Femtoscopic pair correlations of mesons and baryons at RHIC and LHC from HydroKinetic Model

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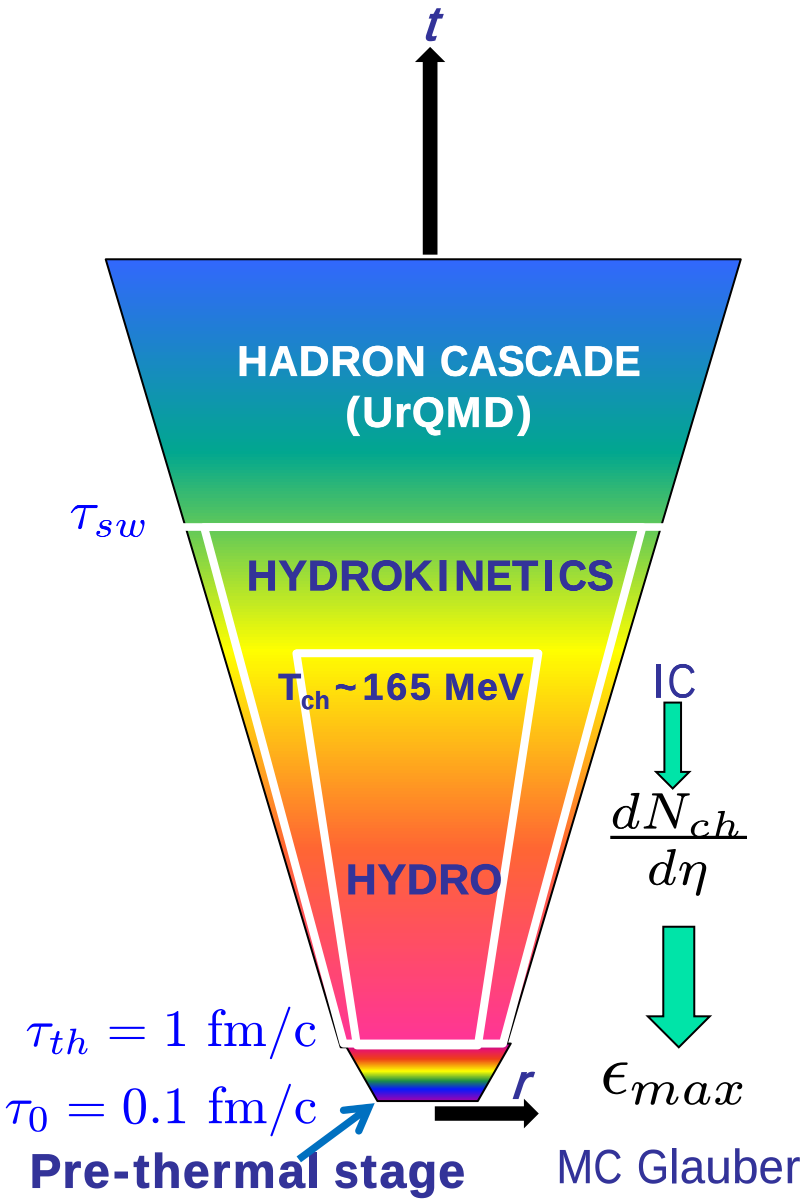
XXIV Quark Matter DARMSTADT May 18 – 24, 2014

HydroKinetic model (HKM)



- HKM is based on solution of relativistic Boltzmann + hydrodynamic equations in the relaxation time approximation for emission function;
- provides evaluation of emission function based on particle escape probabilities

Complete algorithm incorporates the stages:

- thermalization of initially non-thermal matter;
- viscous chemically equilibrium hydro expansion;
- nonequilibrated post chemical freeze-out evolution of decaying hadron fluid;
- a switch to UrQMD cascade at space-time hypersurface.

Yu.S., Akkelin, Hama: PRL 89 (2002) 052301
… + Karpenko: PRC 78 (2008) 034906;
Karpenko, Yu.S. :PRC 81 (2010) 054903,
PLB 688 (2010) 50;
Akkelin, Yu.S. : PRC 81 (2010) 064901.
Karpenko, Yu.S., Werner: PRC 87 (2013) 024914.

Pair femtoscopic correlations: Bose-Einstein/Fermi-Dirac + final state interactions (BE/FD + FSI)

$$C(p,q) = 1 + \lambda \frac{\int d^4x_1 d^4x_2 g_1(x_1,p) g_2(x_2,p) \left(|\psi(\tilde{q},r)|^2 - 1 \right)}{\int d^4x_1 g_1(x_1,p_1) \int d^4x_2 g_2(x_2,p_2)} = 1 + \lambda \left\langle \left(|\psi(\tilde{q},r)|^2 - 1 \right) \right\rangle$$

where $\psi(\tilde{q},r)$ is reduced Bether-Salpeter amplitude, $r = x_1 - x_2, R = (x_1 + x_2)/2$

$$q = p_1 - p_2, p = (p_1 + p_2)/2 \qquad \tilde{q} = q - p(qp)/p^2$$

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For identical bosons (in smoothness approximation) with only Coulomb FSI

Y. Sinyukov, R. Lednicky, S.V. Akkelin, J. Pluta, B. Erazmus, Phys. Lett. B 432 (1998) 248.

$$C(p,q) = 1 - \lambda + \lambda \left\langle \left| \psi^c_{-\mathbf{k}*}(\mathbf{r}^*) \right|^2 \right\rangle (1 + \langle \cos(qx) \rangle)$$

where $\langle \cos(qx) \rangle) = \exp(-q^2_{out} R^2_{out} - q^2_{side} R^2_{side} - q^2_{long} R^2_{long} - ...c.t.)$

Pair femtoscopic correlations: Bose-Einstein / Fermi-Dirac + final state interactions (BE / FD + FSI)

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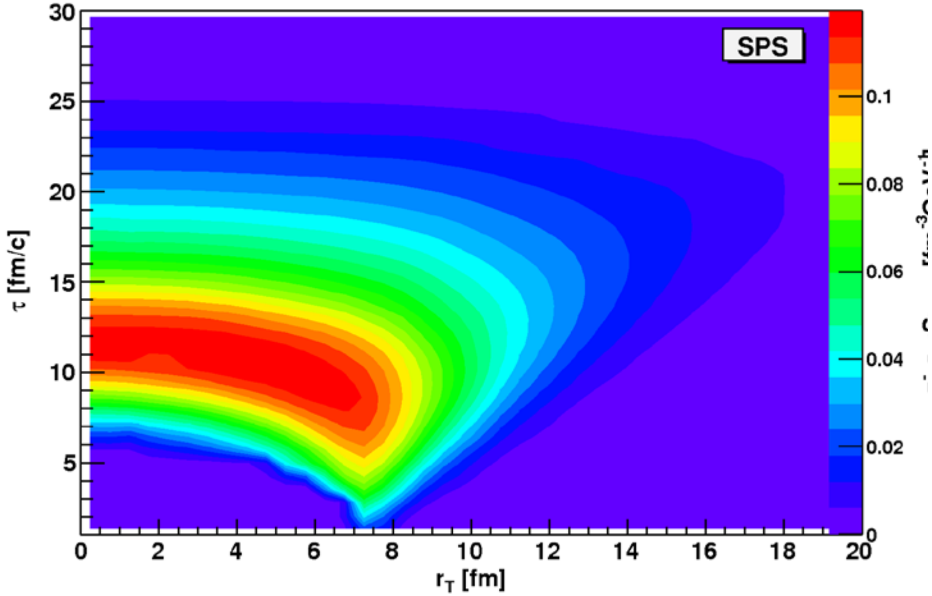
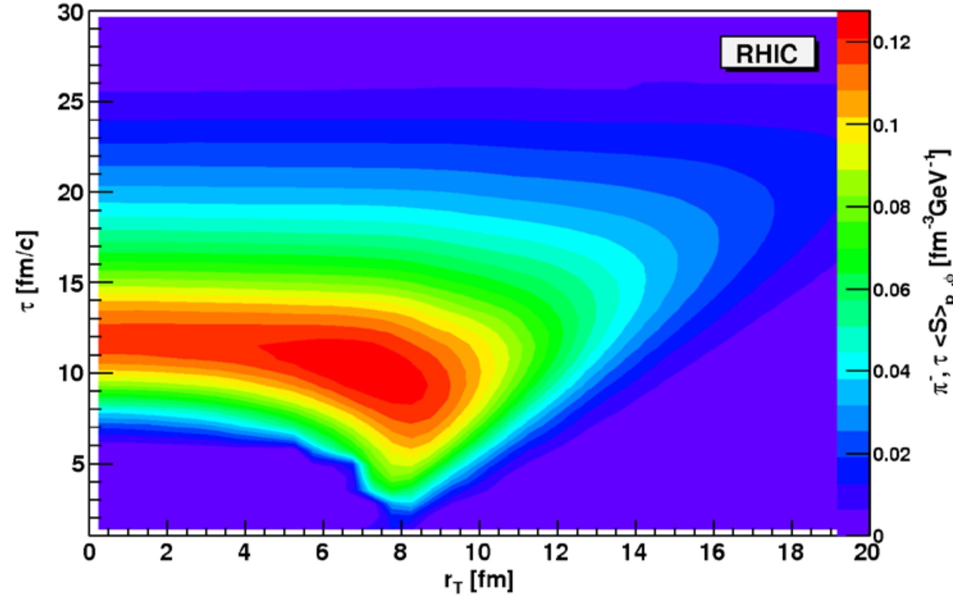
Correlation functions in semi-classical events generators BE correlation:

$$C(\vec{q}) = \frac{\sum_{i \neq j} \delta_\Delta(\vec{q} - p_i + p_j)(1 + cos(p_j - p_i)(x_j - x_i))}{\sum_{i \neq j} \delta_\Delta(\vec{q} - p_i + p_j)}$$

where $\delta_\Delta(x) = 1$ if $|x| < \Delta p/2$ and 0 otherwise, with Δp being the bin size in histograms.

