Quantum JIMWLK From Schwinger-Keldysh QCD

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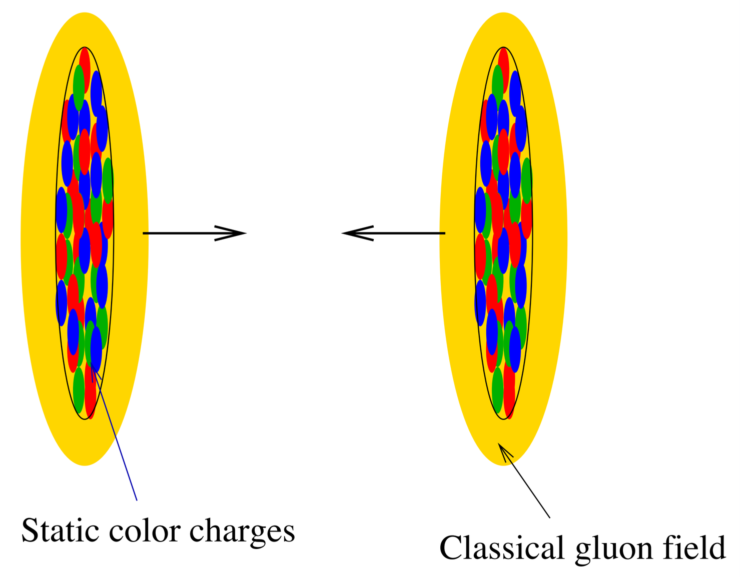
Introduction

- Initial condition of heavy ion physics: Strong colour source + Strong gluon field
- Classical field limit: Large occupation number
- Q: How far can one push Classical description?



- Q: Best way to separate $\mathbf{A}^\mu = \mathcal{A}^\mu_{\rm cl} + \mathbf{a}^\mu = \mathbf{O}(1/g) + \mathbf{O}(1)$?

Schwinger-Keldysh Formalism

- Inclusive and average quantity calculations

$$\langle \mathcal{O}(t)\rangle = \mathrm{Tr}\left(\mathcal{O}\hat\rho(t)\right)$$

- Evolution of the density operator needs 2 evolution ops

$$\hat\rho(t) = e^{-i\hat{H}t}\hat\rho_0 e^{i\hat{H}t} = \sum_n \rho_n e^{-i\hat{H}t}|n\rangle\langle n|e^{i\hat{H}t}$$

- Matrix element:

$$\rho_{lm}(t) = \sum_{p,q} \underbrace{\langle l|e^{-i\hat{H}t}|p\rangle}_{U_{lp}(t)}\langle p|\hat\rho_0|q\rangle \underbrace{\langle q|e^{i\hat{H}t}|m\rangle}_{U^\dagger_{qm}(t)}$$

SK continued

- In path integral form with sources,

$$\rho[J_1, J_2] = \int [d\varphi_1 d\varphi_2 d\phi_f]\langle\phi_f|\hat{U}_{J_1}|\varphi_1\rangle\rho_0[\varphi_1,\varphi_2]\langle\varphi_2|\hat{U}^\dagger_{J_2}|\phi_f\rangle$$
$$= \int [d\varphi_1 d\varphi_2 d\phi_f]\rho_0[\varphi_1,\varphi_2]$$
$$\underbrace{\int_{\varphi_1}^{\phi_f}\mathcal{D}\phi_1 e^{i\int L(\phi_1)+iJ_1\phi_1}}_{\langle\phi_f|\hat{U}_{J_1}|\varphi_1\rangle}\underbrace{\int_{\varphi_2}^{\phi_f}\mathcal{D}\phi_2 e^{-i\int L(\phi_2)-iJ_2\phi_2}}_{(\langle\phi_f|\hat{U}_{J_2}|\varphi_2\rangle)^*}$$

Keldysh Rotation

- Define

$$\phi_1 = \phi_c + \phi_q/2,\quad \phi_2 = \phi_c - \phi_q/2$$

- Lagrangian:

$$L(\phi_1) - L(\phi_2) = \phi_q\underbrace{\left(\frac{\delta L(\phi_c)}{\delta\phi_c} + J_c\right)}_{\text{Classical EoM}} + \frac{1}{3!}\phi_q^3\frac{\delta^3 L(\phi_c)}{\delta\phi_c^3} + J_q\phi_c$$

with the classical source $J_c = (J_1 + J_1)/2$ and $J_q = J_1 - J_2$.

