Nonlinear waves in second order conformal hydrodynamics

H. Marrochio†, D. A. Fogaça†, F. S. Navarra†, and J. Noronha†

University of São Paulo

QUARK MATTER - 2014

† Instituto de Física, Universidade de São Paulo Rua do Matão Travessa R, 187, 05508-090 São Paulo, SP, Brazil

Abstract

In this work [1] we study wave propagation in dissipative relativistic fluids described by a simplified set of the 2nd order viscous conformal hydrodynamic equations. Small amplitude waves are studied within the linearization approximation while waves with large amplitude are investigated using the reductive perturbation method. Our results indicate the presence of a "soliton-like" wave solution in 2nd order conformal hydrodynamics despite the presence of dissipation and relaxation effects. More details in arXiv:1402.5548

1 Introduction and Motivation

We investigate how the presence of a nonzero shear viscosity relaxation time affects wave propagation in relativistic fluids.

The simplest extension of the well-known Navier-Stokes (NS) equations to relativistic fluids is plagued with instabilities and acausal signal propagation in the resulting equations [2, 3, 4].

Currently, most fluid-dynamical simulations of the QGP employ a set of relaxation-type equations similar to those derived by Israel and Stewart (IS) [5] to close the conservation laws.

In the following sections we study the propagation of linear and nonlinear waves in relativistic fluids described by (a simplified set of) of the 2nd order conformal IS equations.

2 Second-order conformal hydrodynamic equations

We focus on the simplest set of equations that can still describe a causal (and stable) conformal dissipative fluid. The simplest set of conformal Israel-Stewart theory equation are:

$$D\varepsilon + (\varepsilon + p)\theta - \pi^{\mu\nu}\sigma_{\mu\nu} = 0 \qquad (1)$$

$$(\varepsilon + p)Du^{\alpha} - \nabla_{\perp}^{\alpha} p + \Delta_{\nu}^{\alpha}\partial_{\mu}\pi^{\mu\nu} = 0 \qquad (2)$$

$$\tau_{\pi}\left(\Delta_{\alpha}^{\mu}\Delta_{\beta}^{\nu}D\pi^{\alpha\beta} + \frac{4}{3}\pi^{\mu\nu}\theta\right) + \pi^{\mu\nu} = 2\eta\sigma^{\mu\nu} \qquad (3)$$

where the dissipative tensor $\pi^{\mu\nu}$ is also a degree of freedom and satisfies a differential equation. Under linear perturbation, one can show that small fluctuations around the equilibrium state in the x direction leads to ($c_s^2 = 1/3$ and $\chi = \frac{4\eta}{3s}$):

$$\frac{\partial^2}{\partial x^2}\delta\varepsilon - 3\frac{\partial^2}{\partial t^2}\delta\varepsilon - 3\tau_{\pi}^0\frac{\partial^3}{\partial t^3}\delta\varepsilon = -\left(\frac{3\chi}{T_0} + \tau_{\pi}^0\right)\frac{\partial}{\partial t}\frac{\partial^2}{\partial x^2}\delta\varepsilon \qquad (4)$$

The Fourier transform of the energy density $\delta\varepsilon(\hat{x},\hat{t}) = \mathcal{A}\, e^{Im[\hat{\omega}]\hat{t}} e^{iRe[\hat{\omega}]\left(\hat{k}\hat{x}/Re[\hat{\omega}]-\hat{t}\right)}$ leads to a dispersion relation for Eq. 4. We consider the coefficients of the strongly coupled $\mathcal{N}=4$ Supersymmetric Yang-Mills (SYM) fluid where $\eta_0/s_0 = 1/(4\pi)$ and $\hat{\tau}_{\pi} = [2 - ln(2)]/(2\pi)$ [6]. In Fig. 1 we plot the group velocity $\hat{v}_g$ and the attenuation coefficient $Im[\hat{\omega}]$ for the three roots $\hat{\omega}_I$, $\hat{\omega}_{II}$ and $\hat{\omega}_{III}$ of (4) dispersion relation. In Fig. 1-a one can clearly notice that the three modes are stable since the imaginary parts of the modes are always negative. In Fig. 1-b there is no causality violation since there is no divergence as $\hat{k}$ increases and the group velocity is bounded by unity. This figure shows that the linear sound wave disturbances around thermodynamical equilibrium in 2nd order hydrodynamics (with the transport coefficients of strongly-coupled $\mathcal{N}=4$ SYM) are causal and stable. A similar study can be done for the shear channel [4].

