New universal parametrization of initial-state fluctuations and its application to event-by-event anisotropy

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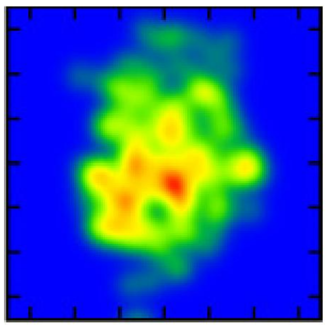
Outline

- Introduce Elliptic Power and Power parameterizations of ε_n fluctuations.
- Apply Elliptic Power and Power to v_n data.

Motivation: fluctuations of harmonics

- Fluctuating initial state and harmonic flow v_n:

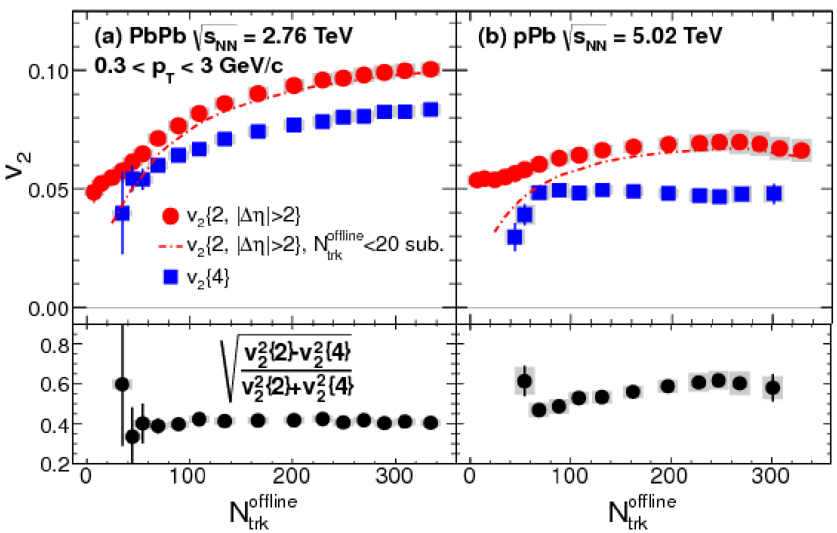


Glauber, KLN, IP-Glasma

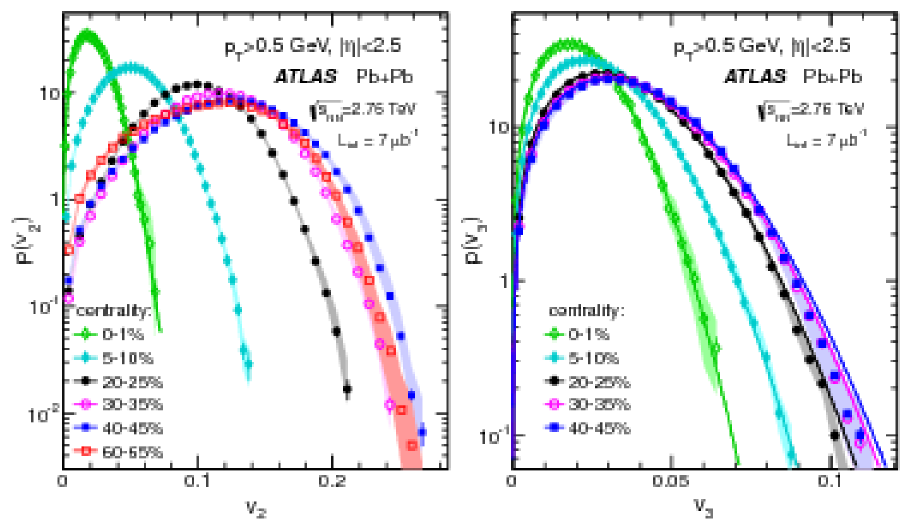
$$\underbrace{\Longleftrightarrow}_{\text{medium exp.}} \qquad \frac{dN}{d\phi_p} \sim 1 + 2\sum_n v_n e^{in(\phi_p - \Psi_n)}$$

- Fluctuations of v_n in experiments:

(CMS)PLB724 (2013)213-240



(ATLAS) JHEP 1311(2013)183



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2 What can we learn from EbyE v_n distribution? In particular, η/s?

