QUARK MATTER 2014

Darmstadt, Germany, 19-24 May, 2014

MULTIPLICITY FLUCTUATION FROM HYDRODYNAMIC NOISE

Tetsufumi Hirano
Sophia Univ.

In collaboration with
R. Kurita, K. Murase and K. Nagai

Outline

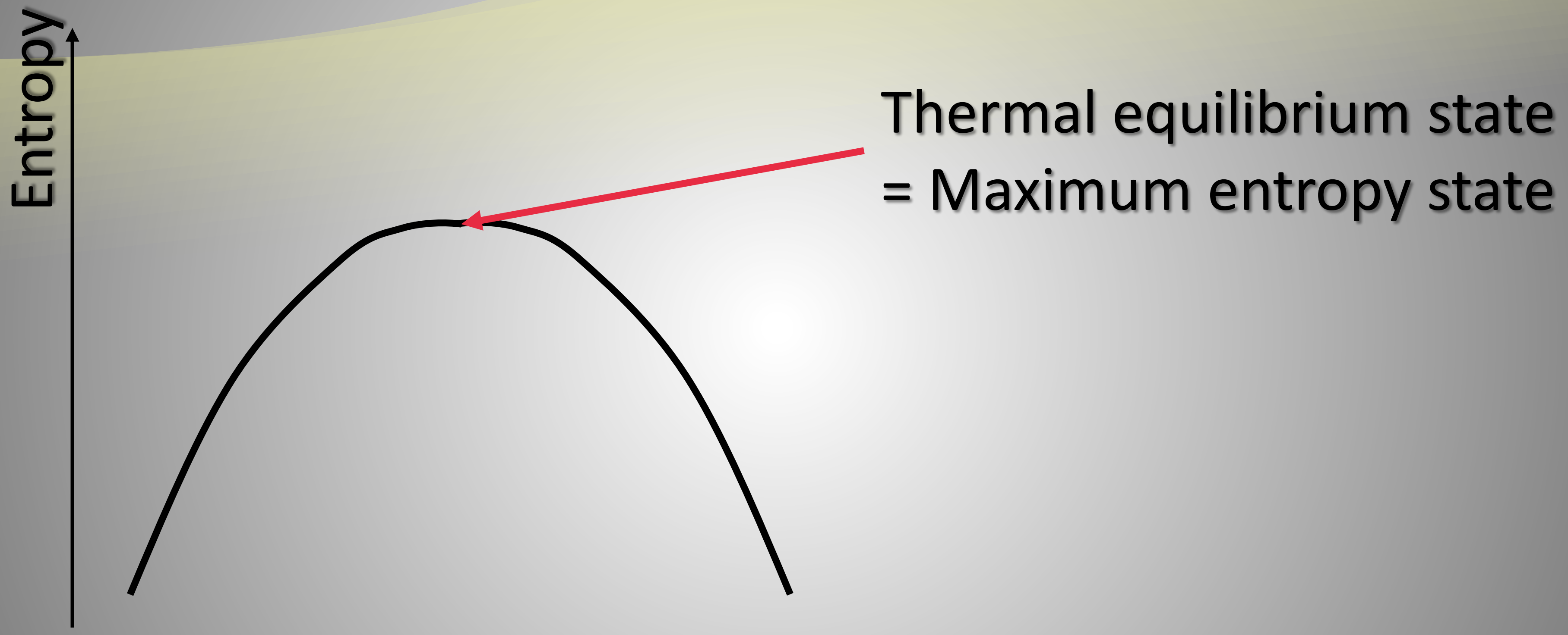
- Introduction
- Fluctuating Hydrodynamics in Bjorken Expansion
- Fluctuation theorem
- Summary

INTRODUCTION

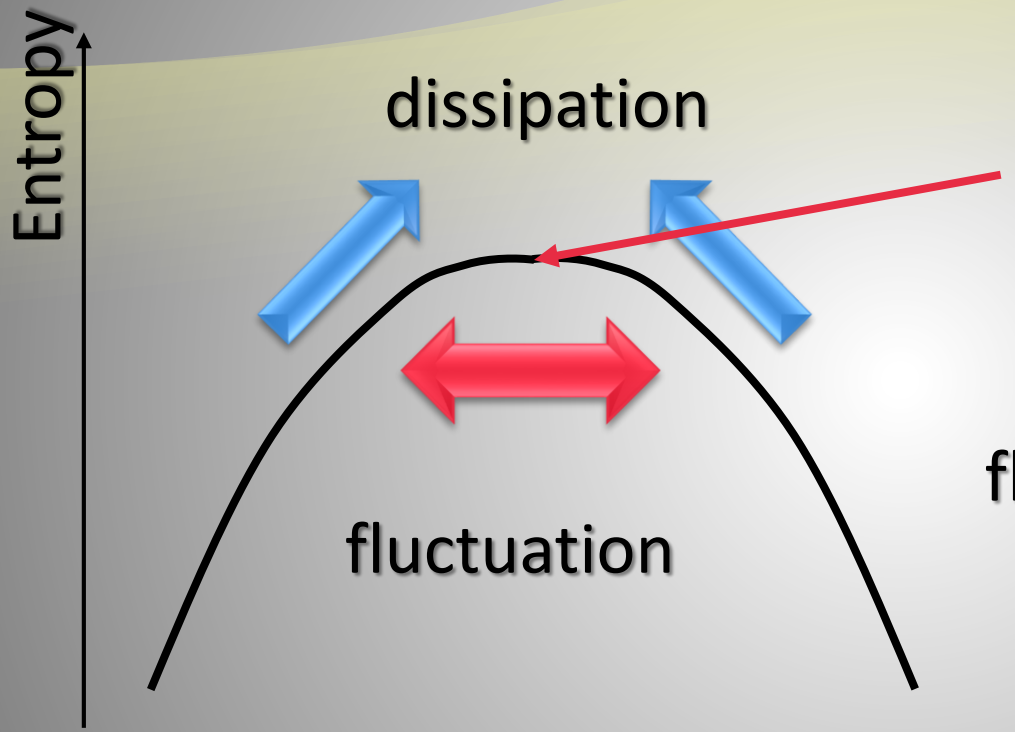
- Conventional hydrodynamics
  → Space-time evolution of (coarse-grained) thermodynamic quantities
- Microscopic information
  → Lost through coarse-graining process
- Does the lost information play an important role in dynamics in <u>small system</u> and/or on <u>an e-by-e basis</u>?
  → Thermal (Hydrodynamic) fluctuation!

