Charmonium spectra and dispersion relation with improved Bayesian analysis in lattice QCD

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I. Background

- Strongly coupled QGP
  Success of ideal hydrodynamic model for QGP at the RHIC
- Dynamical property of QGP medium
  → Real time information is needed
- First-principles calculation (Lattice QCD)
  Analytic continuation imaginary time to real time is difficult (ill-posed problem)
  → Maximum entropy method

Purpose

1. Improve the error of MEM by extending MEM
2. Analyze the dispersion relation of charmonia at finite temperature.

II. MEM

Analytic continuation

Imaginary time correlation function (from Lattice QCD)

$$D(\tau,\vec{p}) = \int d^3x e^{i\vec{p}\cdot\vec{x}} \left\langle J_i(\tau,\vec{x}) J_i^\dagger(0,\vec{0}) \right\rangle$$

$$= \int_0^\infty K(\tau,\omega) A(\omega,\vec{p}) d\omega \qquad K(\tau,\omega) = \frac{e^{-\tau\omega} + e^{-(\beta-\tau)\omega}}{1-e^{-\beta\omega}}$$

Spectral function (which desired)

MEM

$$A_{\text{out}} = \int d\alpha \int [dA]\, A(\omega) P(A,\alpha)$$

$$P(A,\alpha) = [\text{Likelihood function}](A) \times [\text{Prior probability}](A,\alpha)/Z$$

Likelihood function
Information of Lattice QCD

$$\exp(-\chi^2) = \exp\left[-\frac{1}{2}\sum_{i,j}(D(\tau_i)-D_A(\tau_i))\, C_{ij}^{-1}\, (D(\tau_j)-D_A(\tau_j))\right]$$

$$D(\tau_i) = \frac{1}{N_{\text{conf}}}\sum_{m=1}^{N_{\text{conf}}} D^m(\tau_i)$$

$$C_{ij} = \frac{1}{N_{\text{conf}}(N_{\text{conf}}-1)}\sum_{m=1}^{N_{\text{conf}}}(D^m(\tau_i)-D(\tau_i))(D^m(\tau_j)-D(\tau_j))$$

Prior probabilty
Information of pQCD

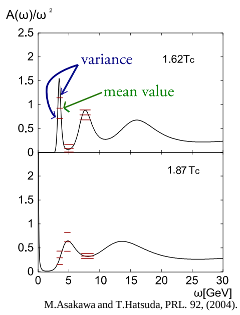
$$\exp\left(\alpha\int_0^\infty \left[A(\omega)-m(\omega)-A(\omega)\log\left(\frac{A(\omega)}{m(\omega)}\right)\right]d\omega\right)$$

$$m(\omega) = m_0\omega^2$$

The error of MEM

$$\langle(\delta A_{\text{out}})^2\rangle_I = \int d\alpha \int [dA] \int_{I\times I} d\omega d\omega' \, \delta A(\omega)\delta A(\omega') P(A,\alpha) \Big/ \int_{I\times I} d\omega d\omega'$$

$$\delta A(\omega) = A(\omega) - A_\alpha(\omega)$$

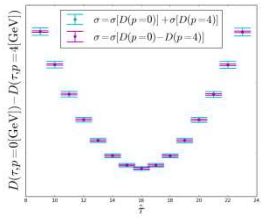


η_c melts between $1.62T_c$ and $1.87T_c$

Usually, the analysis is performed for a single correlator.

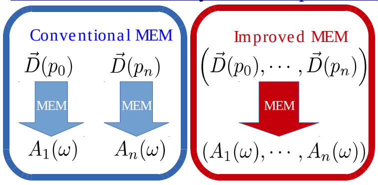
III. Improved MEM

Different correlators measured on same configuration have strong correlations



Take this correlation into Bayesian analysis

Extend the MEM analysis for the product space of correlation functions



The correlation is in the covariance matrix C_{ij}

IV. Results

Lattice setup

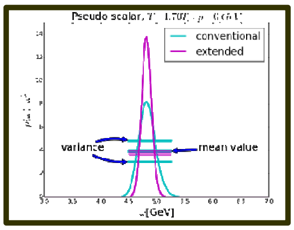
- SU(3) pure gauge theory
- Wilson gauge and standard Wilson quark action

N_τ	T/T_c	N_σ	L_σ[fm]	a_σ[fm]	a_σ/a_τ	β
44	1.70	64	2.50	0.00975	4	7.0
96	0.78	64	2.50	0.00975	4	7.0

C. Nonaka, et al., J. Phys. G: Nucl. Part. Phys. 38, 124109 (2011).

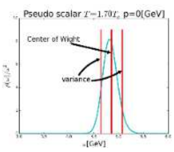
Spectral function

- Analyze two correlators together
- The error of spectra is drastically improved.
- The width of the peak become narrow.
- The peak corrsponds to η_c remains at 1.70Tc



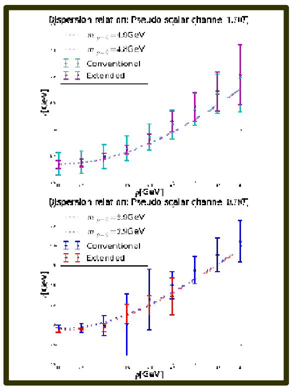
Despersion relation

- The error is estimated as variance of the center of weight.



$$(\text{variance}) = \frac{\sqrt{\langle(\omega\delta A(\omega)/\omega^2)^2\rangle_I}}{\langle A(\omega)/\omega^2\rangle_I}$$

- Line: $\omega = \sqrt{{m_0}^2 + p^2}$
- Big improvement at low momentum region



V. Summary & Future plan

- We extend the MEM analysis to the product space of the correlators to take advantage of more data and the strong correlation among Euclidean correlators with different momenta.
- The error of MEM is drastically improved.
- Make α multi-dimensional.