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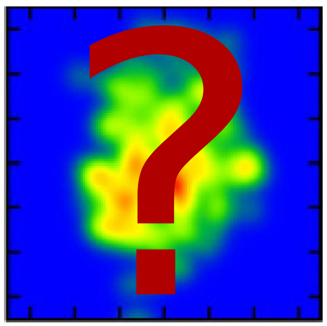

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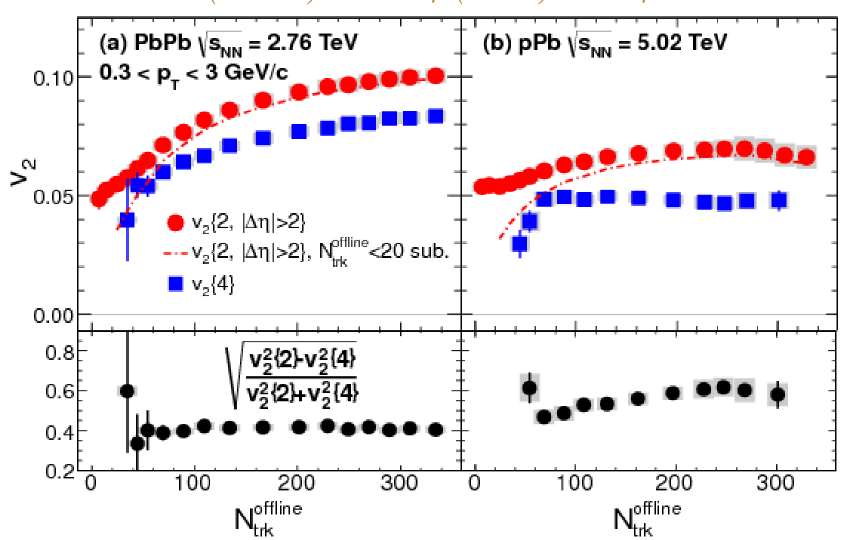

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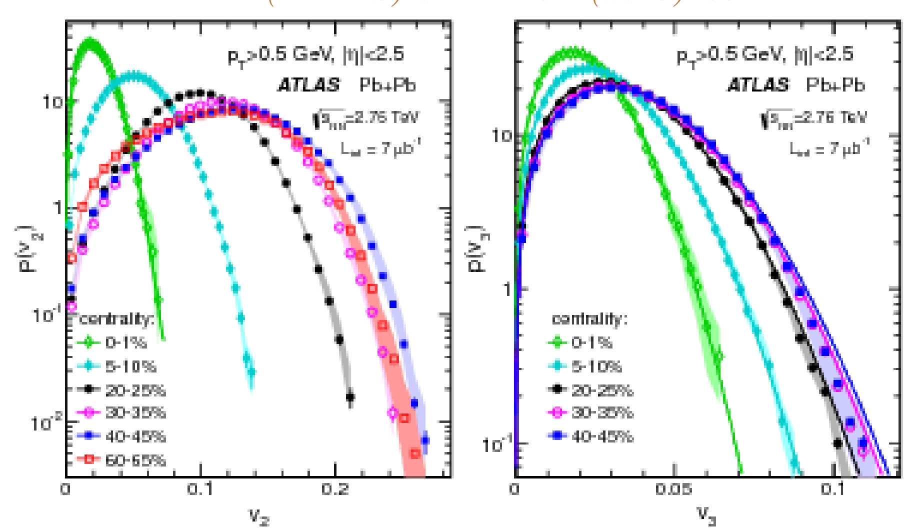

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Initial state eccentricity fluctuations: ε_2

- Eccentricity: $$\varepsilon_2 e^{i2\Phi_2} = -\frac{\{r^2 e^{i2\phi_r}\}}{\{r^2\}} = \varepsilon_x + i\varepsilon_y$$

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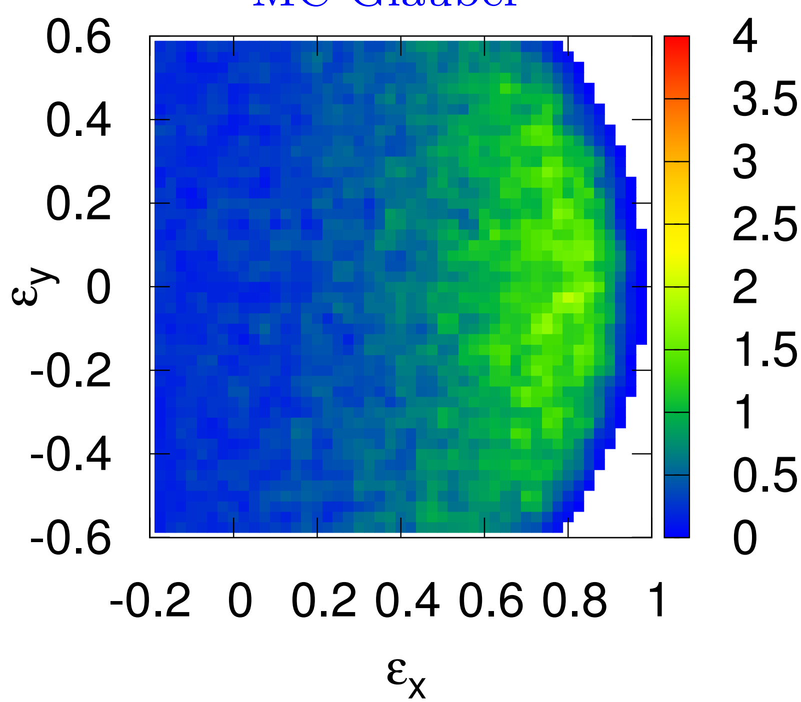
MC-Glauber



