Viscous hydrodynamics for systems undergoing strongly anisotropic expansion*

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presented at
Quark Matter 2014
Darmstadt, 19-24 May 2014

*Supported by the U.S. Department of Energy

Reference: D. Bazow, U. Heinz, M. Strickland, arXiv:1311:6720 [nucl-th]

Motivation

- Large differences between the longitudinal and transverse expansion rates lead to large shear viscous effects (longitudinal/transverse pressure anisotropies) in the early stage of heavy-ion collisions.
- These cause Israel-Stewart theory to break down at early times.
- Anisotropic hydrodynamics (aHydro) deals with the large longitudinal/transverse pressure anisotropy "nonperturbatively"; this improves the performance of hydrodynamics at early times.
- But: aHydro accounts for only one of the 5 independent components of the shear stress tensor, ignoring the others $\Longrightarrow$ unreliable for the computation of elliptic flow which is sensitive to $\pi^{xx} - \pi^{yy}$, for example.
- On the other hand: these 4 remaining components of the viscous stress tensor become never as large as the longitudinal/transverse pressure difference (with smooth initial density profiles they start out as zero, with fluctuating initial conditions they are initially small).
- $\Longrightarrow$ Idea: treat large longitudinal/transverse pressure anisotropy "nonperturbatively" with aHydro, add remaining viscus corrections "perturbatively" à la Israel-Stewart $\Longrightarrow$ vaHydro.
- Expect better performance at all times compared to both aHydro and Israel-Stewart theory.

Kinetic theory vs. hydrodynamics

Both simultaneously valid if weakly coupled and pressure gradients are small

hydro equations remain structurally unchanged for strongly coupled systems

$$p^\mu \partial_\mu f(x,p) = C(x,p) = \frac{p\cdot u(x)}{\tau_{\rm rel}(x)}\Big(f_{\rm eq}(x,p) - f(x,p)\Big)$$

in relaxation time approximation (RTA)

For conformal systems $\tau_{\rm rel}(x) = c/T(x)$.

Macroscopic currents:

$$j^\mu(x) = \int_p p^\mu\, f(x,p) \equiv \langle p^\mu \rangle$$

$$T^{\mu\nu}(x) = \int_p p^\mu\, p^\nu\, f(x,p) \equiv \langle p^\mu p^\nu \rangle$$

where

$$\int_p \dots \equiv \frac{g}{(2\pi)^3}\int \frac{d^3p}{E_p} \dots \equiv \langle \dots \rangle$$

Ideal fluid dynamics (I)

Ideal hydro $\Longleftrightarrow$ $f(x,p) = f_{\rm iso}(x,p) \equiv f_{\rm iso}\left(\frac{p\cdot u(x) - \mu(x)}{T(x)}\right)$

(**Locally isotropic** momentum distribution, **not necessarily exponential or in chemical equilibrium**)

If not in chemical equilibrium, then $\partial_\mu j^\mu \neq 0$.

If not exponential in $(p\cdot u(x){-}\mu(x))/T(x)$, then $C(x,p) \neq 0$, but still $\int_p p^\mu C = 0$ (energy-momentum conservation).

For ideal hydro

$$j^\mu_{\rm id}(x) = n(x)\, u^\mu(x)$$
$$T^{\mu\nu}_{\rm id} = e(x) u^\mu(x) u^\nu(x) - P\big(e(x)\big)\Delta^{\mu\nu}(x)$$

where $\Delta^{\mu\nu} \equiv g^{\mu\nu} - u^\mu u^\nu$ = spatial projector in l.r.f.

Write

$$p^\mu = E u^\mu + p^{\langle\mu\rangle}$$

where $E \equiv u\cdot p$ = energy in l.r.f., $p^{\langle\mu\rangle} \equiv \Delta^{\mu\nu} p_\nu$ = spatial momentum in l.r.f.

Ideal fluid dynamics (II)

Ideal hydro equations follow from

$$\partial_\mu j^\mu = \frac{n_{\rm eq} - n(x)}{\tau_{\rm rel}(x)}$$
$$\partial_\mu T^{\mu\nu} = 0$$

which one can solve for $n(x)$, $e(x)$, $u^\mu(x)$.

Then $T(x)$, $\mu(x)$, $P(x)$ follow from the EOS.

Note: if system is locally isotropic but not in chemical and thermal equilibrium, this can be accounted for by non-equilibrium chemical potentials and a non-equilibrium pressure in the EOS $P(e,n) = P(T,\mu)$. In this case one sees non-zero entropy production $\partial_\mu S^\mu \sim 1/\tau_{\rm rel} \neq 0$.

Israel-Stewart viscous fluid dynamics (I)

$$f(x,p) = f_{\rm iso}\left(\frac{p\cdot u(x) - \mu(x)}{T(x)}\right) + \delta f(x,p)$$

Separation made unique by Landau matching:

First define l.r.f. by $T^{\mu\nu}u_\nu = e u^\mu$ with $u^\nu u_\nu = 1 \Longrightarrow$ fixes flow vector u^μ

Next, require

$$e(x) = e_{\rm iso}(T,\mu) \Rightarrow \langle E^2\rangle = \langle E^2\rangle_{\rm iso}(T,\mu)$$
$$n(x) = n_{\rm iso}(T,\mu) \Rightarrow \langle E\rangle = \langle E\rangle_{\rm iso}(T,\mu)$$

