Self-consistent Cooper-Frye freeze-out of a viscous fluid to particles

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Abstract

Comparing hydrodynamic simulations to heavy-ion data inevitably requires the conversion of the fluid to particles. In dissipative fluids the conversion is ambiguous and particle species are typically given distributions independent of the microscopic dynamics that keep the system near thermal equilibrium. In contrast to this ad-hoc assumption, we compute distributions self-consistently by solving the linearized Boltzmann equation. We find that the momentum dependence of the corrections in all systems investigated is best fit by a power close to $\frac{3}{2}$ rather than the typically used quadratic ansatz. The effects on harmonic flow are also calculated and found to be substantial, thus the form of these corrections should be taken into account when extracting medium properties from experimental data.

Outline of the Problem

Whether one models a heavy-ion collision with hydrodynamics alone or uses a "hybrid" hydrodynamics+hadron transport simulation, a conversion of the fluid description to one of particles is inevitable and typically done using the Cooper-Frye[1] prescrition:

Particle distributions are calculated on a constant T hypersurface σ:

$$E\, dN_i = p^\mu d\sigma_\mu(x)\, d^3p\, (f_i^{eq}(x,\vec{p}) + \delta f_i)$$

$$T^{\mu\nu}(x) \equiv \sum_i \int \frac{d^3p}{E} p^\mu p^\nu [f_i^{eq}(x,\vec{p}) + \delta f_i(x,\vec{p})] = T^{\mu\nu}_{ideal}(x) + \underbrace{\pi^{\mu\nu}(x)}_{shear}$$

Considering only shear stress, the viscous corrections have only **one** freedom left: a function of magnitude of momentum[2], $\chi_i(p/T)$ for each species i.

$$\delta f_i(x,\vec{p}) = \chi_i(p/T)\, f_i^{eq} \frac{p_\mu p_\nu}{T^2} \frac{\pi^{\mu\nu}}{T\eta}$$

Infinitely many functions χ can reproduce the hydrodynamic fields. χ is typically chosen via the "democratic Grad"[3] ansatz where **ALL particle species** receive the same **constant** χ correction and viscous corrections to the distribution functions are all **assumed** quadratic in momentum:

$$\chi_i^{dem\ Grad}(p/T) = \frac{1}{2}\frac{\eta}{s} = CONSTANT$$

This ad hoc choice assigns the same χ_i to different species even though species that interact more should be closer to equilibrium. We instead calculate the χ_i functions from covariant transport theory to maintain consistency with microscopic collision rates in a hadron gas.

Linearized Boltzmann Equation

$$p^\mu \partial_\mu f_i(x,\vec{p}) = S_i(x,\vec{p}) + \sum_{jk\ell} C^{ij\to k\ell}[f_i, f_j, f_k, f_\ell](x,\vec{p})$$

$$C^{ij\to k\ell}[f_i, f_j, f_k, f_\ell](x,\vec{p}_1) \equiv \iiint_{234} \left(\frac{g_i g_j}{g_k g_\ell} f_{3k} f_{4\ell} - f_{1i} f_{2j}\right) \bar{W}_{12\to 34}^{ij\to k\ell}\, \delta^4(p_1+p_2-p_3-p_4)$$

For small deviations from local equilibrium we linearize in δf and solve the resulting integral equations for χ_i variationally with a functional approach similiar to[4,5].

$$p^\mu \partial_\mu f_i^{eq} + p^\mu \partial_\mu \delta f_i = \sum_{jk\ell} \left\{ C^{ij\to k\ell}[\delta f_i, f_j^{eq}, \delta f_k, f_\ell^{eq}] + C^{ij\to k\ell}[f_i^{eq}, \delta f_j, f_k^{eq}, \delta f_\ell] \right\}$$

Results: Pion-Nucleon Gas

We first test the results in a conversion to a simple pion-nucleon system with energy-independent effective cross sections[6] in the **"dynamic Grad"** ansatz where corrections are still quadratic in momentum but can depend on the cross-sections between species, i.e.

