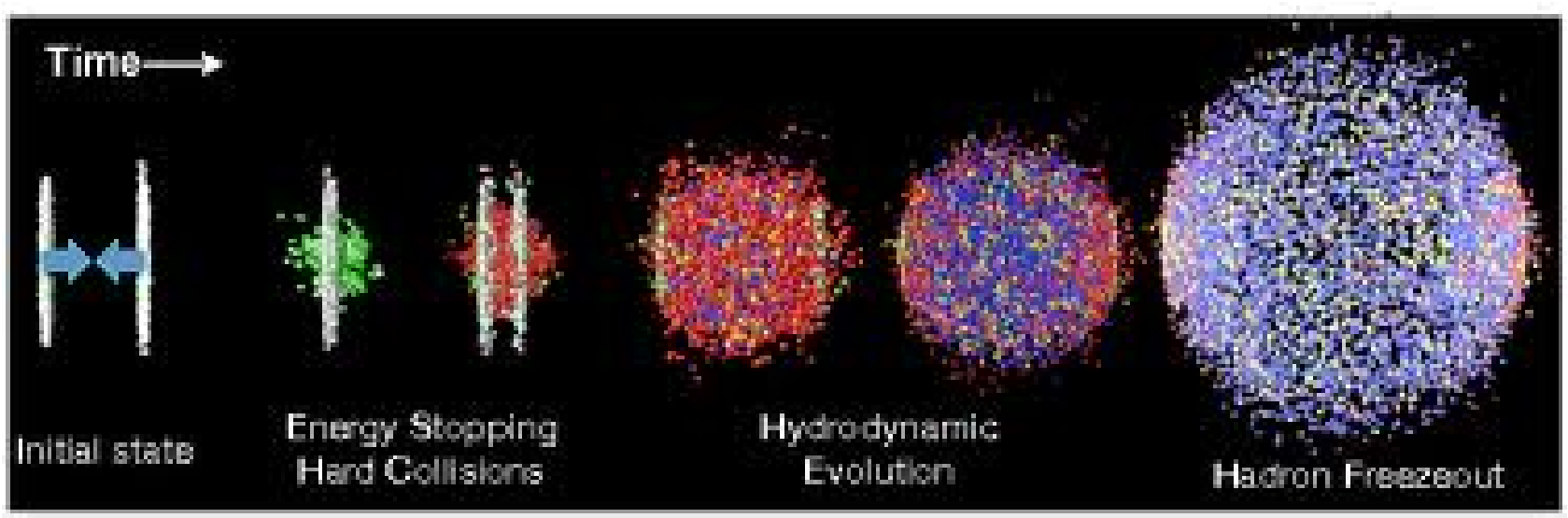
Digital quantum simulation of Kitaev's toric code

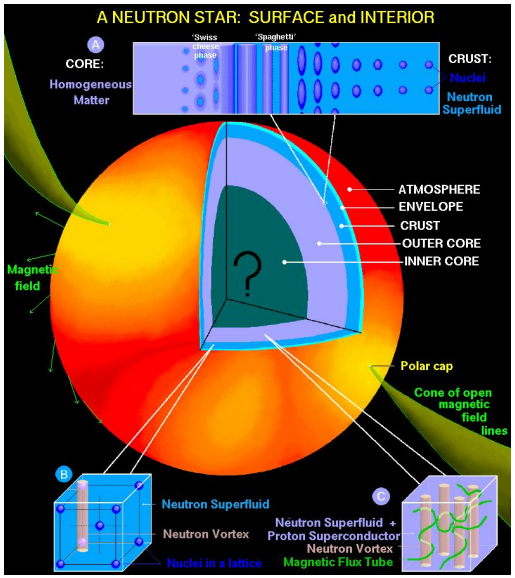




B. P. Lanyon et al., Science 334 (2011) 6052.

Can heavy-ion physics or QCD at non-zero density benefit from quantum simulations in the (very) long run?





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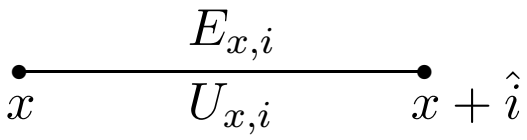
Wilson's $U(1)$ lattice gauge theory



$$U = \exp(i\varphi),\ U^\dagger = \exp(-i\varphi) \in U(1)$$

Electric field operator E

$$E = -i\partial_\varphi,\ [E, U] = U,\ [E, U^\dagger] = -U^\dagger,\ [U, U^\dagger] = 0$$

Generator of $U(1)$ gauge transformations

$$G_x = \sum_i (E_{x-\hat{i},i} - E_{x,i}), \quad [H, G_x] = 0$$

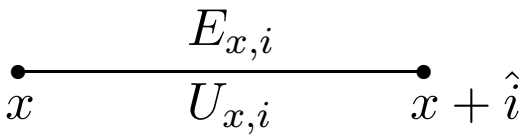
$U(1)$ gauge invariant Hamiltonian

$$H = \frac{g^2}{2} \sum_{x,i} E_{x,i}^2 - \frac{1}{2g^2} \sum_{\square} (U_\square + U_\square^\dagger),$$

$$U_\square = U_{x,i} U_{x+\hat{i},j} U^\dagger_{x+\hat{j},i} U^\dagger_{x,j}$$

operates in an infinite-dimensional Hilbert space per link

$U(1)$ quantum link model



$U = S_1 + iS_2 = S_+, \; U^\dagger = S_1 - iS_2 = S_-$

Electric field operator E

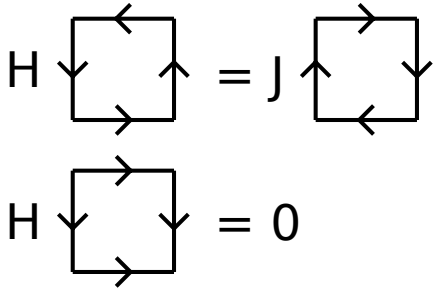
$$E = S_3, \; [E, U] = U, \; [E, U^\dagger] = -U^\dagger, \; [U, U^\dagger] = 2E$$

