QCD in background magnetic fields

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in collaboration with
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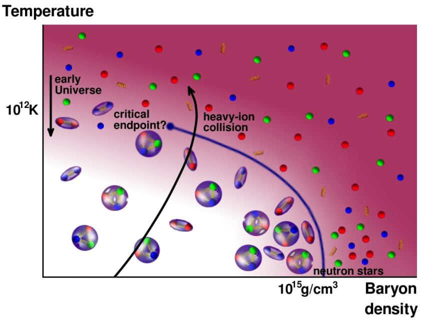
Outline

- introduction: QCD + magnetic fields in nature and in experiments
- approach: through lattice simulations
- results: effects of the magnetic field on the (thermal) QCD vacuum
  - paramagnetism at high temperatures
  - electric polarization around topological objects (chiral magnetic effect)
- conclusions

Introduction

QCD phase diagram

- why is the physics of the quark-gluon plasma interesting?
  - large T: early Universe, cosmological models
  - large ρ: neutron stars
  - large T and/or ρ: heavy-ion collisions, experiment design



QCD phase diagram

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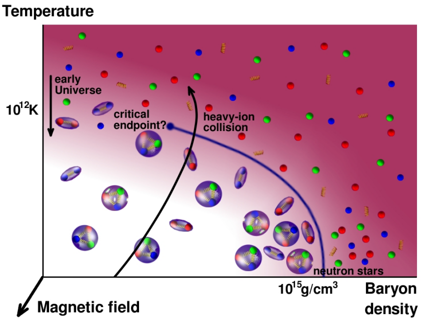
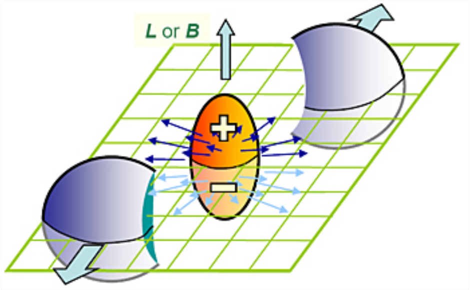


- background B: a new direction to probe the strong interactions (separate quarks from gluons)
- this talk: consider $T-B$ plane

Example: heavy-ion collision



[STAR collaboration, '10]

- off-central collisions: beams generate magnetic fields: strength controlled by $\sqrt{s}$ and impact parameter (centrality)
- strong (but very uncertain) time-dependence
- anisotropic spatial gradients

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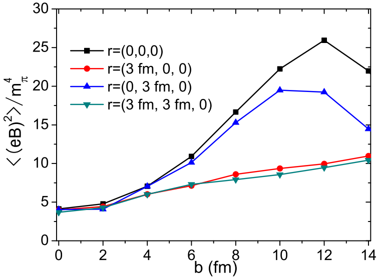
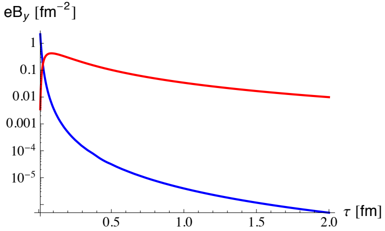


[Bloczynski et al. '12]



[Gursoy et al '13]

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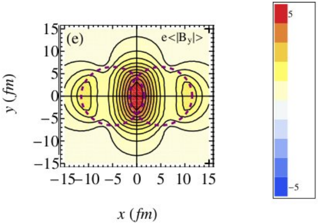
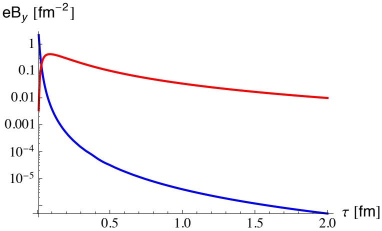


[Deng et al '12]



[Gursoy et al '13]

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Magnetic response I: susceptibility

Magnetic susceptibility

- simplification: constant background magnetic field B
- free energy density in background magnetic field

$$f(B) = -\frac{T}{V} \log \mathcal{Z}(B)$$

- magnetization

$$\mathcal{M} = -\frac{\partial f}{\partial (eB)}, \qquad \mathcal{M}\big|_{B=0} = 0$$

- susceptibility

$$\chi = \frac{\partial \mathcal{M}}{\partial (eB)}\bigg|_{B=0} = -\left.\frac{\partial^2 f}{\partial (eB)^2}\right|_{B=0}$$

