

Domain growth and Fluctuations during Quenched Transition to QGP

¹Ranjita K. Mohapatra and ²Ajit M. Srivastava

¹Physical Research Laboratory, Ahmedabad, ²Institute of Physics, Bhubaneswar, India.

Phys. Rev. C 88, 044901 (2013)



Motivation : To study about non-trivial Z(3) vacuum structure, Z(3) domains and their consequences in QGP phase.

Z(3) CENTER SYMMETRY OF QCD AND POLYAKOV LOOP ORDER PARAMETER

Partition function for a system of gluons in terms of path integral formalism

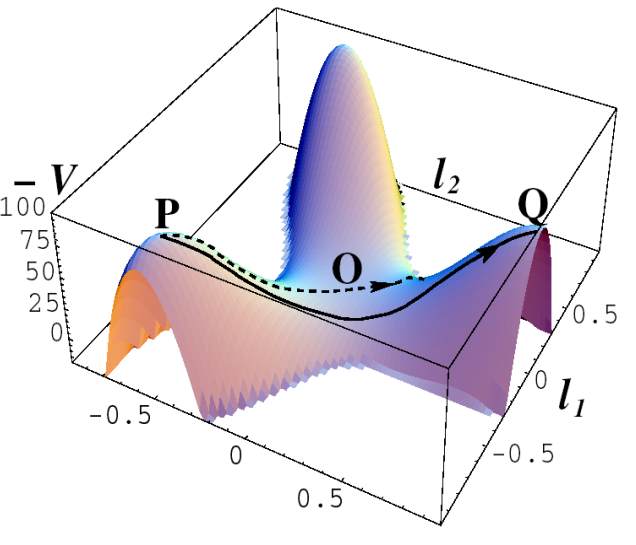
$$Z = \int D[A] \exp(-S[A]) \quad S[A] = \int_0^\beta d\tau \int d^3x (\text{Tr} F_{\mu\nu} F_{\mu\nu})$$

- Gauge fields obey periodic boundary condition $A_\mu(x, \beta) = A_\mu(x, 0)$
- Action is invariant under gauge transformation $A_\mu \rightarrow U(A_\mu + i\partial_\mu)U^{-1}$
- Gauge transformations are also periodic $U(x, \beta) = U(x, 0)$
- Extra symmetry of the theory $U(x, \beta) = ZU(x, 0)$
- Where Z(N) is the centre group of SU(N). (ZU = UZ)

$$Z = e^{2\pi i n / N} \quad n = 0, 1, 2, \dots, N-1$$

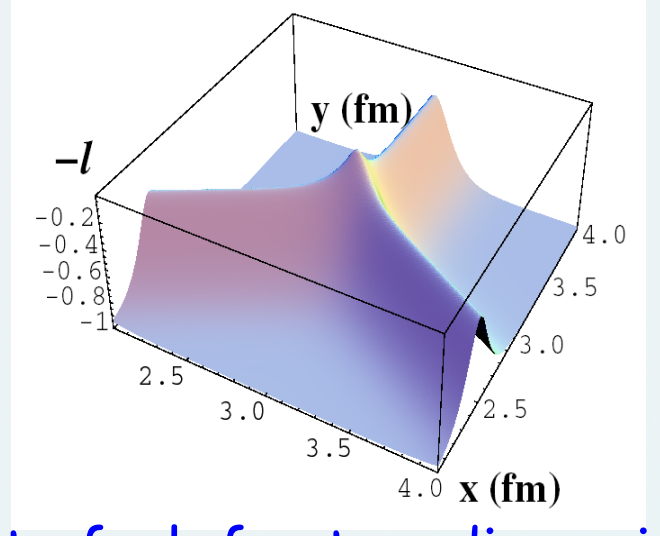
- Finite temperature SU(N) gauge theory has Z(N) symmetry as the Euclidean action is invariant under Z(N) transformation.
- Z(3) symmetry is restored below T_c in the confined phase and spontaneously broken in the deconfined phase giving rise to 3 Z(3) vacua.
- Polyakov Loop Order Parameter : $l(x) = \frac{1}{N} \text{Tr}(P \exp(i g \int_0^\beta A_0(x, \tau) d\tau)) \quad \langle l(x) \rangle \sim e^{-\beta \Delta F}$
- ΔF : change in free energy of the pure glue theory due to addition of an isolated quark
- Under Z(3) symmetry : $l(x) \rightarrow Z l(x)$
- In confined phase, Z(3) symmetry is restored corresponding to $\langle l(x) \rangle = 0$
- In deconfined phase, Z(3) symmetry is spontaneously broken corresponding to finite $\langle l(x) \rangle$

Domain Wall and String



Plot of inverted potential $-V(l)$ in (l_1, l_2) plane

- Domain walls are the solution while going from one vacuum to another vacuum. The solution (solid curve) can never go through origin i.e. l is never zero inside the wall. The intersection of three different interfaces gives rise to string like structure.