Generator of $U(1)$ gauge transformations

$$G_x = \sum_i (E_{x-\hat{i},i} - E_{x,i}), \quad [H, G_x] = 0$$

Gauge invariant Hamiltonian for $S = \frac{1}{2}$

$$H = \frac{g^2}{2} \sum_{x,i} E_{x,i}^2 - \frac{1}{2g^2} \sum_{\square} (U_\square + U_\square^\dagger),$$



defines a gauge theory with a 2-d Hilbert space per link.

D. Horn, Phys. Lett. B100 (1981) 149
P. Orland, D. Rohrlich, Nucl. Phys. B338 (1990) 647
S. Chandrasekharan, UJW, Nucl. Phys. B492 (1997) 455

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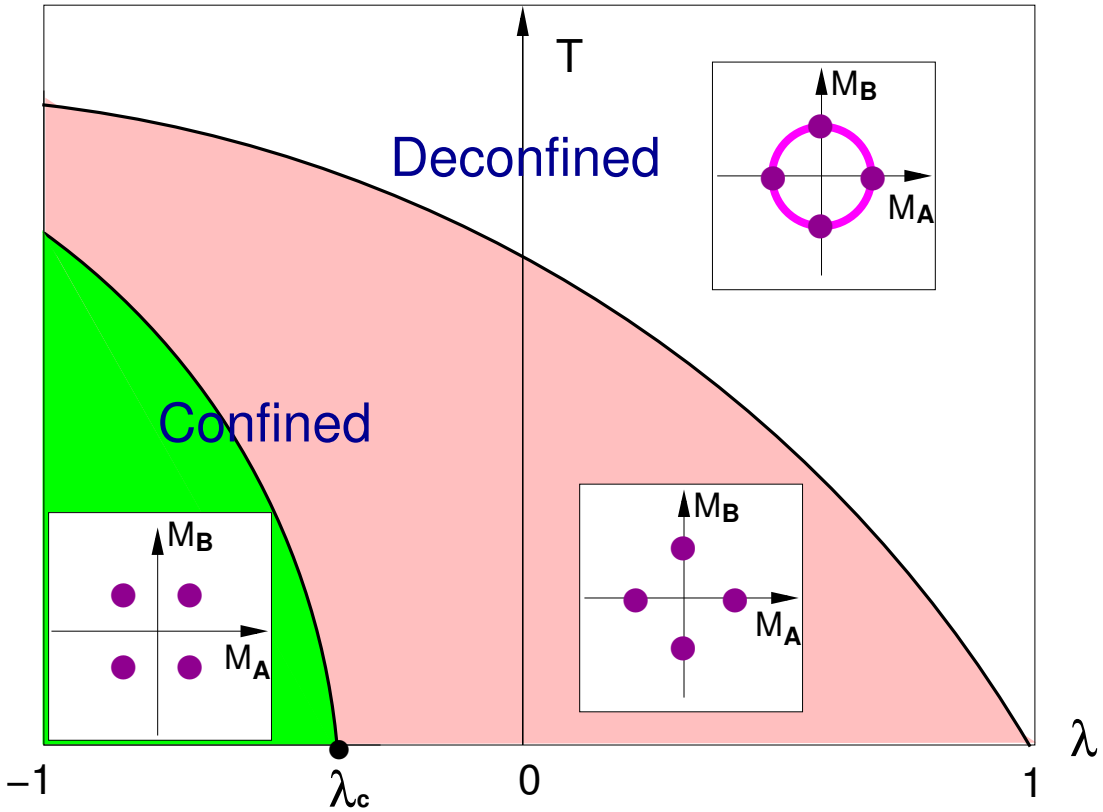
Hamiltonian with Rokhsar-Kivelson term