$\Longrightarrow \quad \langle E\rangle_\delta = \langle E^2\rangle_\delta = 0 \Longrightarrow$ fixes $T(x)$, $\mu(x)$

Viscous decomposition of j^μ, $T^{\mu\nu}$:

$$j^\mu = j^\mu_{\rm id} + V^\mu \qquad\qquad V^\mu = \left\langle p^{\langle\mu\rangle}\right\rangle_\delta$$

$$T^{\mu\nu} = T^{\mu\nu}_{\rm id} - \Pi\Delta^{\mu\nu} + \pi^{\mu\nu} \qquad\qquad \Pi = -\tfrac{1}{3}\left\langle p^{\langle\alpha\rangle}p_{\langle\alpha\rangle}\right\rangle_\delta, \quad \pi^{\mu\nu} = \left\langle p^{\langle\mu}p^{\nu\rangle}\right\rangle_\delta.$$

Here $A^{\langle\mu\nu\rangle} \equiv \Delta^{\mu\nu}_{\alpha\beta}A^{\alpha\beta}$ with $\Delta^{\mu\nu}\alpha\beta = \frac{1}{2}\left(\Delta^\mu{}_\alpha\Delta^\nu{}_\beta + \Delta^\nu{}_\alpha\Delta^\mu{}_\beta\right) - \frac{1}{3}\Delta^{\mu\nu}\Delta_{\alpha\beta}$

$\Longrightarrow \pi^{\mu\nu} = T^{\langle\mu\nu\rangle}$ has 5 independent components (3 for (2+1)-d, 1 for (0+1)-d)

Altogether 9 viscous flow degrees of freedom.

Israel-Stewart viscous fluid dynamics (II)

Israel-Stewart equations of motion for viscous pressures (Israel&Stewart 1979, Muronga 2002):
Define $\dot{F} \equiv DF \equiv u^\mu \partial_\mu F,\ \theta \equiv \partial{\cdot}u, \sigma^{\mu\nu} = \partial^{\langle\mu} u^{\nu\rangle}$:

$$\dot{\Pi} = -\frac{1}{\tau_\Pi}\left[\Pi + \zeta\theta + \Pi\zeta T \partial_\mu \left(\frac{\tau_\Pi u^\mu}{2\zeta T}\right)\right] \equiv -\frac{1}{\tau'_\Pi}\left[\Pi + \zeta'\theta\right],$$

$$\dot{\pi}^{\langle\mu\nu\rangle} \equiv \Delta^{\mu\nu}_{\alpha\beta} D\pi^{\alpha\beta} = -\frac{1}{\tau_\pi}\left[\pi^{\mu\nu} - 2\eta\sigma^{\mu\nu} + \pi^{\mu\nu}\eta T \partial_\mu \left(\frac{\tau_\pi u^\mu}{2\eta T}\right)\right] \equiv -\frac{1}{\tau'_\pi}\left[\pi^{\mu\nu} - 2\eta'\sigma^{\mu\nu}\right],$$

where η, ζ are shear and bulk viscosity (first order transp. coeffs.), τ_Π, τ_π are shear and bulk pressure relaxation times (second order transp. coeffs.), and

$$\tau'_\Pi = \frac{\tau_\Pi}{1+\gamma_\Pi},\ \tau'_\pi = \frac{\tau_\pi}{1+\gamma_\pi},\ \zeta' = \frac{\zeta}{1+\gamma_\Pi},\ \eta' = \frac{\eta}{1+\gamma_\pi}$$

$$\gamma_\Pi = \frac{1}{2}\zeta T \partial_\mu \left(\frac{\tau_\Pi u^\mu}{2\zeta T}\right) \longrightarrow \frac{4}{3}\tau_\Pi\theta,$$

$$\gamma_\pi = \frac{1}{2}\eta T \partial_\mu \left(\frac{\tau_\pi u^\mu}{2\eta T}\right) \longrightarrow \frac{4}{3}\tau_\pi\theta,$$

where the arrow indicates the conformal limit.

Israel-Stewart viscous fluid dynamics (III)

The Israel-Stewart equations are not the most general form of second order equations of motion for the viscous pressures. For a complete set of second-order terms, together with the associated transport coefficients computed from Boltzmann theory, see Denicol, Molnar, Niemi, Rischke, EPJA 48 (2012) 170 (DMNR).

Problem with applying IS theory to heavy-ion collisions:
for early times, as $\tau \to 0$,

$$\tau^2 \sigma^{\eta\eta} = -(\sigma^{xx} + \sigma^{yy}) \to -\frac{2}{3\tau}$$

$\Longrightarrow$ very large viscous corrections! $\Longrightarrow$ δf no longer small.

This problem is caused by the rapid self-similar longitudinal expansion.