Emission functions for top SPS, RHIC and LHC energies

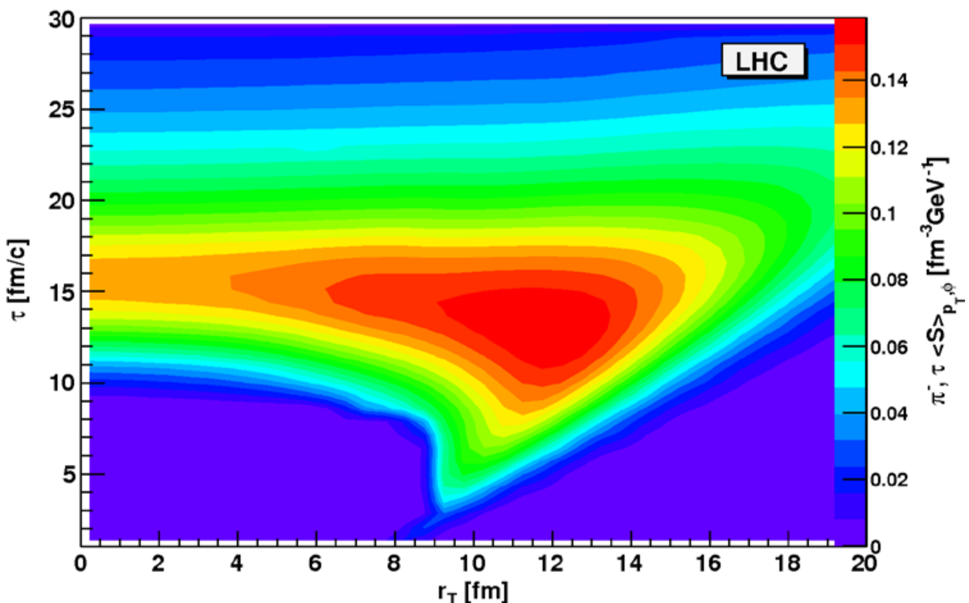




$$R_{out}^2 \approx R_{side}^2 + v^2\langle\triangle t^2\rangle_p - 2v\langle\triangle x_{out}\triangle t\rangle_p, v = \frac{p_T}{p^0}$$

HBT puzzle

$$\frac{R_{out}}{R_{side}} \approx 1$$



HKM prediction: solution of the HBT Puzzle

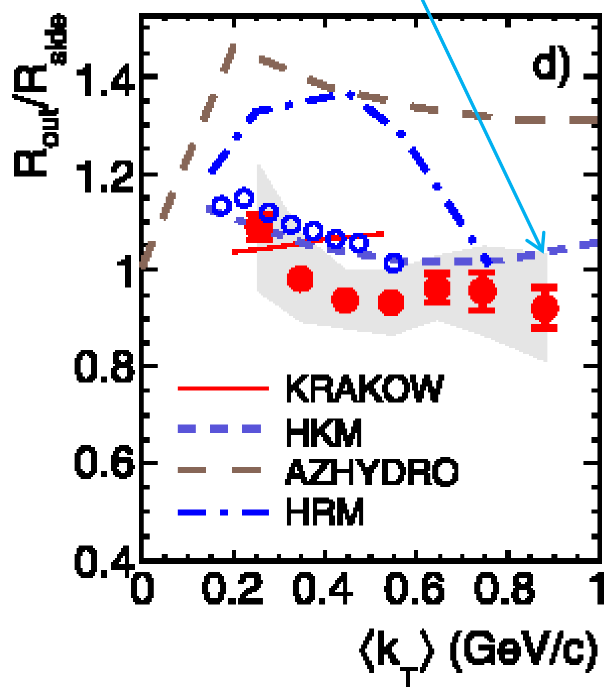
Two-pion Bose–Einstein correlations in central Pb–Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV ☆ ALICE Collaboration Physics Letters B 696 (2011) 328-

ALICE

Quotations:

HKM

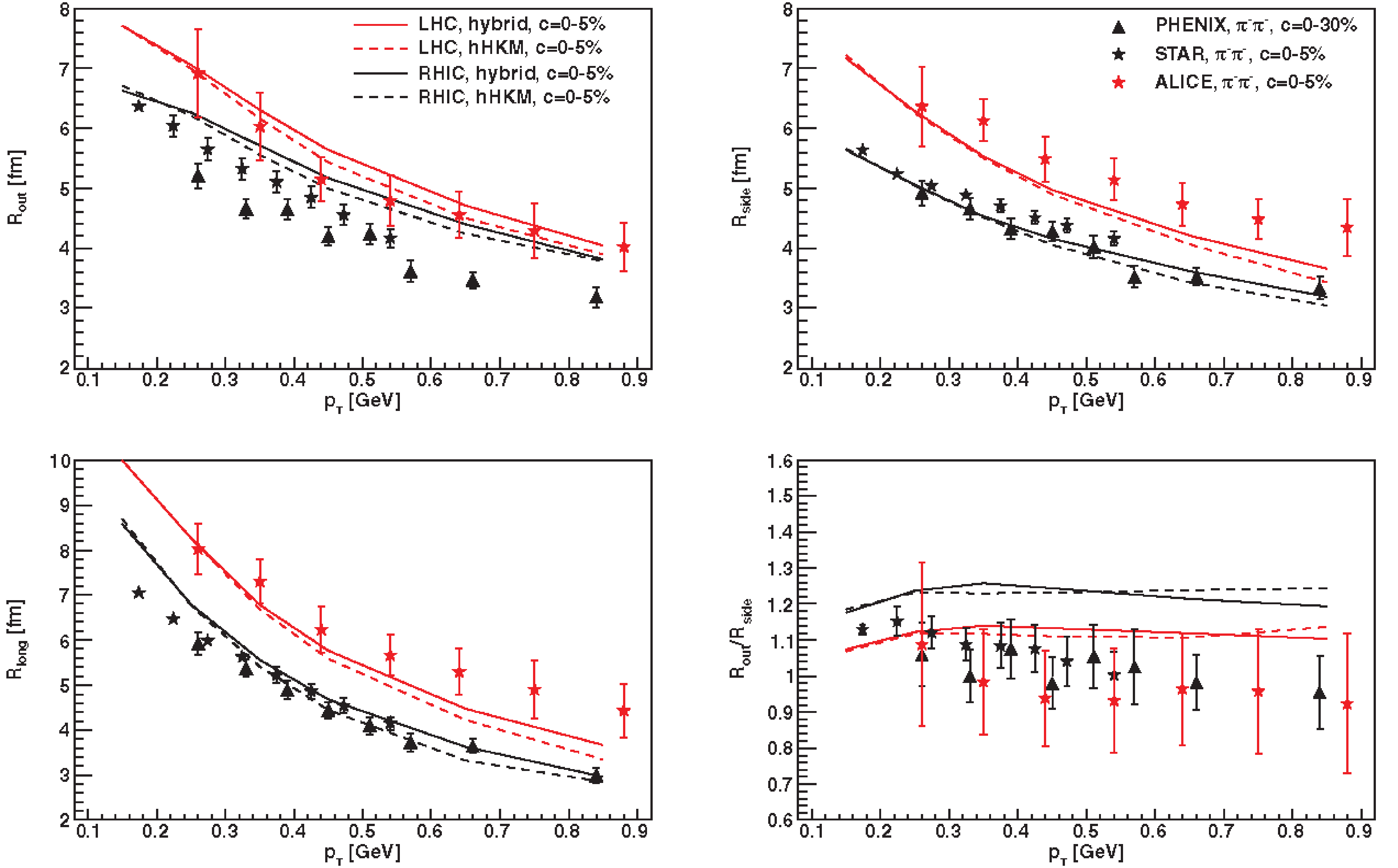
Available model predictions are compared to the experimental data in Figs. 2-d and 3. Calculations from three models incorporating a hydrodynamic approach, AZHYDRO [45], KRAKOW [46,47], and HKM [48,49], and from the hadronic-kinematics-based model HRM [50,51] are shown. An in-depth discussion is beyond the scope of this Letter but we notice that, while the increase of the radii between RHIC and the LHC is roughly reproduced by all four calculations, only two of them (KRAKOW and HKM) are able to describe the experimental R_{out}/R_{side} ratio.



[48] I.A. Karpenko, Y.M. Sinyukov, Phys. Lett. B 688 (2010) 50.
[49] N. Armesto, et al. (Eds.), J. Phys. G 35 (2008) 054001.

Pion interferometry radii for central RHIC and LHC events

Karpenko, Yu.S., Werner: PRC 87 (2013) 024914.