Calzetta, Kapusta, Muller, Stephanov, Moore, Murase, Young, …

Fluctuation Dissipation Relation



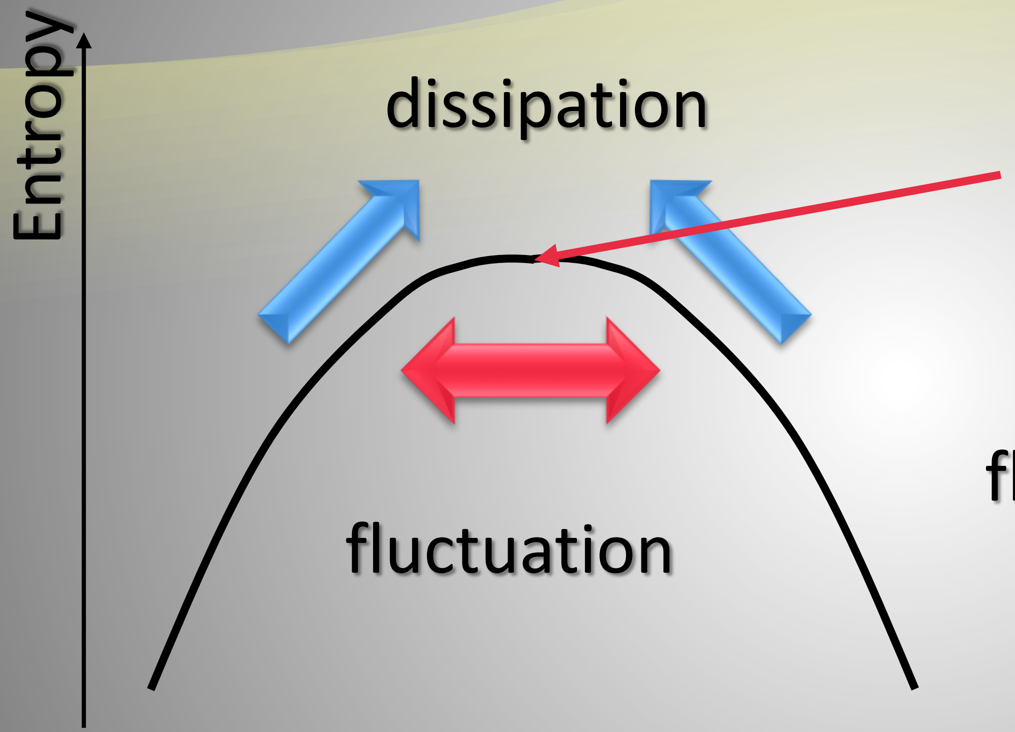
Fluctuation Dissipation Relation



Thermal equilibrium state
= Maximum entropy state

Balance between
fluctuation and dissipation
→ Stability of thermal
equilibrium state

Fluctuation Dissipation Relation



Thermal equilibrium state
= Maximum entropy state

Balance between
fluctuation and dissipation
→ Stability of thermal
equilibrium state

$$\langle \delta\Pi(x)\delta\Pi(x')\rangle = TG^*(x,x')$$

G^* : Symmetrized correlation function

$\delta\Pi$: Thermal (hydrodynamic) fluctuation

Fluctuating Hydrodynamics in Bjorken Expansion

Bjorken Expansion with Viscosity and Fluctuation

Equation of motion

$$\frac{de}{d\tau} = -\frac{e + P(e)}{\tau}\left(1 - \frac{\pi - \Pi}{sT}\right)$$

Constitutive equations

Shear: $$\tau_\pi \frac{d\pi}{d\tau} + \pi = \frac{4\eta}{3\tau} + \xi_\pi$$

Bulk: $$\tau_\Pi \frac{d\Pi}{d\tau} + \Pi = -\frac{\zeta}{\tau} + \xi_\Pi$$

e : Energy density

P : Pressure

s : Entropy density

π : Shear stress

Π : Bulk pressure

η : Shear viscosity

ζ : Bulk viscosity

τ_π, τ_Π : Relaxation time

Bjorken Expansion with Viscosity and Fluctuation

Equation of motion

$$\frac{de}{d\tau} = -\frac{e+P(e)}{\tau}\left(1-\frac{\pi-\Pi}{sT}\right)$$

Constitutive equations

Shear: $$\tau_\pi \frac{d\pi}{d\tau} + \pi = \frac{4\eta}{3\tau} + \xi_\pi$$

Bulk: $$\tau_\Pi \frac{d\Pi}{d\tau} + \Pi = -\frac{\zeta}{\tau} + \xi_\Pi$$

ξ: Gaussian white noise*

*K. Murase, TH, arXiv:1304.3243;
K. Murase, Poster H-21

e : Energy density

P : Pressure

s : Entropy density

π : Shear stress

Π : Bulk pressure

η : Shear viscosity

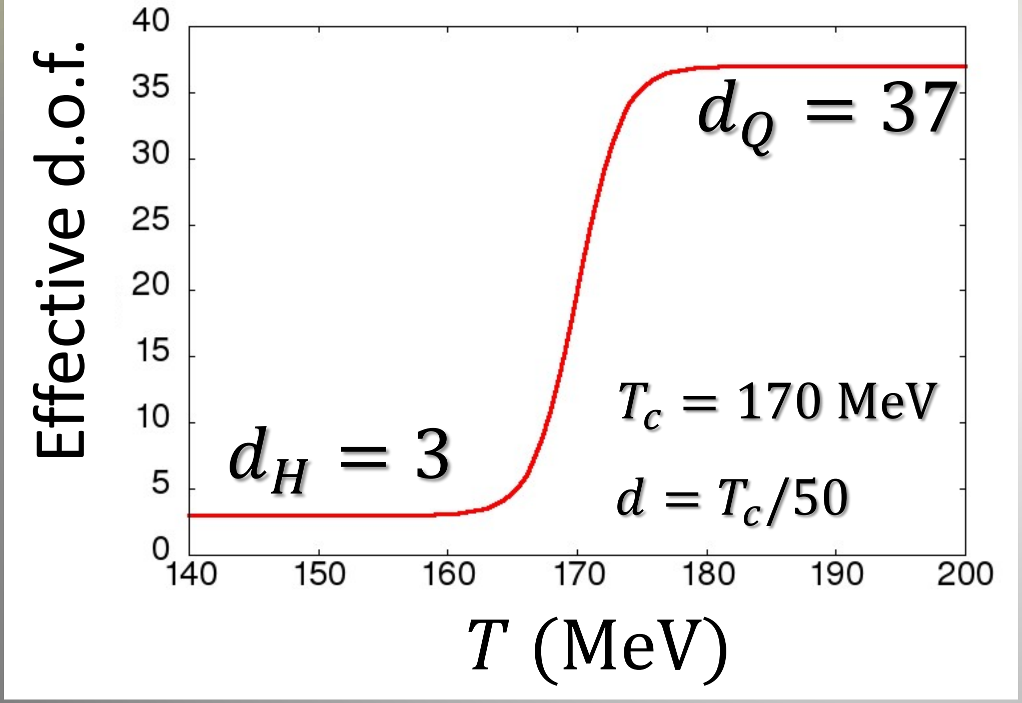
ζ : Bulk viscosity

τ_π, τ_Π : Relaxation time

Hydrodynamic noises!