* 2D distribution from MC-Glauber:

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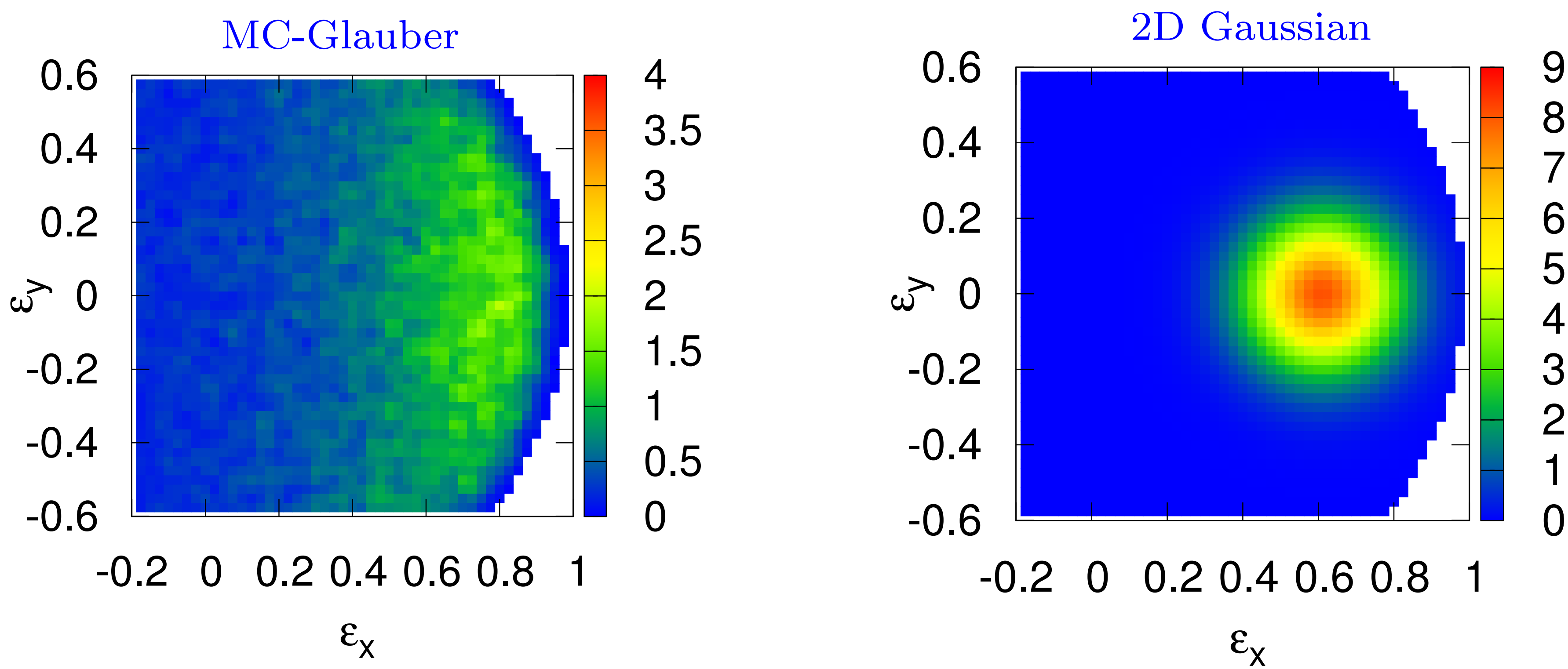

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* 2D Gaussian with non-zero mean. $\Rightarrow$ Bessel-Gaussian. *(S. Voloshin et al, PLB 659)*

Elliptic Power distribution and Power distribution

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- **Elliptic Power distribution** : (e.g. assuming N independent point-like sources)

$$P_{\mathrm{EP}}(\varepsilon_x, \varepsilon_y) = \frac{\alpha}{\pi}(1-\varepsilon_0^2)^{\alpha+\frac{1}{2}} \frac{(1-\varepsilon_x^2-\varepsilon_y^2)^{\alpha-1}}{(1-\varepsilon_0\varepsilon_x)^{2\alpha+1}}, \qquad \text{with } \varepsilon_x^2+\varepsilon_y^2 < 1$$