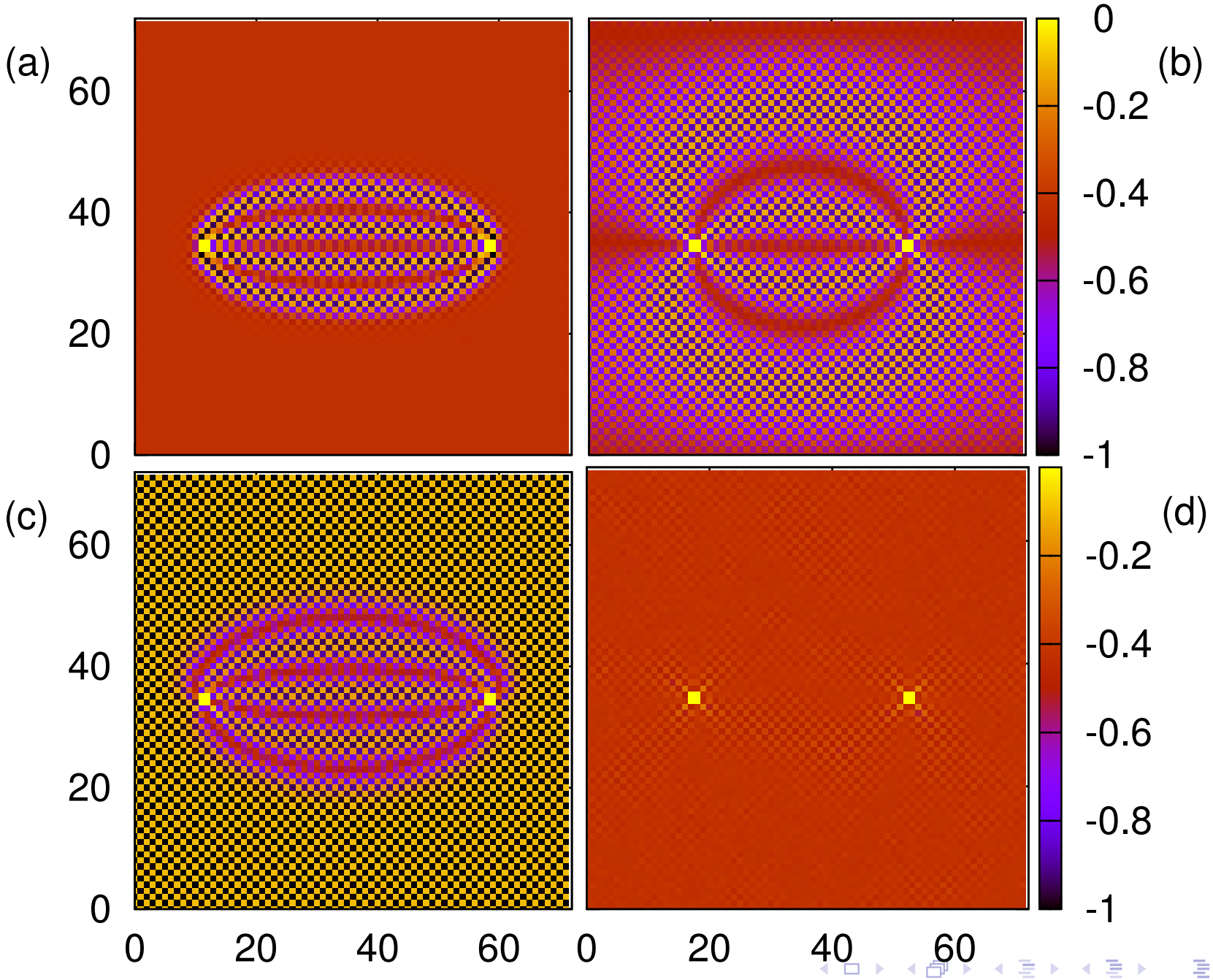
$$H = -J\left[\sum_{\square}(U_{\square} + U_{\square}^{\dagger}) - \lambda \sum_{\square}(U_{\square} + U_{\square}^{\dagger})^2\right]$$

Phase diagram



D. Banerjee, F.-J. Jiang, P. Widmer, UJW, JSTAT (2013) P12010.

Energy density of charge-anti-charge pair $Q = \pm 2$: $L^2 = 72^2$



Hamiltonian for staggered fermions and $U(1)$ quantum links

$$H = -t \sum_x \left[\psi_x^\dagger U_{x,x+1} \psi_{x+1} + \text{h.c.} \right] + m \sum_x (-1)^x \psi_x^\dagger \psi_x + \frac{g^2}{2} \sum_x E_{x,x+1}^2$$

Bosonic rishon representation of the quantum links

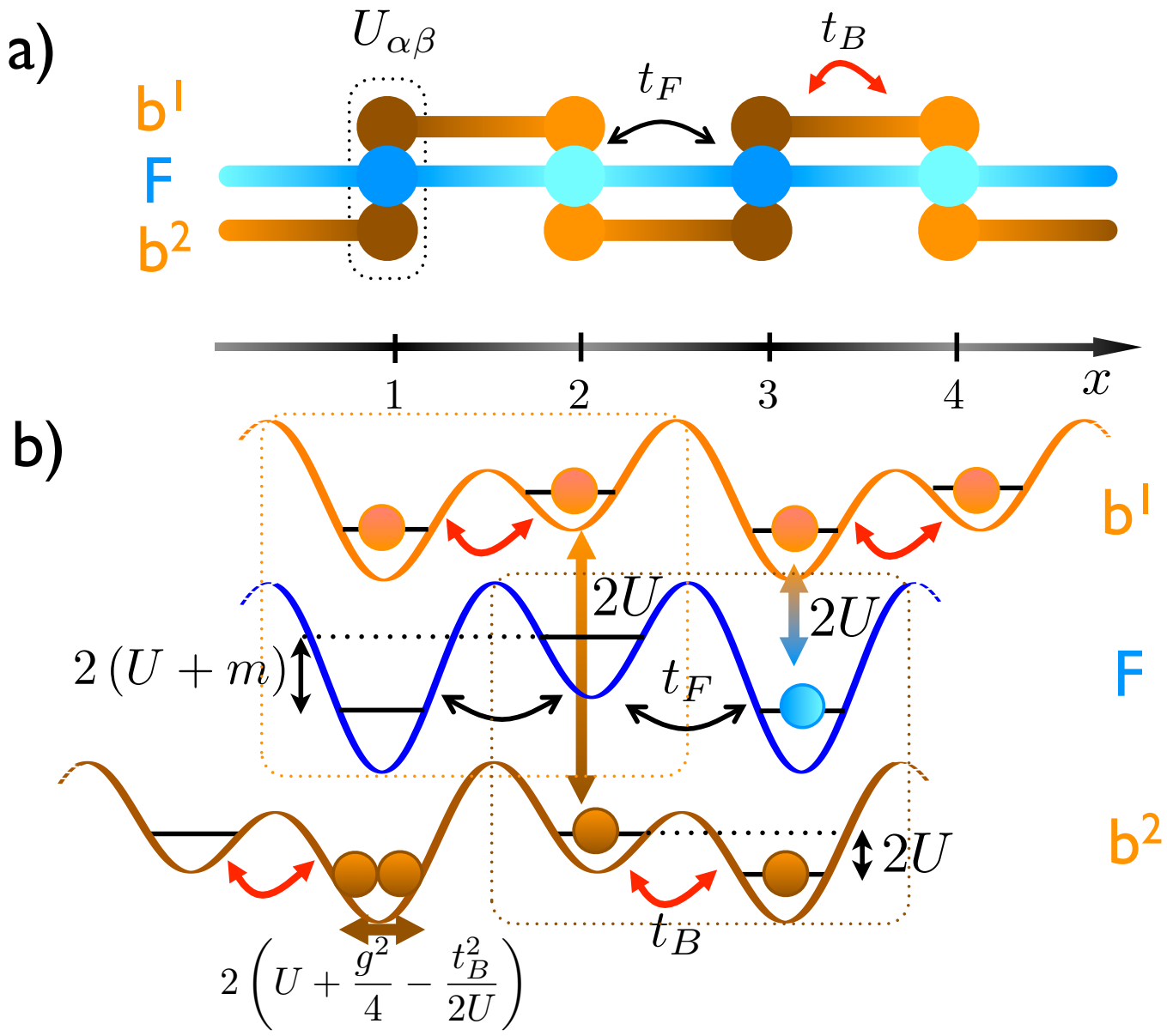
$$U_{x,x+1} = b_x b_{x+1}^\dagger, \; E_{x,x+1} = \frac{1}{2} \left(b_{x+1}^\dagger b_{x+1} - b_x^\dagger b_x \right)$$