$$\delta f_i \sim c_i(\sigma_{ij}) p^2$$

These distribution functions are used in the Cooper-Frye prescription to calculate spectra and elliptic flow after running AZHYDRO[6] 0.2p2 for a typical RHIC collision at b = 7 fm. The shear stress tensor is calculated from the flow gradients using the Navier-Stokes estimate with viscosity to entropy ratio taken to be $\eta/s = 0.1$:

$$\pi^{\mu\nu}_{NS} = \eta[\nabla^\mu u^\nu + \nabla^\nu u^\mu - \tfrac{2}{3}\Delta^{\mu\nu}(\partial u)]$$

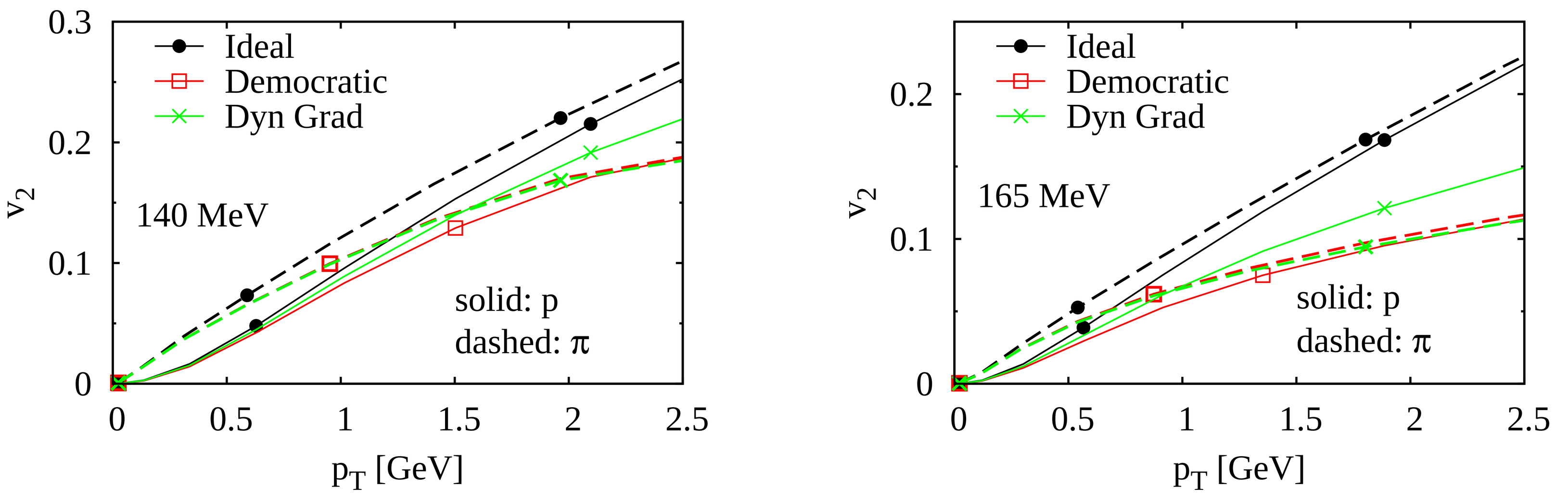

FIGURE 1: Elliptic flow $v_2(p_T)$ for a pion-nucleon gas calculated from linearized kinetic theory. Standard "democratic Grad" (open boxes) are compared to our self-consistent (crosses) calculations. The left plot is for conversion on a T=140 MeV hypersurface while the right is 165 MeV.

Self-consistent viscous corrections in a pion-nucleon gas lead to an elliptic flow splitting at moderately high momenta, with proton $v_2 \sim 30\%$ larger than pion v_2. If the decoupling is done at lower temperature, the effects of viscosity diminish as the systems gradients are reduced.