- sign distinguishes between
  - paramagnets ($\chi > 0$): like magnetic field
  - diamagnets ($\chi < 0$): repel magnetic field
- additive renormalization

$$\chi_r = \chi - \chi\big|_{T=0}$$

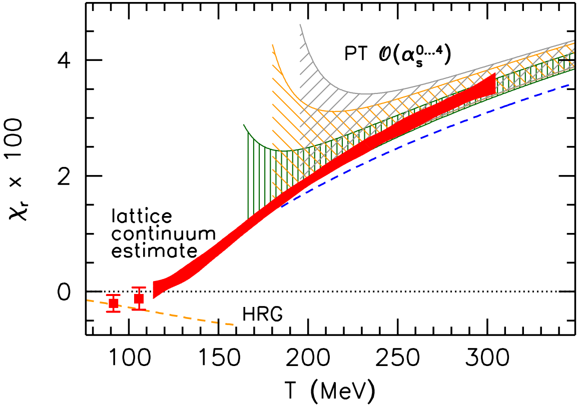
Magnetic susceptibility

- direct lattice simulation at nonzero B is possible (no sign problem)
- complication: B in a finite periodic volume is quantized

$$\Phi = qB \cdot L^2 = 2\pi N_b, \qquad N_b \in \mathbb{Z}$$

- in principle χ is ill-defined
- to circumvent this problem:
  - generalized integral method to determine $f(B,T)$
  - numerical differentiation to calculate χ
    [Bali et al. '13, Bali et al. in preparation]

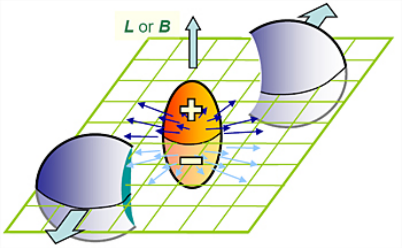
Magnetic susceptibility

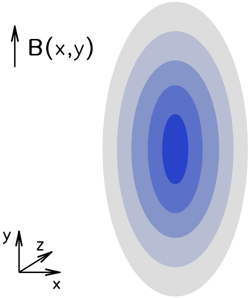


- high T: paramagnetic free quarks $\Leftrightarrow$ low T: diamagnetic pions [Bali, Bruckmann, Endrődi, Katz, Schäfer, in preparation]
- surprisingly good agreement with PT at not-so-high T

Paramagnetism - heavy ions

- strong paramagnetism at high $T \to$ free energy minimal where B is maximal
  - in non-uniform magnetic fields: deformation of QGP





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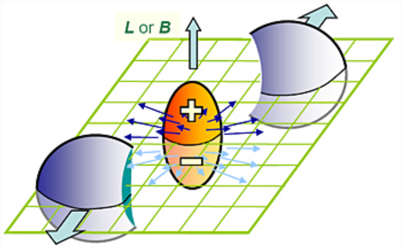
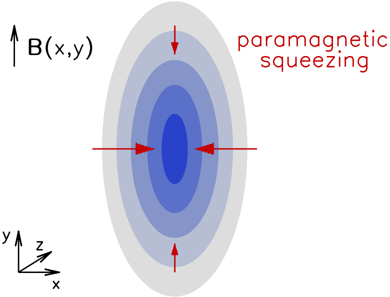
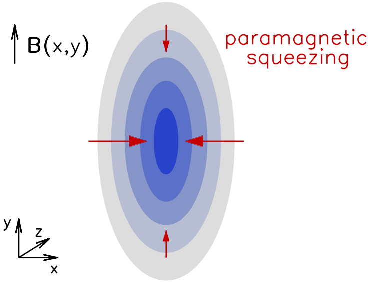


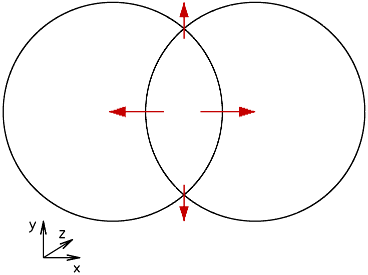


- free energy minimization squeezes QCD matter anisotropically
  [Bali,Bruckmann,Endrődi,Schäfer '13]

Squeezing versus elliptic flow

- elliptic flow: anisotropic pressure gradients due to initial geometry





- ▶ competition between squeezing and elliptic flow
- ▶ crude estimate: squeezing contributes $5 - 50\%$, depending on beam energy [Bali,Bruckmann,Endrődi,Schäfer '13]
- ▶ need a more sophisticated model which takes into account $B(x, y, t)$ and compares the two effects carefully