Anisotropic hydrodynamics (aHydro) (I)

Martinez and Strickland 2009, 2010; Florkowski and Ryblewski 2010

A non-perturbative method to account for large shear viscous effects stemming from large difference between longitudinal and transverse expansion rates.

$$f(x,p) = f_{\rm iso}\left(\frac{\sqrt{p_\mu \Xi^{\mu\nu}(x) p_\nu} - \tilde{\mu}(x)}{\Lambda(x)}\right) \equiv f_{\rm RS}(x,p)$$

where $\Xi^{\mu\nu}(x) = u^\mu(x)u^\nu(x) + \xi(x) z^\mu(x) z^\nu x$. (Romatschke&Strickland 2003)
3 flow and 3 "thermodynamic" parameters: $u^\mu(x)$; $\Lambda(x)$, $\tilde{\mu}(x)$, $\xi(x)$.

aHydro decomposition:

$$j^\mu_{\rm RS} = n_{\rm RS} u^\mu, \qquad T^{\mu\nu}_{\rm RS} = e_{\rm RS} u^\mu u^\nu - P_T \Delta^{\mu\nu} + (P_L - P_T) z^\mu z^\nu,$$

where, for massless partons ($m = 0$), the effects of local momentum anisotropy can be factored out:

$$n_{\rm RS} = \langle E \rangle_{\rm RS} = \mathcal{R}_0(\xi) n_{\rm iso}(\Lambda, \tilde{\mu}),$$
$$e_{\rm RS} = \langle E^2 \rangle_{\rm RS} = \mathcal{R}(\xi) e_{\rm iso}(\Lambda, \tilde{\mu}),$$
$$P_{T,L} = \langle p^2_{T,L} \rangle_{\rm RS} = \mathcal{R}_{T,L}(\xi) P_{\rm iso}(\Lambda, \tilde{\mu}).$$

(See paper for $\mathcal{R}$-functions.) The isotropic pressure is obtained from a locally isotropic EOS,

$$P_{\rm iso}(\Lambda, \tilde{\mu}) = P_{\rm iso}(e_{\rm iso}(\Lambda, \tilde{\mu}), n_{\rm iso}(\Lambda, \tilde{\mu}))$$

For massless noninteracting partons, $P_{\rm iso}(\Lambda, \tilde{\mu}) = \frac{1}{3} e_{\rm iso}(\Lambda, \tilde{\mu})$ independent of chemical composition.

Anisotropic hydrodynamics (aHydro) (II)

Martinez and Strickland 2009, 2010; Florkowski and Ryblewski 2010

If we want to compare aHydro with ideal and IS viscous hydro, we need to assign the locally anisotropic system an appropriate temperature $T(x) = T\Big(\xi(x), \Lambda(x), \tilde{\mu}(x)\Big)$ and chemical potential $\mu(x) = \mu\Big(\xi(x), \Lambda(x), \tilde{\mu}(x)\Big)$, and think of $f_{\rm RS}(\xi, \Lambda)$ as an expansion around the locally isotropic distribution $f_{\rm iso}(T)$. This is done by "dynamical Landau matching": We demand that $e_{\rm RS}(\xi, \Lambda, \tilde{\mu}) = e_{\rm iso}(T, \mu)$ and $n_{\rm RS}(\xi, \Lambda, \tilde{\mu}) = \mathcal{R}_0(\xi) n_{\rm iso}(T, \mu)$.

For example, using a Boltzmann distribution for $f_{\rm iso}(x,p)$ with $\mu = \tilde{\mu} = 0$, one finds (Martinez & Strickland 2010)

$$T = \Lambda \mathcal{R}^{1/4}(\xi)$$

With this matching we can write

$$T^{\mu\nu}_{\rm RS} = T^{\mu\nu}_{\rm id} - (\Delta P + \Pi_{\rm RS})\Delta^{\mu\nu} + \pi^{\mu\nu}_{\rm RS}$$

where

$$\Delta P + \Pi_{\rm RS} = -\frac{1}{3}\int_p p_\alpha \Delta^{\alpha\beta} p_\beta (f_{\rm RS} - f_{\rm iso}) \qquad (= 0 \text{ for } m = 0),$$

$$\pi^{\mu\nu}_{\rm RS} = \int_p p^{\langle\mu} p^{\nu\rangle} (f_{\rm RS} - f_{\rm iso}) = (P_T - P_L)\, \frac{x^\mu x^\nu + y^\mu y^\nu - 2 z^\mu z^\nu}{3}$$

We see that $\pi^{\mu\nu}_{\rm RS}$ has only one independent component, $P_T - P_L$, so aHydro leaves 4 of the 5 components of $\pi^{\mu\nu}$ unaccounted for.

Anisotropic hydrodynamics (aHydro) (III)

Martinez and Strickland 2009, 2010; Florkowski and Ryblewski 2010

For massless particles we have

$$\frac{P_T - P_L}{P_{\rm iso}(e)} = \mathcal{R}_T(\xi) - \mathcal{R}_L(\xi),$$

so the EOM for $\pi^{\mu\nu}_{\rm RS}$ can be replaced by an EOM for ξ.

For $m \neq 0$, to separate ΔP from the viscous pressure Π, we need an "anisotropic EOS" for

$$\frac{\Delta P}{P_{\rm iso}} \equiv \frac{2P_T + P_L}{3P_{\rm iso}} - 1.$$

Viscous anisotropic hydrodynamics (vaHydro) (I)

$$f(x,p) = f_{\rm RS}(x,p) + \delta\tilde{f}(x,p) = f_{\rm iso}\left(\frac{\sqrt{p_\mu \Xi^{\mu\nu}(x) p_\nu} - \tilde{\mu}(x)}{\Lambda(x)}\right) + \delta\tilde{f}(x,p)$$

Landau matching: $T^\mu_{\ \nu} u^\nu = e u^\mu$ with $u^\mu u_\mu = 1 \Longrightarrow$ fixes u^μ

no contribution to e, n from $\delta\tilde{f}$: $\langle E\rangle_{\tilde{\delta}} = \langle E\rangle_{\tilde{\delta}} = 0 \Longrightarrow$ fixes $\Lambda, \tilde{\mu}$.