HKM predictions for kaon femtoscopy at RHIC

Freeze-out Dynamics via Charged Kaon Femtoscopy in $\sqrt{s_{NN}}$=200 GeV Central Au+Au Collisions

STAR Collaboration

PRC 88 (2013) 3
arXiv:1302.3168 [nucl-ex]

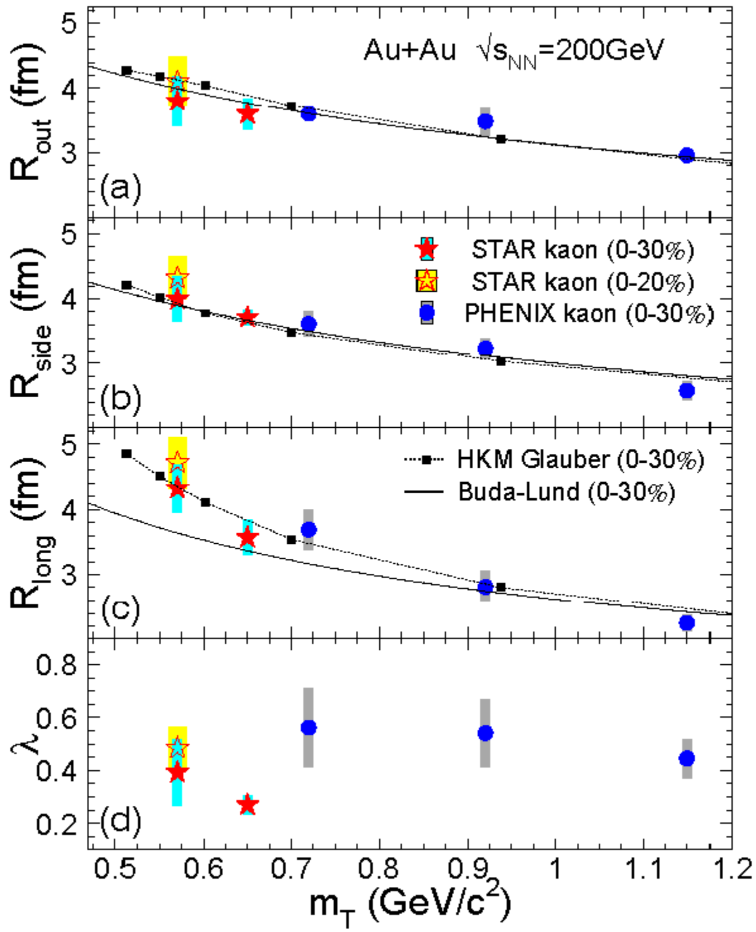


FIG. 4. Transverse mass dependence of Gaussian radii (a) R_{out}, (b) R_{side} and (c) R_{long} for mid-rapidity kaon pairs from the 30% most central Au+Au collisions at $\sqrt{s_{NN}}$=200 GeV. STAR data are shown as solid stars; PHENIX data [10] as solid circles (error bars include both statistical and systematic uncertainties). Hydro-kinetic model [23] with initial Glauber condition and Buda-Lund model [22] calculations are shown by solid squares and solid curves, respectively.

Quotations:

Our measurement at $0.2 \leq k_T \leq 0.36$ GeV/c clearly favours the HKM model as more representative of the expansion dynamics of the fireball.

....

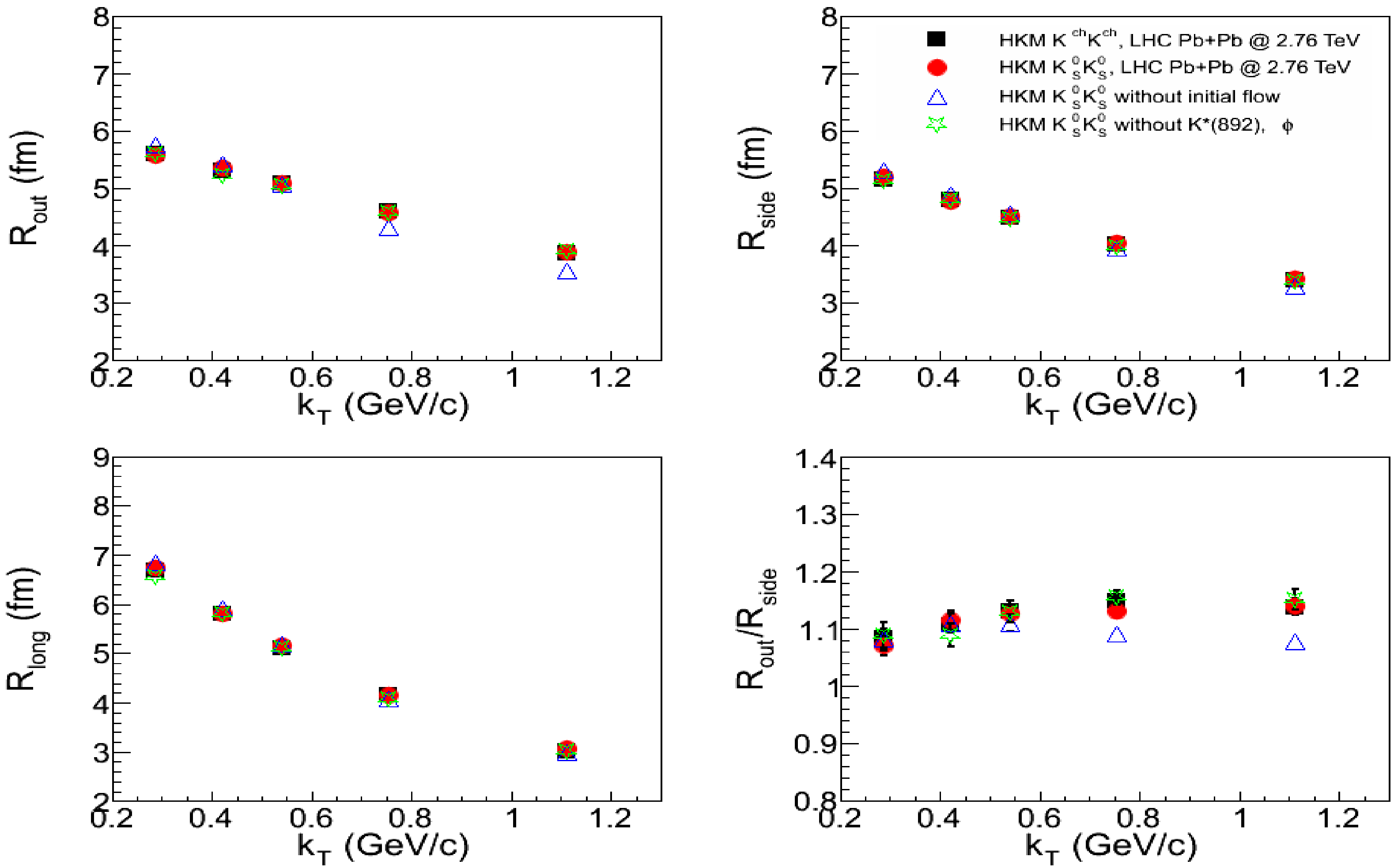
In the outward and sideward directions, this decrease is adequately described by m_T–scaling. However, in the longitudinal direction, the scaling is broken. The results are in favor of the hydro-kinetic predictions [23] over pure hydrodynamical model calculations.

....

[23] I. A. Karpenko and Y. M. Sinyukov, Phys. Rev. C **81** (2010) 054903.

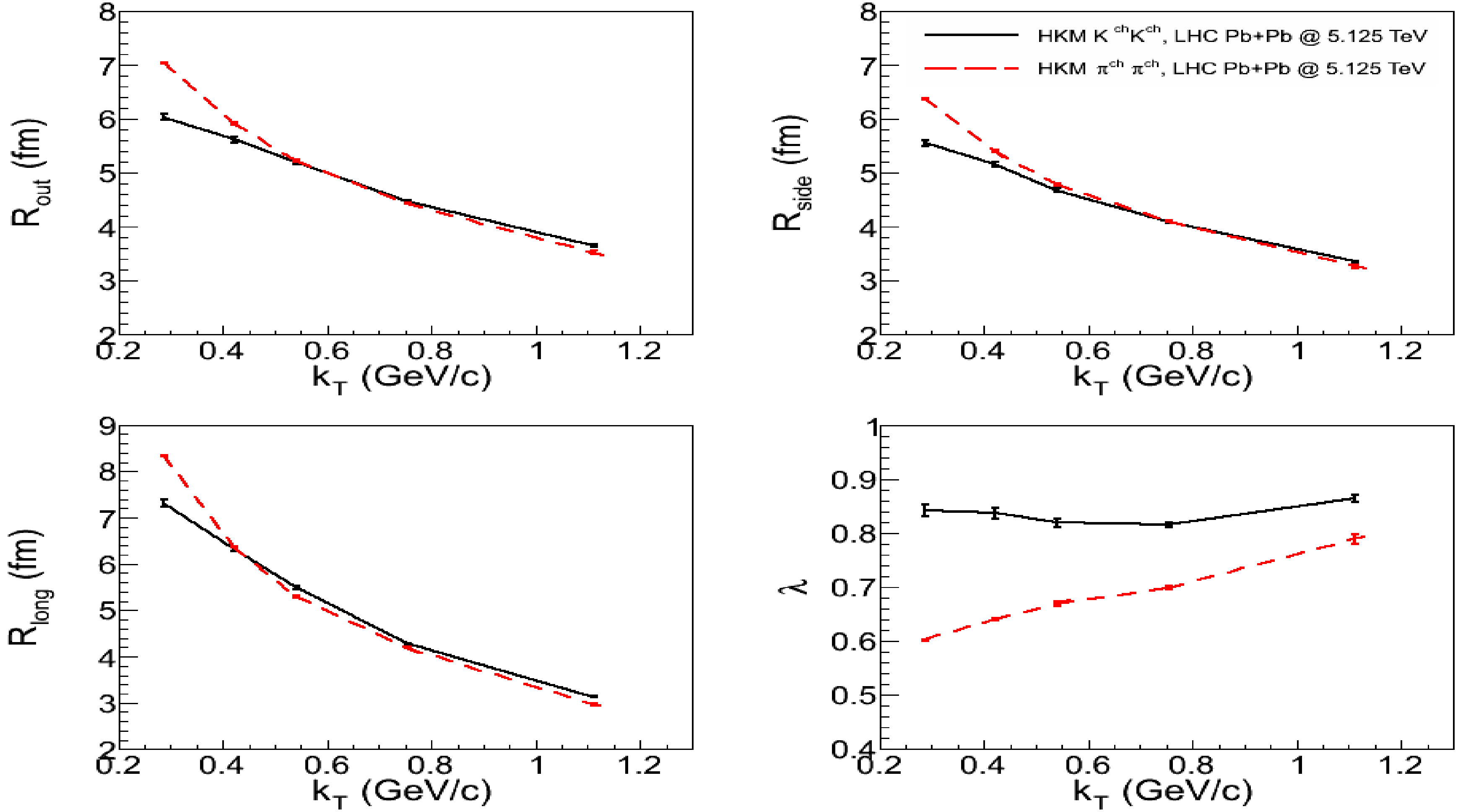
HKM predictions for kaon 3D femtoscopy at LHC

Shapoval, Braun-Munzinger, Karpenko, Yu.S. arXiv:1404.4501, Nucl. Phys. A, 2014



HKM predictions for the dependence of $K^{ch}K^{ch}$ and $K^0_SK^0_S$ 3D interferometry radii R_i on k_T for $\sqrt{s} = 2.76$ GeV Pb+Pb LHC collisions, $c = 0-5\%$, $|\eta| < 0.8$, $0.14 < p_T < 1.5$ GeV/c.

k_T- scaling at k_T > 0.4-0.5 GeV for pion and kaon HBT radii at LHC full energy



Prediction for the k_T-dependence of $K^{ch}K^{ch}$ and $\pi^-\pi^-$ 3D interferometry radii for $\sqrt{s_{NN}} = 5.125$ TeV Pb+Pb LHC collisions, $c = 0 - 5\%$, $|\eta| < 0.8$, $0.14 < p_T < 1.5$ GeV/c. For $\sqrt{s_{NN}} = 2.76$ TeV Pb+Pb LHC case the corresponding radii values are $4 - 7\%$ lower for kaons and $2 - 4\%$ lower for pions.