Surface plot of $-l$ for 4-dimensional lattice in x-y plane. String ($l=0$) is attached to the three interfaces.

Effective potential based on Polyakov loop

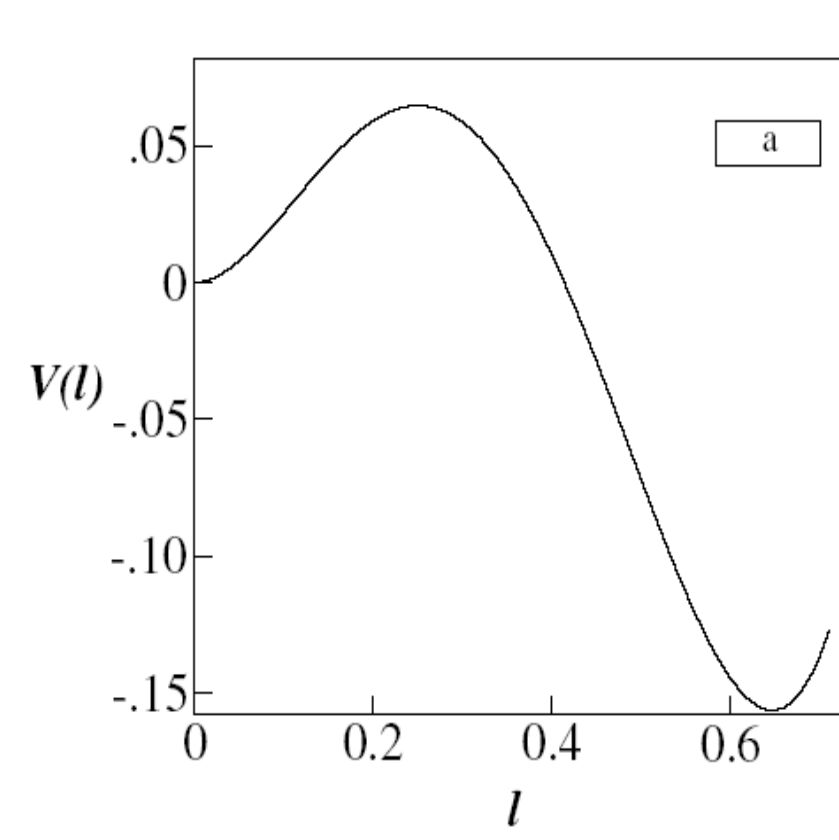
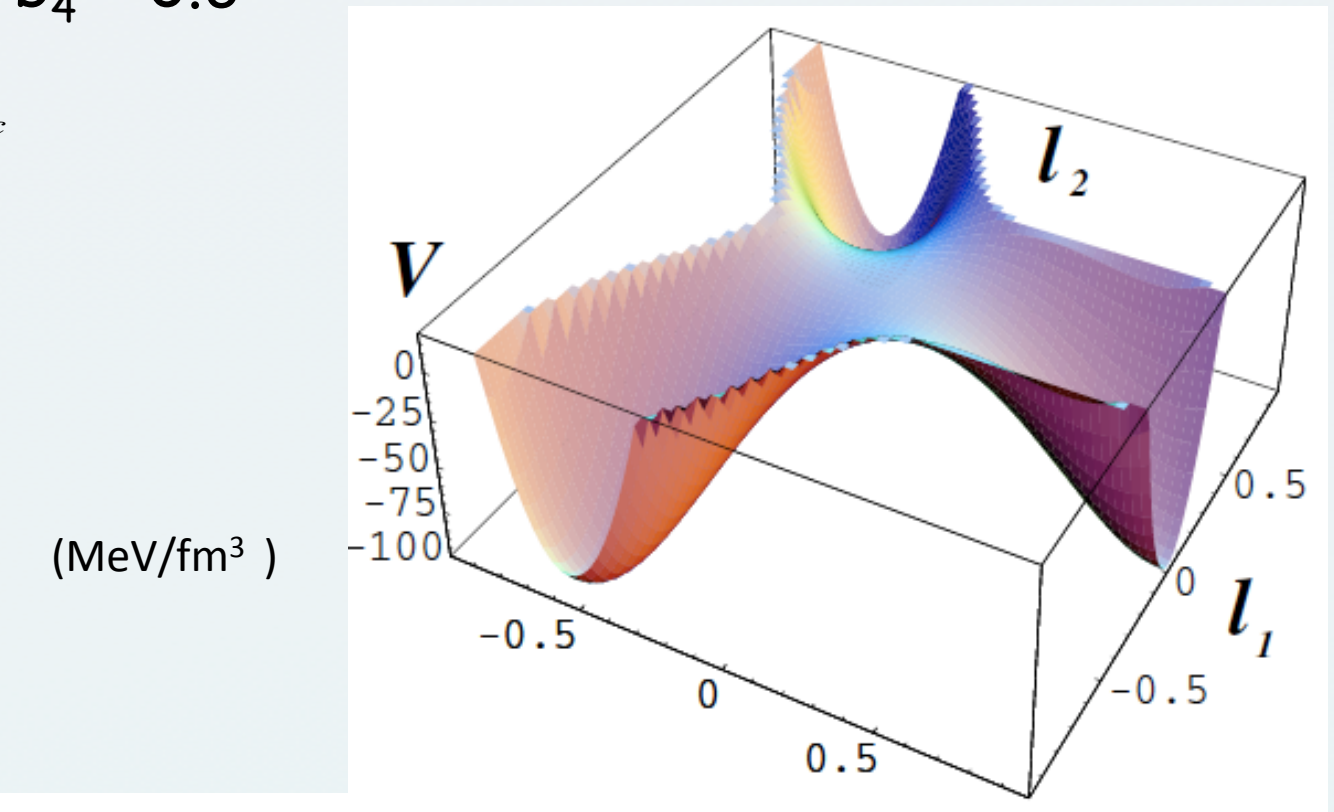
- Effective Lagrangian density $L = \frac{N}{g^2} |\partial_\mu l|^2 T^2 - V(l) \quad V(l) = \left(-\frac{b_2}{2} |l|^2 - \frac{b_3}{6} (l^3 + l^{*3}) + \frac{1}{4} |l|^4 \right) b_4 T^4$
- Normalized such that $l_0 \rightarrow 1$ as $T \rightarrow \infty$. With $l = |l| e^{i\theta}$ b_3 gives $\cos(3\theta)$ leading to Z(3) degenerate vacuum structure.
- Coefficients b_2, b_3, b_4 are chosen to fit lattice results for energy density and pressure in pure SU(3) gauge theory.