no contribution to $P_T{-}P_L$ from $\delta\tilde{f}$: $\frac{x_\mu x_\nu + y_\mu y_\nu - 2 z_\mu z_\nu}{2} \langle p^{\langle\mu} p^{\nu\rangle}\rangle_{\tilde{\delta}} = 0 \Longrightarrow$ fixes ξ.

vaHydro decomposition:

$$j^\mu = j^\mu_{\rm RS} + \tilde{V}^\mu, \qquad \tilde{V}^\mu = \left\langle p^{\langle\mu\rangle}\right\rangle_{\tilde{\delta}},$$

$$T^{\mu\nu} = T^{\mu\nu}_{\rm RS} - \tilde{\Pi}\Delta^{\mu\nu} + \tilde{\pi}^{\mu\nu}, \qquad \tilde{\Pi} = -\frac{1}{3}\left\langle p^{\langle\alpha\rangle} p_{\langle\alpha\rangle}\right\rangle_{\tilde{\delta}}, \quad \tilde{\pi}^{\mu\nu} = \left\langle p^{\langle\mu} p^{\nu\rangle}\right\rangle_{\tilde{\delta}},$$

$u_\mu \tilde{\pi}^{\mu\nu} = \tilde{\pi}^{\mu\nu} u_\nu = (x_\mu x_\nu + y_\mu y_\nu - 2 z_\mu z_\nu)\tilde{\pi}^{\mu\nu} = \tilde{\pi}^\mu_\mu = 0 \Longrightarrow \tilde{\pi}^{\mu\nu}$ has 4 degrees of freedom.

Strategy: solve hydrodynamic equations for aHydro (which treat $P_T{-}P_L$ nonperturbatively) with added viscous flows from $\delta\tilde{f}$, together with IS-like "perturbative" equations of motion for $\tilde{\Pi}$, $\tilde{V}^\mu$, $\tilde{\pi}^{\mu\nu}$.

Viscous anisotropic hydrodynamics (vaHydro) (II)

Hydrodynamic equations of motion:

$\partial_\mu j^\mu = C \equiv \int_p C(x,p) \Longrightarrow \dot{n}_{\rm RS} = -n_{\rm RS}\theta - \partial_\mu \tilde{V}^\mu + \frac{n_{\rm RS}-n_{\rm iso}}{\tau_{\rm rel}}$ in RTA

$\partial_\mu T^{\mu\nu} = 0 \Longrightarrow$

$$\dot{e} = -(e{+}P_T)\theta_\perp - (e{+}P_L)\frac{u_0}{\tau} - \tilde{\Pi}\theta + \tilde{\pi}^{\mu\nu}\sigma_{\mu\nu},$$

$$(e{+}P_T{+}\tilde{\Pi})\dot{u}_\perp = -\partial_\perp(P_T{+}\tilde{\Pi}) - u_\perp(\dot{P}_T{+}\dot{\tilde{\Pi}}) - u_\perp(P_T{-}P_L)\frac{u_0}{\tau} + \left(\frac{u_x\Delta^1{}_\nu + u_y\Delta^2{}_\nu}{u_\perp}\right)\partial_\mu\tilde{\pi}^{\mu\nu},$$

$$(e{+}P_T{+}\tilde{\Pi})u_\perp\dot{\phi}_u = -D_\perp(P_T{+}\tilde{\Pi}) - \frac{u_y\partial_\mu\tilde{\pi}^{\mu 1} - u_x\partial_\mu\tilde{\pi}^{\mu 2}}{u_\perp},$$

where $\theta_\perp = \partial_\tau u_0 + \nabla_\perp\cdot\boldsymbol{u}_\perp$ and $D_\perp = (u_x\partial_y - u_y\partial_x)/u_\perp$.

To derive **equations of motion for $\tilde{\Pi}$, $\tilde{\boldsymbol{V}}^\mu$, and $\tilde{\pi}^{\mu\nu}$**, we follow DMNR (2012). Ignoring heat conduction by setting $\tilde{\mu}=0$ and taking $m=0$ we find

$$\dot{\tilde{\pi}}^{\mu\nu} = -2\dot{u}_\alpha\tilde{\pi}^{\alpha(\mu}u^{\nu)} - \frac{1}{\tau_{\rm rel}}\Big[(P{-}P_T)\Delta^{\mu\nu} + (P_L{-}P_T)z^\mu z^\nu + \tilde{\pi}^{\mu\nu}\Big] + \mathcal{K}_0^{\mu\nu} + \mathcal{L}_0^{\mu\nu} + \mathcal{H}_0^{\mu\nu\lambda}\dot{z}_\lambda$$
$$+\mathcal{Q}_0^{\mu\nu\lambda\alpha}\nabla_\lambda u_\alpha + \mathcal{X}_0^{\mu\nu\lambda}u^\alpha\nabla_\lambda z_\alpha - 2\lambda^0_{\pi\pi}\tilde{\pi}^{\lambda\langle\mu}\sigma^{\nu\rangle}_{\lambda} + 2\tilde{\pi}^{\lambda\langle\mu}\omega^{\nu\rangle}_{\lambda} - 2\delta^0_{\pi\pi}\tilde{\pi}^{\mu\nu}\theta$$

(see Bazow, UH, Strickland, arXiv:1311.6720v2 for details).

Test of vaHydro: (0+1)-dimensional expansion (I)

For (0+1)-d (longitudinally boost-invariant) expansion, the BE can be solved exactly in RTA (Florkowski, Ryblewski, Strickland, PRC88 (2013) 024903), and the solution can be used to test the various macroscopic hydrodynamic approximation schemes.