Results: Multicomponent Hadron Gas

We now convert the fluid to a hadron gas that includes resonances up to the $\Omega(1672)$ under two simple models for cross-sections between different species: 1) All species receive the same σ_{ij} = 30mb with every other species and 2) Cross sections follow the additive quark model (AQM) with meson and baryon asymmetry: $\sigma_{MM} : \sigma_{MB} : \sigma_{BB} = 4 : 6 : 9$.

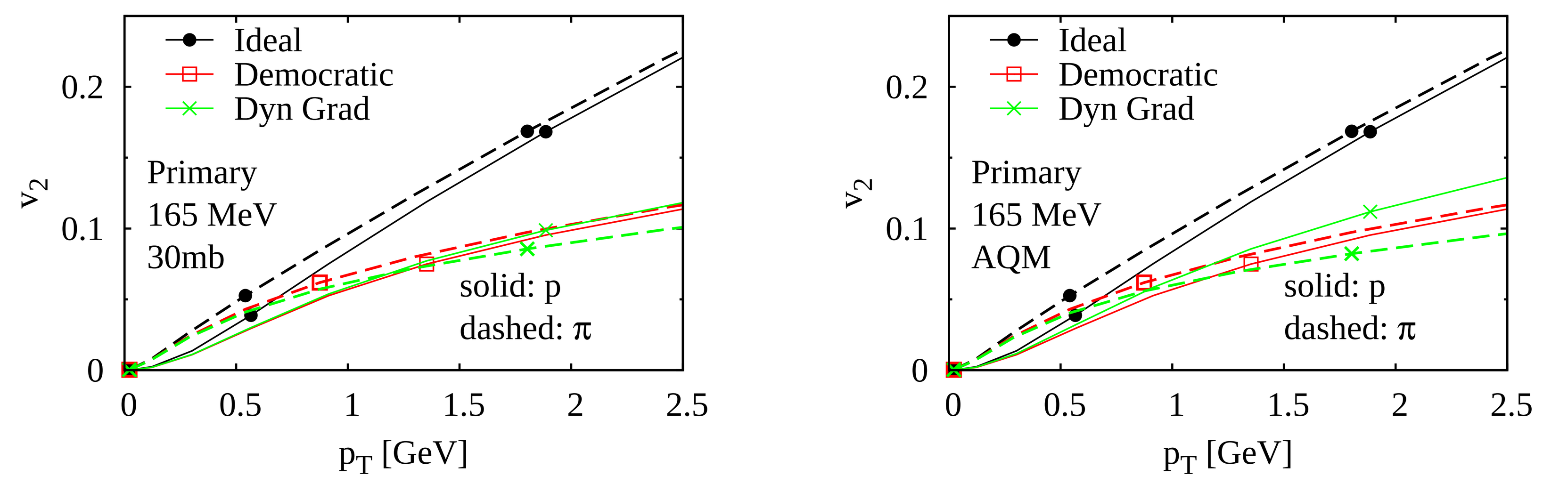

FIGURE 2: Elliptic flow in a hadron gas for constant and AQM cross sections in the dynamic Grad ansatz with T_{dec}=165 MeV.

