Effective lattice Polyakov loop theory for investigations of dense nuclear matter

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The effective theory and its derivation from QCD

- QCD partition function with Yang-Mills action S_g and quark fermion matrix Q for N_f number of flavors on the lattice (Wilson fermions)

$$Z = \int [dU_\mu]\ \exp\left[-S_g(\beta)\right] \prod_{f=1}^{N_f} \det\left[Q^f(\kappa)\right]\ , \quad -S_g = \frac{\beta}{2N_c}\sum_p \left[\mathrm{tr}\, U_p + \mathrm{tr}\, U_p^\dagger\right]\ ,$$

- parameters:
  - $\beta = \frac{2N_c}{g_0^2}$; $\kappa = \frac{1}{2(4+m_0)}$; bare gauge coupling g_0 and bare quark mass m_0
  - number of lattice sites in τ direction N_τ; lattice spacing a; temperature $T = \frac{1}{aN_\tau}$
- effective Polyakov loop action S_{eff} obtained from an integration of spatial links U_k:

$$\exp[-S_{\mathrm{eff}}] \equiv \int [dU_k] \exp\left[-S_g\right] \prod_{f=1}^{N_f} \det\left[Q^f\right]\ ,$$

- dimensional reduction from $3+1D$ to $3D$
  $U_\mu(x,t) \to U_0(x) \to$ Polyakov loops $L(x)$

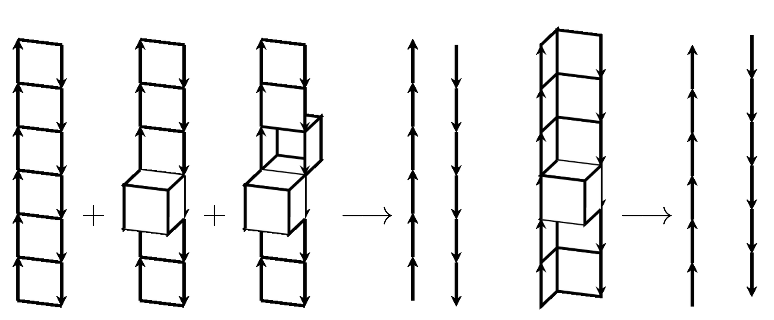
$$L(\mathbf{x}) = \mathrm{Tr}\, W(\mathbf{x}) = \mathrm{Tr}[\prod_{\tau=0}^{N_\tau} U_0(\mathbf{x},\tau)] = \mathcal{P} e^{ig\int_0^{\frac{1}{T}} d\tau A_0(\mathbf{x},\tau)}$$

- remaining path integral

$$Z = \int [dL] e^{-S_{\mathrm{eff}}[L]}$$

The strong coupling and hopping parameter expansion

Effective Yang-Mills action



$$S_{\mathrm{eff}} = \lambda_1 S_{\text{nearest neighbors}} + \lambda_2 S_{\text{next to nearest neighbors}} + \ldots$$

- strong coupling expansion parameter $u = \frac{\beta}{18} + \ldots < 1$
- ordering principle for the interactions: higher representations and long distances are suppressed ($\lambda_1 = O(u^{N_\tau})$, $\lambda_2 = O(u^{2N_\tau})$)
- strong coupling approach suggests logarithmic form of the nearest neighbor interactions

$$e^{-S_{\mathrm{eff}}} \approx \prod_{<i,j>\ \text{nearest n.}} \left(1 + 2\lambda_1 \mathrm{Re}\left(L_i L_j^\dagger\right)\right)$$

Effective quark action

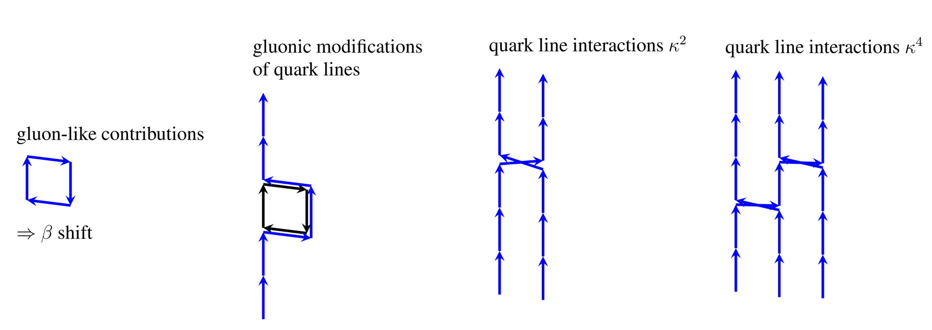
- Wilson-Dirac operator: $Q = 1 - \kappa H[U]$ in quark action

$$S_q = -N_f\, \mathrm{Tr}\log(1-\kappa H) = N_f \sum_l \frac{\kappa^l}{l}\, \mathrm{Tr}\, H^l$$