Setting homogeneous initial conditions in r and η_s and zero transverse flow, $\tilde{\pi}^{\mu\nu}$ reduces to a single non-vanishing component $\tilde{\pi}$: $\tilde{\pi}^{\mu\nu} = \mathrm{diag}(0, -\tilde{\pi}/2, -\tilde{\pi}/2, \tilde{\pi})$ at $z = 0$.

We use the factorization $n_{\mathrm{RS}}(\xi\Lambda) = \mathcal{R}_0(\xi) n_{\mathrm{iso}}(\Lambda)$ etc. to get EOMs for $\dot{\xi}$, $\dot{\Lambda}$, $\dot{\tilde{\pi}}$:

$$\frac{\dot{\xi}}{1+\xi} - 6\frac{\dot{\Lambda}}{\Lambda} = \frac{2}{\tau} + \frac{2}{\tau_{\mathrm{rel}}}\left(1 - \sqrt{1+\xi}\,\mathcal{R}^{3/4}(\xi)\right) \,,$$

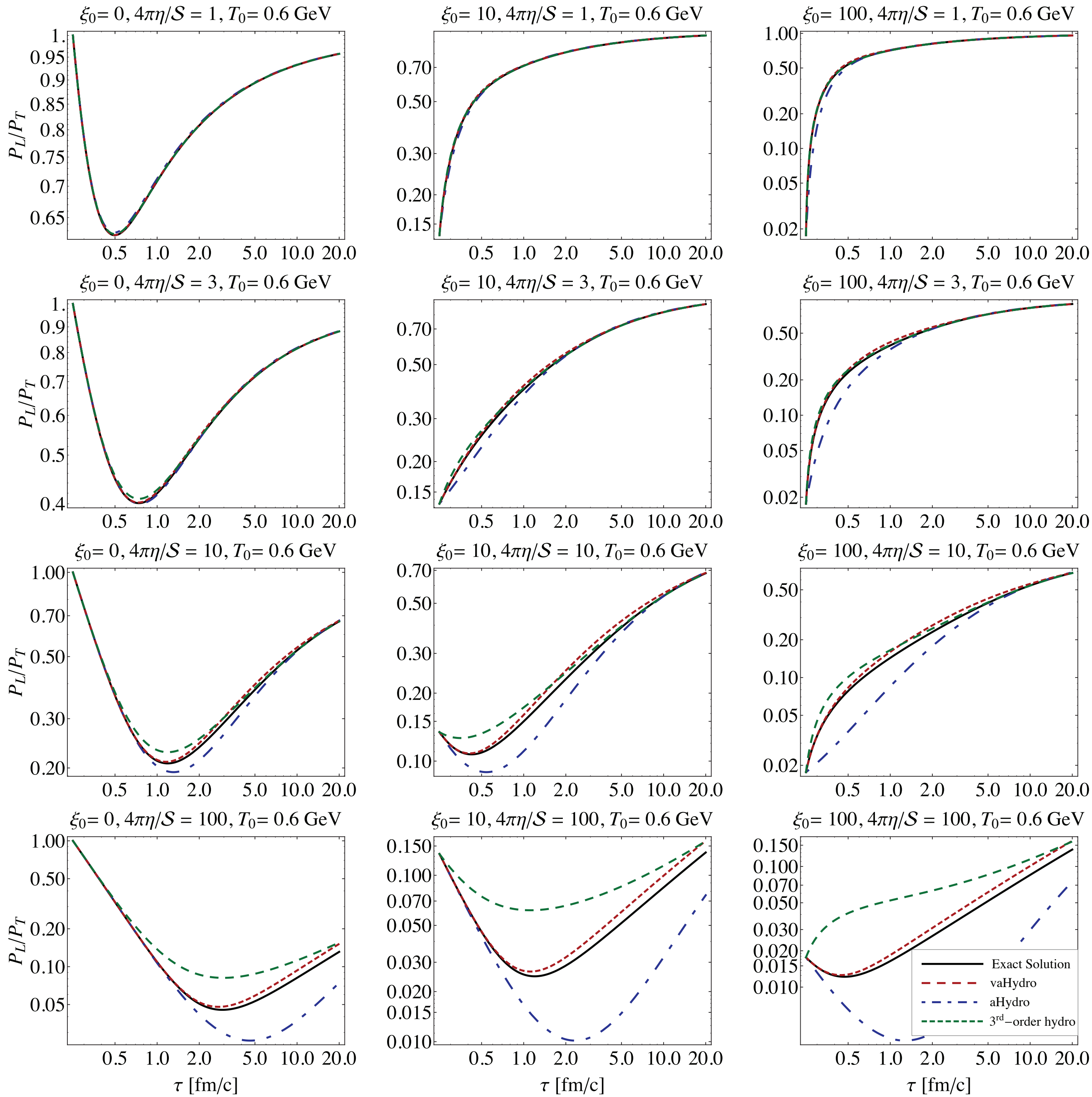
$$\mathcal{R}'(\xi)\dot{\xi} + 4\mathcal{R}(\xi)\frac{\dot{\Lambda}}{\Lambda} = -\left(\mathcal{R}(\xi) + \frac{1}{3}\mathcal{R}_L(\xi)\right)\frac{1}{\tau} + \frac{\tilde{\pi}}{e_{\mathrm{iso}}(\Lambda)\tau},$$