HKM predictions for kaon 1D femtoscopy at LHC @ 2.76 TeV

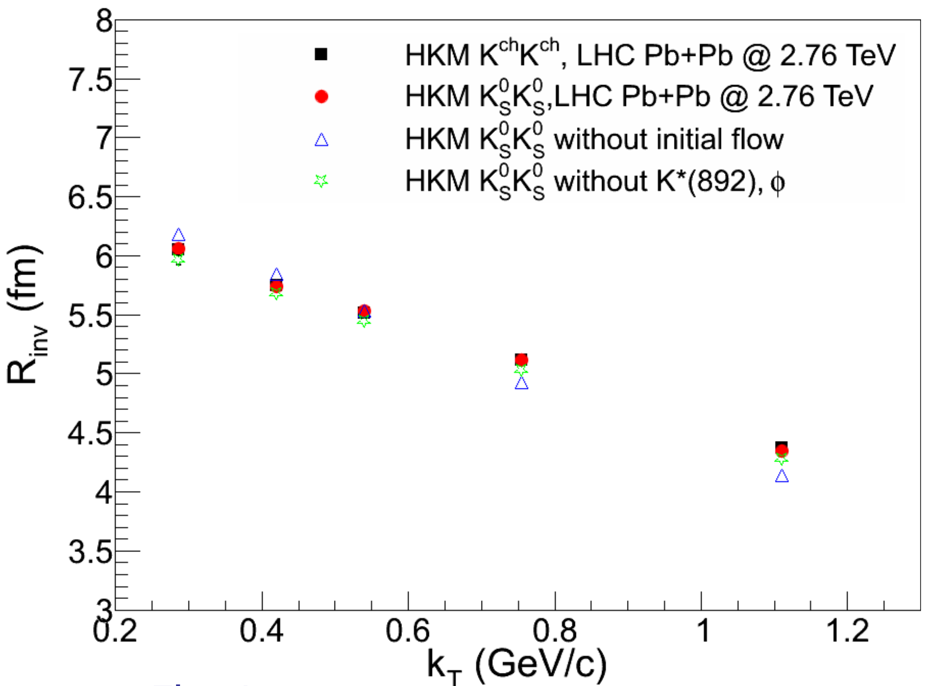


Fig. 1

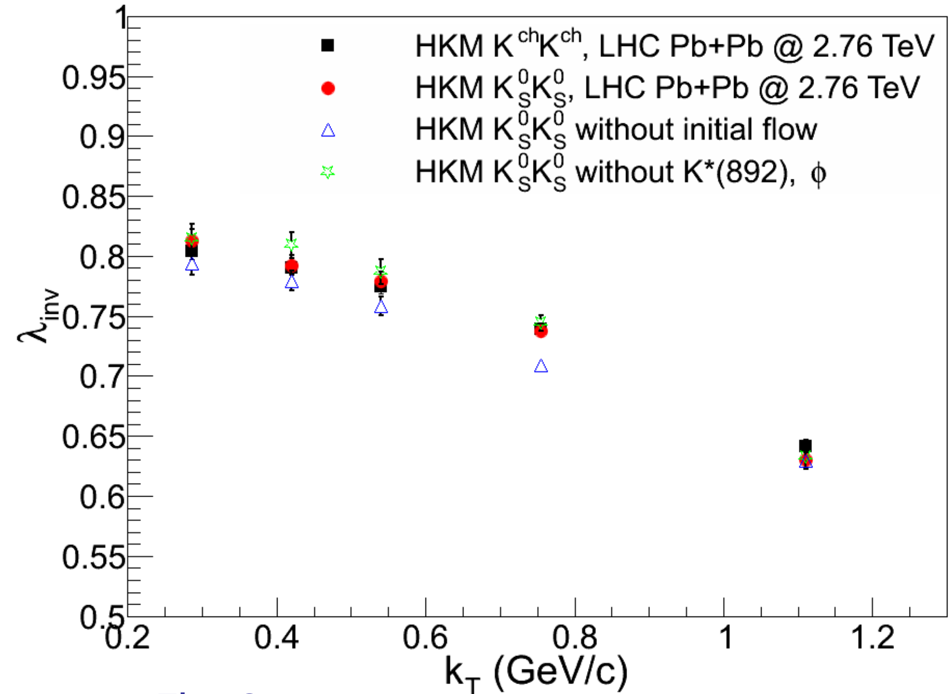


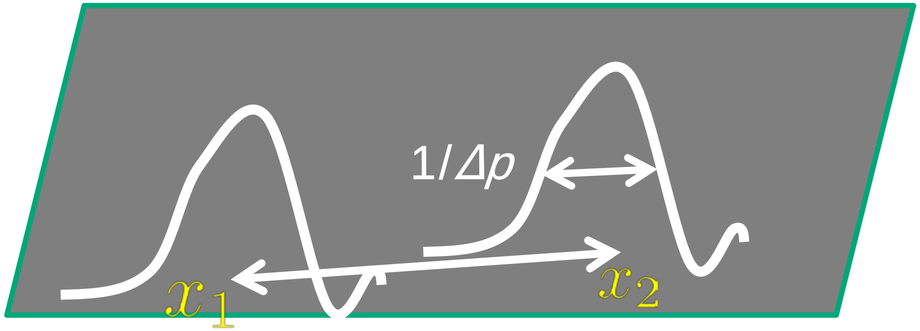
Fig. 2

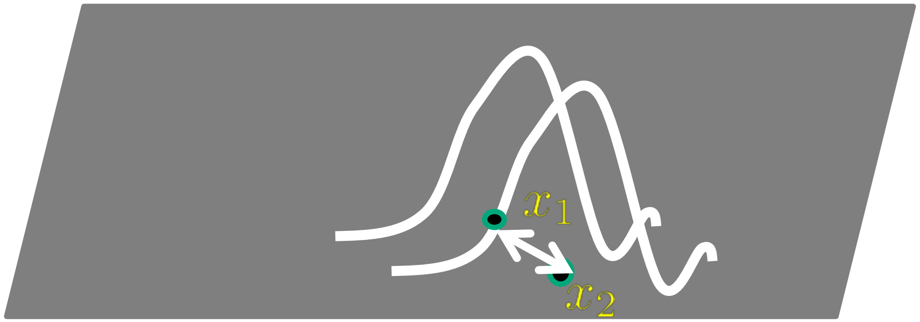
FIG. 1. The HKM prediction for the dependence of $K^{ch}K^{ch}$ and $K^0_SK^0_S$ interferometry radii R_{inv} on k_T for $\sqrt{s_{NN}} = 2.76$ TeV Pb+Pb LHC collisions, $c = 0-5\%$, $|\eta| < 0.8$, $0.14 < p_T < 1.5$ GeV$/c$.

FIG. 2. HKM prediction for the dependence of $K^{ch}K^{ch}$ and $K^0_SK^0_S$ parameter λ_{inv} on k_T.

Uncertainty principle and distinguishability of emitters

The distance between the centers of emitters $\Delta x = x_1 - x_2$ is larger than their sizes related to the widths of the emitted wave packets $1/\Delta p$.





Distinguishable emitters $\Delta x \gg 1/\Delta p$
The states are orthogonal

Indistinguishable emitters $\Delta x \ll 1/\Delta p$
The states are almost the same

Wave function for emitters

$$\psi_{x_i}(\mathbf{p},\mathbf{t}) = e^{i\mathbf{p}\mathbf{x_i}} \tilde{f}(\mathbf{p}) e^{-iE\tau}$$

Spectrum:

$$f(\vec{p}) = \tilde{f}^2(\vec{p}) = const$$

$$f(\vec{p}) = \tilde{f}^2(\vec{p}) = \frac{1}{(2\pi p_0^2)^{3/2}} e^{-\frac{\vec{p}^2}{2p_0^2}}$$

Overlap integral of the wave packages:

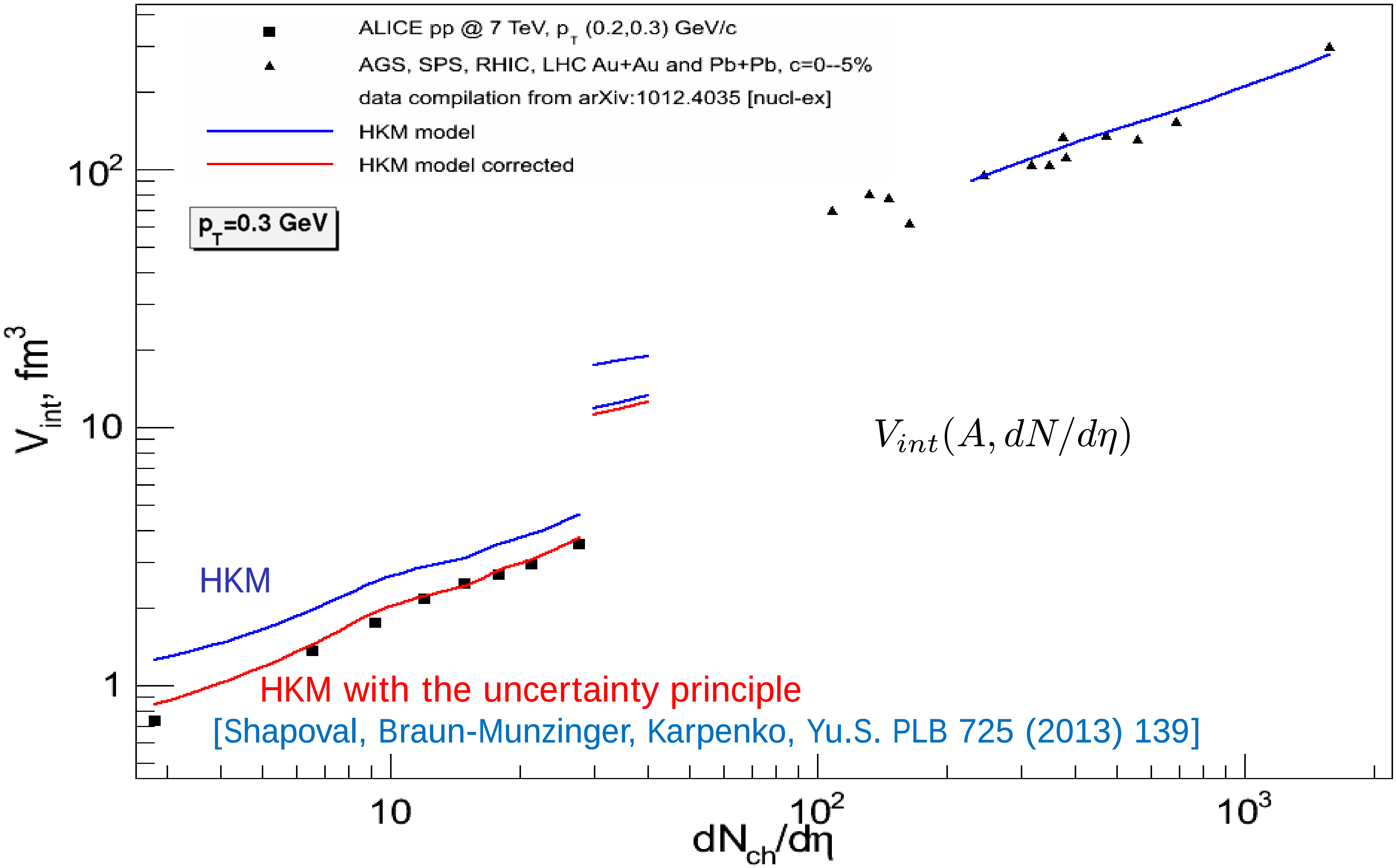
$$I_{\mathbf{xx'}} = \delta^3(\mathbf{x_1} - \mathbf{x_2})$$

$$t = t'$$

$$I_{\mathbf{xx'}} = e^{-\frac{p_0^2(\mathbf{x_1}-\mathbf{x_2})^2}{2}}$$

“Correlation femtoscopy of small systems”: Yu.S., Shapoval: Phys. Rev. D 87, 094024 (2013)

Interferometry volume in pp and central pPb, AA collisions in HKM in view of the uncertainty principle



[Shapoval, Braun-Munzinger, Karpenko, Yu.S. PLB 725 (2013) 139]

Source function S(r*)

The correlation function in smoothness approximation

$$C(p,q) = 1 + \frac{\int d^4x_1 d^4x_2 g_1(x_1,p_1) g_2(x_2,p_2)\left(|\psi(\tilde{q},r)|^2 - 1\right)}{\int d^4x_1 g_1(x_1,p_1)\int d^4x_2 g_2(x_2,p_2)}$$

where $\psi(\tilde{q},r)$ is reduced Bether-Salpeter amplitude, $r = x_1 - x_2, R = (x_1+x_2)/2$

$$q = p_1 - p_2, p = (p_1+p_2)/2 \qquad \tilde{q} = q - p(qp)/p^2$$

The relative distance distribution function

$$s(r,p_1,p_2) = \frac{\int d^4R g_1(R+r/2,p_1) g_2(R-r/2,p_2)}{\int d^4R g_1(R,p_1)\int d^4R g_2(R,p_2)} =$$

$$= \int d^4R \tilde{g_1}(R+r/2, \frac{2m_1}{m_1+m_2}p)\tilde{g_2}(R-r/2,\frac{2m_2}{m_1+m_2}p) = s(r,p)$$

Main contribution to $C(p_1,p_2)$ is at $\mathbf{v}_1 \approx \mathbf{v}_2$

$$\overbrace{C(\mathbf{q}^*,\mathbf{p}{=}\mathbf{0}) - 1}^{R(\mathbf{q})} = \int d^3r^* \underbrace{\int dt^* s(r^*,\mathbf{0})}_{S(\mathbf{r}^*)}(|\psi(\mathbf{r}^*,\mathbf{q}^*)|^2 - 1) = \int d^3r^* S(\mathbf{r}^*) K(\mathbf{r}^*,\mathbf{q}^*)$$

Cartesian correlation moments

Cartesian Harmonics

$$A_{\mathbf{l}}(\Omega) = \sum_{m_i=0}^{l_i/2} \left(-\frac{1}{2}\right)^m \frac{(2l-2m-1)!!}{(2l-1)!!} \frac{l_x!}{(l_x-2m_x)!m_x!} \times$$
$$\frac{l_y!}{(l_y-2m_y)!m_y!} \frac{l_z!}{(l_z-2m_z)!m_z!} n_x^{l_x-2m_x} n_y^{l_y-2m_y} n_z^{l_z-2m_z}.$$

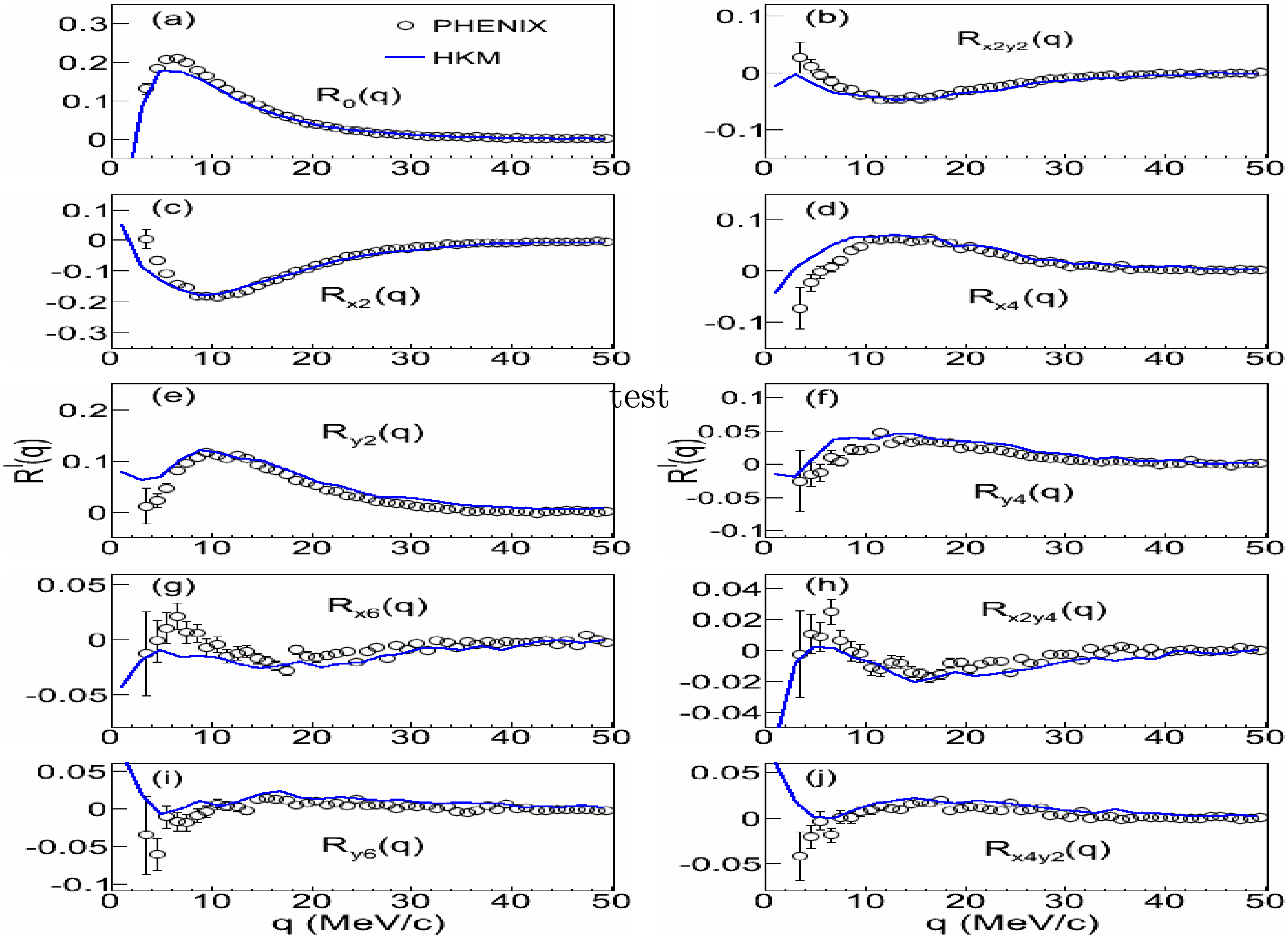
Here $\mathbf{n} = \{n_x, n_y, n_z\}$ is a unit vector in the Ω direction, $l_x + l_y + l_z = l$, $m_x + m_y + m_z = m$, $(-1)!! = 1$

Correlation moments

$$R^l_{\alpha_1 \ldots \alpha_l}(q) = \frac{(2l+1)!!}{l!} \int \frac{d\Omega_{\mathbf{q}}}{4\pi} A^l_{\alpha_1 \ldots \alpha_l}(\Omega_{\mathbf{q}})\, R(\mathbf{q}).$$