Microscopic Hubbard model Hamiltonian

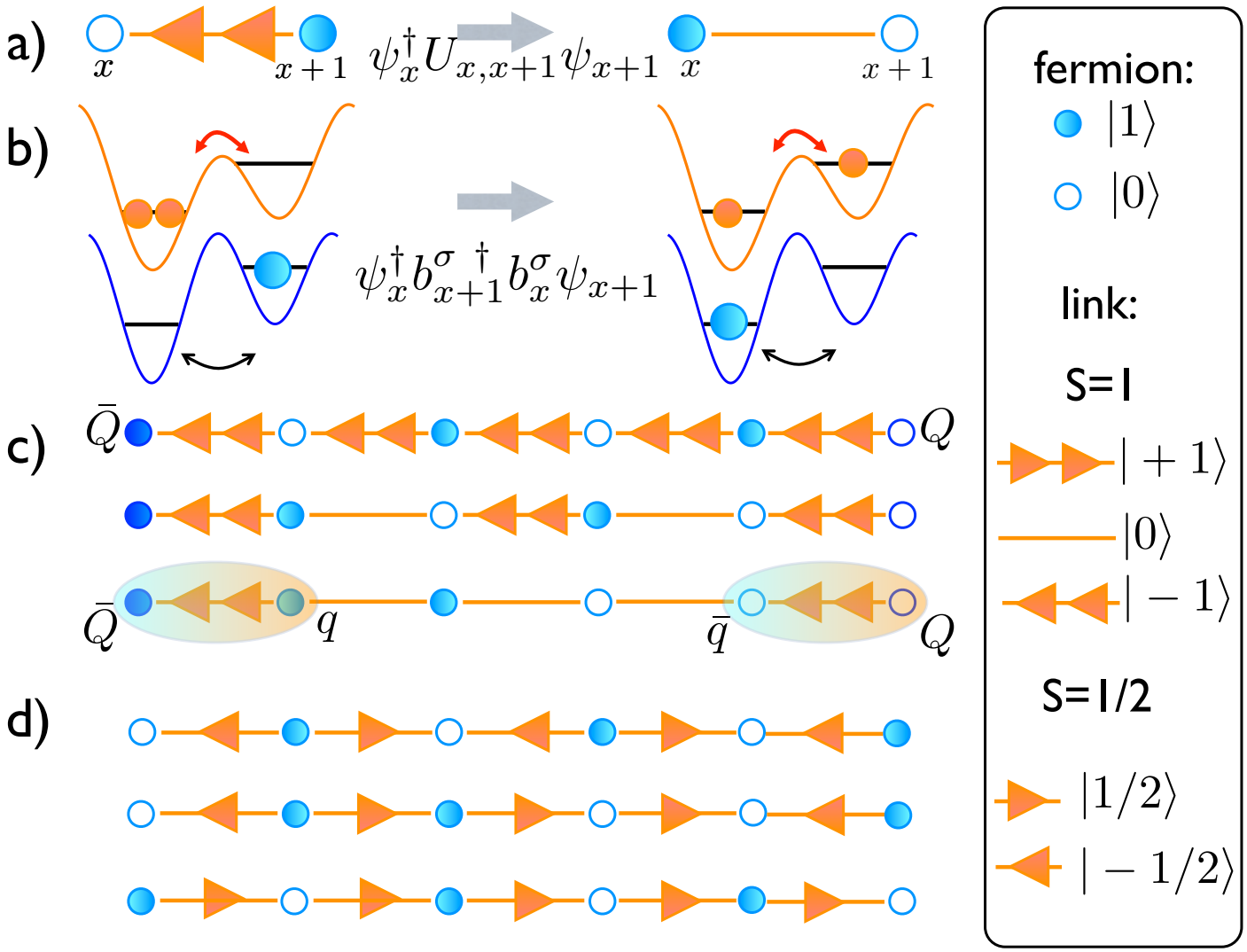
$$\begin{aligned} \widetilde{H} &= \sum_x h_{x,x+1}^B + \sum_x h_{x,x+1}^F + m \sum_x (-1)^x n_x^F + U \sum_x \widetilde{G}_x^2 \\ &= -t_B \sum_{x \text{ odd}} b_x^{1\dagger} b_{x+1}^1 - t_B \sum_{x \text{ even}} b_x^{2\dagger} b_{x+1}^2 - t_F \sum_x \psi_x^\dagger \psi_{x+1} + \text{h.c.} \\ &+ \sum_{x,\alpha,\beta} n_x^\alpha U_{\alpha\beta} n_x^\beta + \sum_{x,\alpha} (-1)^x U_\alpha n_x^\alpha \end{aligned}$$