- Path Integral with $J_q = 0$ and the vacuum average:

$$\rho[J_c] = \int [d\varphi_c^{\rm init} d\pi_c^{\rm init}] \underbrace{W[\varphi_c^{\rm init}, \pi_c^{\rm init}]}_{\text{Wigner transform of } \rho_{\rm vac}[\varphi_1,\varphi_2]}$$
$$\int \mathcal{D}\phi_c\mathcal{D}\phi_q \exp\left(i\int \phi_q E[\phi_c, J_c] + O(\phi_q^3)\right)$$

- Classical EoM $E[\phi_c, J_c]$ naturally emerges
- Quantum effects are in $W[\varphi_c^{\rm init}, \pi_c^{\rm init}]$ and $O(\phi_q^3)$

Schwinger-Keldysh QCD

- With $L = \frac{1}{4}G^{\mu\nu}G_{\mu\nu}$, $J = O(1/g)$ and ignoring the cubic term,

$$\rho[J_c] = \int [da_c d\pi_c]\, W[a_c, \pi_c] \int \mathcal{D}A\, \delta[[D_\mu, G^{\mu\nu}]_a - J^\nu_a]\delta[A^0]$$

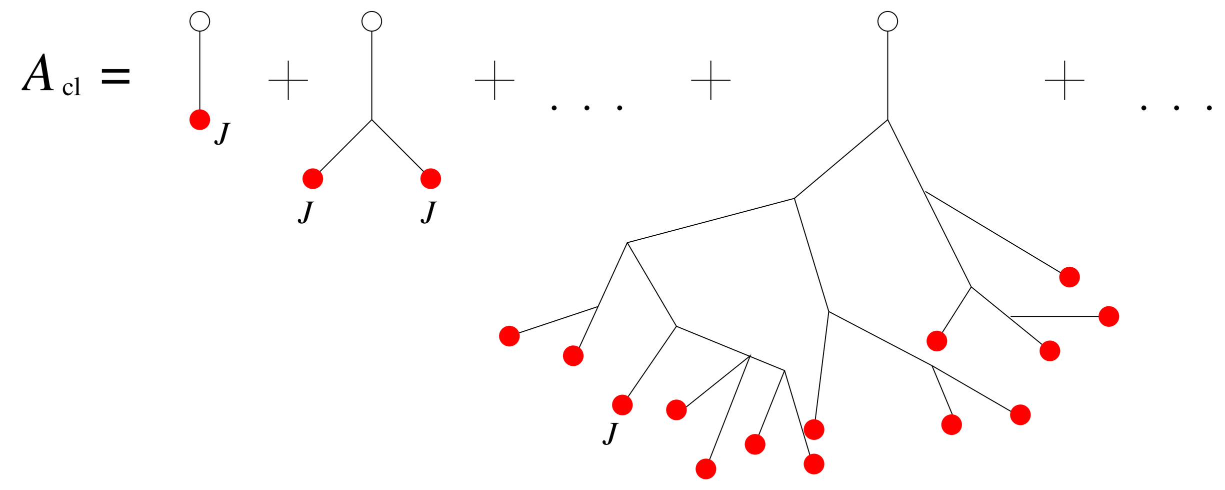
where $D^\mu = \partial^\mu - igA^\mu$ and $G^{\mu\nu} = \frac{i}{g}[D^\mu, D^\nu]$ with $A^\mu \to a^\mu_c$ and $\dot{A}^\mu \to \pi^\mu_c$ as $t \to t_{\rm init}$.

- Fully classical evolution with an explicit gauge condition
- Initial vacuum density functional contains quantum zero-point oscillations:

$$W[a_c, \pi_c] = \exp\left(-\frac{1}{2}\sum_k (E_k a_k^2 + \pi_k^2/E_k)\right)$$

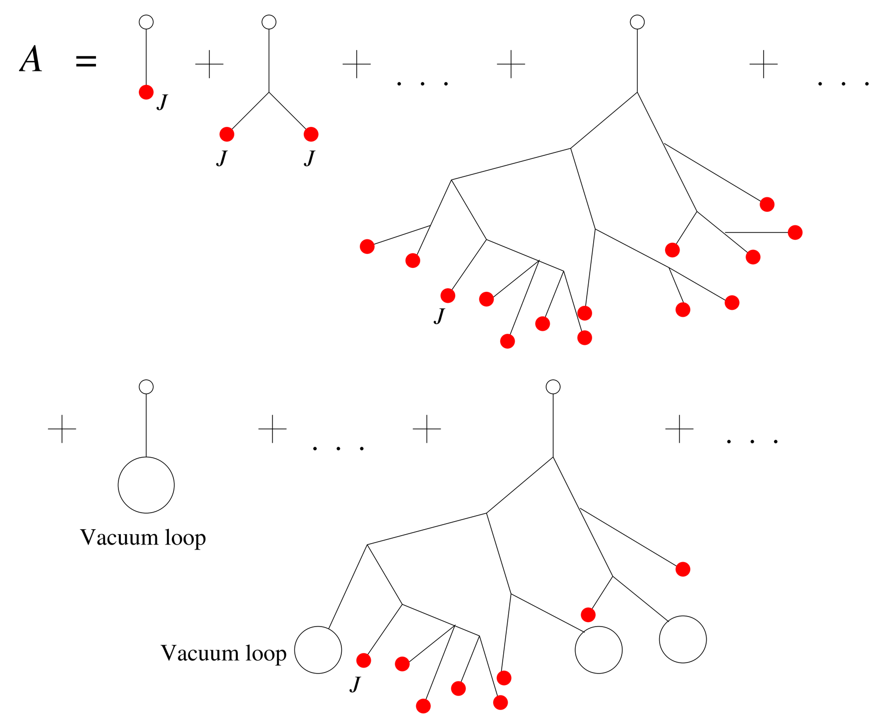
- Can formulate this with $x^\pm = (t \pm z)/\sqrt{2}$ and two colliding sources $\Longrightarrow$ Firm theoretical basis for Glasma