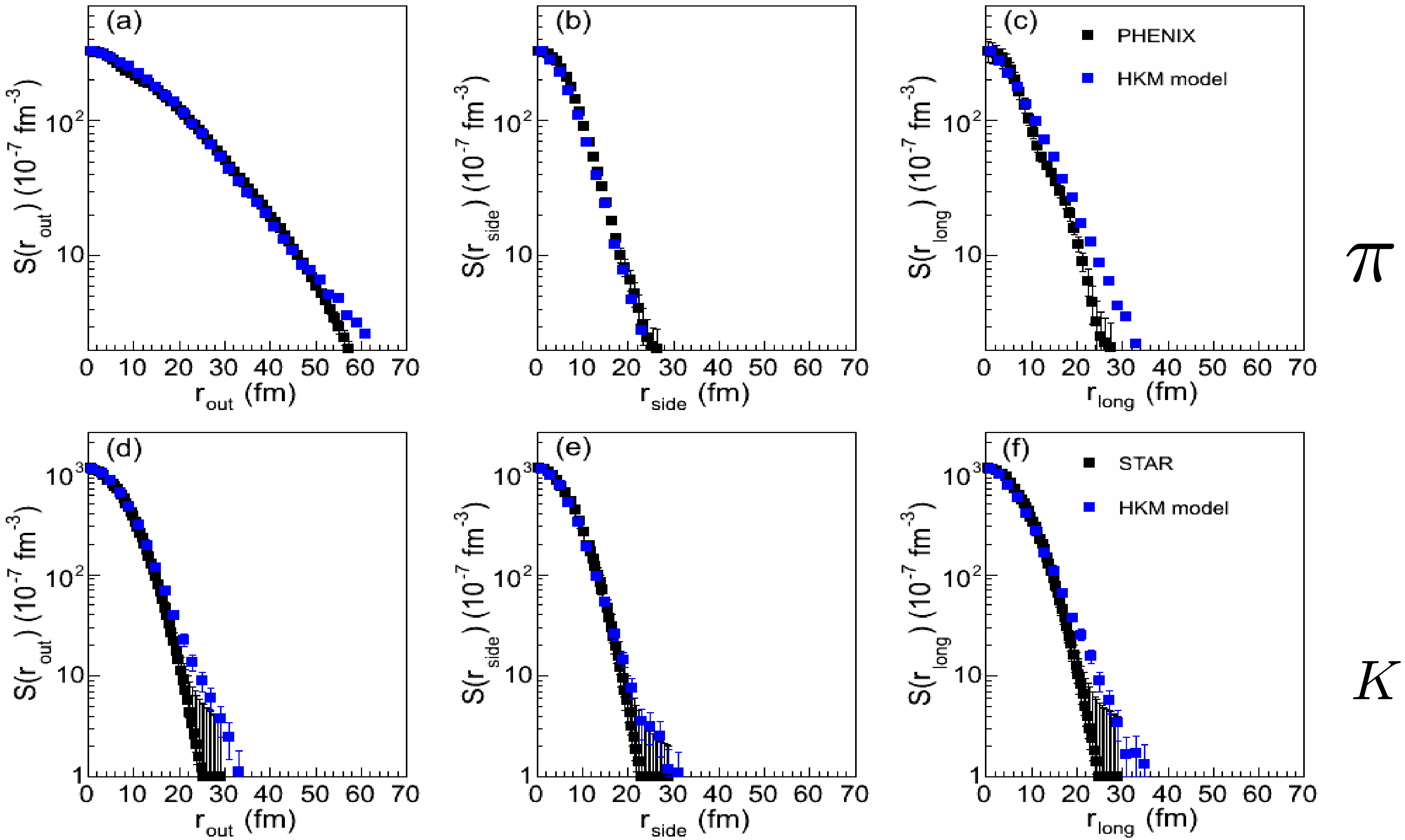
$R_{xl_xyl_y}$ denotes the Cartesian correlation moment corresponding to $l = l_x + l_y$, $\alpha_1 = \ldots = \alpha_{l_x} = x$, $\alpha_{l_x+1} = \ldots = \alpha_l = y$

Correlation moments in HKM and PHENIX at $\sqrt{s}$ = 200 GeV

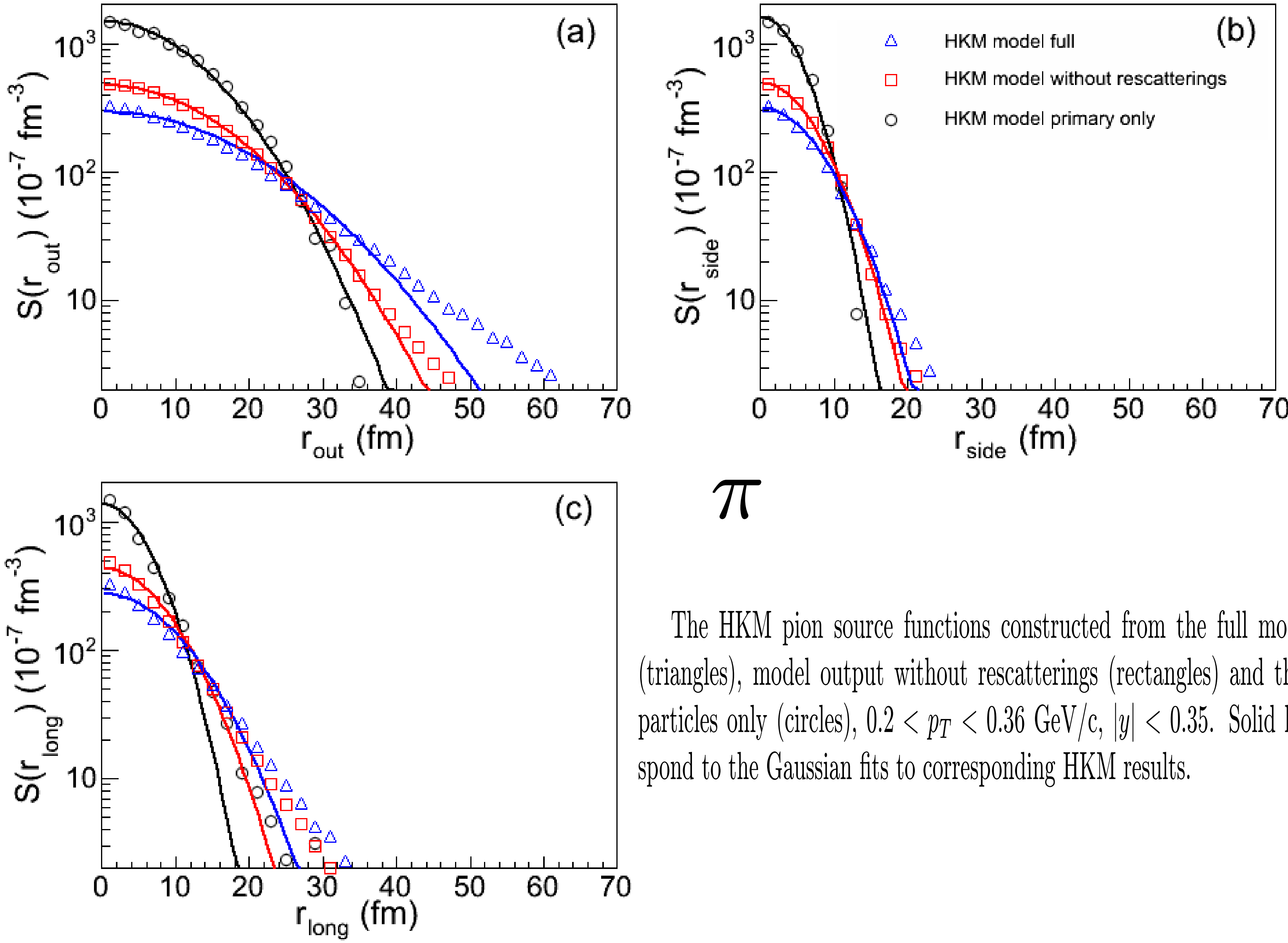


Source function for pions (top) and kaons (bottom) at RHIC

Shapoval, Yu.S, Karpenko PRC **88**, 064904 (2013)



An analysis of the heavy tail origination at RHIC



The HKM pion source functions constructed from the full model output (triangles), model output without rescatterings (rectangles) and the primary particles only (circles), $0.2 < p_T < 0.36$ GeV/c, $|y| < 0.35$. Solid lines correspond to the Gaussian fits to corresponding HKM results.

Baryon-(anti)baryon correlations

Shapoval, Erazmus, Lednicky, Yu.S. – **arXiv: 1405.3594**

The correlation for the fraction of correctly identified primary baryon pairs is:

$$C(k^*) = \frac{C_{meas}(k^*) - 1}{\text{Pair Purity}} + 1$$

At RHIC the "Pair Purity" = $\lambda = 0.175 \pm 0.025$

$$C(k^*) = \left\langle \left| \Psi^S_{-\mathbf{k}^*}(\mathbf{r}^*) \right|^2 \right\rangle$$

for $p\,\Lambda \oplus \bar{p}\,\bar{\Lambda}$ & $p\,\bar{\Lambda} \oplus \bar{p}\,\Lambda$ pairs

Lednicky and Lyuboshitz analytical model

In typical nuclear collisions the source radius can be considered much larger than the range of the strong interaction potential, so $\Psi^S_{-\mathbf{k}^*}$ at small k^* can be approximated by s-wave solution :

$$\Psi^S_{-\mathbf{k}^*}(\mathbf{r}^*) = e^{-i\mathbf{k}^* \cdot \mathbf{r}^*} + \frac{f^S(k^*)}{r^*} e^{ik^* \cdot r^*}$$

For Gaussian source $S(\mathbf{r}^*) \propto e^{-\frac{\mathbf{r}^{*2}}{4r_0^2}}$

$$C(k^*) = 1 + \sum_S \rho_S \left[\frac{1}{2} \left| \frac{f^S(k^*)}{r_0} \right|^2 \left(1 - \frac{d_0^S}{2\sqrt{\pi} r_0} \right) + \frac{2\Re f^S(k^*)}{\sqrt{\pi} r_0} F_1(2k^* r_0) - \frac{\Im f^S(k^*)}{r_0} F_2(2k^* r_0) \right]$$

