Approach to equilibrium:
Universal properties in expanding gauge and scalar field theories

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Motivation

The thermalization process of quantum systems with far-from-equilibrium initial conditions may involve the approach to a nonthermal fixed point (attractor). In this case, the system *partially forgets its initial conditions* and the nonequilibrium dynamics of the single-particle distribution f becomes *self-similar*, described by dynamical scaling exponents and a (stationary) scaling function. [1-3]

Different microscopic theories may even share the same exponents and functional forms. Such universal behavior far from equilibrium has been predicted for systems ranging from early-universe inflaton dynamics, gluon evolution in ultrarelativistic heavy-ion collisions to table-top experiments with cold atoms.

Questions

It has been demonstrated recently that longitudinally expanding non-Abelian plasmas, relevant for heavy-ion collisions in the limit of high energy and weak coupling, approach a nonthermal fixed point. [1,2]

Is this behavior part of a general universality class far from equilibrium?

Does the scalar field theory also lie in this universality class?

Theory & method

Longitudinally expanding **scalar O(4) theory** (classical EOM):

$$\left(\frac{1}{\tau}\partial_\tau \tau\, \partial_\tau - \partial_T^2 - \frac{1}{\tau^2}\,\partial_\eta^2 - \frac{\lambda}{24}\,\varphi_b\varphi_b\right)\varphi_a(\tau,\boldsymbol{x_T},\eta) = 0\,, \qquad \lambda \ll 1\,,$$

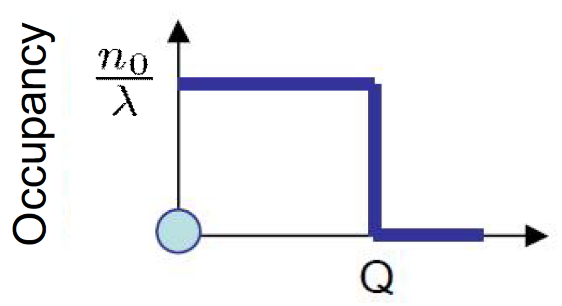
with transverse coordinates $\mathbf{x_T}$, spatial rapidity η and weak coupling λ .

Our numerical method: **classical-statistical lattice simulations**

- Solve classical equation of motion on the lattice.
- Sampling with quantum initial conditions introduces fluctuations.
- This accurately describes quantum field theory for large occupation numbers f (I) or large macroscopic fields ϕ (II) (classicality condition).