D. Banerjee, M. Dalmonte, M. Müller, E. Rico, P. Stebler, UJW, P. Zoller, Phys. Rev. Lett. 109 (2012) 175302.

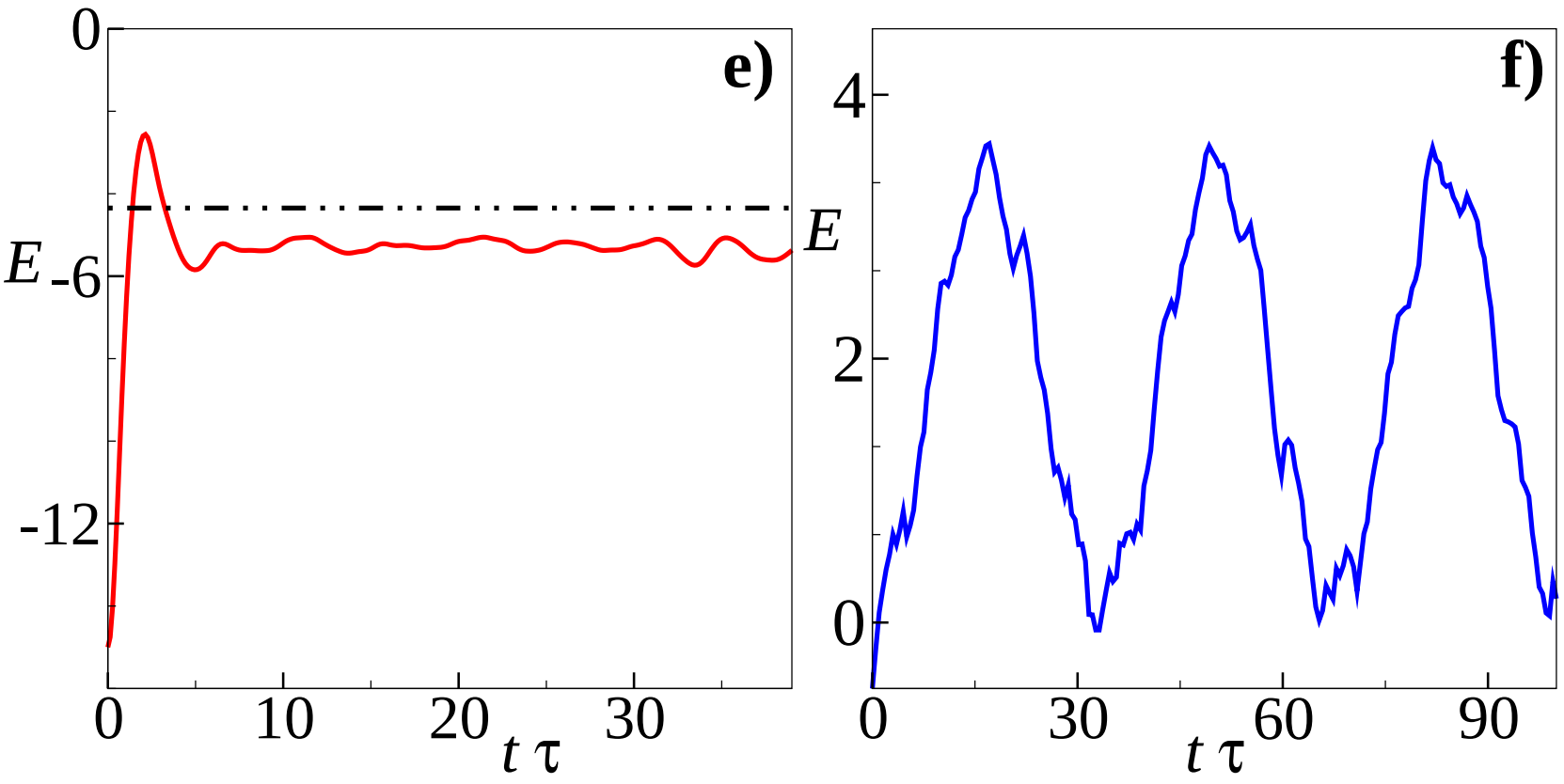
Optical lattice with Bose-Fermi mixture of ultra-cold atoms



From string breaking to false vacuum decay



Quantum simulation of the real-time evolution of string breaking and of coherent vacuum oscillations



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$U(N)$ quantum link operators

$$U^{ij} = S_1^{ij} + iS_2^{ij},\ U^{ij\dagger} = S_1^{ij} - iS_2^{ij},\ i,j \in \{1,2,\ldots,N\},\ [U^{ij}, U^{ij\dagger}] \neq 0$$

$SU(N)_L \times SU(N)_R$ gauge transformations of a quantum link

$$[L^a, L^b] = if_{abc}L^c,\ [R^a, R^b] = if_{abc}R^c,\ a,b,c \in \{1,2,\ldots,N^2-1\}$$

$$[L^a, R^b] = [L^a, E] = [R^a, E] = 0$$

Infinitesimal gauge transformations of a quantum link

$$[L^a, U] = -\lambda^a U,\ [R^a, U] = U\lambda^a,\ [E, U] = U$$

Algebraic structures of the $U(N)$ quantum link model

$U^{ij},\ U^{ij\dagger},\ L^a,\ R^a, E,\ 2N^2 + 2(N^2-1) + 1 = 4N^2 - 1$ $SU(2N)$ generators