- expansion around heavy quark limit, $\kappa = \frac{1}{2(4+m_0)} \ll 1$
- static quarks: only propagation in τ direction $\Rightarrow$ Polyakov loop L

$$\det(1 + T^- + T^+) = \prod_n (1 + cL_n + c^2 L_n^\dagger + c^3)^2 (1 + \bar{c}L_n^\dagger + \bar{c}^2 L_n + \bar{c}^3)^2$$

- higher orders: spatial propagation $\Rightarrow$ non-trivial interactions of Polyakov loops e. g.



- chemical potential μ
- quarks L (T^+) get factors $e^{a\mu}$: $c = (2\kappa e^{a\mu})^{N_\tau}$
- anti-quarks $L^\dagger$ (T^-) get factors $e^{-a\mu}$: $\bar{c} = (2\kappa e^{-a\mu})^{N_\tau}$

$\Rightarrow$ interactions up to $\kappa^n + u^m$, $m+n=4$ included

Low temperature limit in the heavy dense regime

- low temperature: N_τ large
- heavy: $\kappa \ll 1$
- dense: $2\kappa e^{a\mu} \approx 1$; $\bar{c} \approx 0$

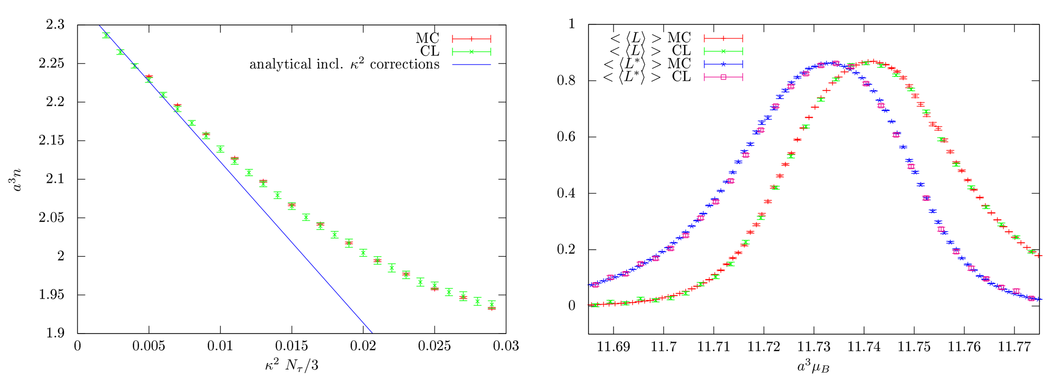
$\Rightarrow$ dominated by short range quark line interactions

Circumvent the sign problem: Numerical and analytic investigations of the effective theory

- sign problem: complex fermion determinant prevents lattice simulations at larger chemical potential

Non-perturbative effects from complex Langevin and standard MC simulations

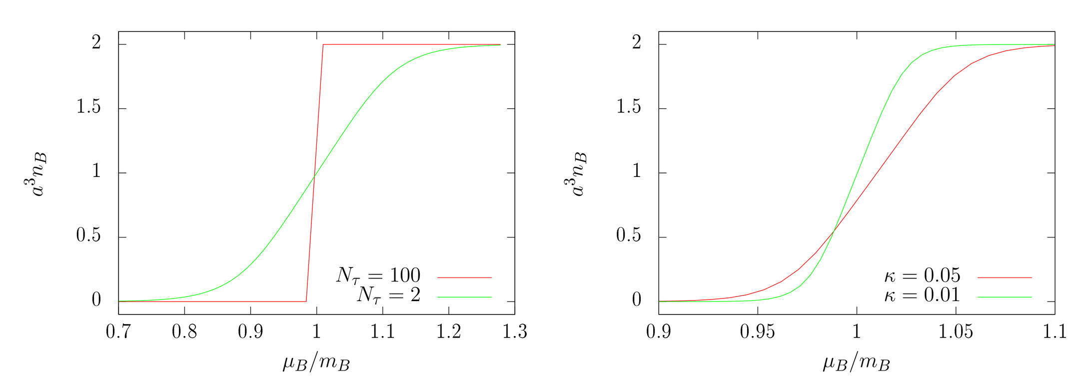
- effective theory inherits only mild version of sign problem
- solution 1: standard MC simulations and reweighting
- solution 2: complex Langevin algorithm
- correctness criteria checked, consistent results