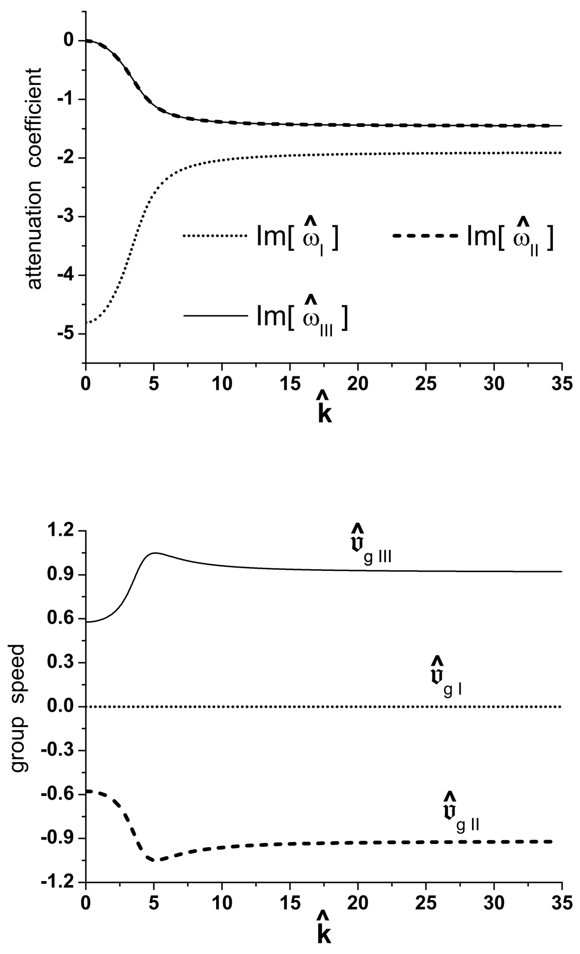

Fig. 1 - Stability and causality properties of disturbances around equilibrium for IS hydrodynamics (described by Eq. (4)) with transport coefficients from strongly-coupled $\mathcal{N}=4$ SYM.

3 Nonlinear wave equations in conformal IS

The effects from a relaxation timescale $\hat{\tau}_{\pi}$ have not yet been studied in the context of nonlinear wave propagation. In order to investigate its effects in the study of nonlinear waves, we shall use the Reductive Perturbation Method (RPM) [7]. The RPM was used to study nonlinear waves in relativistic and non-relativistic hydrodynamics in [8]. Our goal in this section is to find the nonlinear wave equation that governs the perturbation of the energy density in a hot dissipative and causal fluid described by IS hydrodynamics.

The procedure consists of changing the variables x and t to the stretched coordinates X and Y, and following an expansion in a small dimensionless parameter σ:

$$X = \sigma^{1/2}\frac{1}{L}\left(x - \frac{t}{\sqrt{3}}\right) \quad \text{and} \quad Y = \sigma^{3/2}\frac{t}{\sqrt{3}L} \qquad (5)$$

$$\eta = \sigma^{1/2}\tilde{\eta} \quad \text{and} \quad \tau_{\pi} = \sigma^{1/2}\tilde{\tau}_{\pi}\,. \qquad (6)$$

$$\hat{\varepsilon} = \frac{\varepsilon}{\varepsilon_0} = 1 + \sigma\varepsilon_1 + \sigma^2\varepsilon_2 + \sigma^3\varepsilon_3 + \ldots \qquad (7)$$

$$\hat{v}_x = \frac{v_x}{c_s} = \sigma v_1 + \sigma^2 v_2 + \sigma^3 v_3 + \ldots \qquad (8)$$

and

$$\hat{\pi}^{xx} = \frac{\pi^{xx}}{p_0} = \sigma\pi_1^{xx} + \sigma^2\pi_2^{xx} + \sigma^3\pi_3^{xx} + \ldots\,. \qquad (9)$$

The set of differential equations obtained from the RPM method is given by

$$\frac{\partial}{\partial t}\hat{\varepsilon}_1 + \frac{1}{\sqrt{3}}\frac{\partial}{\partial \hat{x}}\hat{\varepsilon}_1 + \frac{1}{2\sqrt{3}}\hat{\varepsilon}_1\frac{\partial}{\partial \hat{x}}\hat{\varepsilon}_1 = \frac{\chi}{2}\frac{\partial^2}{\partial \hat{x}^2}\hat{\varepsilon}_1 \qquad (10)$$

and

$$\frac{\partial}{\partial t}\hat{\varepsilon}_2 + \frac{1}{\sqrt{3}}\frac{\partial}{\partial \hat{x}}\hat{\varepsilon}_2 + \frac{1}{2\sqrt{3}}\hat{\varepsilon}_1\frac{\partial}{\partial \hat{x}}\hat{\varepsilon}_2 - \frac{\chi}{2}\frac{\partial^2}{\partial \hat{x}^2}\hat{\varepsilon}_2 + \frac{1}{2\sqrt{3}}\hat{\varepsilon}_2\frac{\partial}{\partial \hat{x}}\hat{\varepsilon}_1 + \frac{\chi}{4}\left(\frac{\partial}{\partial \hat{x}}\hat{\varepsilon}_1\right)^2$$