Magnetic response II: topology

Broken rotational symmetry

- magnetic field $B = F_{xy}$ induces the rotational symmetry breaking expectation value [Ioffe, Smilga '84] (to leading order in F_{xy})

$$\left\langle \bar{\psi}_f \sigma_{xy} \psi_f \right\rangle \propto q_f F_{xy}, \qquad \sigma_{\mu\nu} = \frac{1}{2i}[\gamma_\mu, \gamma_\nu]$$

- ▸ magnetic field produces spin-polarization

- in a topological background pseudoscalar channels open up

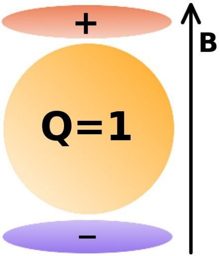
$$\left\langle \bar{\psi}_f \sigma_{zt} \psi_f \right\rangle_Q \propto q_f F_{xy}$$

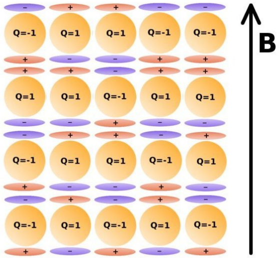
- ▸ magnetic field + Q produces electric polarization (compare chiral magnetic effect [Kharzeev et al '09])

From topology to electric dipoles

- in a *locally* fluctuating topological background

$$\left\langle \int \mathrm{d}^4x \, q_{\mathrm{top}}(x) \cdot \bar{\psi}_f \sigma_{zt} \psi_f(x) \right\rangle \propto q_f F_{xy}$$





- ▸ magnetic field induces *local* correlation between topology and electric polarization [Buividovich et al. '10, Bali et al. '14]

Local CP-violation

- consider the dimensionless combination
  [Bali, Bruckmann, Endrődi, Fodor, Katz, Schäfer '14]

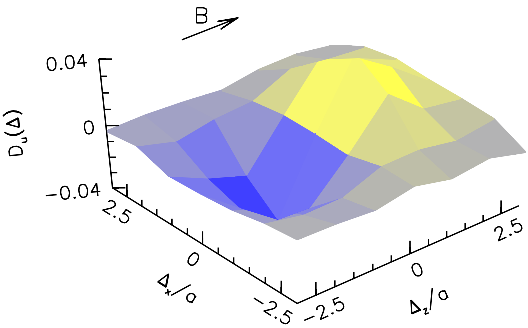
$$C_f = \frac{\left\langle q_{\rm top}(x) \cdot \bar{\psi}_f \sigma_{zt} \psi_f(x) \right\rangle}{\sqrt{\left\langle q^2_{\rm top}(x) \right\rangle} \left\langle \bar{\psi}_f \sigma_{xy} \psi_f(x) \right\rangle}$$

  - model description: self-dual gluonic background and $m_f \approx 0$
    $C_f \sim 1 \Rightarrow B$-polarization equals E-polarization for unit topology
  - lattice simulation: physical m_π, continuum extrapolation
    $C_f \sim 0.13 \Rightarrow$ non-perturbative QCD interactions prevent full electric polarization of the quarks

Electric charge separation

- evidence for extended electric charge dipole moment
  [Bali, Bruckmann, Endrődi, Fodor, Katz, Schäfer '14]

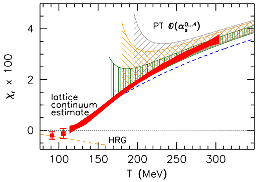
$$D_f(\Delta) = \left\langle \int \mathrm{d}^4x \, q_{\rm top}(x) \cdot \bar{\psi}_f \gamma_0 \psi_f(x+\Delta) \right\rangle \propto q_f B, \qquad \text{if } \Delta \parallel B$$



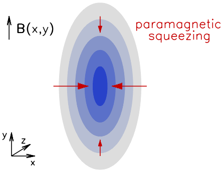
Conclusions

Summary

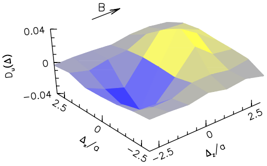
- magnetic fields significantly affect the thermal QCD vacuum



  - strong paramagnetism at high temperatures



  - possible implication for heavy-ion collisions: paramagnetic squeezing



  - induced electric polarization around topological objects (weaker than usual model assumptions would give)