$$\dot{\tilde{\pi}} = -\frac{1}{\tau_{\mathrm{rel}}}\left[\left(\mathcal{R}(\xi) - \mathcal{R}_{\mathrm{L}}(\xi)\right)P_{\mathrm{iso}}(\Lambda) + \tilde{\pi}\right] - \frac{38}{21}\frac{\tilde{\pi}}{\tau}$$
$$+12\left[\frac{\dot{\Lambda}}{\Lambda}\left(\mathcal{R}_{\mathrm{L}}(\xi) - \frac{1}{3}\mathcal{R}(\xi)\right) + \left(\frac{1+\xi}{\tau} - \frac{\dot{\xi}}{2}\right)\left(\mathcal{R}_{-1}^{zzzz}(\xi) - \frac{1}{3}\mathcal{R}_1^{zz}(\xi)\right)\right]P_{\mathrm{iso}}(\Lambda),$$

τ_{rel} and η/s are related by (Denicol, Koide, Rischke, PRL 105 (2010))

$$\tau_{\mathrm{rel}} = 5\frac{\eta/s}{T} = 5\frac{\eta/s}{\mathcal{R}^{1/4}(\xi)\Lambda}$$

We solve these equations and compare with the exact solution:



ξ0= 0, 4πη/S = 1, T0= 0.6 GeV
ξ0= 10, 4πη/S = 1, T0= 0.6 GeV
ξ0= 100, 4πη/S = 1, T0= 0.6 GeV
ξ0= 0, 4πη/S = 3, T0= 0.6 GeV
ξ0= 10, 4πη/S = 3, T0= 0.6 GeV
ξ0= 100, 4πη/S = 3, T0= 0.6 GeV
ξ0= 0, 4πη/S = 10, T0= 0.6 GeV
ξ0= 10, 4πη/S = 10, T0= 0.6 GeV
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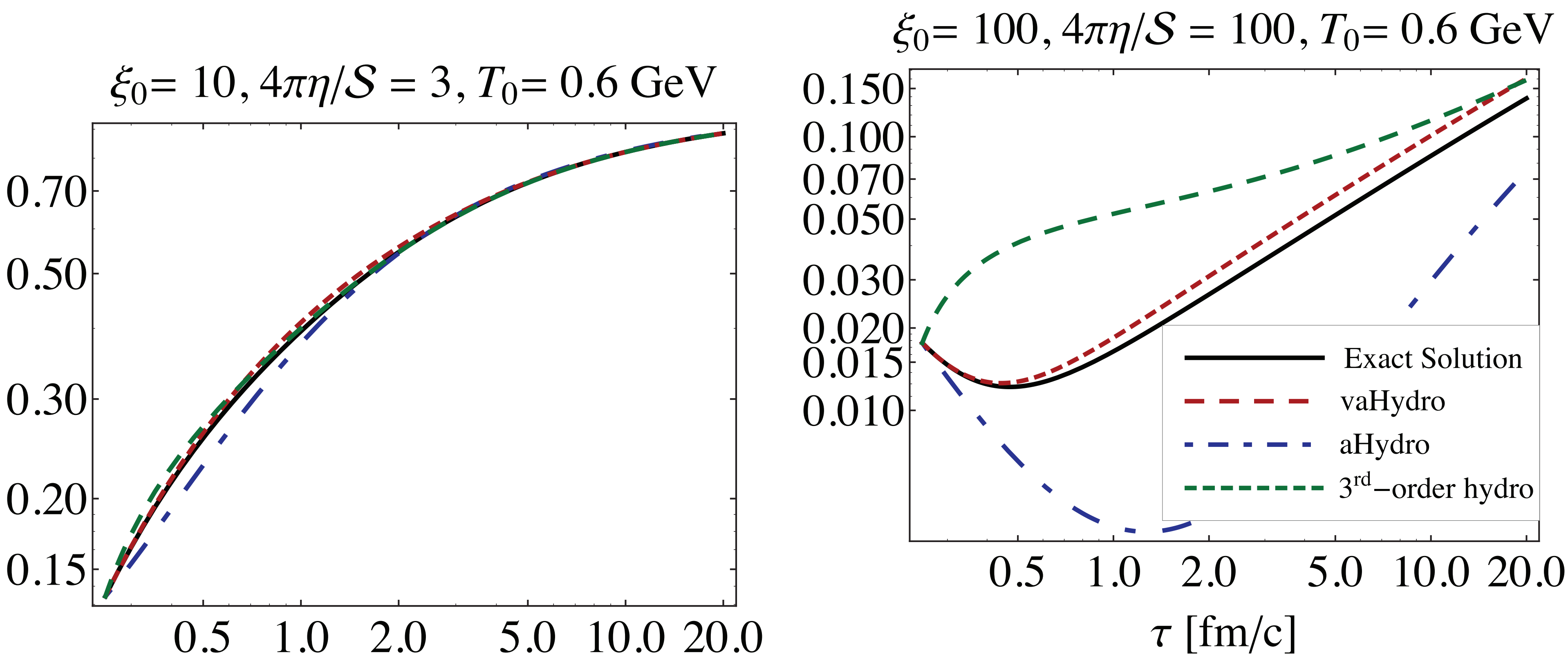
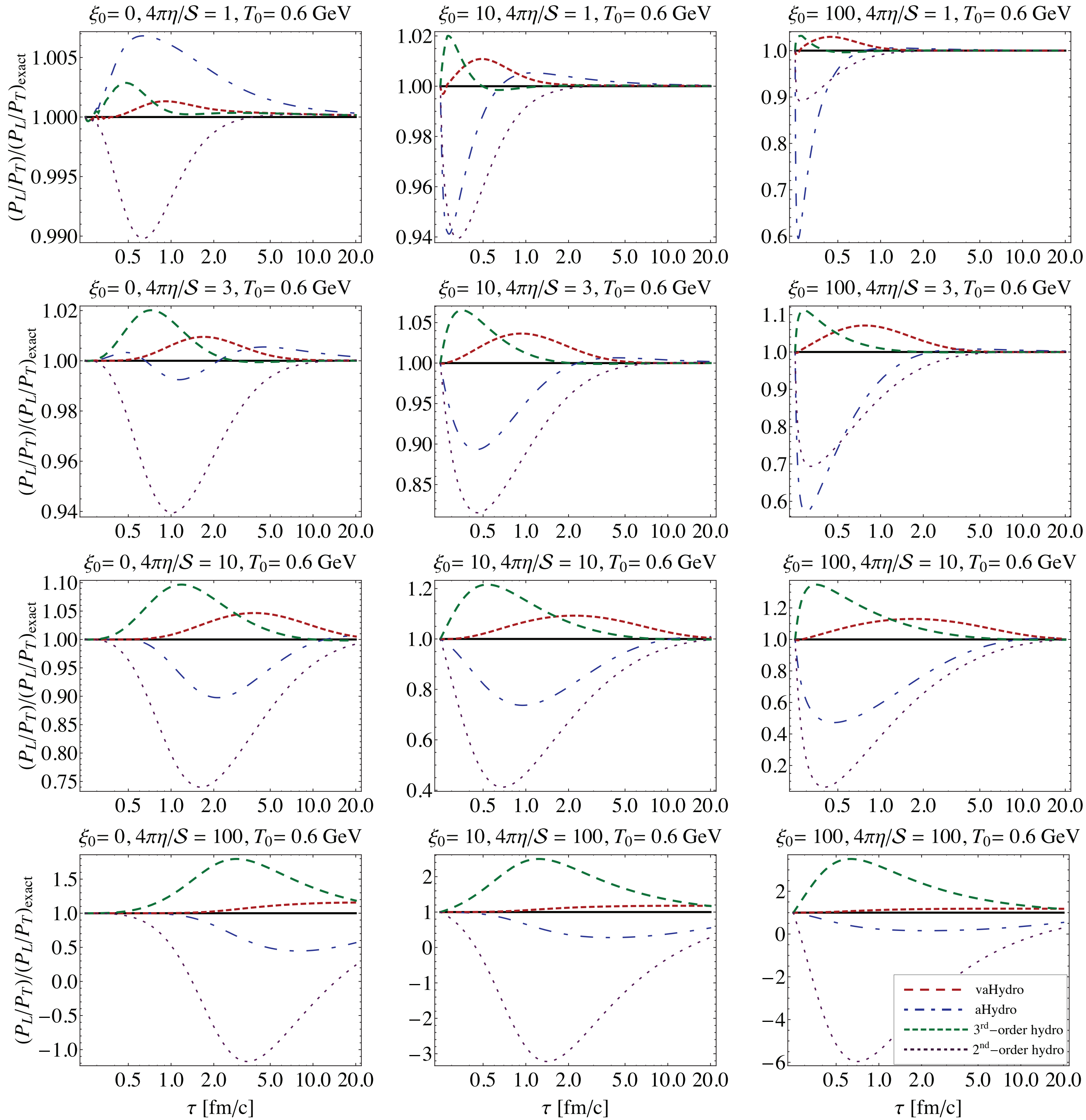
ξ0= 0, 4πη/S = 100, T0= 0.6 GeV
ξ0= 10, 4πη/S = 100, T0= 0.6 GeV
ξ0= 100, 4πη/S = 100, T0= 0.6 GeV
PL/PT
τ [fm/c]
Exact Solution
vaHydro
aHydro
3rd−order hydro