$$+\frac{\chi}{2}\hat{\varepsilon}_1\frac{\partial^2}{\partial \hat{x}^2}\hat{\varepsilon}_1 + \frac{1}{4}\hat{\varepsilon}_1\frac{\partial}{\partial \hat{t}}\hat{\varepsilon}_1 + \frac{1}{4\sqrt{3}}\hat{\varepsilon}_1\frac{\partial}{\partial \hat{x}}\hat{\varepsilon}_1 + \frac{\chi}{2}\left[\frac{\chi\sqrt{3}}{12} - \frac{\hat{\tau}_{\pi}}{\sqrt{3}}\right]\frac{\partial^3}{\partial \hat{x}^3}\hat{\varepsilon}_1 = 0\,, \qquad (11)$$

where $\hat{\varepsilon}_1 \equiv \sigma\varepsilon_1$ and $\hat{\varepsilon}_2 \equiv \sigma^2\varepsilon_2$.

Eq. 10 is the well-known Burgers equation, while Eq. 11 is a differential equation that depends non-trivially on the evolution of ϵ_1.

4 Numerical Results

The evolution of the energy density profile is developed for the following initial condition:

$$\hat{\varepsilon}_1(\hat{x},0) = A_1\, sech^2\left(\frac{\hat{x}}{B_1}\right) \qquad (12)$$

and inserting the obtained numerical solution of (10) into (11) with the initial profile for $\hat{\varepsilon}_2$

$$\hat{\varepsilon}_2(\hat{x},0) = A_2\, sech^2\left(\frac{\hat{x}}{B_2}\right). \qquad (13)$$

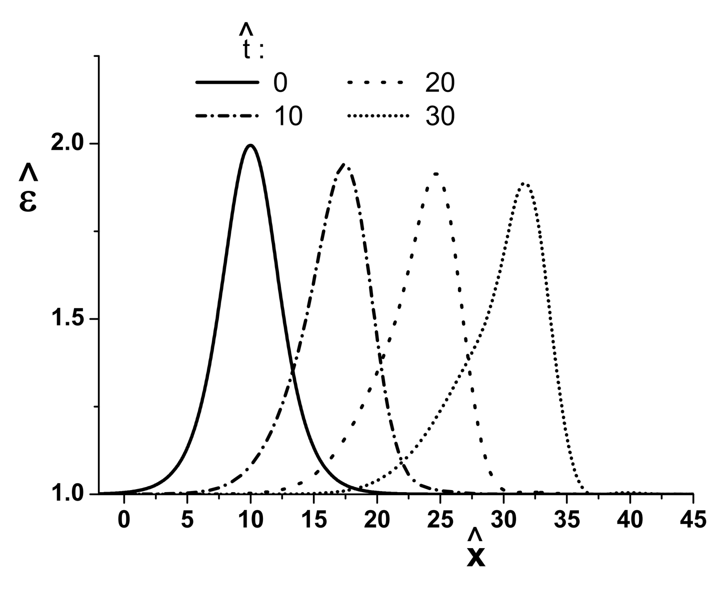

Fig 2 - Numerical solutions for the energy density disturbances in the nonlinear regime in Eqs. (29) for $\eta_0/s_0 = 1/(4\pi)$ and $\hat{\tau}_{\pi} = [2 - ln(2)]/(2\pi)$. The perturbations with these initial profiles mimic soliton behavior.