Model for Equation of State

$$d_{\mathrm{eff}} = d_H \frac{1 - \tanh\frac{T - T_c}{d}}{2} + d_Q \frac{1 + \tanh\frac{T - T_c}{d}}{2}$$



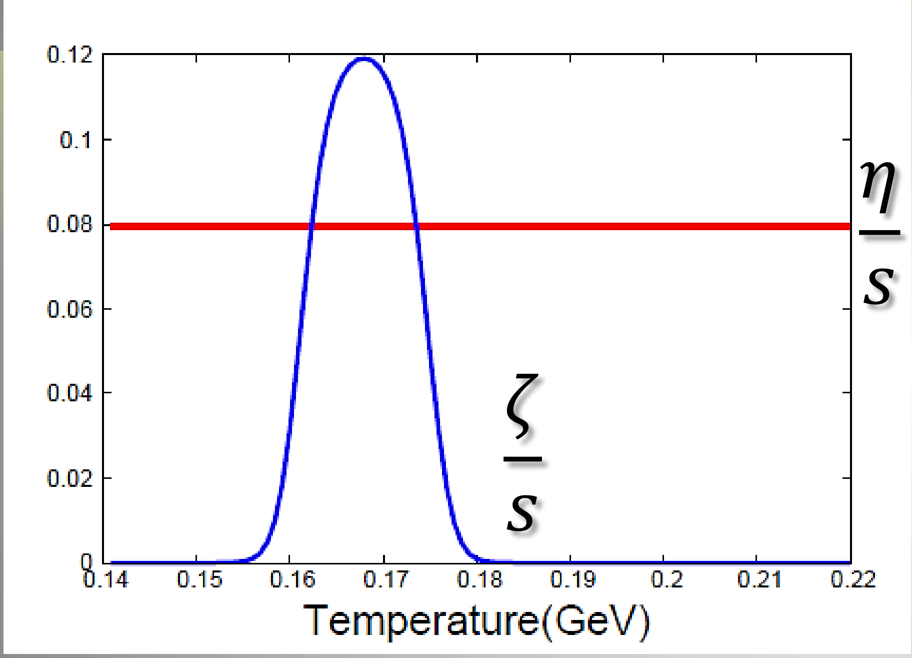
$$s(T) = d_{\mathrm{eff}} \frac{4\pi^2}{90} T^3$$

$$p(T) = \int_0^T s(T')dT'$$

$$e = Ts - p$$

D.H.Rischke, nucl-th/9809044

Models for Transport Coefficients



Shear viscosity

$$\frac{\eta}{s} = \frac{1}{4\pi}$$

P.Kovtun *et al*., PRL94, 111601 (2005)

Bulk viscosity

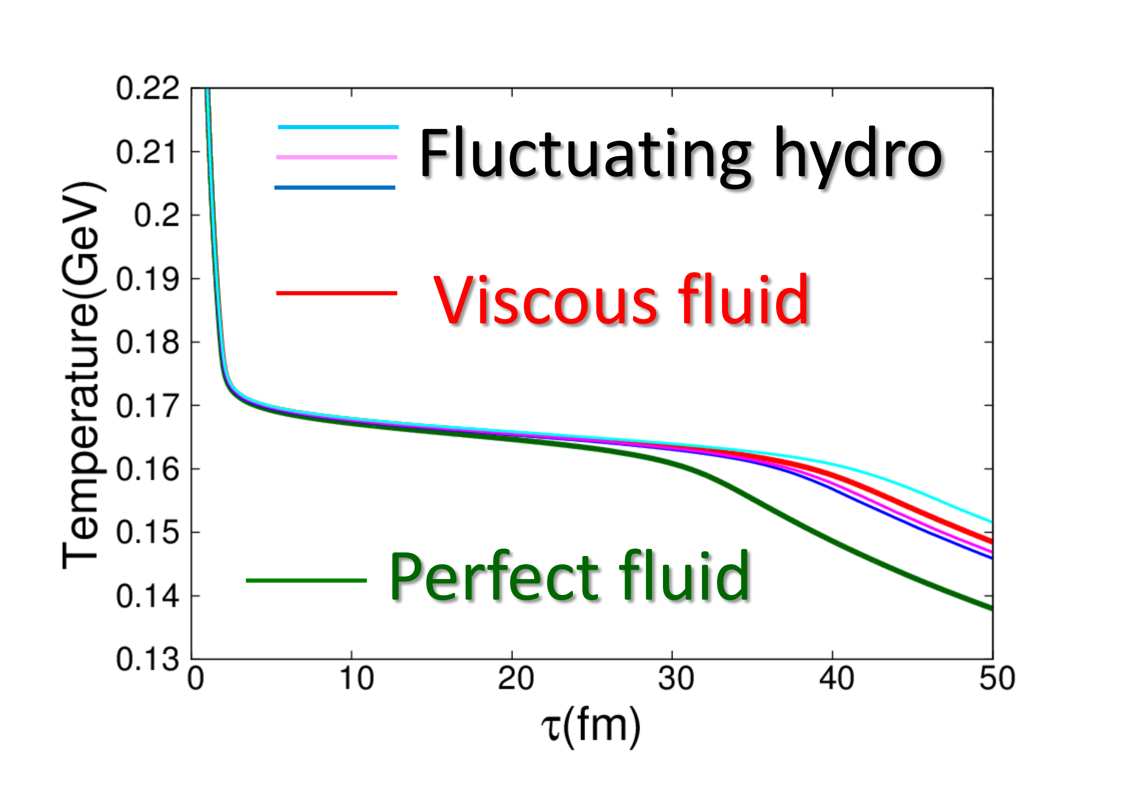
$$\frac{\zeta}{s} = 15\left(\frac{1}{3} - c_s^2\right)^2 \frac{\eta}{s}$$

S.Weinberg, Astrophys.J.168, 175 (1971)

Relaxation time $\tau_\pi = \tau_\Pi = \frac{3\eta}{2p}$

Caveat: Just for demonstration!