$$b_2(r) = \left(1 - \frac{1.11}{r} \right) \left(1 + \frac{0.265}{r} \right)^2 \left(1 + \frac{0.300}{r} \right)^3 - 0.487$$

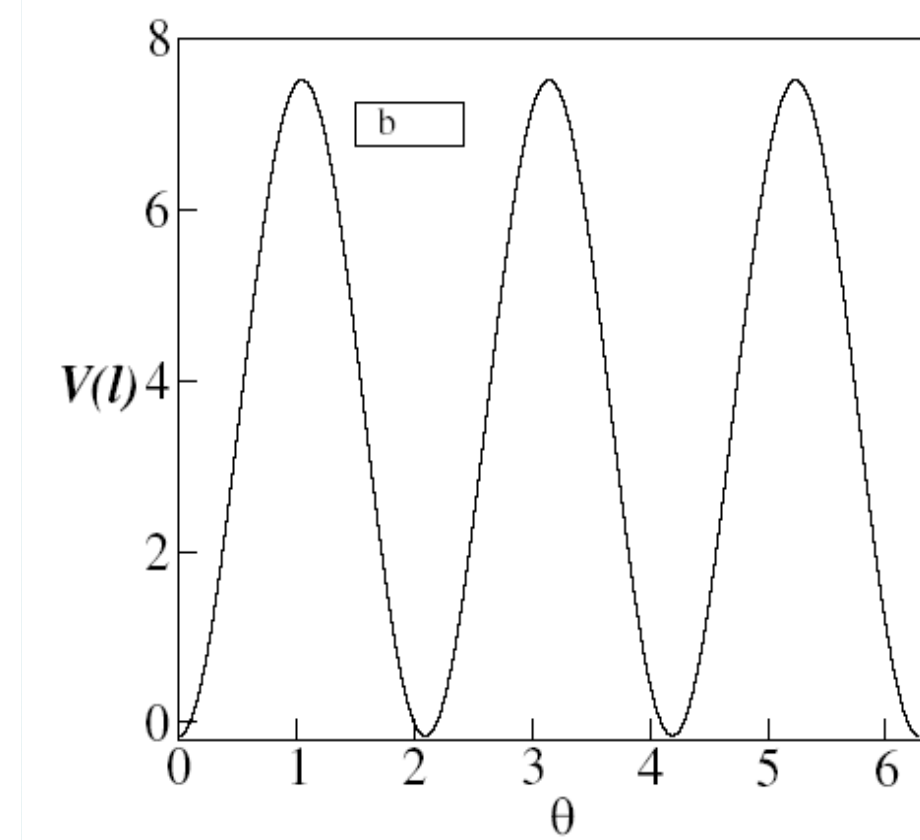
$$b_3 = 2.0, b_4 = 0.6 \quad r = T/T_c$$

R Pisarski, Phys Rev D, 62, 111501(2000)

- This potential represents a weakly first order phase transition with $T_c = 182$ MeV.
- There are 3 degenerate vacua corresponding to the phases at $\theta = 0, 2\pi/3, 4\pi/3$.