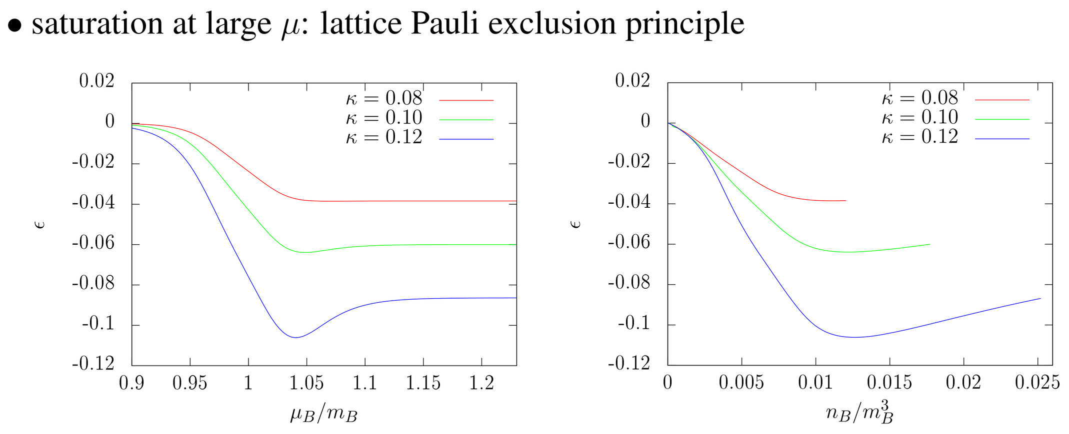
Analytic expansion of the effective theory

- small effective couplings: perturbative expansion of effective theory
- expansion parameter: effective coupling λ_1 and κ^2 two quark line interaction

Nuclear liquid gas transitions in the heavy dense regime of QCD



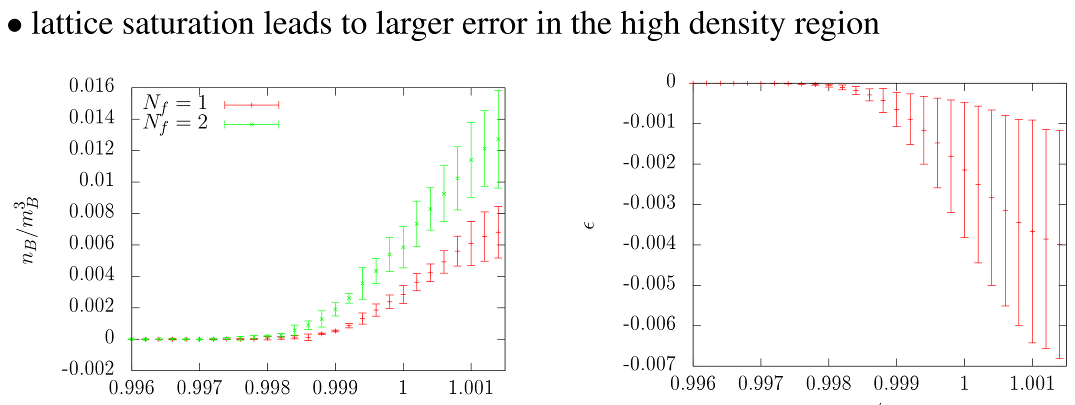
- n_B zero at small μ : "silver blaze"
- $\mu_B \approx m_B$ baryons are excited (step function at $T=0$)
- saturation at large μ: lattice Pauli exclusion principle



- onset below $\mu_B = m_B$ due to nuclear binding energy
- energy density: e
- binding energy per nucleon: $\epsilon = \frac{e - n_B m_B}{n_B m_B}$
- effect of attractive quark-quark interaction: ϵ negative, decreases with meson mass