Time Evolution of Temperature



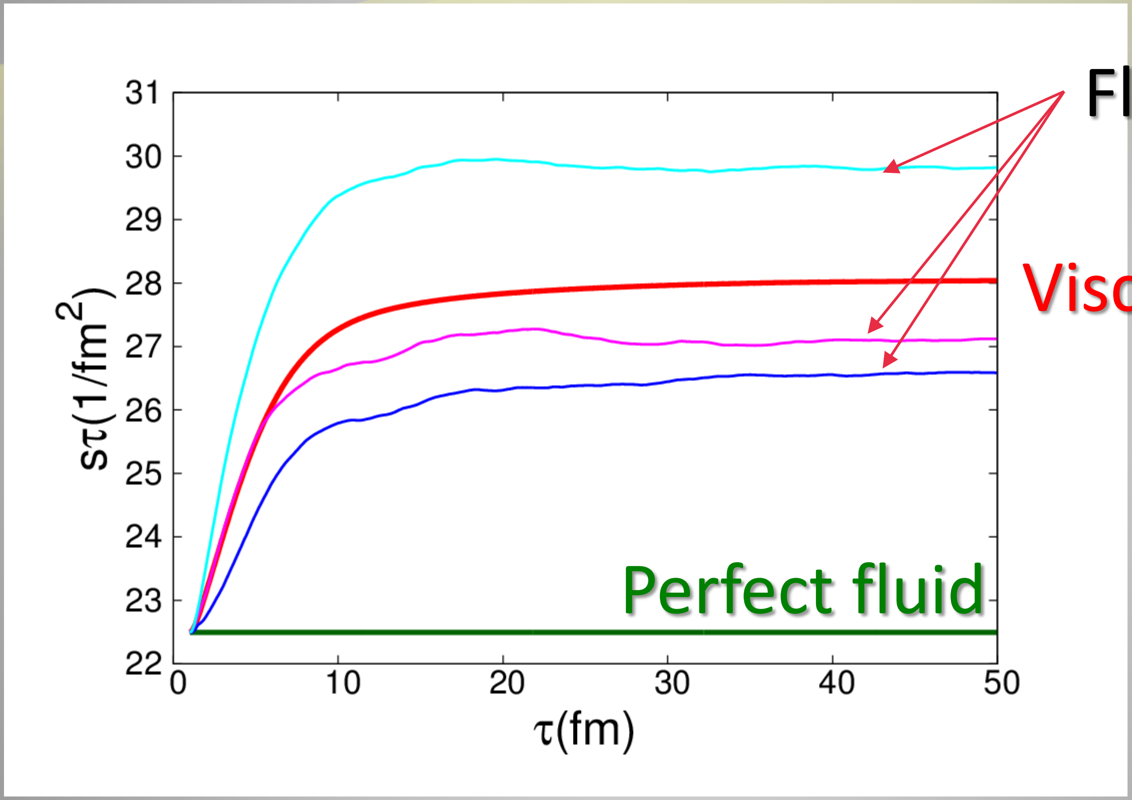
Initial conditions

$\tau_0 = 1\ \mathrm{fm}$

$T_0 = 0.22\ \mathrm{GeV}$

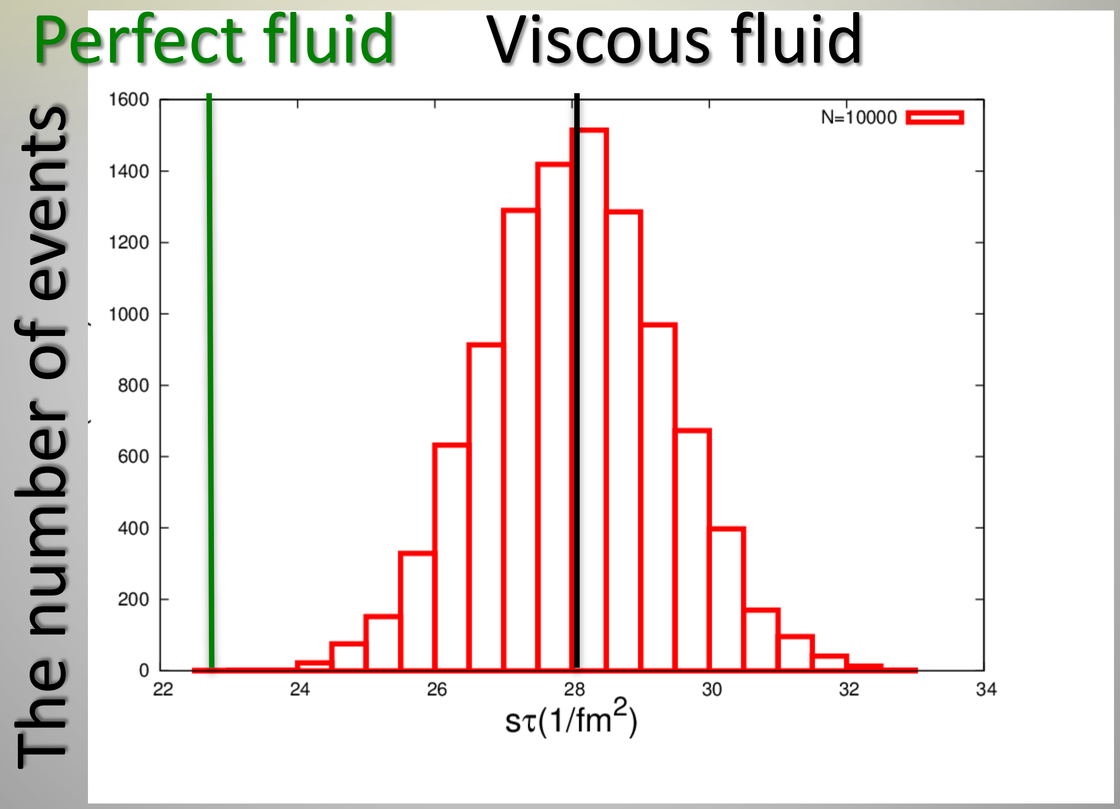
$\pi_0 = \Pi_0 = 0$

Time Evolution of Entropy



Fixed initial condition
→ Final entropy fluctuation
due to hydrodynamic noises

Entropy Distribution



Fluctuating entropy around mean value

Fluctuation Theorem

D.J.Evans et al., PRL71,2401(1993); G.Gallavotti and E.G.D.Cohen, PRL74, 2694(1995);
D.J.Evans and D.J.Searles, PRE52, 5839(1995).

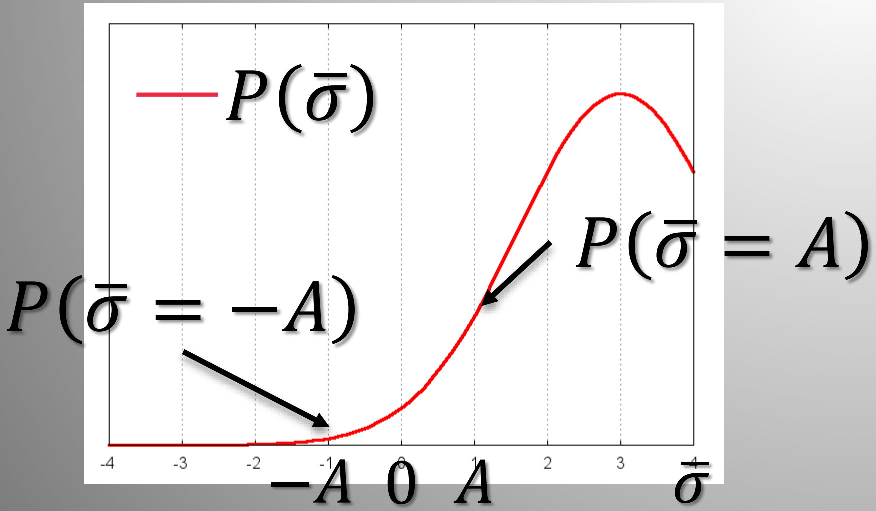
(Transient) Fluctuation Theorem*

$$\frac{P(\bar{\sigma}=A)}{P(\bar{\sigma}=-A)}=e^{At}$$

$P(\bar{\sigma})$: Probability distribution of entropy production rate

$$\bar{\sigma}(t)=\frac{1}{t}\int_0^t \sigma(s)ds$$

Entropy production rate averaged over $0<s<t$



Probability of negative entropy production
← Quantified through fluctuation theorem!