Plot of $V(l)$ in $\theta = 0$ direction



Plot of $V(l)$ w.r.t θ

- Plots of $V(l)$ (in units of T_c^4) for $T=185$ MeV.
- Barrier vanishes at $T = 250$ MeV

Numerical Techniques:

- We carry out 2+1 dimensional simulation of C-D phase transition as a quench.
- We use 2000X2000 lattice (physical size 20 fm).
- Quench the system from low temp to 400 MeV in 1 fm time and then the temp decreases due to longitudinal expansion.

$$T(\tau) = T(\tau_0) \left(\frac{\tau_0}{\tau} \right)^{1/3}$$

- Initial field configuration constitutes a small patch around $l = 0$.

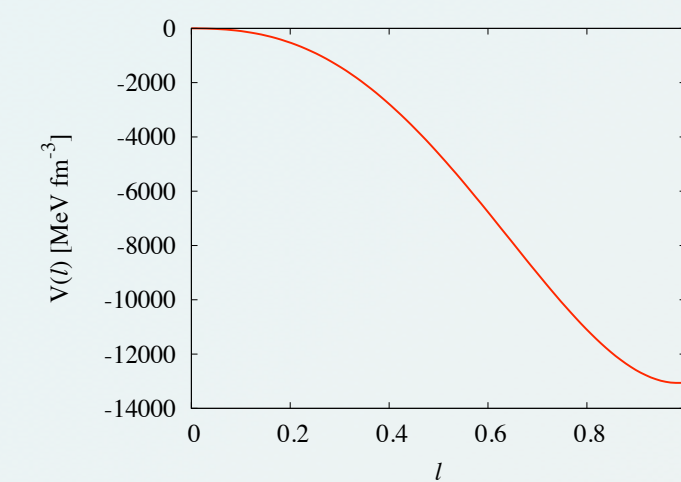
- Field evolution is numerically implemented by a stabilized Leapfrog algorithm of 2nd order accuracy both in space and time.

$$\ddot{l}_i + \frac{\dot{l}_i}{\tau} - \frac{\partial^2 l_i}{\partial x^2} - \frac{\partial^2 l_i}{\partial y^2} = -\frac{\partial V(l)}{\partial l_i}, \quad i=1,2 \quad \text{with } \dot{l}_i = 0 \text{ at } \tau = 0$$

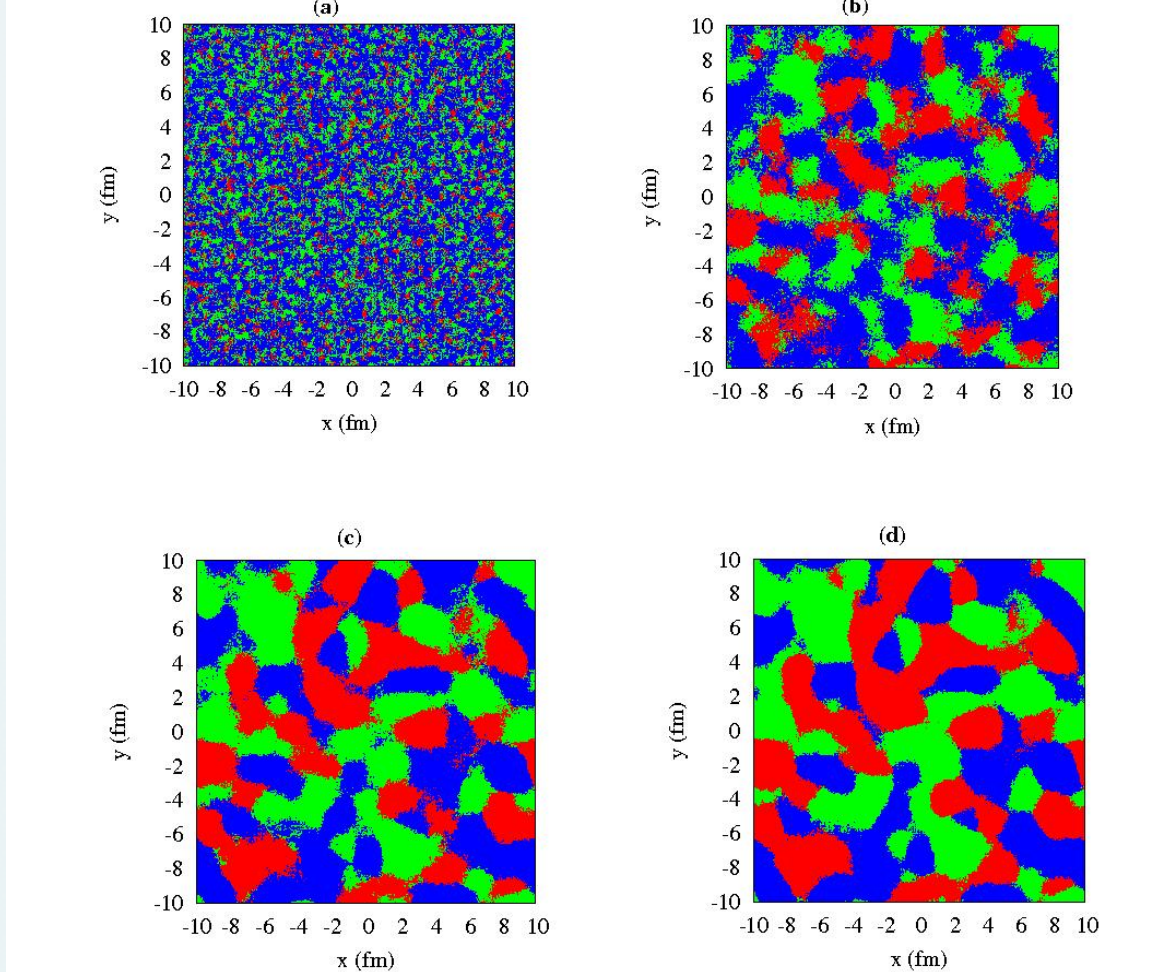
Domain Growth during Quenched transition to QGP