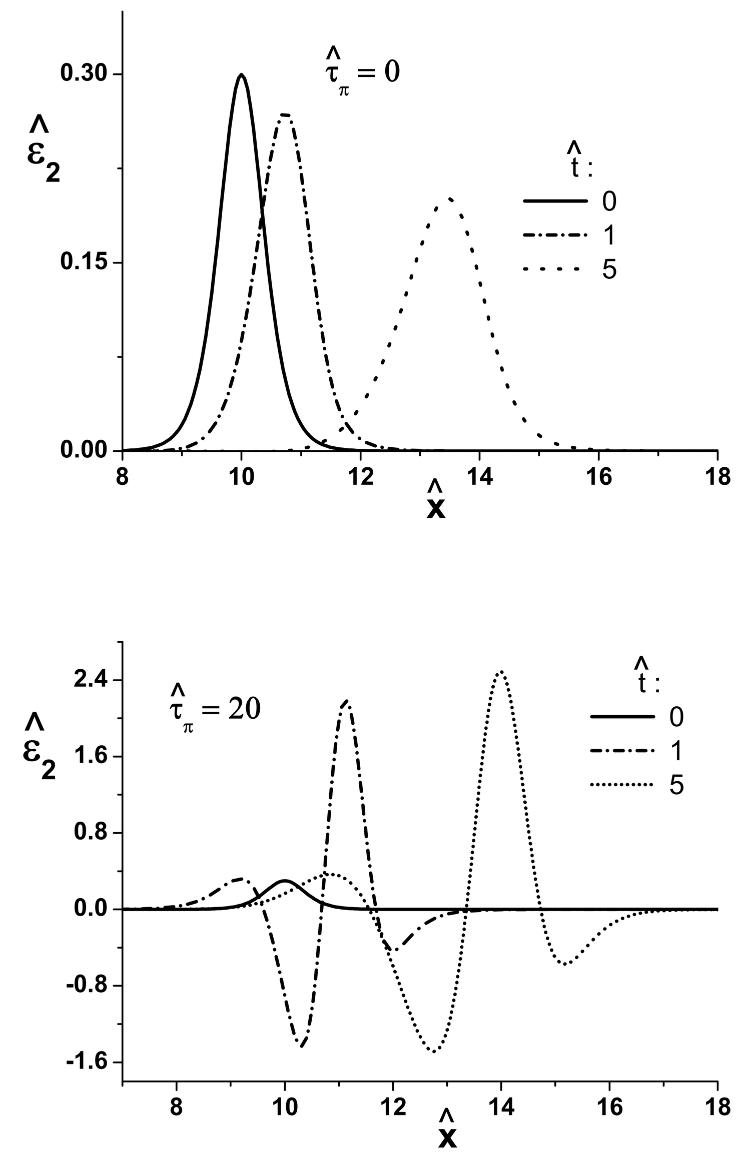

Fig 3 - Numerical solutions for the energy density disturbance in the nonlinear regime in Eq. (30) for $\eta_0/s_0 = 1/(4\pi)$ and two choices of $\hat{\tau}_{\pi}$. For large values of the relaxation coefficient, the energy perturbation $\hat{\varepsilon}_2$ acquires large amplitude and becomes inconsistent as a small disturbance.

The main contribution from $\hat{\tau}_{\pi}$ in the energy density profile is a dispersive effect, which also adds to the solitonic behavior. However, we noticed it is possible to find a breakdown of the expansion for large values of $\hat{\tau}_{\pi}$ in relation to η_0/s_o. Our nonlinear treatment of the wave equation for the energy density in hydrodynamics indicates that in a consistent microscopic theory $\hat{\tau}_{\pi}$ and η_0/s_0 must be of comparable magnitude (this is valid, for instance, in the case of kinetic theory calculations).

5 Conclusions

We derived a system of coupled differential equations which describes nonlinear wave perturbations in the energy density of 2nd order conformal fluids. Our semi-analytical treatment provides a simple (yet nontrivial) picture of how the relaxation time coefficient affects the propagation of sound waves perhaps in a more transparent way than in a complex numerical hydrodynamical simulation.

6 Acknowledgments

This work was partially financed by the Brazilian funding agencies FAPESP,CNPq and CAPES.

References

[1] D. A. Fogaça, H. Marrochio, F. S. Navarra and J. Noronha, arXiv:1402.5548 [nucl-th]. Submitted to publication in Physics Review C.

[2] W.A. Hiscock and L. Lindblom, Ann. Phys. (N.Y.) **151**, 466 (1983); Phys. Rev. D **31**, 725 (1985); Phys. Rev. D **35**, 3723 (1987); Phys. Lett. A **131**, 509 (1988).

[3] G. S. Denicol, T. Kodama, T. Koide and P. Mota, J. Phys. G **35**, 115102 (2008).

[4] S. Pu, T. Koide and D. H. Rischke, Phys. Rev. D **81**, 114039 (2010).

[5] W. Israel, Annals Phys. **100**, 310 (1976); W. Israel and J. M. Stewart, Annals Phys. **118**, 341 (1979).

[6] R. Baier, P. Romatschke, D. T. Son, A. O. Starinets and M. A. Stephanov, *(JHEP)* **04**, 100 (2008).

[7] R. C. Davidson, *Methods in Nonlinear Plasma Theory*, Academic Press, New York and London, (1972).

[8] D. A. Fogaça, F. S. Navarra and L. G. Ferreira Filho, *Solitons: Interactions, Theoretical and Experimental Challenges and Perspectives* (Nova Science Publishers, New York, 2013); arXiv:1212.6932 [nucl-th].