*Milestone in non-equilibrium statistical physics since linear response theory

Entropy Fluctuation

Consequence of fluctuation theorem in Bjorken expansion*

$$\frac{\Delta S_{\mathrm{fin}}}{S_{\mathrm{fin}}} = \frac{\sqrt{2\langle\Delta(\tau s)\rangle}}{\tau_0 s_0 + \langle\Delta(\tau s)\rangle} \frac{1}{\sqrt{\Delta\eta_s \Delta x \Delta y}}$$

$\langle\Delta(\tau s)\rangle$: Average entropy production per volume of local thermal system

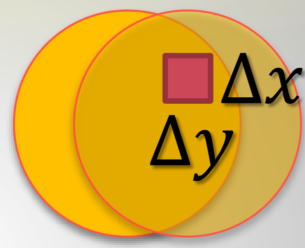
$\Delta V = \tau\Delta\eta_s\Delta x\Delta y$: Volume of local thermal system

*For brief derivation, see backup slide

Entropy Fluctuation (contd.)

The number of independent local thermal system:

$$N = \frac{S(b)}{\Delta x \Delta y}$$



<u>Theorem</u>: Upper bound of total entropy fluctuation in Bjorken expansion

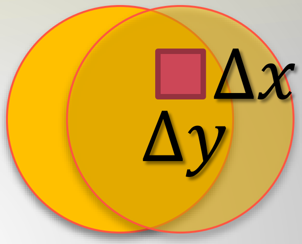
$$\frac{\Delta S_{\mathrm{tot}}}{S_{\mathrm{tot}}} = \frac{1}{\sqrt{N}} \frac{\Delta S_{\mathrm{fin}}}{S_{\mathrm{fin}}} = \frac{\sqrt{2\langle \Delta(\tau s) \rangle}}{\tau_0 s_0 + \langle \Delta(\tau s) \rangle} \frac{1}{\sqrt{\Delta \eta_s S(b)}}$$

$$\leq \frac{1}{\sqrt{2\tau_0 s_0}} \frac{1}{\sqrt{\Delta \eta_s S(b)}} = \frac{1}{\sqrt{2 S_{\mathrm{ini}}}}$$

Entropy Fluctuation (contd.)

The number of independent local thermal system:

$$N = \frac{S(b)}{\Delta x \Delta y}$$

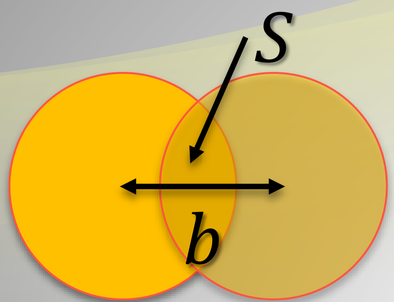


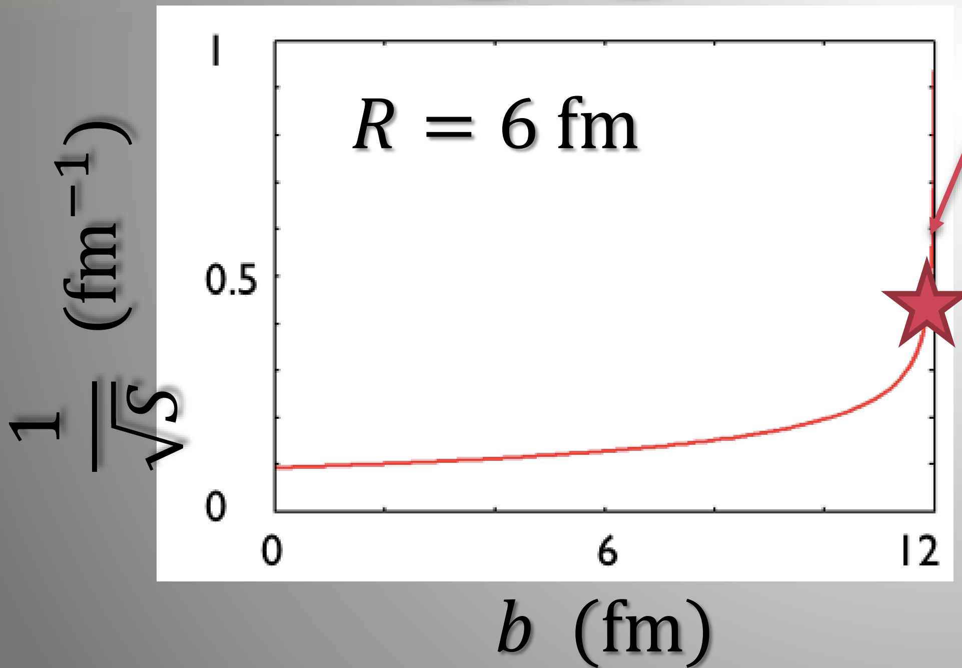
<u>Theorem</u>: Upper bound of total entropy fluctuation in Bjorken expansion

$$\frac{\Delta S_{\mathrm{tot}}}{S_{\mathrm{tot}}} = \frac{1}{\sqrt{N}} \frac{\Delta S_{\mathrm{fin}}}{S_{\mathrm{fin}}} = \frac{\sqrt{2\langle \Delta(\tau s) \rangle}}{\tau_0 s_0 + \langle \Delta(\tau s) \rangle} \frac{1}{\sqrt{\Delta \eta_s S(b)}}$$

$$\leq \frac{1}{\sqrt{2\tau_0 s_0}} \frac{1}{\sqrt{\Delta \eta_s S(b)}} = \frac{1}{\sqrt{2 S_{\mathrm{ini}}}}$$

Centrality Dependence of Entropy Fluctuation





p-p collision

$$\sigma_{\rm in} = 61 \text{ mb at LHC}$$
$$\sigma_{\rm in} = 42 \text{ mb at RHIC}$$