- The phase transition can be viewed as a quench due to the early thermalization to a QGP state.
- The barrier between true vacuum and false vacuum ($l = 0$) vanishes at temperature above 250 MeV.
- Quench the system from low temperature to high temperature (~ 400 MeV) in very short time (less than 1 fm), but the field still sits around $l = 0$ and just rolls down to true vacuum after that. Z(3) domains form and coarsens.

Plot of $V(l)$ in $\theta = 0$ direction for $T = 400$ MeV. No barrier between true and false vacuum.



Evolution of Z(3) Domains



- Red $\rightarrow \theta = 0$ vacuum
- Green $\rightarrow \theta = 2\pi/3$
- Blue $\rightarrow \theta = 4\pi/3$

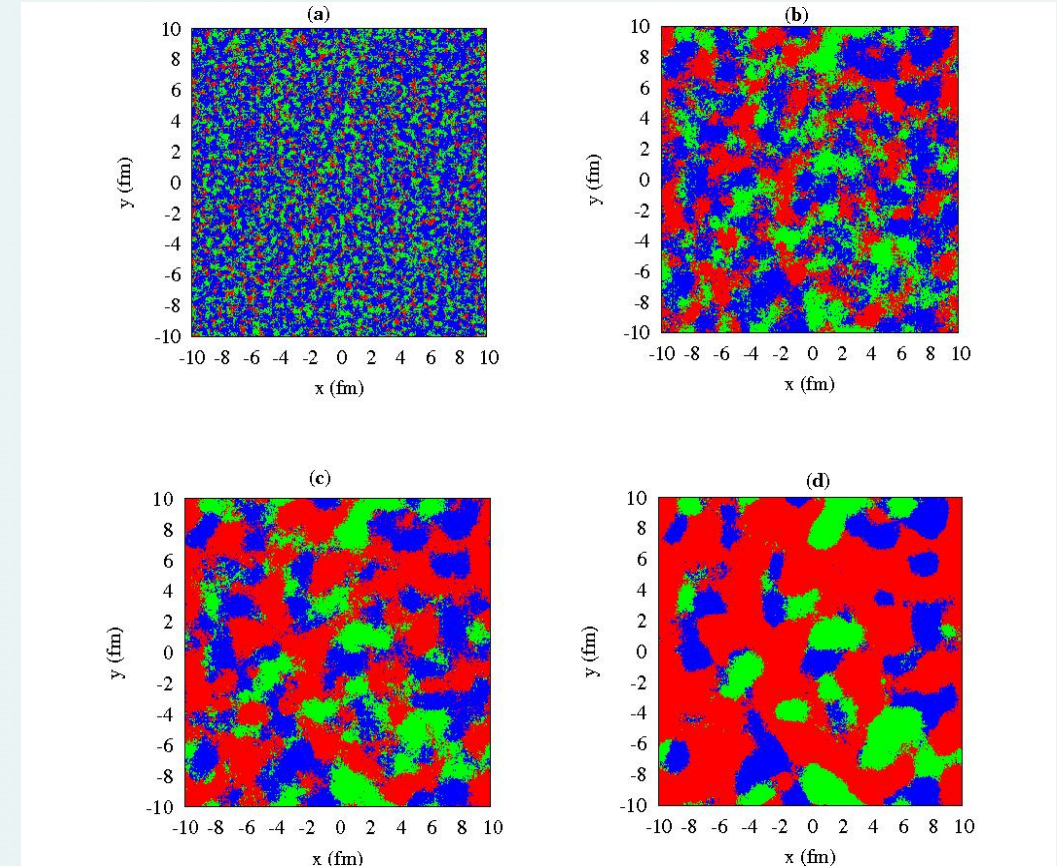
- At $\tau = 1.2, 2.0, 2.4$ and 2.8 fm respectively. Here Temp are 376, 317, 298 and 283 MeV and corresponding magnitudes of l are 0.04, 0.08, 0.2 and 0.4 respectively.