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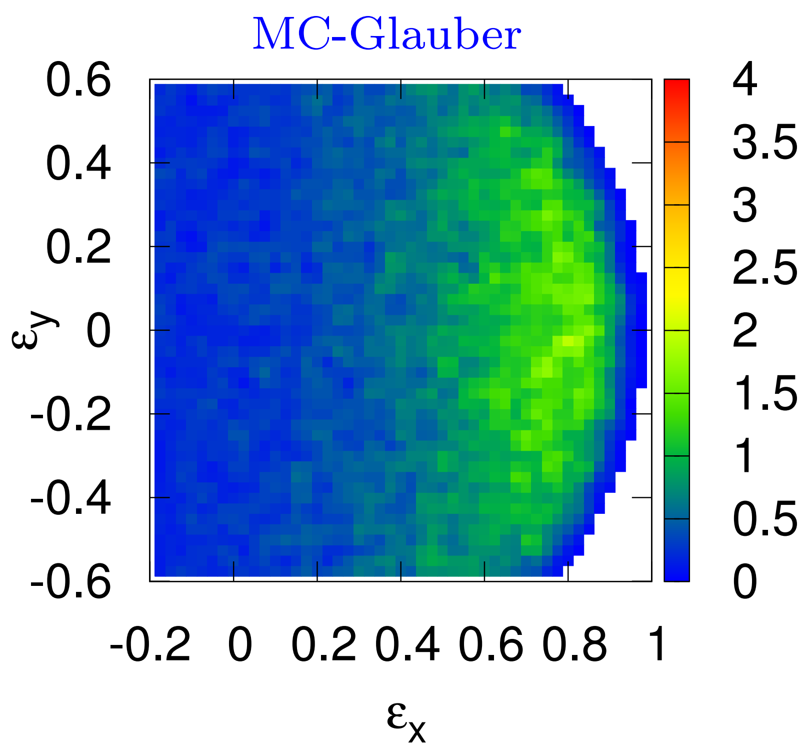