Test of vaHydro: (0+1)-dimensional expansion (II)

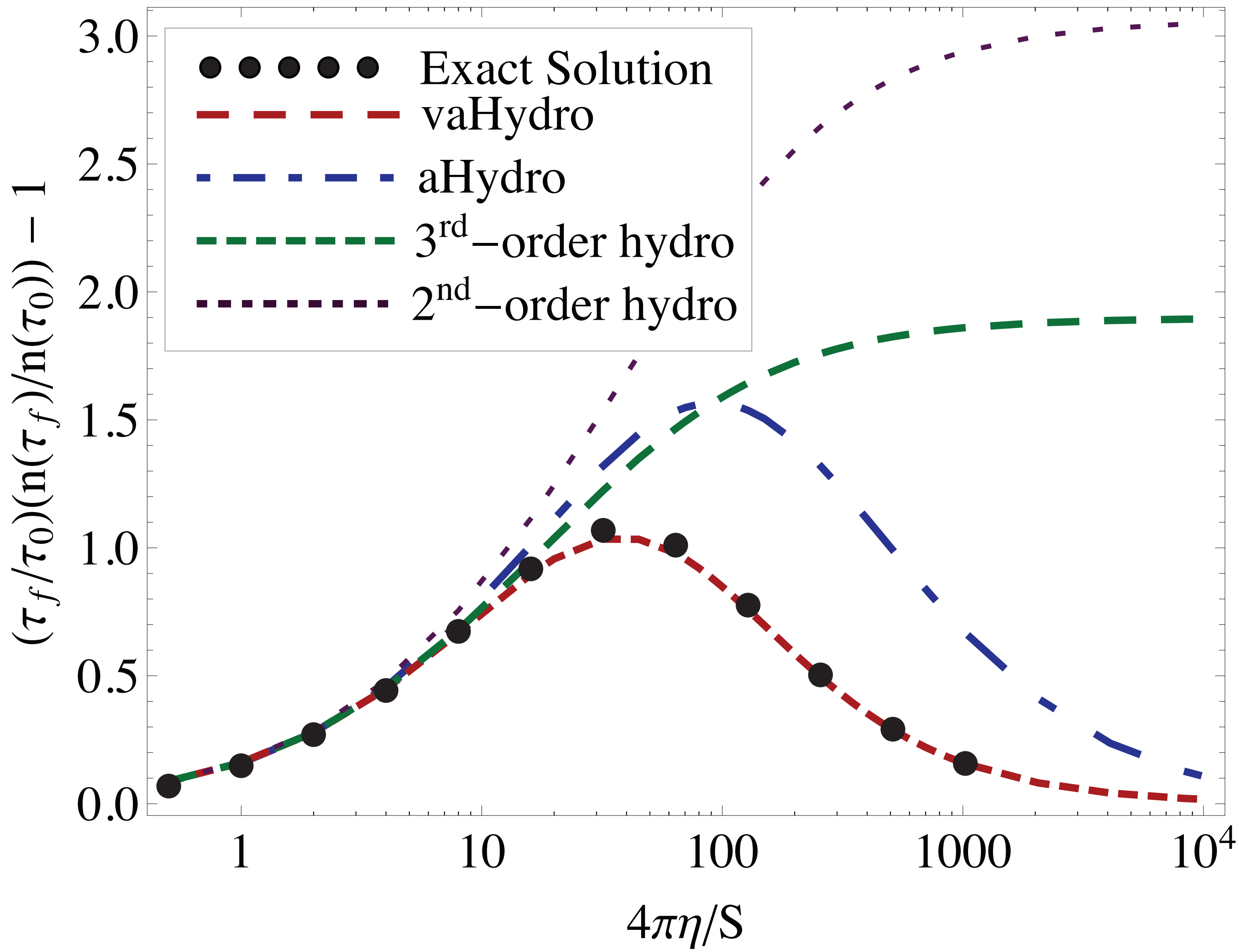
Pressure anisotropy P_L/P_T vs. τ:




ξ0= 0, 4πη/S = 1, T0= 0.6 GeV
ξ0= 10, 4πη/S = 1, T0= 0.6 GeV
ξ0= 100, 4πη/S = 1, T0= 0.6 GeV
ξ0= 0, 4πη/S = 3, T0= 0.6 GeV
ξ0= 10, 4πη/S = 3, T0= 0.6 GeV
ξ0= 100, 4πη/S = 3, T0= 0.6 GeV
ξ0= 0, 4πη/S = 10, T0= 0.6 GeV
ξ0= 10, 4πη/S = 10, T0= 0.6 GeV
ξ0= 100, 4πη/S = 10, T0= 0.6 GeV
ξ0= 0, 4πη/S = 100, T0= 0.6 GeV
ξ0= 10, 4πη/S = 100, T0= 0.6 GeV
ξ0= 100, 4πη/S = 100, T0= 0.6 GeV
(PL/PT)/(PL/PT)exact
τ [fm/c]
vaHydro
aHydro
3rd−order hydro
2nd−order hydro

Test of vaHydro: (0+1)-dimensional expansion (III)

Total entropy (particle) production $\frac{n(\tau_f)\cdot\tau_f}{n(\tau_0)\cdot\tau_0} - 1$



Conclusions

- For early times and/or near the transverse edge in heavy-ion collision fireballs, rapid longitudinal expansion generates large inverse Reynolds numbers for the shear pressure, $\mathrm{R}_\pi^{-1} = \sqrt{\pi^{\mu\nu}\pi_{\mu\nu}}/P_{\mathrm{iso}}$, causing Israel-Stewart second order viscous hydrodynamics to break down.
- The large local pressure anisotropies caused by a large difference in longitudinal and transverse expansion rates can be treated efficiently by using the non-perturbative aHydro approach which is based on an expanseion around a locally spheroidally deformed distribution f_{RS}.
- This strongly reduces the shear inverse Reynolds numbers $\tilde{\mathrm{R}}_\pi^{-1} = \sqrt{\tilde{\pi}^{\mu\nu}\tilde{\pi}_{\mu\nu}}/\mathcal{P}_{\mathrm{iso}}$ associated with the remaining shear stress tensor $\tilde{\pi}^{\mu\nu}$ resulting from the much smaller deviation $\delta\tilde{f}$ of the local distribution function from f_{RS}.
- vaHydro combines the advantages of aHydro with a complete (although perturbative) second-order treatment of all remaining viscous effects à la Israel-Stewart.
- In a test of (0+1)-d expansion, which maximizes the difference between longitudinal and transverse expansion rates, against an exact solution of the Boltzmann equation, vaHydro outperforms all other known hydrodynamic approximation schemes by a considerable margin.
- This should open the door in (3+1)-d systems to match microscopic pre-equilibrium theories to viscous hydrodynamics at earlier times than possible with IS-theory and its variants.