- Small η/s due to small size of domains (M Asakawa et al, PRL 110, 202301, 2013)

Small vs large explicit symmetry breaking

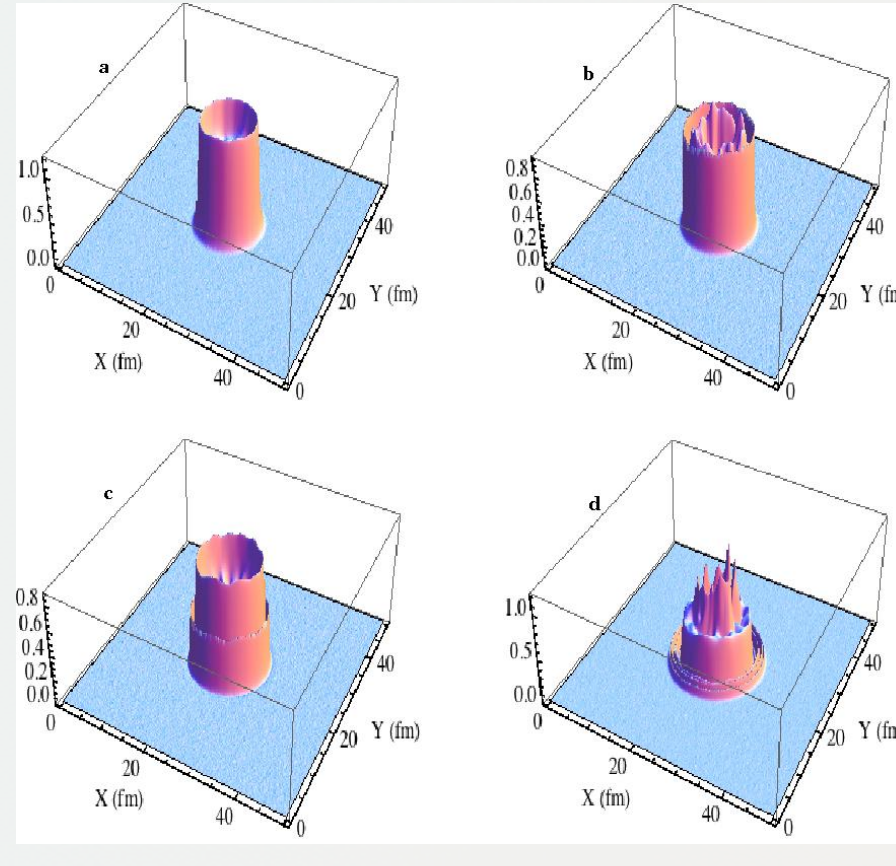
- Small explicit symmetry breaking : Initial patch of l shifts towards $\theta = 0$ vacuum, still overlapping with the initial equilibrium value of l . l rolls down to every vacuum, but $\theta = 0$ vacuum is more dominant.
- Large explicit symmetry breaking : l shifts more towards $\theta = 0$ vacuum, so initial patch rolls down entirely $\theta = 0$ direction. No formation of Z(3) domains or interfaces or strings.
- However, we see huge oscillations of the field before reaching to true vacuum.
- This may affect elliptic flow anisotropy.

Small explicit symmetry breaking : evolution of domains



- $\theta = 0$ domain is dominant compared to other 2 domains
- At $\tau = 1.2, 1.6, 2.0$ and 2.4 fm (temp 376, 342, 317 and 298 MeV) respectively.

Large explicit symmetry breaking : Huge oscillations of field



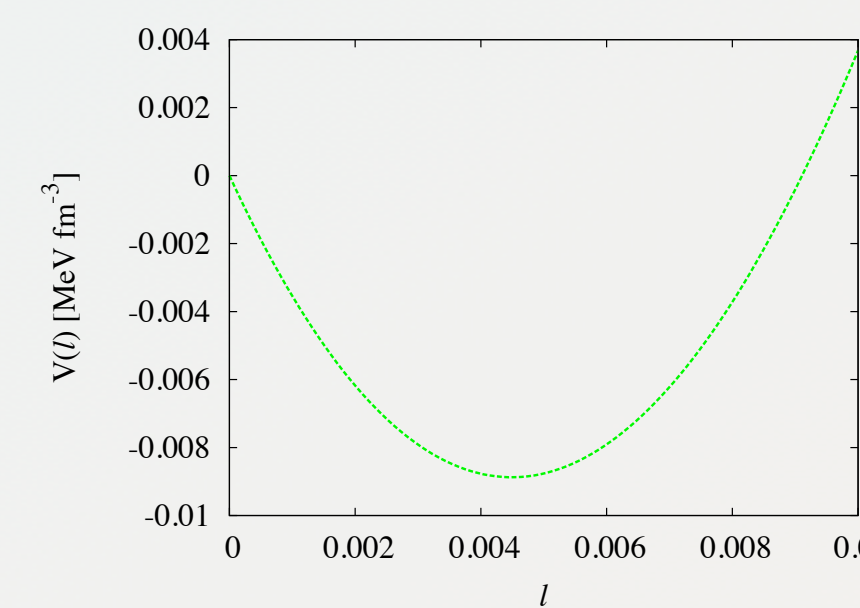
- Field rolls down everywhere towards $\theta = 0$ vacuum. Plots are at $\tau = 2.6, 3.6, 4.4$ and 6.2 fm respectively. Huge oscillations of field will affect flow anisotropy.