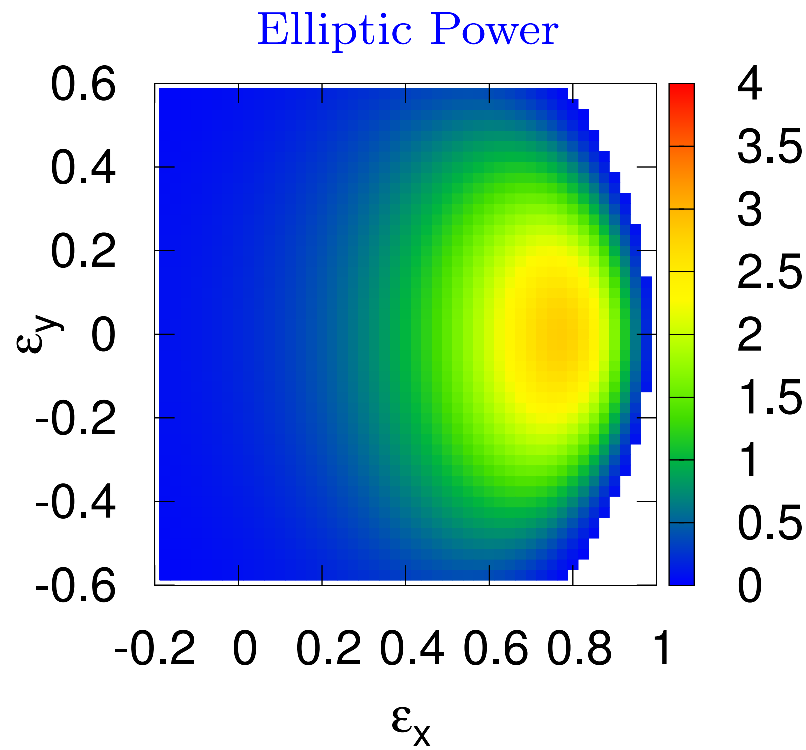

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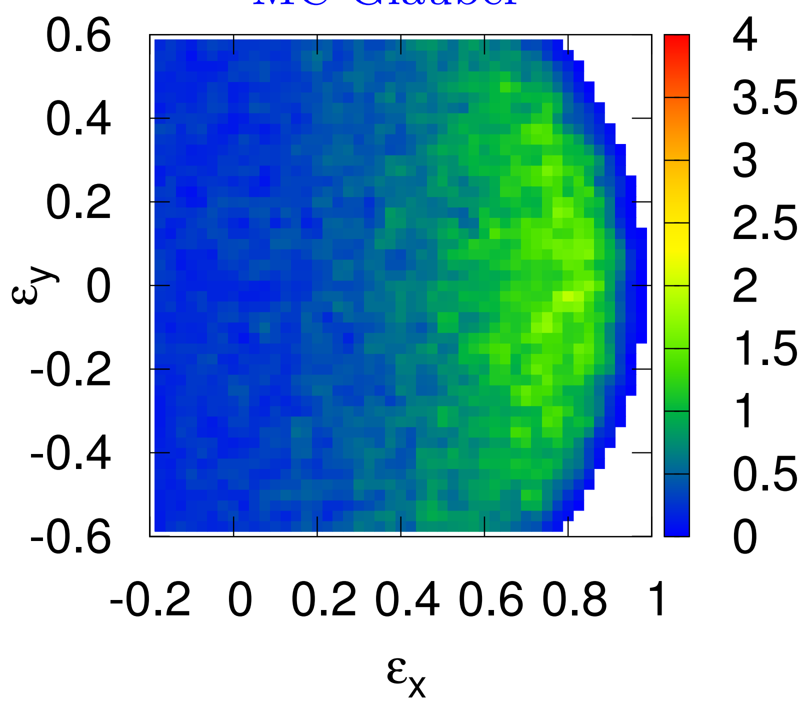
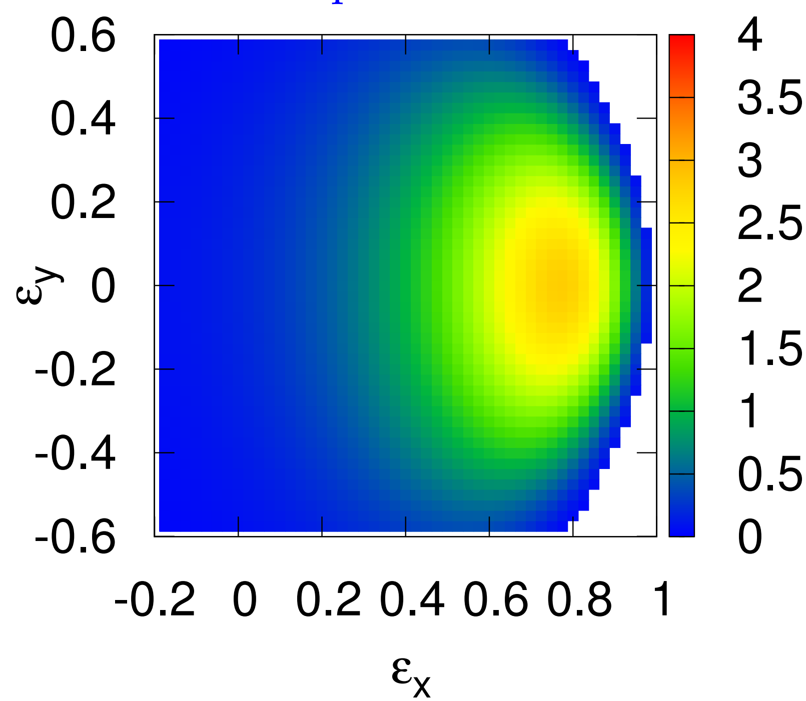