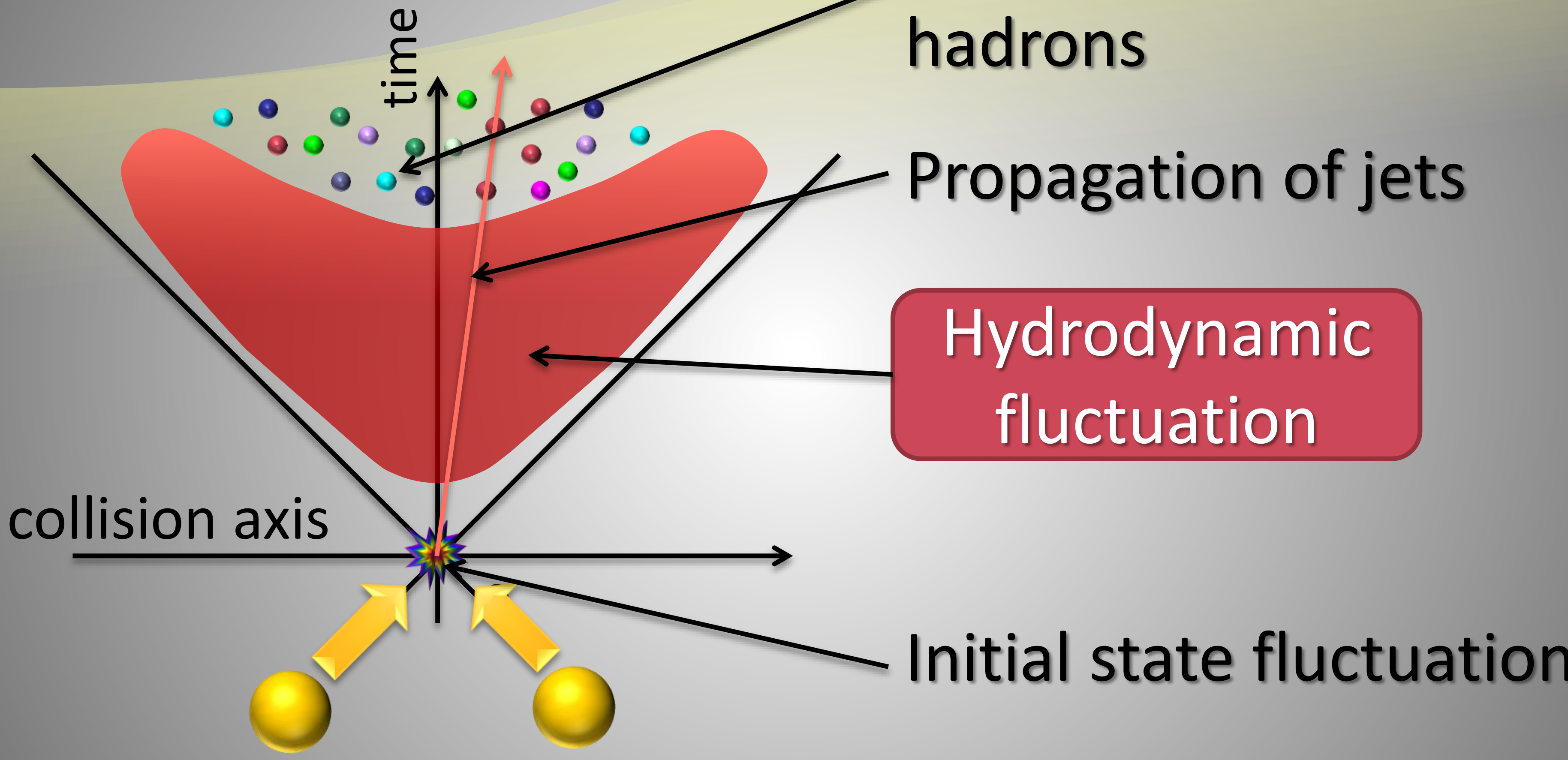
$$\frac{1}{\sqrt{\sigma_{\rm in}}} \sim 0.4 - 0.5$$

Enhancement of fluctuation effects in small system

Summary and Outlook

- Hydrodynamic fluctuation in 1D Bjorken expansion
- Final entropy (multiplicity) fluctuation due to hydro noises
- Fluctuation theorem in 1D Bjorken expansion → Upper bound of fluctuation
- More quantitative analysis needed
  - Fluctuation of flow velocity?
  - Higher harmonics in 2D expansion?

Introduction

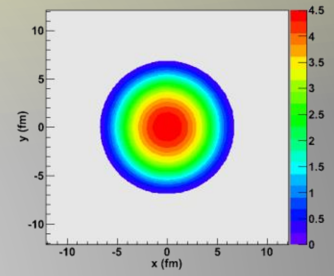


Fluctuations appear everywhere!

Size of Coarse-Grained System

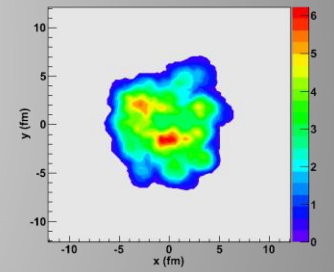
- Hydro at work to describe elliptic flow (~ 2001)

$d \lesssim 5$ fm



- E-by-e hydro at work to describe higher harmonics (~ 2010)

$d \lesssim 1$ fm



- Hydro at work in p-p and/or p-A??? (2012-)

$d \lesssim 1$ fm?

Is fluctuation important in such a small system?

Green-Kubo Formula

$$\eta = \lim_{\omega\to 0}\lim_{q\to 0}\frac{1}{2\omega}\int dtdx\, e^{i(\omega t - qx)}$$

$$\times\left\langle\left[T_{xy}(t,x),T_{xy}(0,0)\right]\right\rangle$$

Slow dynamics → How slow?

Macroscopic time scale ~ $1/\omega$ ← $t_{\text{"macro"}}$

Microscopic time scale ~ τ

cf.) Long tail problem (liquid in 2D, glassy system, super-cooling, etc.)

Relaxation and Causality

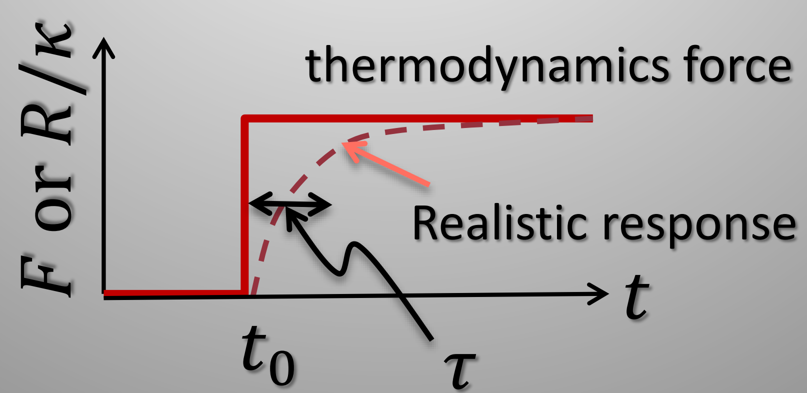
Constitutive equations at Navier-Stokes level

$$\pi^{\mu\nu} = 2\eta\partial^{<\mu}u^{\nu>},$$
$$\Pi = -\varsigma\partial_\mu u^\mu,$$
$$\ldots$$

Instantaneous response violates causality

→ Critical issue in relativistic theory

→ Relaxation plays an essential role



Causal Hydrodynamics

Linear response to thermodynamic force

$$\Pi(t) = \int dt' G_R(t, t') F(t')$$

Retarded Green function (as an example)

$$G_R(t, t') = \frac{\kappa}{\tau} \exp\left(-\frac{t - t'}{\tau}\right) \theta(t - t')$$