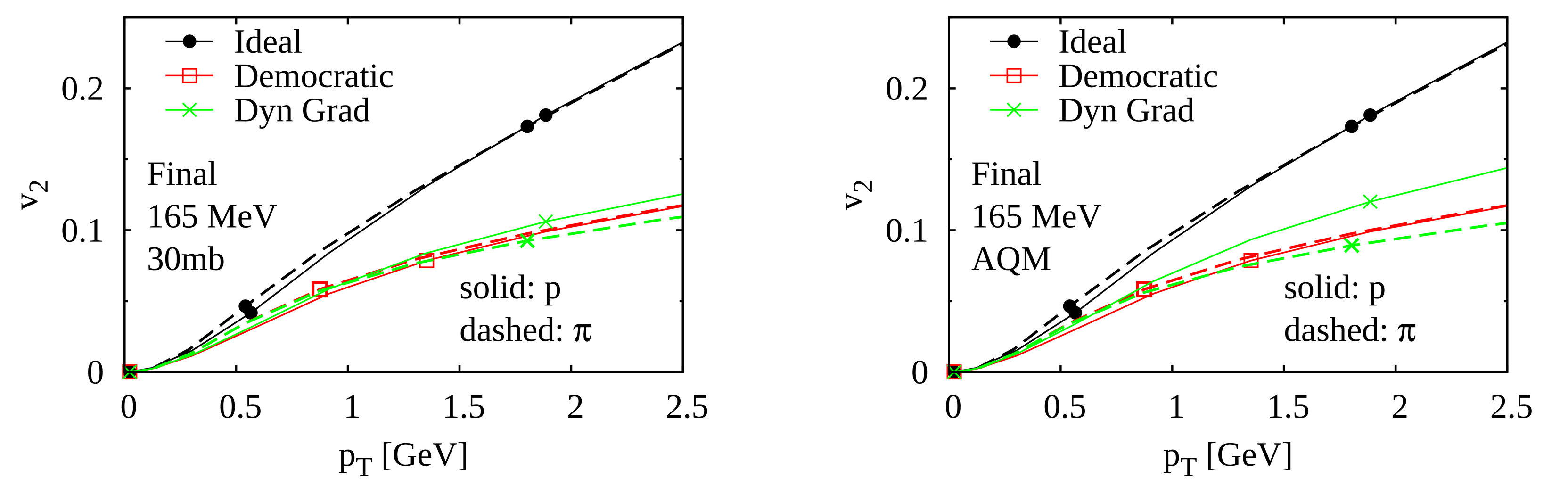

FIGURE 3: Elliptic flow of stable pions and nucleons after resonances have been decayed by the AZHYDRO code RESO.

Results: Hadron Gas $\delta f \sim c_i p^{\frac{3}{2}}$

Our analytic functional solutions prefer viscous corrections that grow more slowly than quadratic, closer to $\delta f \sim c_i p^{\frac{3}{2}}$.