Elliptic Power

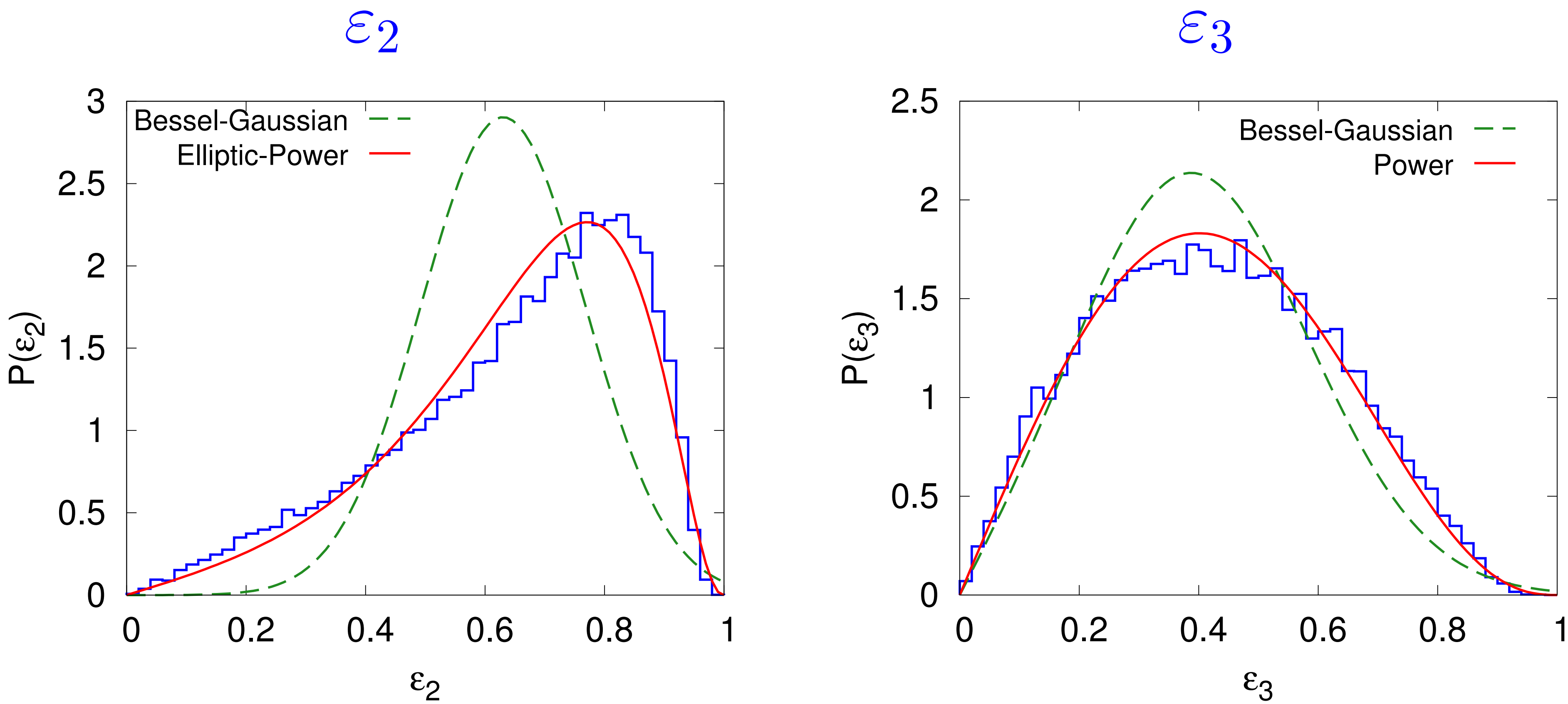


- **Power distribution** (e.g. ε_3 in AA, ε_n in p-Pb) : fluctuation-driven with $\varepsilon_0 = 0$

$$P_{\mathrm{Power}}(\varepsilon_x, \varepsilon_y) = \frac{\alpha}{\pi}(1-\varepsilon_x^2-\varepsilon_y^2)^{\alpha-1} \qquad \Leftarrow \qquad P_{\mathrm{EP}}(\varepsilon_0 \to 0)$$

Test of Elliptic Power and Power

- Test of Elliptic Power parameterization: MC-Glauber PbPb with centrality 75%-80%.



- Significant improvement with new parameterizations.

  * ε_2 fit $\rightarrow \chi^2/\text{dof} \sim 8$(Elliptic Power) and 88(Bessel-Gaussian), ε_3 fit $\rightarrow \chi^2/\text{dof} \sim 4$(Power) and 36(Bessel-Gaussian).

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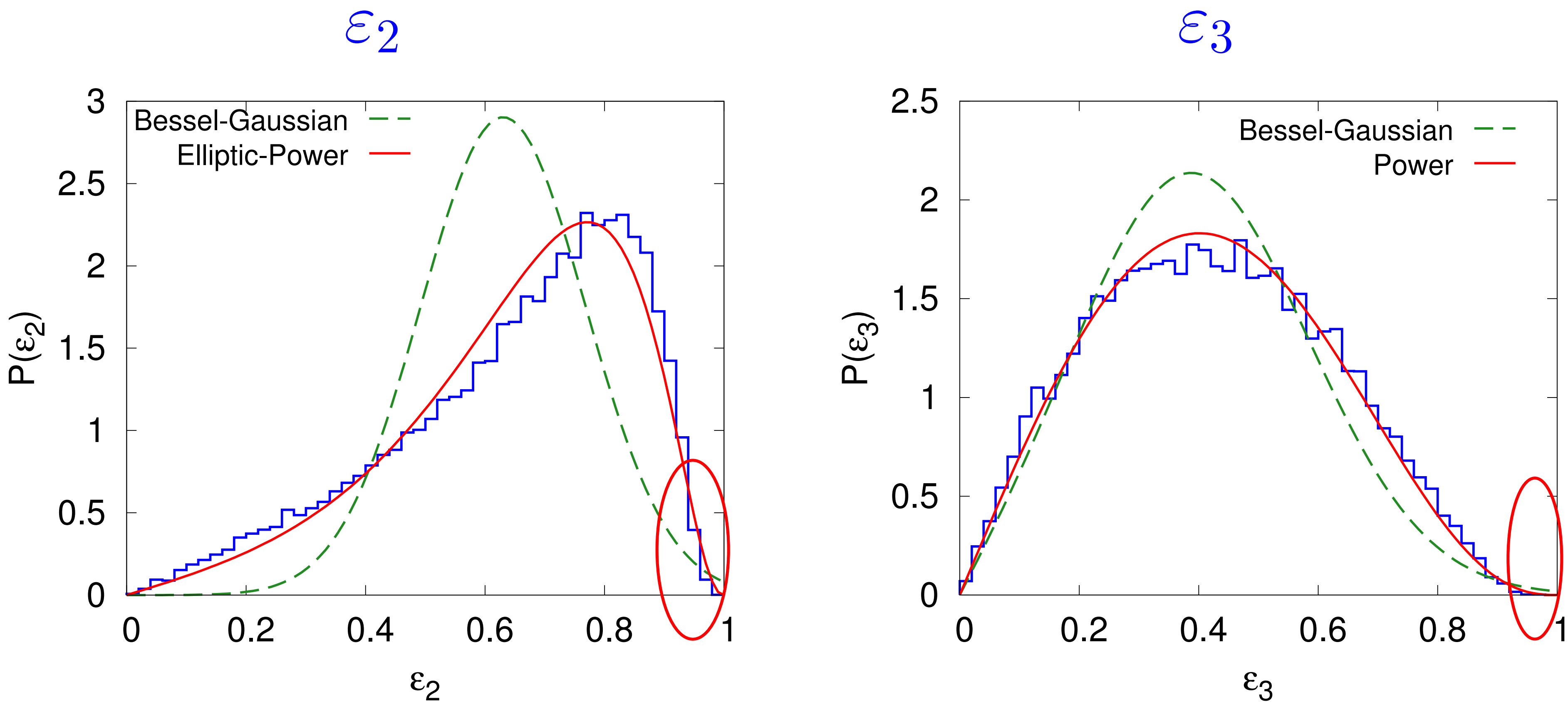