$SU(N)$ gauge invariant Hamiltonian

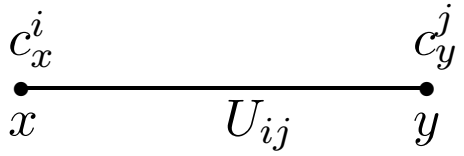
$$H = \frac{g^2}{2}\sum_{x,i}(L_{x,i}^2 + R_{x,i}^2) - \frac{1}{2g^2}\sum_{\square}\mathrm{Tr}(U_\square + U_\square^\dagger) - \gamma\sum_{x,i}(\det U_{x,i} + \det U_{x,i}^\dagger)$$

R. Brower, S. Chandrasekharan, UJW, Phys. Rev. D60 (1999) 094502

Fermionic rishons at the two ends of a link

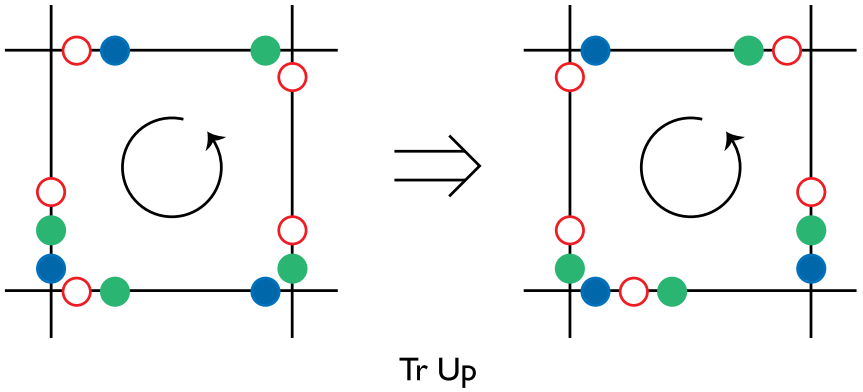
$$\{c_x^i, c_y^{j\dagger}\} = \delta_{xy}\delta_{ij}, \ \{c_x^i, c_y^j\} = \{c_x^{i\dagger}, c_y^{j\dagger}\} = 0$$

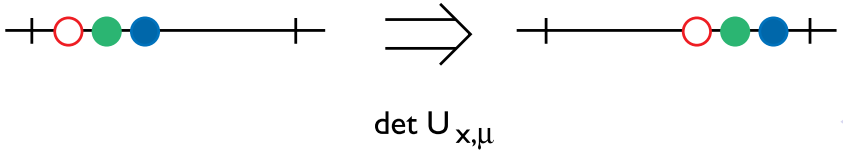
Rishon representation of link algebra



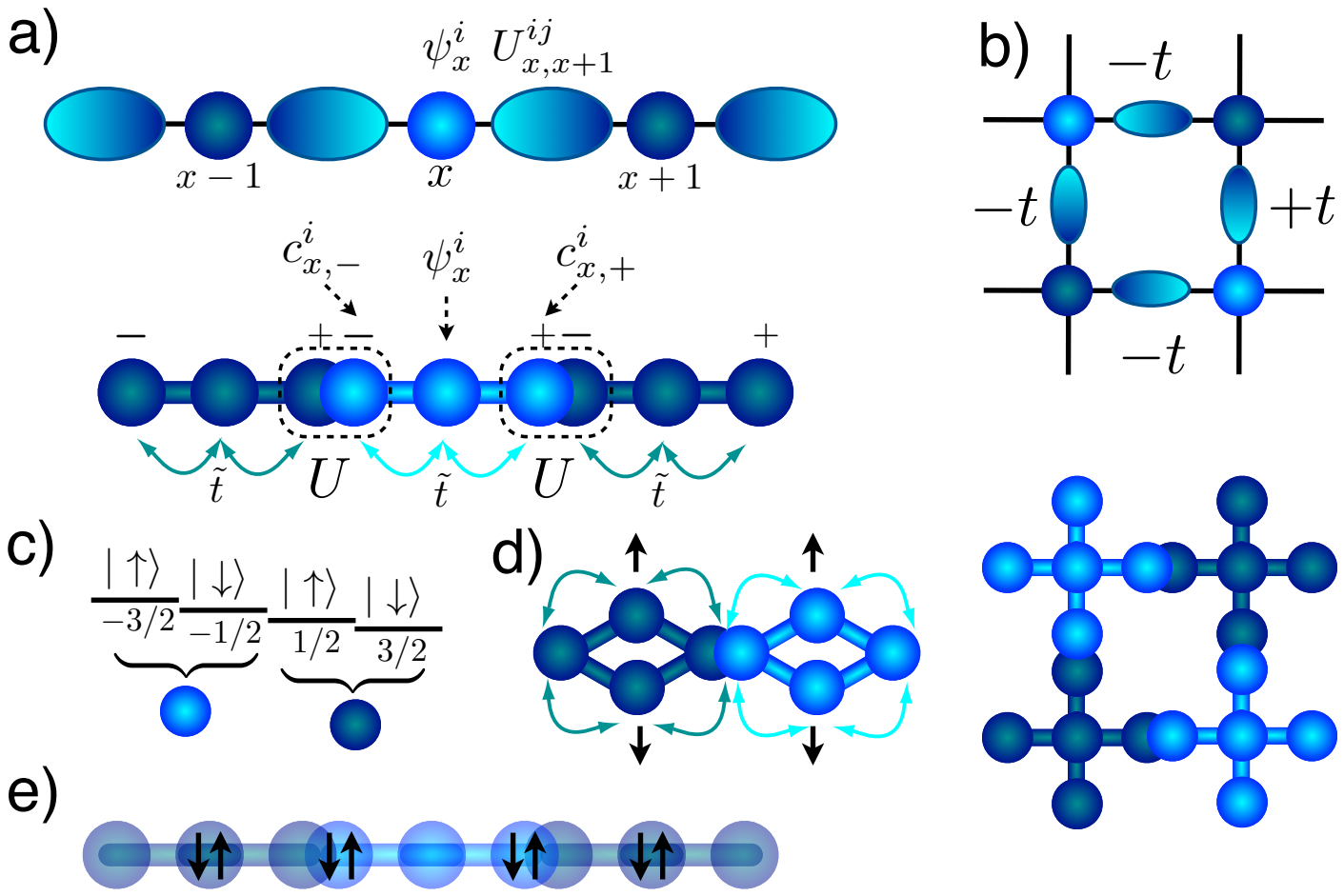
$$U_{xy}^{ij} = c_x^i c_y^{j\dagger}, \ L_{xy}^a = c_x^{i\dagger}\lambda_{ij}^a c_x^j, \ R_{xy}^a = c_y^{i\dagger}\lambda_{ij}^a c_y^j, \ E_{xy} = \frac{1}{2}(c_y^{i\dagger}c_y^i - c_x^{i\dagger}c_x^i)$$