Effect of Quarks

- Z(N) symmetry is explicitly broken giving rise to one true vacuum and two meta stable vacua in presence of quarks.
- The effect of explicit symmetry breaking is realized by adding a linear term in the effective potential.

Shift of $l = 0$ vacuum

$$V(l) = \left(-b_1 \left(\frac{l+l^*}{2} \right) - \frac{b_2}{2} |l|^2 - \frac{b_3}{6} (l^3 + l^{*3}) + \frac{1}{4} |l|^4 \right) b_4 T^4$$



The false vacuum shifts by a small amount $\epsilon = 0.0044$ For $b_1 = 0.005$ and $T = 190$ MeV

Plot of $V(l)$ in $\theta = 0$ direction

Modeling of transverse expansion

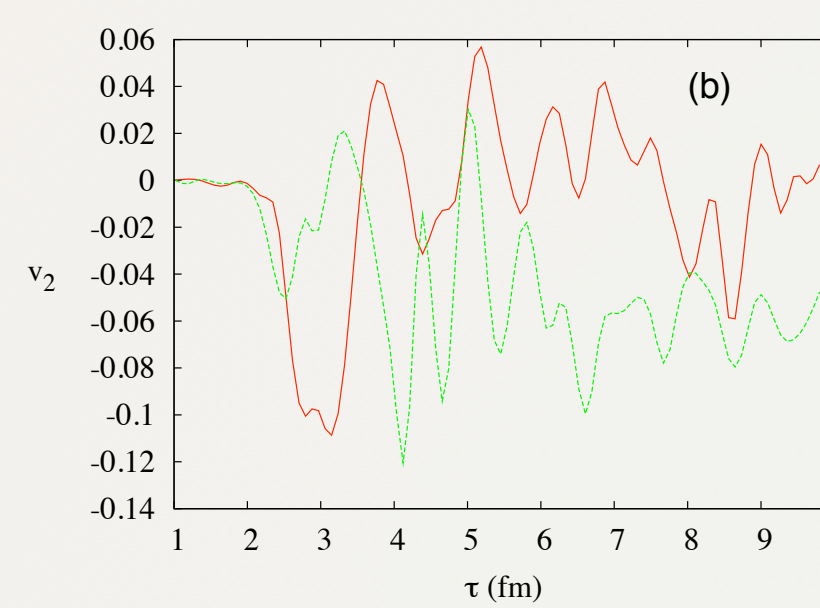
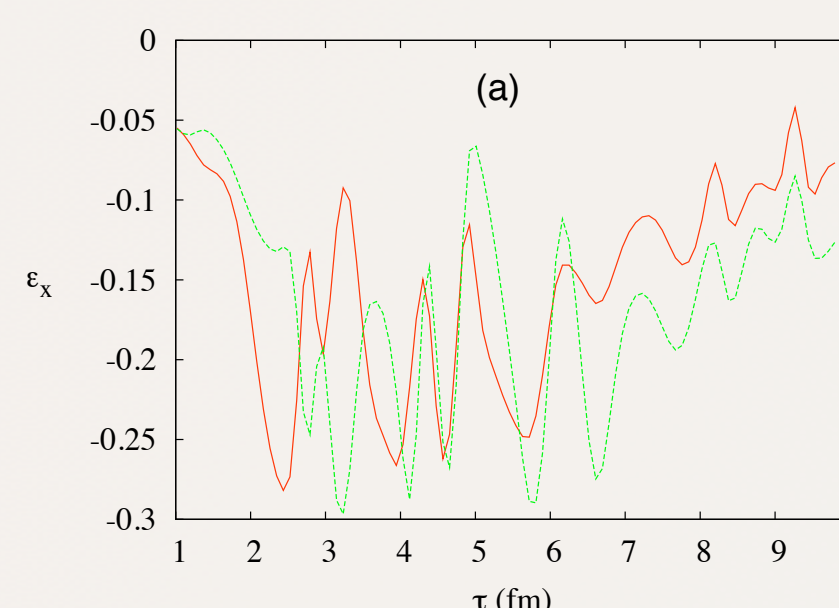
- A study of momentum anisotropy development with (quench) and without (equilibrium) the presence of large oscillations of the order parameter field.
- Wood Saxon temp profile with X and Y elliptical shape for non central collision.
- Transverse size increases with uniform acceleration of $0.015c$ per fm.
- This expanding background of temp profile represents expanding QGP in which the evolution of the order parameter field is studied.
- Interpretation of the simulation is that we study long wavelength modes of l which are coupled to a background of short wavelength modes which are in thermal equilibrium.
- In case of equilibrium, the field sits at the vacuum expectation value.