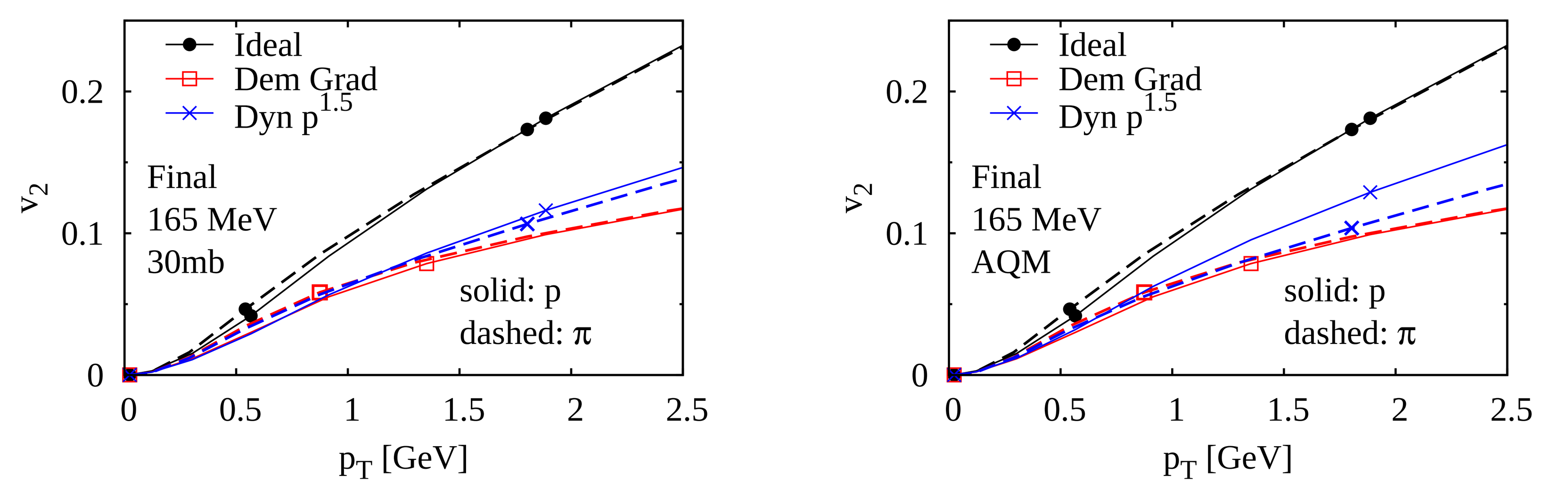

FIGURE 4: Elliptic flow of stable pions and nucleons after resonances have decayed assuming $\delta f \sim c_i p^{\frac{3}{2}}$.

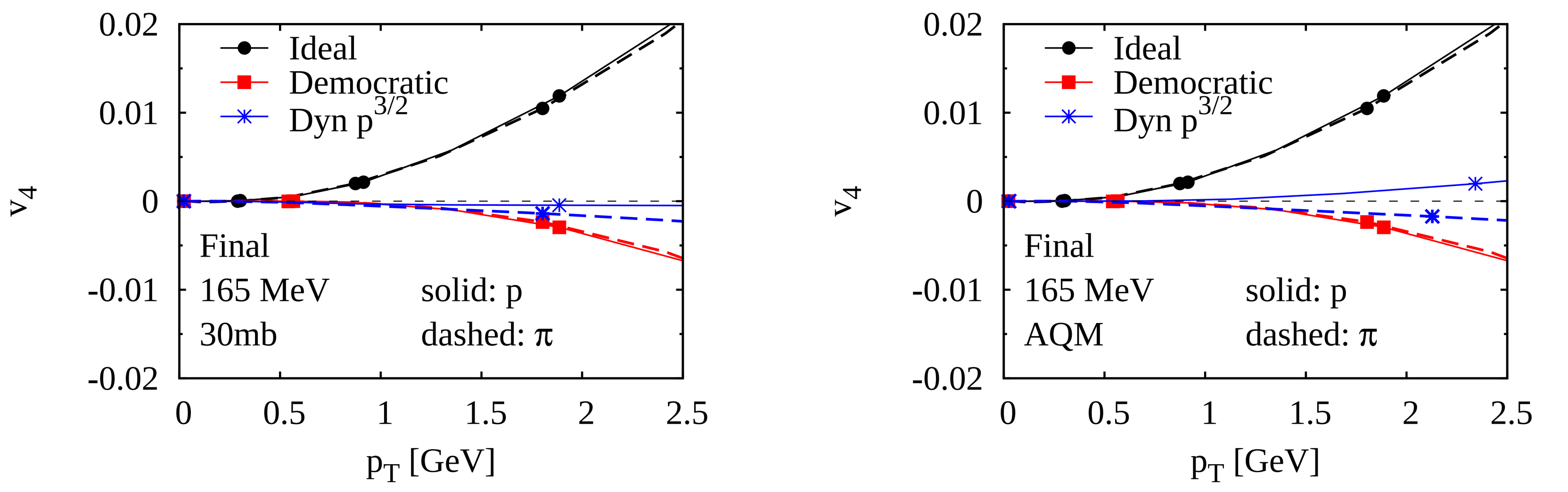

FIGURE 5: v_4 of stable pions and nucleons after resonances have decayed assuming $\delta f \sim c_i p^{\frac{3}{2}}$.

Identified particle observables are affected by the viscous particle distributions used in Cooper-Frye. Only after proper treatment of this ambiguety can reliable medium properties be extracted from data.

References & Acknowledgments

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