Can a "rishon abacus" implemented with ultra-cold atoms be used as a quantum simulator?



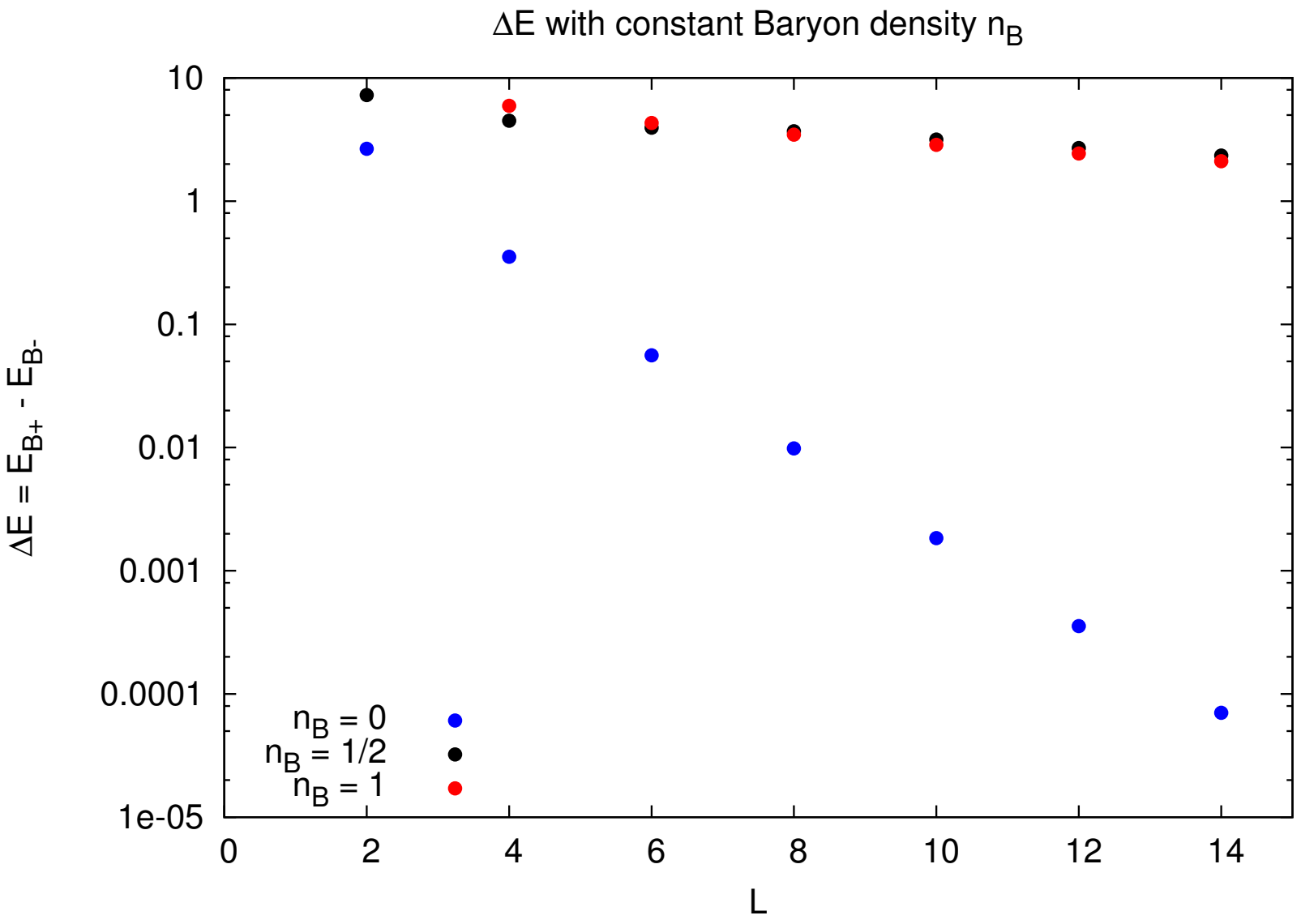


Optical lattice with ultra-cold alkaline-earth atoms (^{87}Sr or ^{173}Yb) with color encoded in nuclear spin



D. Banerjee, M. Bögli, M. Dalmonte, E. Rico, P. Stebler, UJW, P. Zoller, Phys. Rev. Lett. 110 (2013) 125303

Restoration of chiral symmetry at baryon density $n_B \geq \frac{1}{2}$



Other proposals for digital quantum simulators for Abelian and non-Abelian quantum link models

M. Müller, I. Lesanovsky, H. Weimer, H. P. Büchler, P. Zoller,
Phys. Rev. Lett. 102 (2009) 170502; Nat. Phys. 6 (2010) 382.
L. Tagliacozzo, A. Celi, P. Orland, M. Lewenstein,
Nature Communications 4 (2013) 2615.
L. Tagliacozzo, A. Celi, A. Zamora, M. Lewenstein,
Ann. Phys. 330 (2013) 160.