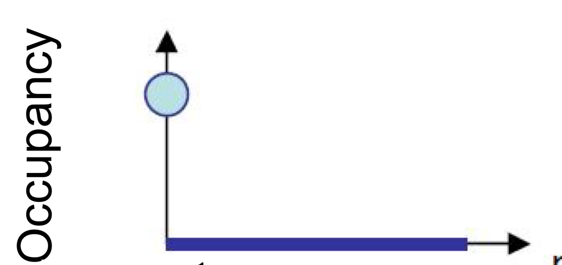
Initial conditions

(I) Fluctuation dominated



$$\phi = 0\,, \quad f(p \le Q) = \frac{n_0}{\lambda}$$

(II) Condensate driven



$$\phi \sim \frac{1}{\sqrt{\lambda}}\,, \quad f = 0$$

Thermal-like distribution

Hard momenta:
Stationary integrated distribution

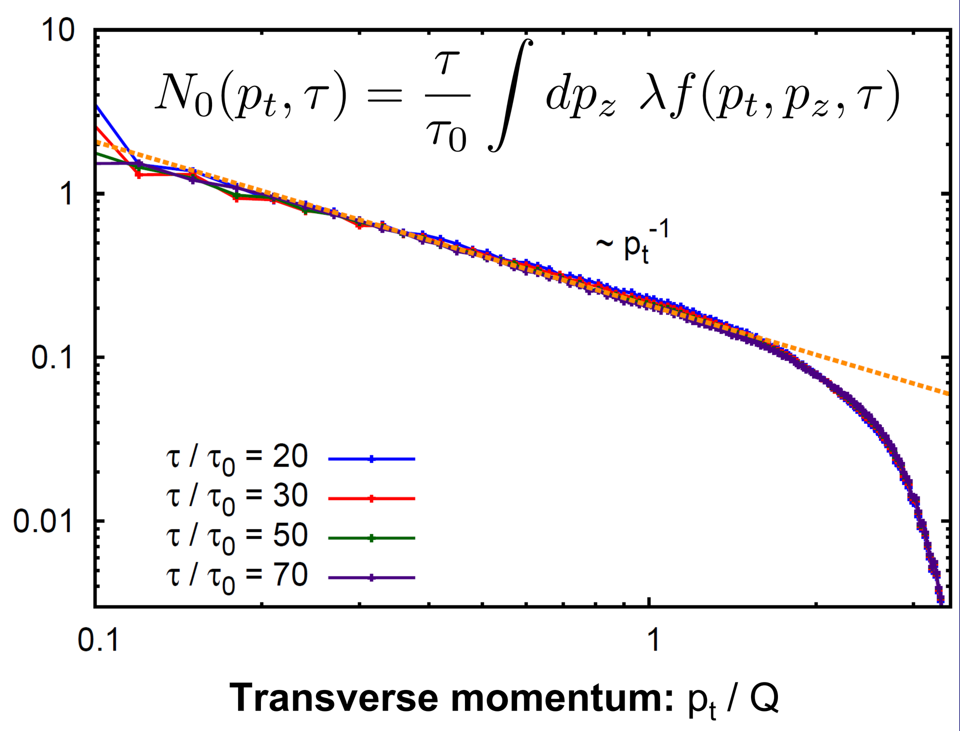


Fig. 1

Universal scaling

Self-similar evolution $f(\tau,p_t,p_z) = \tau^\alpha f_s(\tau^\beta p_t, \tau^\gamma p_z)$

with transverse and longitudinal momenta p_t and p_z , dynamical scaling exponents α, β, γ and scaling function f_s .

Universal scaling function f_s

At characteristic hard momenta ($p_t = p_t^*$ for scalars), gauge and scalar theories share the same scaling function for different initial conditions.