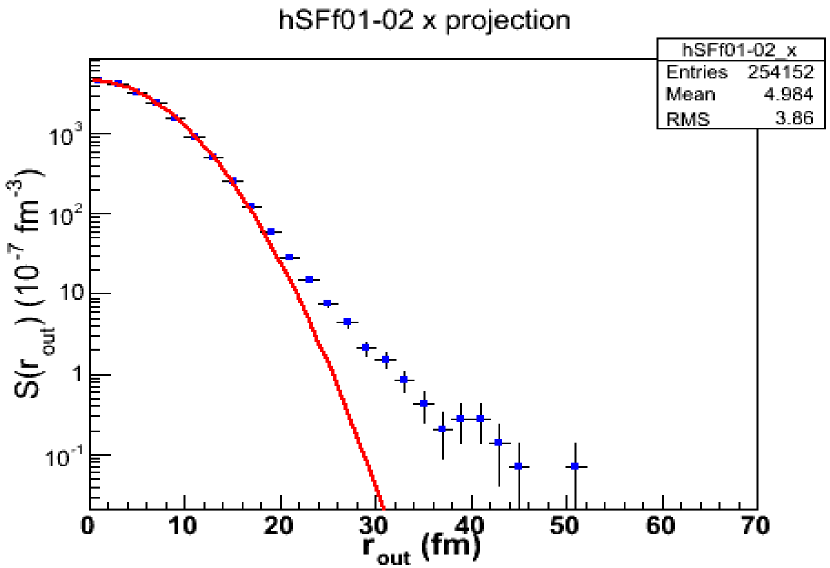
$$F_1(z) = \int_0^z dx e^{x^2 - z^2}/z,$$

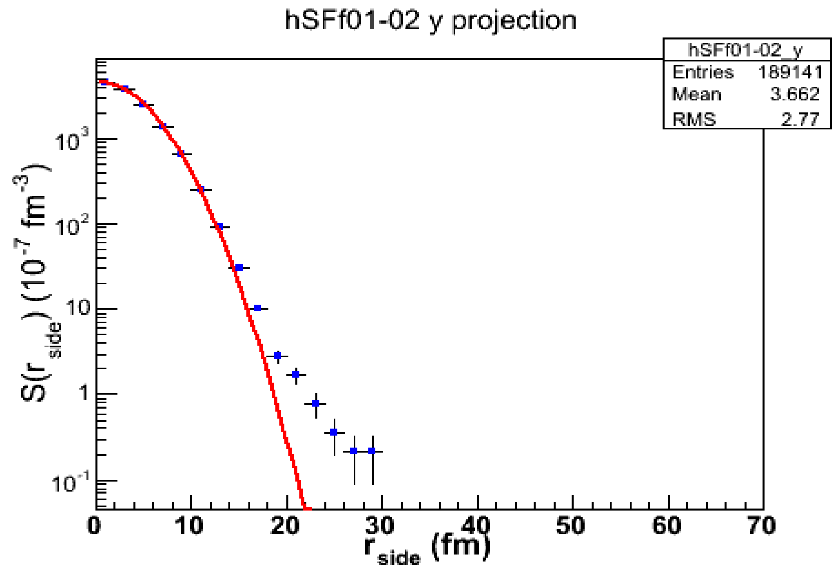
$$F_2(z) = (1 - e^{-z^2})/z,$$

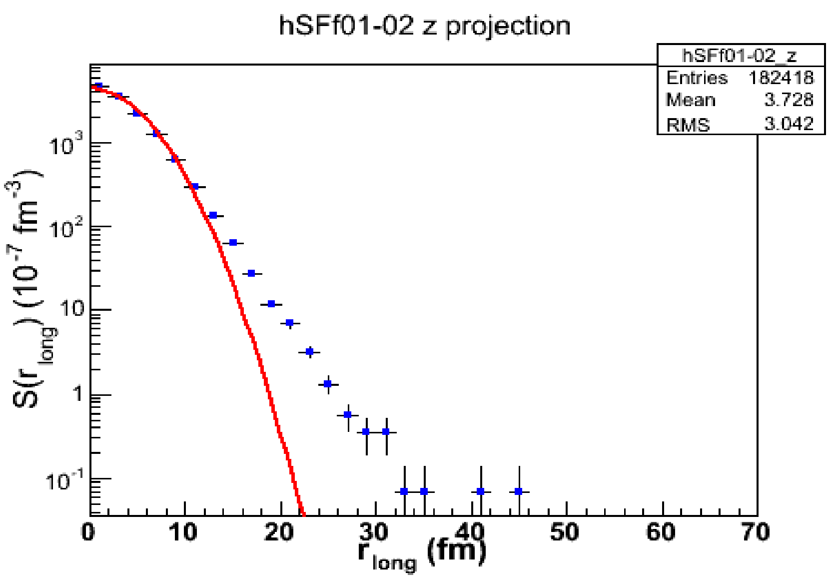
$$f^S(k^*) = \left(\frac{1}{f_0^S} + \frac{1}{2} d_0^S k^{*2} - ik^* \right)^{-1}.$$

Parameters: scattering lengths f_0^S, effective radii d_0^S and the source radius r_0.

The Λp HKM source function projections at RHIC top energy







The experimental and HKM source radius values are:

$$r_0^{exp} = 3.09 \pm 0.30^{+0.17}_{-0.25} \pm 0.2 \text{ fm}$$

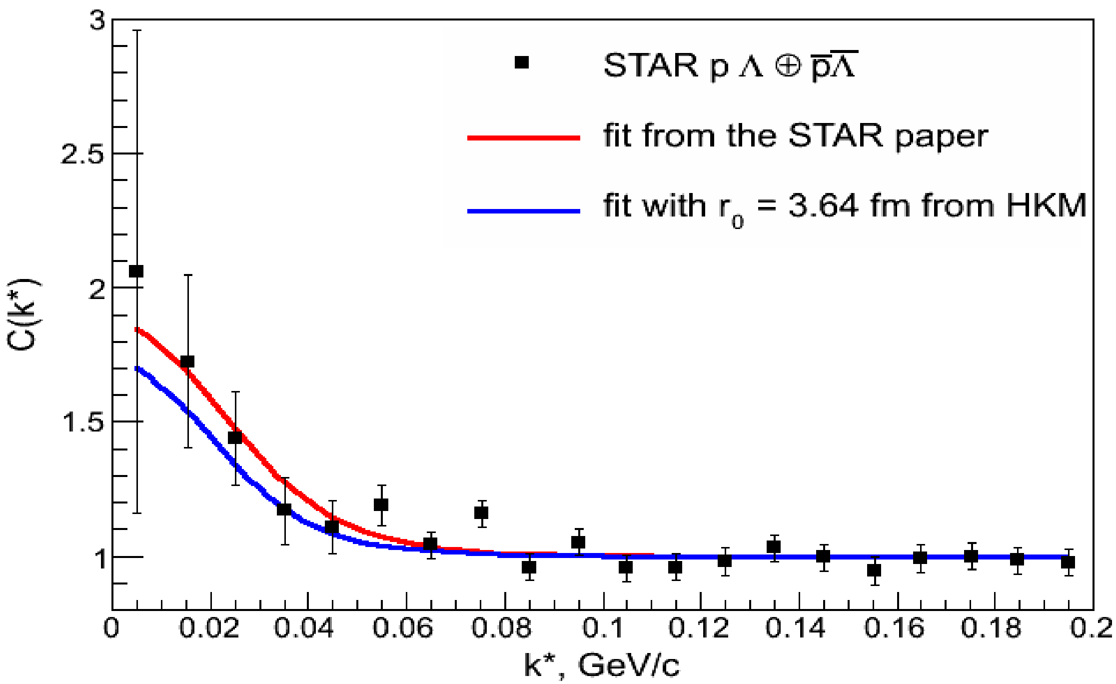
$$r_0^{\text{HKM}} = 3.637 \pm 0.001 \text{ fm}$$

The $p-\Lambda \oplus \bar{p}-\bar{\Lambda}$ correlation function

F. Wang, S. Pratt, Phys. Rev. Lett. 83, 3138 (1999)

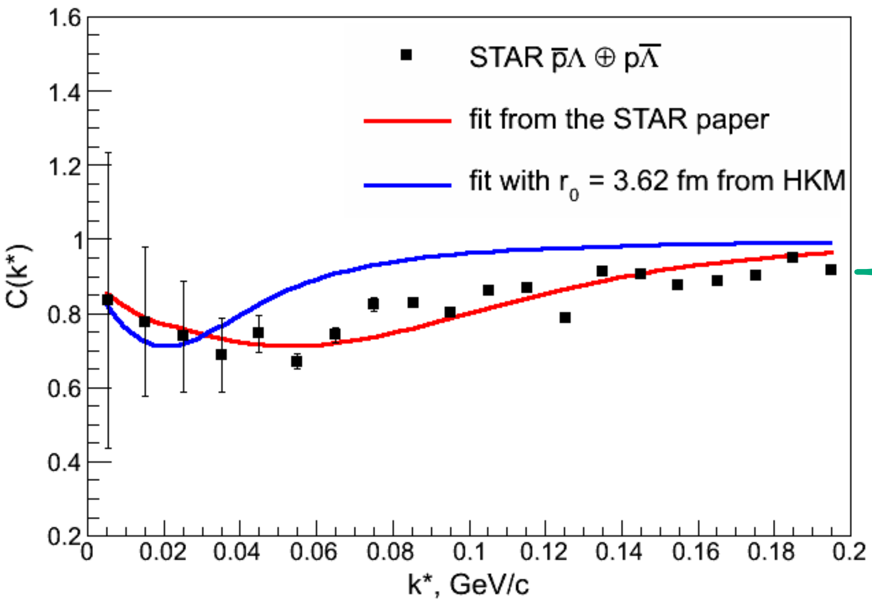
$$f_0^s = 2.88 \text{ fm}, \; f_0^t = 1.66 \text{ fm}, \; d_0^s = 2.92 \text{ fm}, \; d_0^t = 3.78 \text{ fm}$$

$$r_0^{exp} = 3.09 \pm 0.30^{+0.17}_{-0.25} \pm 0.2 \text{ fm} \quad r_0^{\text{HKM}} = 3.637 \pm 0.001 \text{ fm}$$



The $p-\Lambda \oplus \bar{p}-\bar{\Lambda}$ correlation function measured by STAR (black markers), the corresponding STAR fit within the Lednicky and Lyuboshitz analytical model (red line) and HKM fit within the same model with the source radius r_0 obtained from the HKM calculations (blue line).