Convergence and continuum limit

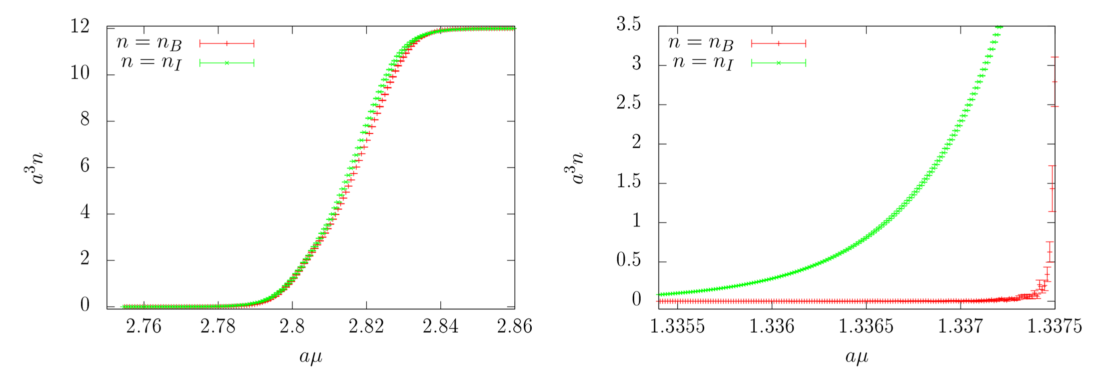
- estimate truncation error: compare κ^2 and κ^4 results
- continuum limit $a \to 0$ at fixed $\frac{m_B}{T}$ and $T = \frac{1}{aN_\tau}$ requires larger values of κ
- combined error: truncation error and uncertainty of continuum extrapolation
- lattice saturation leads to larger error in the high density region



$T = 10$ MeV $\qquad N_f = 2$, $T = 10$ MeV

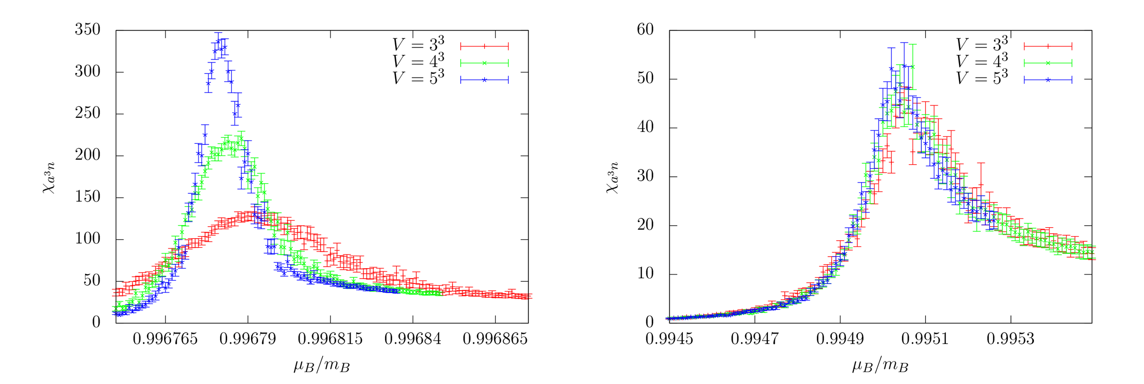
- main features persist in the continuum limit
- heavy quark limit: small binding energy, smooth crossover

Isospin chemical potential



- $\mu_I = \mu_u = -\mu_d$
- pion condensation: $\mu_I = m_\pi/2$
- transitions coincide for static quarks: $m_B/3 = m_\pi/2$
- effect of quark interactions: gap between the two transitions

First studies beyond the heavy mass regime



$N_\tau = 500$, $\kappa = 0.12$ $\qquad N_\tau = 250$, $\kappa = 0.12$

- at large quark masses, higher temperatures: onset transition is a smooth crossover
- at lower masses, lower temperatures: transition becomes first order

$\Rightarrow$ transition between crossover and first order correctly reproduced by effective theory

- conservative estimate of reliable region in current truncation: small difference between $O(\kappa^2)$ and $O(\kappa^4)$
- so far interesting parameters outside this region, but κ^4 approximation might still be reasonable

Conclusions and further directions

- systematic derivation of effective Polyakov loop theory by a combined strong coupling and hopping parameter expansion
- useful tool at finite chemical potential, "solution" to the sign problem
- heavy dense low temperature regime: effective theory reproduces the features of full QCD

Improvements of the effective action: Yang-Mills contribution

- outside heavy dense low temperature regime: gluonic interactions become relevant

$\Rightarrow$ need further improvements of effective theory

- in confined region: ordering principle of effective couplings suggested by strong coupling still valid
- improvement of the effective couplings: include non-perturbative input form simulations of full theory

Improvements of the effective action: quark contribution

- interesting for QCD: lower mass
- higher orders in the κ expansion necessary
- investigations of relevant gluon-quark interactions at higher temperatures

Further investigations

- further investigations of validity outside the heavy dense low temperature regime

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