Classical Field



Only the solution of $[D, G] = J$

Semi-Classical



- Vacuum loops due to $W[a_c, \pi_c]$.
- Point: Vacuum loops can be represented by the $\langle J(x)J(x)\rangle$ correlator.
- Can replace $W[a_c, \pi_c] \Longrightarrow W[J]$.

Observables in Semi-Classical SK-QCD

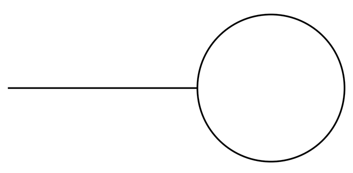
$$\langle\mathcal{O}\rangle = \int [dJ] W[J]\mathcal{O}(A_c[J])$$

- With the classical solution A_c and the distribution $W[J]$ providing the spectrum of color sources and quantum fluctuations.
- Renormalization group equation for the vacuum loop $\Longrightarrow$ JIMWLK equation for $W[J]$ by matching $\langle AA\rangle$ with $\langle JJ\rangle$ and $\langle A\rangle$ with $\langle J\rangle$.

$$\frac{\partial W}{\partial Y} = -\frac{1}{2\pi}\int_{u,v}\frac{\delta}{\delta a_c(u)}\eta(u|v)\frac{\delta}{\delta a_c(v)}W$$

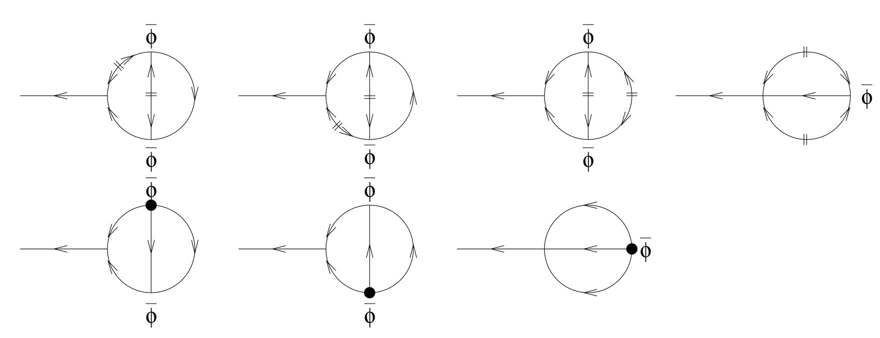
with

$$\eta(u|v) \sim \int_{x,y} G_T(u|x)\left\langle D_\mu a^\mu_c(x) D_\nu a^\nu_c(y)\right\rangle G_T(y|v)$$

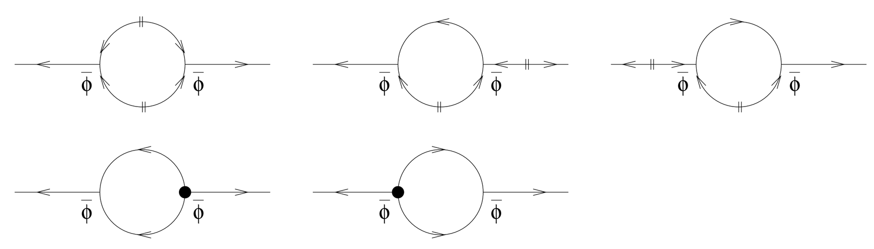
- JIMWLK equation is in Y since  $\sim \log(k_{\rm cut-off}) \sim Y$

NNLO

For $\langle a\rangle$



For $\Sigma_{\rm sym}$



- Dotted vertex: Quantum 3-point correlation
- No fully semi-classical description possible
- Further study: Can one still write down an equation for $W[Y|J]$?

Refs

- S. Jeon, Annals Phys. **340**, 119 (2014)
- L. D. McLerran and R. Venugopalan, Phys.Rev. **D49**, 2233 (1994)
- J. Jalilian-Marian, A. Kovner, L. D. McLerran, and H. Weigert, Phys.Rev. **D55**, 5414 (1997)
- E. Iancu, A. Leonidov, and L. D. McLerran, Nucl.Phys. **A692**, 583 (2001)
- K. Dusling, F. Gelis, and R. Venugopalan, Nucl.Phys. **A872**, 161 (2011)

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