Elliptic flow anisotropy

$$\text{Spatial eccentricity } \epsilon_s = \frac{\int \rho(y^2 - x^2) dx dy}{\int \rho(y^2 + x^2) dx dy}$$

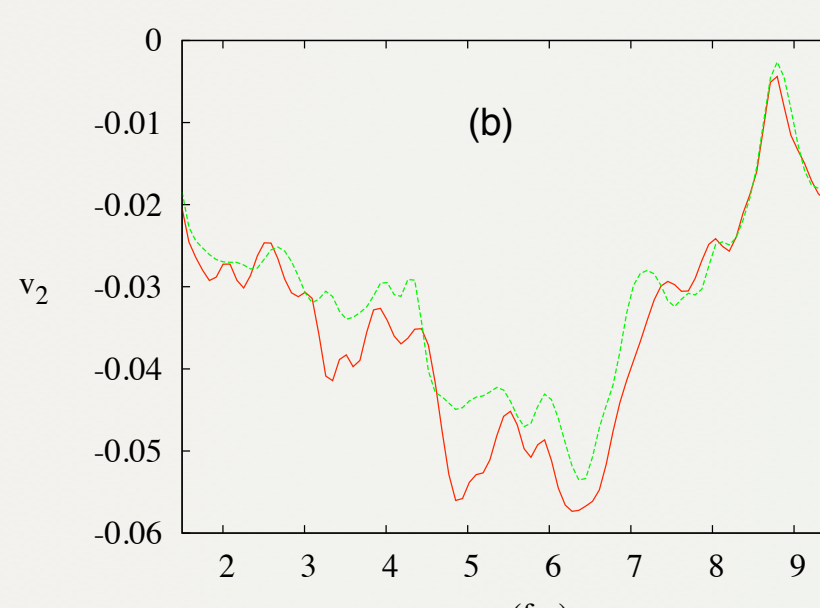
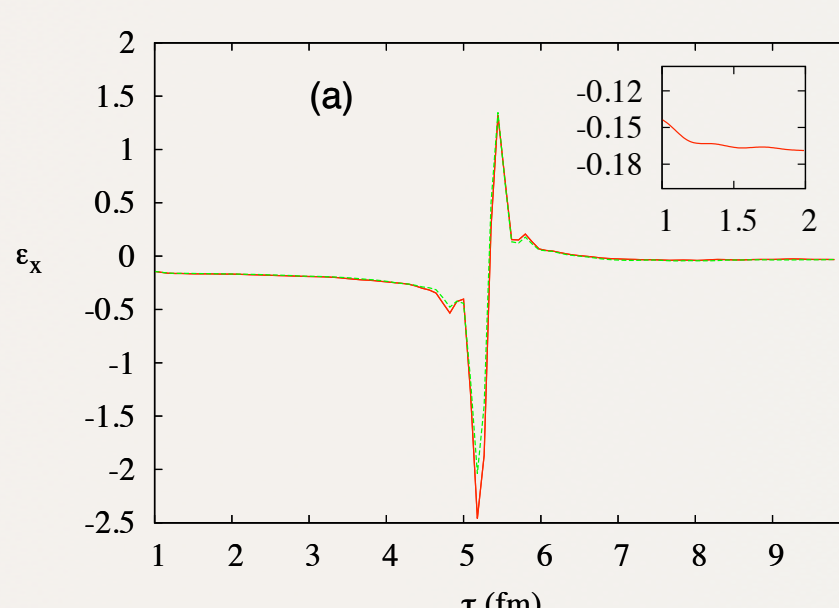
- Calculate T_{0x} and T_{0y} components of energy momentum tensor.
- Calculate $\theta = \tan^{-1}(T_{0y}/T_{0x})$ and Net momentum density = $\text{Sqrt}(T_{0x}^2 + T_{0y}^2)$
- Now calculate elliptic flow coefficient by Fourier expanding net momentum density w.r.t θ .

Quench case : initial elliptical shaped temp profile with eccentricity 0.5



Corresponds to two different realizations of the initial random field configuration.

Equilibrium case : initial elliptical shaped temp profile with eccentricity 0.5



Initial value of $\epsilon_s = -0.14$

Conclusion

- During quench, Z(3) domains are produced and coarsens in time with small explicit symmetry breaking.
- Due to large explicit symmetry breaking, field roll down to true vacuum only and no Z(3) domains appear.
- There are huge oscillations of the field and it would affect the flow anisotropy.
- Huge oscillations in flow anisotropy in quench case compared to equilibrium case.
- So, effect of quench should be taken care of in hydrodynamic simulations.