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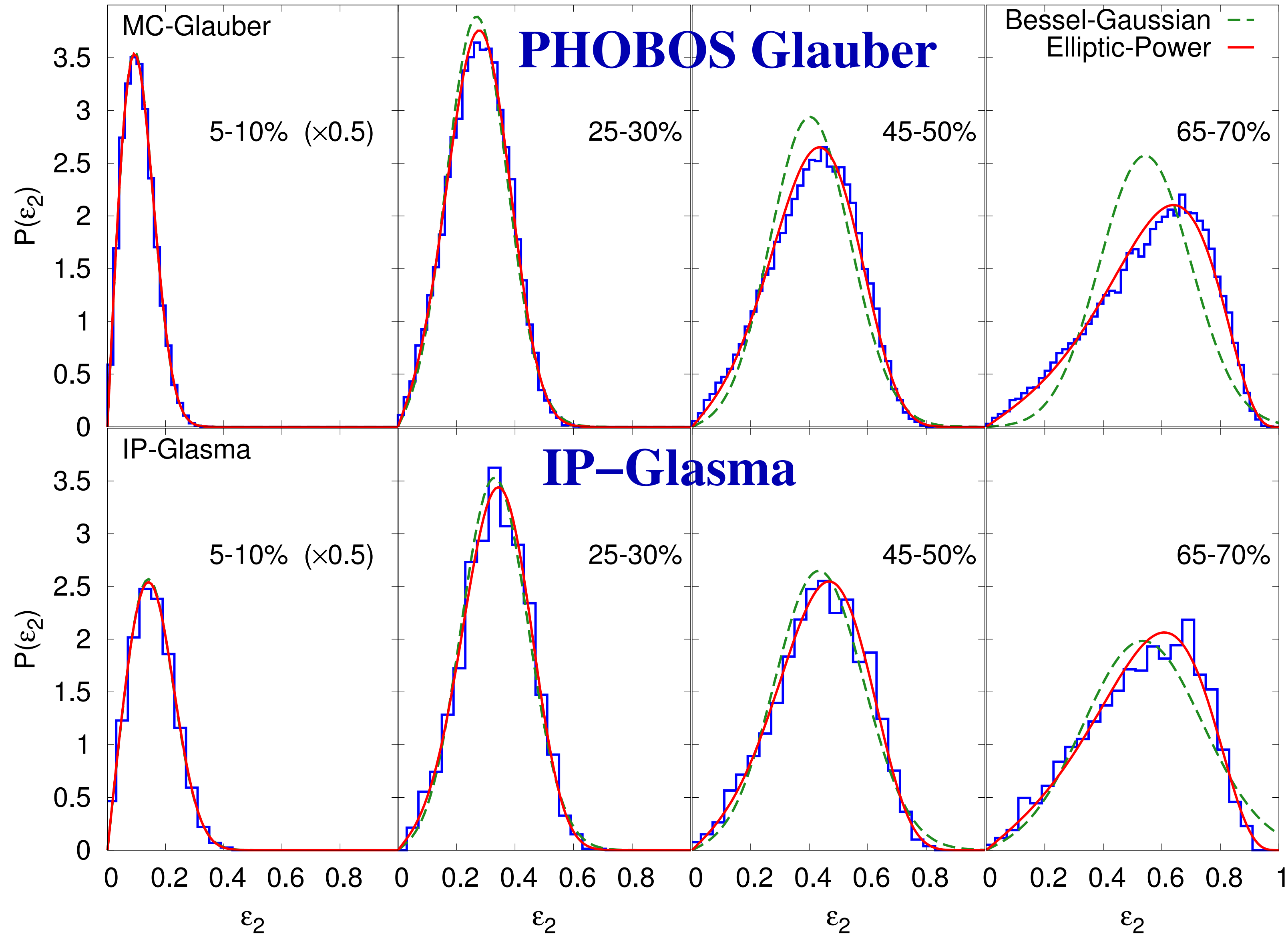
Elliptic Power distribution: test of universality



PHOBOS Glauber (B.Alver et al, arXiv:0805.4411) *IP-Glasma (B.Schenke et al, arXiv:1312.5588)*

* Elliptic Power (and Power) universally parameterizes fluctuations of ε_n.