Other proposals for analog quantum simulators for Abelian and non-Abelian gauge theories with and without matter

H. P. Büchler, M. Hermele, S. D. Huber, M. P. A. Fisher, P. Zoller,
Phys. Rev. Lett. 95 (2005) 040402.
E. Kapit, E. Mueller, Phys. Rev. A83 (2011) 033625.
E. Zohar, B. Reznik, Phys. Rev. Lett. 107 (2011) 275301.
E. Zohar, J. Cirac, B. Reznik, Phys. Rev. Lett. 109 (2012) 125302;
Phys. Rev. Lett. 110 (2013) 055302; Phys. Rev. Lett. 110 (2013) 125304.

Review on quantum simulators for lattice gauge theories

UJW, Annalen der Physik 525 (2013) 777, arXiv:1305.1602.

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• Quantum simulators promise new insights into strongly correlated materials in condensed matter physics.

• Quantum link models provide an alternative formulation of lattice gauge theory with a finite-dimensional Hilbert space per link, which allows implementations with ultra-cold atoms in optical lattices.

• Quantum simulator constructions have already been presented for the $U(1)$ quantum link model as well as for $U(N)$ and $SU(N)$ quantum link models with fermionic matter.

• This would allow the quantum simulation of the real-time evolution of string breaking as well as the quantum simulation of "nuclear" physics and dense "quark" matter, at least in qualitative toy models for QCD.

• Accessible effects may include chiral symmetry restoration, baryon superfluidity, or color superconductivity at high baryon density, as well as the quantum simulation of "nuclear" collisions.

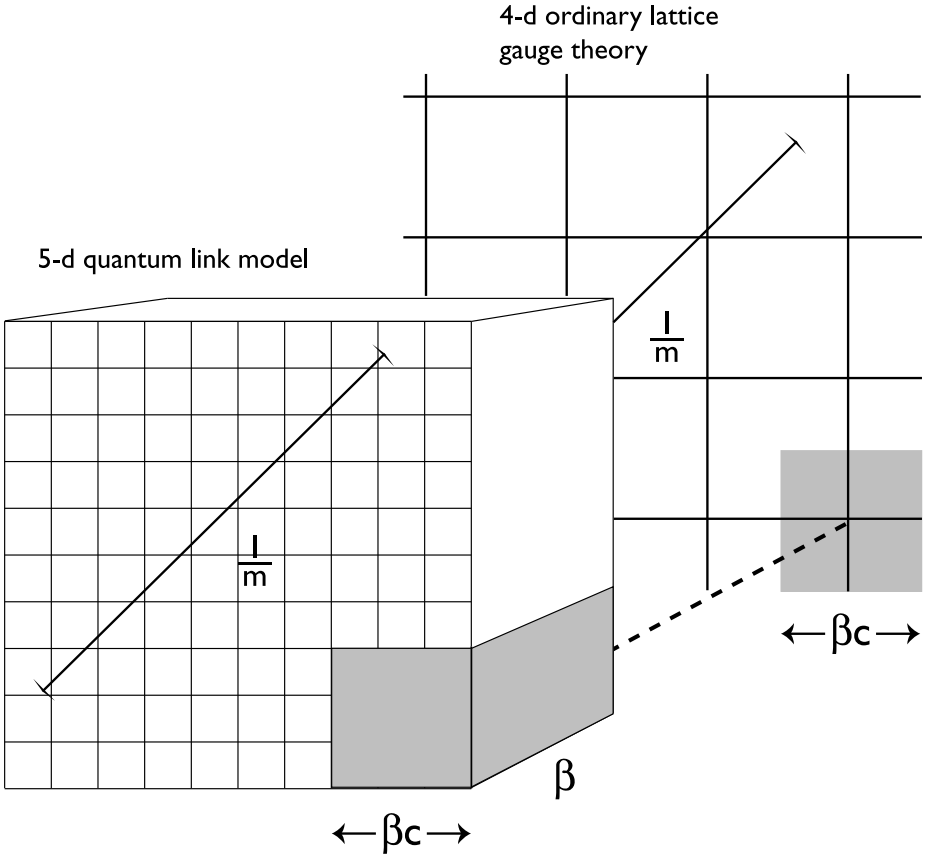
• The path towards quantum simulation of QCD will be a long one. However, with a lot of interesting physics along the way.

Low-energy effective action of a quantum link model

$$S[G_\mu] = \int_0^\beta dx_5 \int d^4x \, \frac{1}{2e^2} \left(\mathrm{Tr}\ G_{\mu\nu} G_{\mu\nu} + \frac{1}{c^2} \mathrm{Tr}\ \partial_5 G_\mu \partial_5 G_\mu \right), \ G_5 = 0$$

undergoes dimensional reduction from $4+1$ to 4 dimensions

$$S[G_\mu] \to \int d^4x \, \frac{1}{2g^2} \mathrm{Tr}\ G_{\mu\nu} G_{\mu\nu}, \ \frac{1}{g^2} = \frac{\beta}{e^2}, \ \frac{1}{m} \sim \exp\left(\frac{24\pi^2 \beta}{11 N e^2} \right)$$



Probability Distribution of the Order Parameters: $L^2 = 24^2$