The $\bar{p}-\Lambda \oplus p-\bar{\Lambda}$ correlation function



STAR: $\Re f = -2.03 \pm 0.96^{+1.37}_{-0.12}$ fm,

$\Im f = 1.01 \pm 0.92^{+2.43}_{-1.11}$ fm,

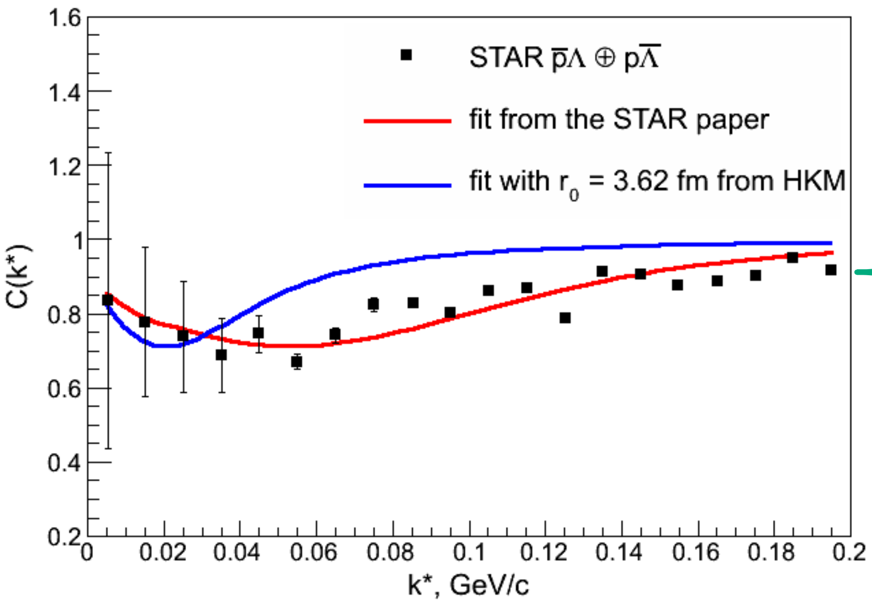
$r_0 = 1.50 \pm 0.05^{+0.10}_{-0.12} \pm 0.3$ fm

(red line)

HKM: $r_0^{\mathrm{HKM}} = 3.621 \pm 0.001$ fm

The best fit – blue line

The $\bar{p}-\Lambda \oplus p-\bar{\Lambda}$ correlation function



$$\text{STAR: } \Re f = -2.03 \pm 0.96^{+1.37}_{-0.12} \text{ fm},$$

$$\Im f = 1.01 \pm 0.92^{+2.43}_{-1.11} \text{ fm},$$

$$r_0 = 1.50 \pm 0.05^{+0.10}_{-0.12} \pm 0.3 \text{ fm}$$

(red line)

$$\text{HKM: } r_0^{\text{HKM}} = 3.621 \pm 0.001 \text{ fm}$$

The best fit – blue line

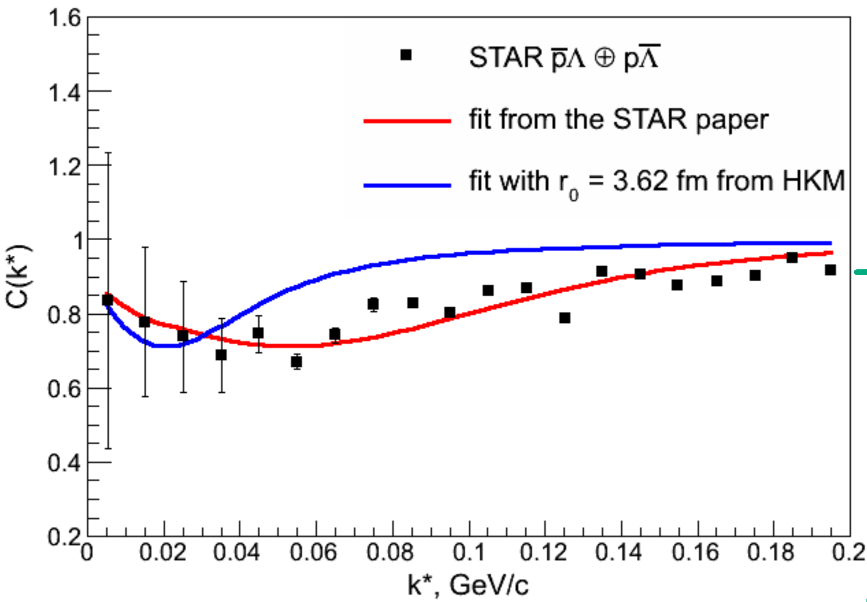
$$C_{meas}(k^*) = \lambda C(k^*) + (1-\lambda)$$

Residual correlations

$$C_{meas}(k^*) = \lambda C(k^*) + (1-\lambda)(1-\beta e^{-4k^{*2}R^2})$$

$$r_0^{\text{HKM}} = 3.621 \pm 0.001 \text{ fm}$$

The $\bar{p}-\Lambda \oplus p-\bar{\Lambda}$ correlation function



$$\text{STAR: } \Re f = -2.03 \pm 0.96^{+1.37}_{-0.12} \text{ fm},$$
$$\Im f = 1.01 \pm 0.92^{+2.43}_{-1.11} \text{ fm},$$
$$r_0 = 1.50 \pm 0.05^{+0.10}_{-0.12} \pm 0.3 \text{ fm}$$

(red line)

$$\text{HKM: } r_0^{\text{HKM}} = 3.621 \pm 0.001 \text{ fm}$$

The best fit – blue line

$$C_{meas}(k^*) = \lambda(k^*)C(k^*) + (1-\lambda(k^*))$$

Residual correlations

$$\lambda(k^*) = a\lambda_{exp}(k^*)$$

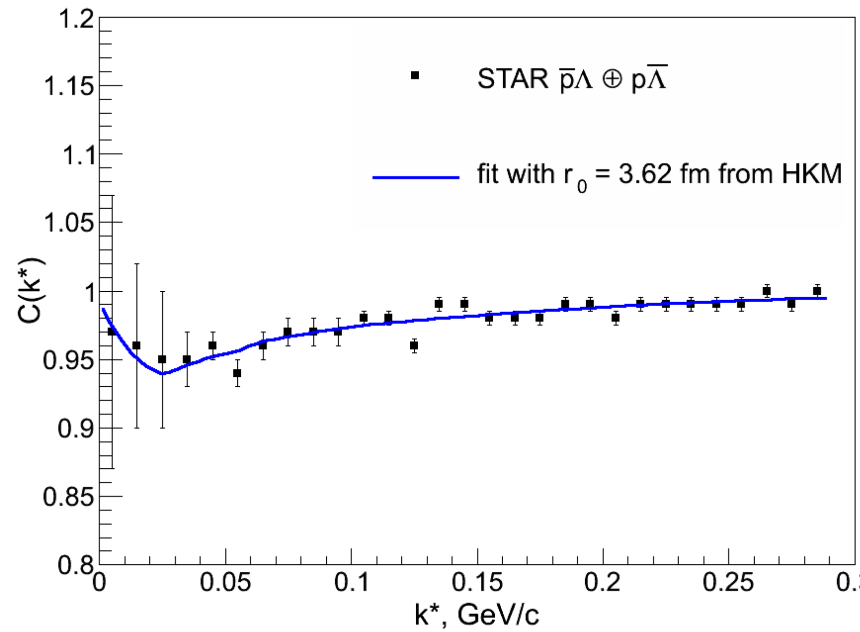
$$C_{meas}(k^*) = \lambda(k^*)C(k^*) + (1-\lambda(k^*))(1-\beta e^{-4k^{*2}R^2})$$

$$r_0^{\text{HKM}} = 3.621 \pm 0.001 \text{ fm}$$

$$a = 1.28 \pm 0.84, \ \Re f_0 = -0.05 \pm 0.68 \text{ fm},$$

$$\Im f_0 = 1.41 \pm 1.07 \text{ fm}, \ \beta = 0.029 \pm 0.005$$

$$R = 0.45 \text{ fm} \pm 0.06 \text{ fm}$$



Conclusion

- Pion interferometry radii are well described at the top RHIC and LHC energies. The kaon radii are agreed with RHIC experiment. The predictions for LHC current and full energies demonstrate similarity of neutral and charged kaon radii, and pion- kaon k_T-scaling at k_T>0.4-0.5 Gev. The HKM prediction for LHC: $R_{out}/R_{side} \approx 1$ is conformed by ALICE Coll. in 2011.

- The comparison of V_{int} vs $dN/d\eta$ for pp, pPb and AA collisions shows that interferometry volume depends not only on multiplicity but also on initial size of colliding systems.

- Emission picture obtained in the HKM describes not only Gaussian interferometry radii, but also the detailed emission characteristics - non-Gaussian source functions. The heavy tails in long- and out- projections of the source function are caused mostly by the particle rescattering at the afterburner - hadronic cascade stage.

- The strong interaction characteristics in baryon-(anti) baryon systems, such as the scattering lengths, can be studied in heavy ion collisions using the source functions for the corresponding baryon pairs.

- It is demonstrated that the residual correlations are played an important role in formation of baryon correlations, in particular, $\bar{p}-\Lambda \ominus p-\bar{\Lambda}$ pairs.

Thank you for your attention!