Differential form

$$\dot{\Pi}(t) = -\frac{\Pi(t) - \kappa F(t)}{\tau}, \qquad v_{\mathrm{signal}} = \sqrt{\frac{\kappa}{\tau}} < c$$

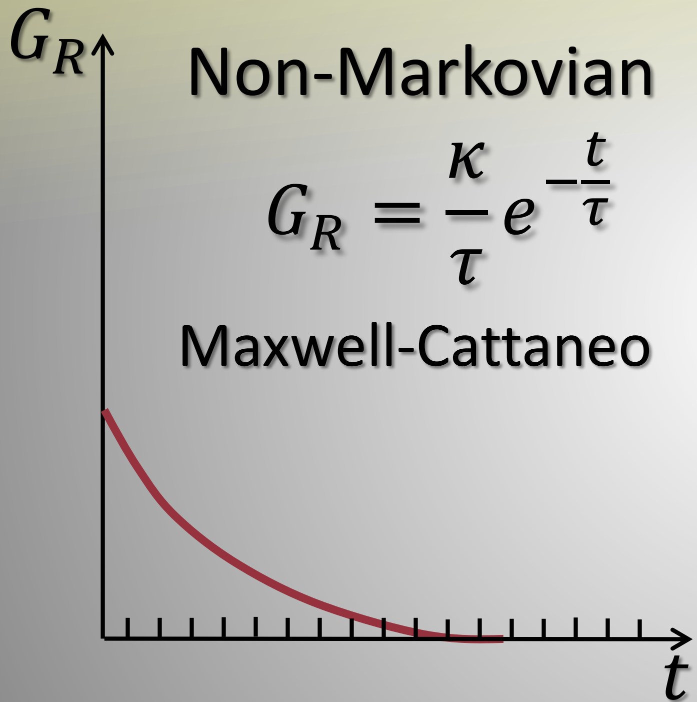
⇨ Maxwell-Cattaneo Eq. (simplified Israel-Stewart Eq.)

Coarse-Graining in Time

Non-Markovian

$$G_R = \frac{\kappa}{\tau} e^{-\frac{t}{\tau}}$$

Maxwell-Cattaneo



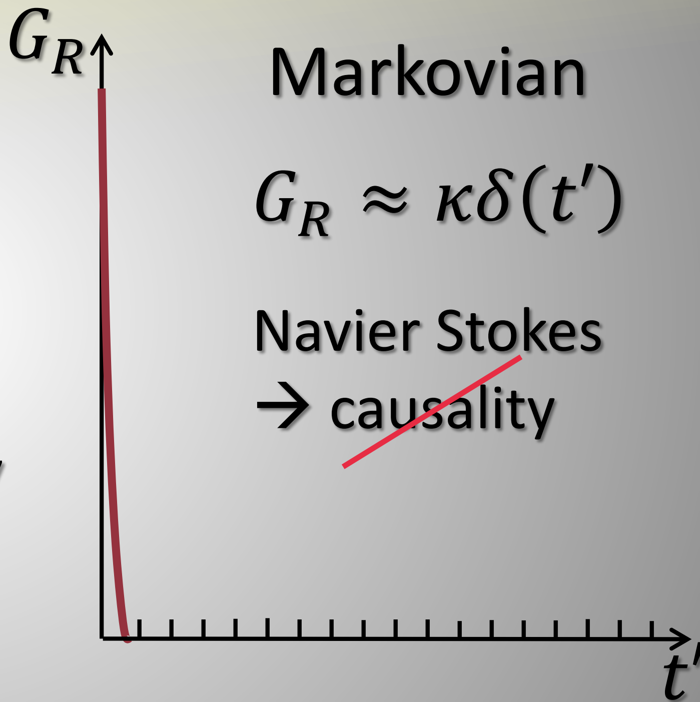
coarse graining ???

$t \to t'$

Markovian

$$G_R \approx \kappa\delta(t')$$

Navier Stokes
→ ~~causality~~

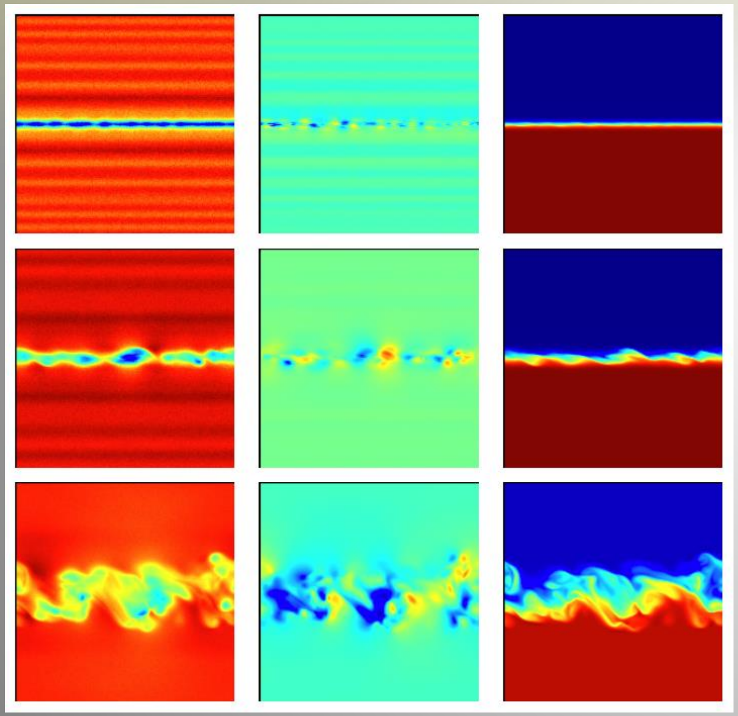


Existence of upper bound in coarse-graining time (or lower bound of frequency) in relativistic theory???

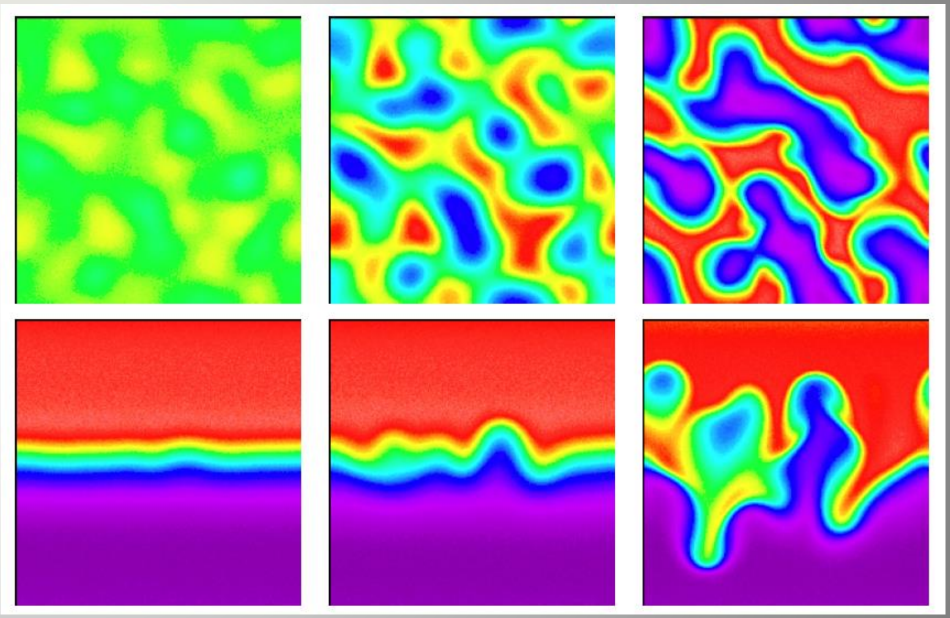
Figures from J.B.Bell *et al*., ESAIM 44-5 (2010)1085

Need Fluctuation?