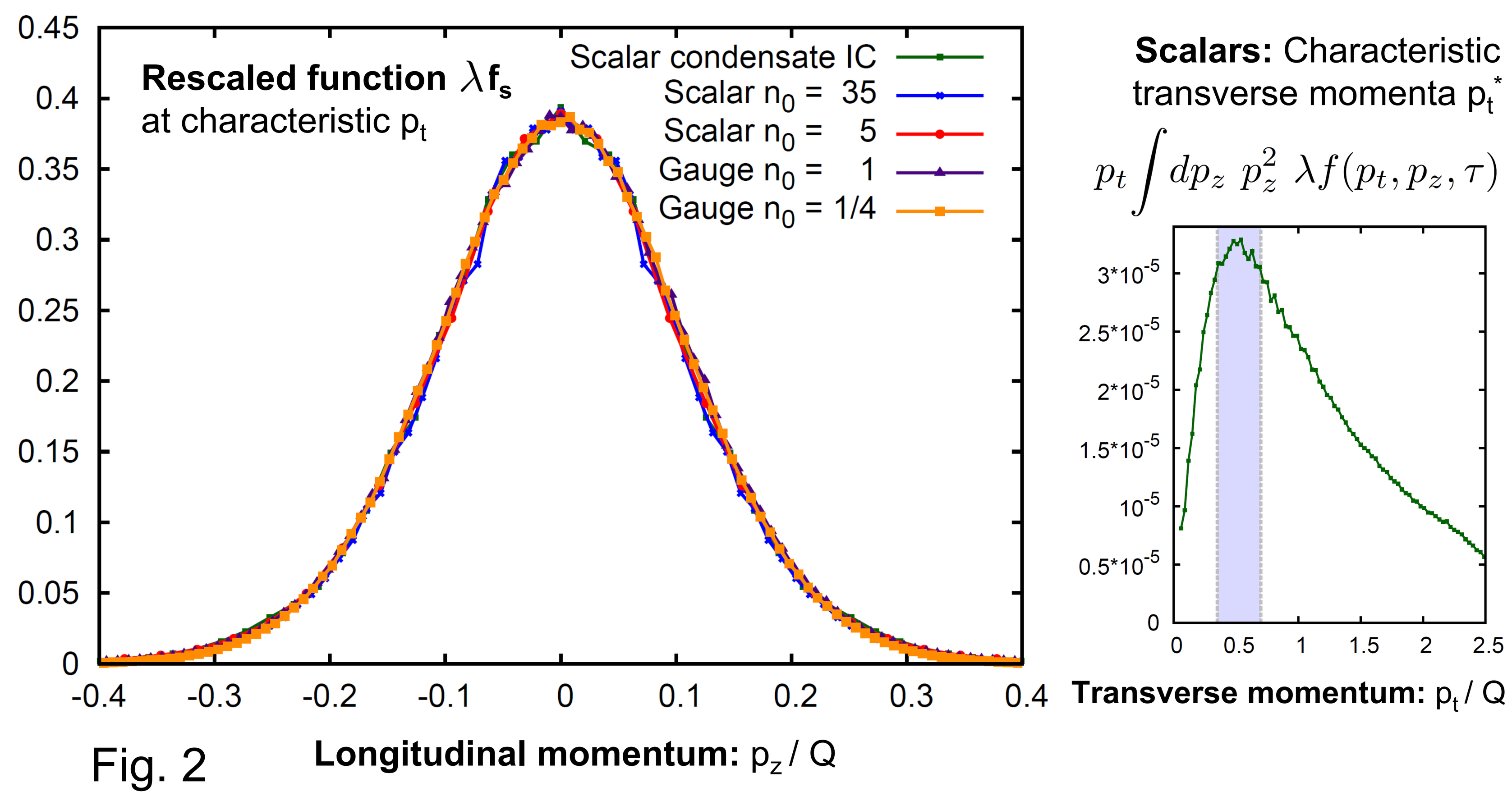


Fig. 2

Scaling exponents

$$N_0(p_t,\tau) = \text{const}$$

Fig.1 $\Rightarrow$ $\alpha - \gamma \simeq -1$, $\beta \simeq 0$

$$\lambda f(p_t^*, p_z = 0) \sim \tau^\alpha$$

Fig.3 $\Rightarrow$ $\alpha \simeq -2/3$, $\beta \simeq 0$, $\gamma \simeq 1/3$

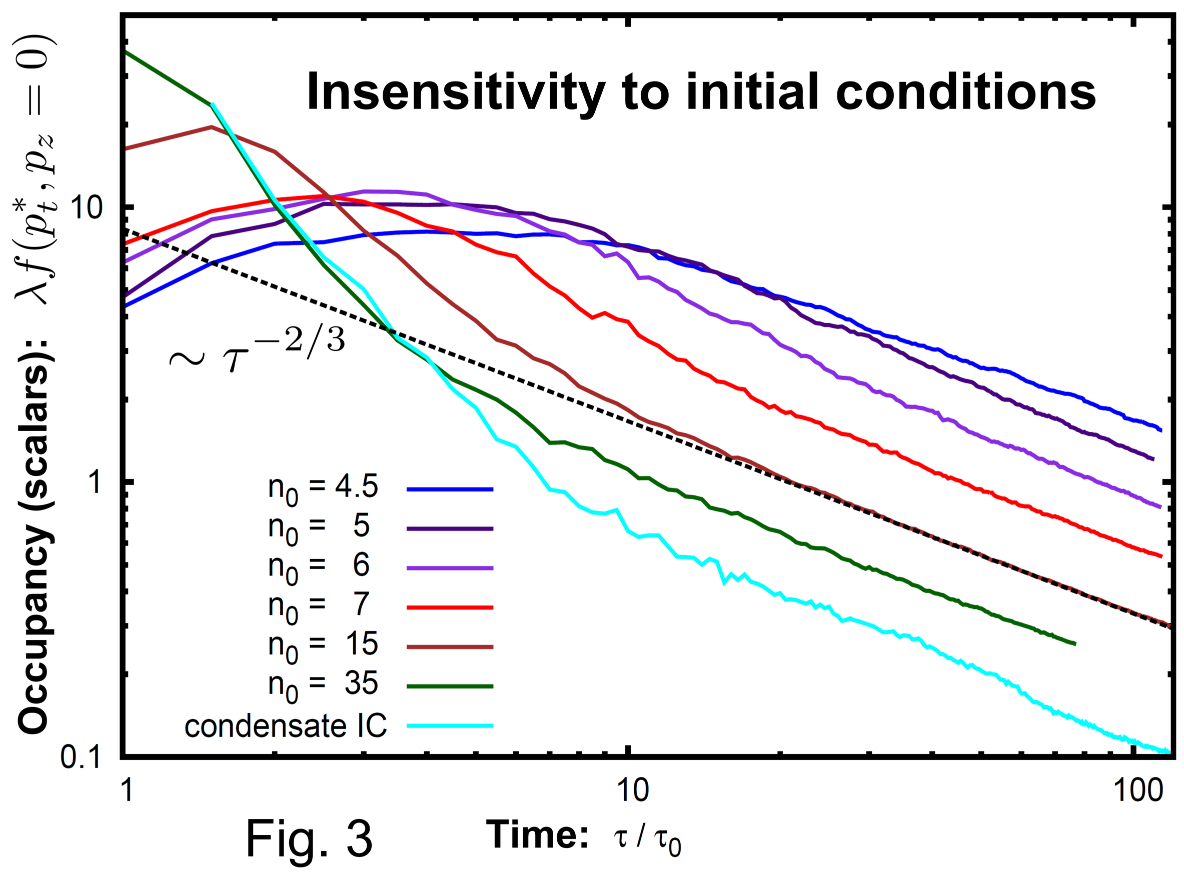


Fig. 3

Same dynamical scaling exponents and same functional form as for gauge theory, independent of initial conditions! [4]

Isotropic distribution & Bose condensate

Similar properties of longitudinally expanding and non-expanding (isotropic) scalar theories:

- Isotropic distribution at soft momenta for large occupancies $\lambda f \gtrsim 1$.
- Leads to Bose-Einstein condensation via inverse particle cascade.

Conclusion & Outlook

Scalar and non-Abelian gauge theories in longitudinally expanding backgrounds share the same universal properties at characteristic (hard) momenta. These are insensitive to initial conditions.

Our findings indicate a more general structure underlying longitudinally expanding systems than anticipated.

(Anticipated by the ***„bottom-up“*** thermalization scenario [Baier, Mueller, Schiff and Son, PLB 502 (2001) 51-58])

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