I. Background

- **Strongly coupled QGP**
  - Success of ideal hydrodynamic model for QGP at the RHIC
- **Dynamical property of QGP medium**
  - Real time information is needed
- **First-principles calculation (Lattice QCD)**
  - Analytic continuation imaginary time to real time is difficult (ill-posed problem)
  - Maximum entropy method

**Purpose**
1. Improve the error of MEM by extending MEM
2. Analyze the dispersion relation of charmonia at finite temperature.

II. MEM

**Analytic continuation**

Imaginary time correlation function (from Lattice QCD)

\[
\langle \mathcal{A}(\tau, \mathbf{r}) \rangle = \int d^3 \mathbf{r} \mathcal{A}(\tau, \mathbf{r}) \mathcal{A}^*(\tau, \mathbf{r})
\]

\[
= \int_0^\infty K(\tau, \mathbf{r}) A(\mathbf{r}) P(\mathbf{r}, \tau) d\omega
\]

MEM

\[
A_{\text{new}} = \int d\omega \int [\mathcal{A}] A(\omega) P(\omega, \alpha)
\]

\[
P(\omega, \alpha) = |\text{Likelihood function}|(A) \times |\text{Prior probability}|(A, \alpha)/Z
\]

\[
\text{Likelihood function (Information of Lattice QCD)}
\]

\[
\text{Prior probability (Information of pQCD)}
\]

**The error of MEM**

\[
\langle (\delta A_{\text{out}})^2 \rangle = \int d\omega \int \mathcal{A}(\omega) \mathcal{A}(\omega') P(\omega, \alpha)
\]

\[
= \int d\omega \int \mathcal{A}(\omega) \mathcal{A}(\omega') P(\omega, \alpha)
\]

\[
\delta A(\omega) = A(\omega) - A_0(\omega)
\]

Usually, the analysis is performed for a single correlation.

III. Improved MEM

Different correlators measured on same configuration have strong correlations

Take this correlation into Bayesian analysis

Extend the MEM analysis for the product space of correlation functions

The correlation is in the covariance matrix \( C_{ij} \)

IV. Results

**Lattice setup**

- SU(3) pure gauge theory
- Wilson gauge and standard Wilson quark action

**Spectral function**

- Analyze two correlators together
- The error of spectra is drastically improved.
  - The width of the peak become narrow.
  - The peak corresponds to \( \eta_c \) remains at 1.70Tc

**Dispersion relation**

- The error is estimated as variance of the center of weight.
  
  \[
  \text{variance} = (\langle (\omega A(\omega) / \omega^2 \rangle) - (\omega A(\omega) / \omega^2 \rangle)^2
  \]

- Line: \( \omega = \sqrt{m_c^2 + p^2} \)
- Big improvement at low momentum region

V. Summary & Future plan

- We extend the MEM analysis to the product space of the correlators to take advantage of more data and the strong correlation among Euclidean correlators with different momenta.
- The error of MEM is drastically improved.
- Make a multi-dimensional.