Ex.) Seeds for instabilities



Kelvin-Helmhortz instability



Rayleigh-Taylor instability

Non-linearity, instability, dynamic critical phenomena,…

Relativistic Fluctuating Hydrodynamics

Causal Hydrodynamics

Linear response to thermodynamic force

$$\Pi(t) = \int dt' G_R(t,t') F(t')$$

	Dissipative current Π	Thermodynamic force F
Shear	Shear stress tensor	Gradient of flow
Bulk	Bulk pressure	Divergence of flow
Diffusion	Diffusion current	Gradient of chemical potential

G_R → Transport coefficient, relaxation time, …

Relativistic Fluctuating Hydrodynamics (RFH)

- Generalized Langevin Eq. for currents

$$\Pi(x) = \int d^4x' G_R(x,x')F(x') + \delta\Pi(x)$$

- Fluctuation-Dissipation Relation (F.D.R.)

$$\langle \delta\Pi(x)\delta\Pi(x')\rangle = TG^*(x,x')$$

G^* : Symmetrized correlation function

$\delta\Pi$: Hydrodynamic fluctuation

K. Murase and TH, arXiv:1304.3243[nucl-th]

Colored Noise in Relativistic System

$$G_R(t,t') = \frac{\kappa}{\tau}\exp\left(-\frac{t-t'}{\tau}\right)\theta(t-t')$$

$$\left\langle \delta\Pi^*_{\omega,\boldsymbol{k}}\delta\Pi_{\omega',\boldsymbol{k}'}\right\rangle = 2\kappa\frac{(2\pi)^4\delta(\omega-\omega')\,\delta^{(3)}(\boldsymbol{k}-\boldsymbol{k}')}{1+\omega^2\tau^2}$$

→ Colored noise!

→ (Indirect) consequence of causality

K. Murase and TH, arXiv:1304.3243[nucl-th]

Integral Form vs. Differential Form

Integral form

$$\Pi(x) = \int d^4x' G_R(x, x') F(x') + \delta\Pi(x)$$

Differential form

$$\tau \frac{d\Pi(x)}{dt} + \Pi(x) = F(x) + \xi(x)$$

$\xi = \mathcal{L}(d/dt)\delta\Pi$: white noise

→ No memory effect nor colored noise

→ Practically convenient

K. Murase and TH, arXiv:1304.3243[nucl-th]

Discretization

$$\tau_\pi \frac{d\pi}{d\tau} + \pi = \frac{4\eta}{3\tau} + \xi_\pi$$

Discretized

$$\pi(\tau + \Delta\tau) = \pi(\tau) + \left(-\frac{\pi}{\tau_\pi} + \frac{4\eta}{3\tau_\pi \tau} + \frac{\xi_\pi}{\tau_\pi} \right) \Delta\tau$$

Some Details about Noises

F.D.R. for shear viscosity

$$\left\langle {\xi_\pi}^{\mu\nu}(x){\xi_\pi}^{\alpha\beta}(x')\right\rangle = 4T\eta\Delta^{\mu\nu\alpha\beta}\delta^{(4)}(x-x')$$

$$\Delta^{\mu\nu\alpha\beta} = \frac{1}{2}\left(\Delta^{\mu\alpha}\Delta^{\nu\beta} + \Delta^{\mu\beta}\Delta^{\nu\alpha}\right) - \frac{1}{3}\Delta^{\mu\nu}\Delta^{\alpha\beta}$$

$$\Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu$$

Bjorken expansion case $u^\mu = (\cosh\eta_s, 0, 0, \sinh\eta_s)$

$$\pi = \pi^{00} - \pi^{33} \Longrightarrow \xi_\pi = \xi_\pi^{00} - \xi_\pi^{33}$$

Some Details about Noises (contd.)

Need F.D.R. for $\xi_\pi = \xi_\pi^{00} - \xi_\pi^{33}$

$$\langle \xi_\pi(x)\xi_\pi(x') \rangle = \langle \xi_\pi^{00}\xi_\pi^{00} \rangle - 2\langle \xi_\pi^{00}\xi_\pi^{33} \rangle + \langle \xi_\pi^{33}\xi_\pi^{33} \rangle$$

Magnitude of fluctuation in discretized space-time

$$\Longrightarrow \frac{4T\eta}{\Delta\tau\Delta V}\left(\Delta^{0000} - 2\Delta^{0033} + \Delta^{3333}\right) = \frac{8T\eta}{3\Delta\tau\Delta V}$$

Width of Gaussian white noise

$$\Delta V = \tau\Delta\eta_s\Delta x\Delta y, \ \Delta\eta_s = 1, \Delta x = \Delta y = 1 \text{ (fm)}$$

Entropy Production Rate in Bjorken Expansion

Our approach

$$\frac{d(s\tau)}{d\tau} = \frac{\pi - \Pi}{T} \lesseqgtr 0$$

Entropy production
→ Not positive definite
due to hydrodynamic noises
→ 2nd law of thermodynamics
on (ensemble) average

Conventional method

$$\frac{d(s\tau)}{d\tau} = \frac{\tau}{T}\left(\frac{3\pi^2}{4\eta} + \frac{\Pi^2}{\zeta}\right) \geq 0$$

Constitutive equations
designed to obey 2nd law
of thermodynamics

Derivation of Theorem

Suppose entropy production rate obeys Gaussian

$$P(\bar{\sigma}) = \frac{1}{\sqrt{2\pi a^2}} \exp\left[-\frac{(\bar{\sigma} - \langle\bar{\sigma}\rangle)^2}{2a^2}\right]$$

From fluctuation theorem,

$$\frac{2\langle\bar{\sigma}\rangle}{a^2} = t \quad \Longrightarrow ta = \sqrt{2\langle\bar{\sigma}\rangle t}$$

$$P(S_{\mathrm{fin}}) \propto \exp\left[-\frac{(S_{\mathrm{fin}} - \langle S_{\mathrm{fin}}\rangle)^2}{2\left(\sqrt{2\langle\bar{\sigma}\rangle t}\right)^2}\right]$$

Evolution of Bulk Pressure