Self-consistent Cooper-Frye freeze-out of a viscous fluid to particles

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Abstract
Comparing hydrodynamic simulations to heavy-ion data inevitably requires the conversion of the fluid to particles. In dissipative fluids the conversion is ambiguous and particle species are typically given distributions independent of the microscopic dynamics that keep the system near thermal equilibrium. In contrast to this ad-hoc assumption, we compute distributions self-consistently by solving the linearized Boltzmann equation. We find that the momentum dependence of the corrections in all systems investigated is best fit by a power close to \( \frac{1}{2} \) rather than the typically used quadratic ansatz. The effects on harmonic flow are also calculated and found to be substantial, thus the form of these corrections should be taken into account when extracting medium properties from experimental data.

Outline of the Problem
Whether one models a heavy-ion collision with hydrodynamics alone or uses a "hybrid" hydrodynamic-kinetic-hadronic simulation, a conversion of the fluid description to one of particles is inevitable and typically done using the Cooper-Frye prescription:

\[
\frac{df}{d\sigma} = \frac{1}{\sqrt{2\pi T}} e^{-\chi_T(p)/T} \delta(p - m) \delta(T - T_{eq})
\]

Particle-distributions are calculated on a constant T hyper-surface:

\[
\chi_T(p) = \frac{1}{2} \int d\sigma \frac{d^3p}{(2\pi)^3} \ln \left(1 + \frac{\sigma}{\delta f}\right)
\]

Considering only shear stress, the viscous corrections have only one freedom left: a function of magnitude of momentum \( f \chi_T(p) / T \) for each species.

\[
\chi_{dyn\; Grad}(p) = \frac{\chi_T(p)}{T} = \text{CONSTANT}
\]

This ad hoc choice assigns the same \( \chi_{dyn\; Grad} \) to different species even though species that interact more should be closer to equilibrium. We instead calculate the \( \chi \) functions from covariant transport theory to maintain consistency with microscopic collision rates in a hadron gas.

Linearized Boltzmann Equation
For small deviations from local equilibrium we linearize in \( \delta f \) and solve the resulting integral equations for \( \chi \) variationally with a functional approach similar to [2,5]:

\[
\frac{d\delta f}{d\sigma} = \frac{\chi}{\pi T} \frac{d\sigma}{d\epsilon} \left( \frac{p}{m} - \frac{\epsilon}{T} \right)
\]

Results: Multicomponent Hadron Gas
We now convert the fluid to a hadron gas that includes resonances up to the D(600) under two simple models for crosssections between different species: 1) All species receive the same \( \sigma_g = 3\pi b^2 \) with every other species and 2) Cross sections follow the additive quark model (AQM) with meson and baryon asymmetry: \( \sigma_{MM\; AQM} = 4 \cdot 6 \cdot 9 \).

Results: Pion-Nucleon Gas
We first test the results in a conversion to a simple pion-nucleon system with energy-independent effective crosssections[8] in the "Dynamic Grad" ansatz where corrections are still quadratic in momentum but can depend on the crosssections between species, i.e.

\[
\delta f_{\pi N} \sim \epsilon_\pi \sigma_{\pi N} p^2
\]

These distribution functions are used in the Cooper-Frye prescription to calculate spectra and elliptic flow after running AZHYDRO[6] \( 0.2 p^2 \) for a typical RHIC collision at \( b = 7 \text{ fm} \). The shear stress tensor is calculated from the flow gradients using the Navier-Stokes estimate with viscosity to entropy ratio taken to be \( \eta/s = 0.2 \pi^2 \) for a typical RHIC collision at \( b = 7 \text{ fm} \). The shear stress tensor is calculated from the flow gradients using the Navier-Stokes estimate with viscosity to entropy ratio taken to be \( \eta/s = 0.2 \pi^2 \). The shear stress tensor is calculated from the flow gradients using the Navier-Stokes estimate with viscosity to entropy ratio taken to be \( \eta/s = 0.2 \pi^2 \).

Figure 1: Elliptic flow \( v_2(p_T) \) for a pion-nucleon gas calculated from linearized kinetic theory. Standard "dynamic Grad" (open boxes) are compared to our self-consistent (crosses) calculations. The left plot is for conversion on a T=165 MeV hyper-surface while the right is at 165 MeV.

Self-consistent viscous corrections in a pion-nucleon gas lead to an elliptic flow splitting at moderately high momenta, with proton \( v_2 \sim 30\% \) larger than pion \( v_2 \). The decoupling is done at lower temperature, the effects of viscosity diminish as the systems gradients are reduced.

Results: Hadron Gas \( \delta f \sim \epsilon p^2 \)
Our analytic functional solutions prefer viscous corrections that grow more slowly than quadratic, closer to \( \delta f \sim \epsilon p^2 \).

Figure 2: Elliptic flow in a hadron gas for constant and AQM cross sections in the Dynamic Grad ansatz with \( T_{eq}=165 \text{ MeV} \).

Figure 3: Elliptic flow of stable pions and nucleons after resonance decays by the AZHYDRO code RESO.

Figure 4: Elliptic flow of stable pions and nucleons after resonance decays have been decayed by the AZHYDRO code RESO.

Figure 5: \( v_2(p_T) \) for pions and nucleons after resonance decays have been decayed assuming \( \delta f \sim \epsilon p^2 \).

Identified particle observables are affected by the viscous particle distributions used in Cooper-Frye. Only after proper treatment of this ambiguity can reliable medium properties be extracted from data.

References & Acknowledgments
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