Applications to data

- With Elliptic Power and Power characterizing initial state fluctuations,

- Linear eccentricity scaling: ($n = 2$ and 3) *H.Niemi et al.,Phys.Rev. C87 (2013) 054901*

$$v_n = \underbrace{\kappa_n}_{\text{medium resp.: } \eta/s} \times \underbrace{\varepsilon_n}_{\text{Elliptic Power or Power}}$$

- Ignore fluctuations in medium response, i.e., κ_n does not fluctuate.

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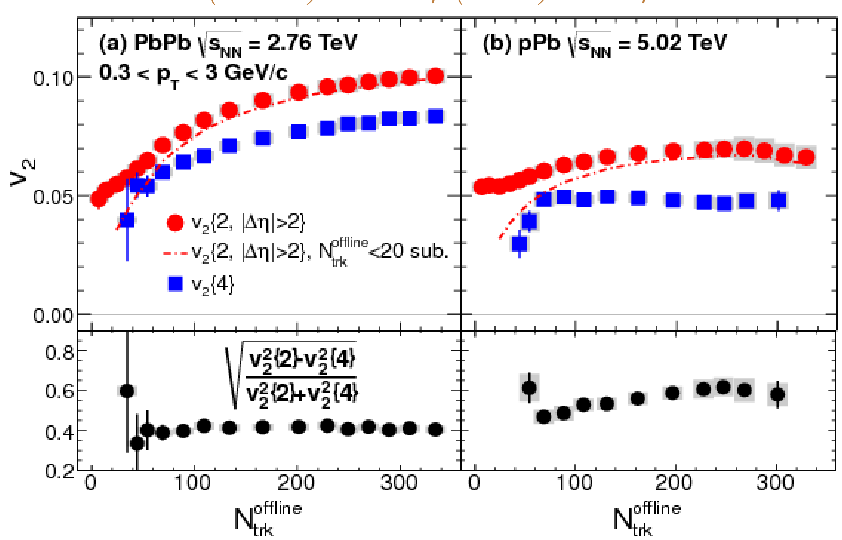

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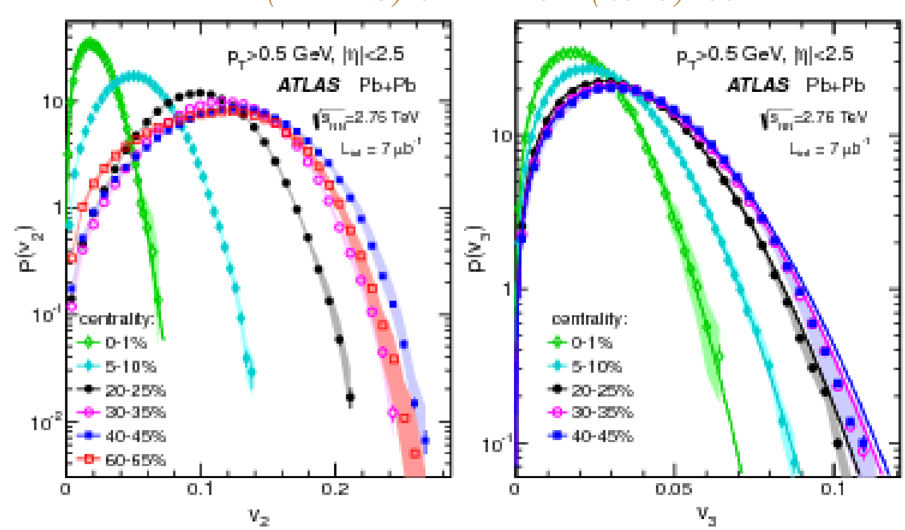

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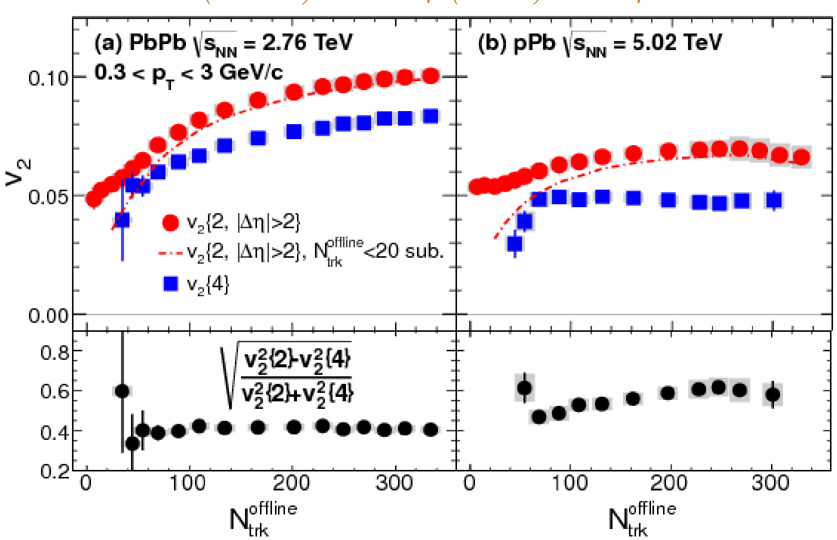

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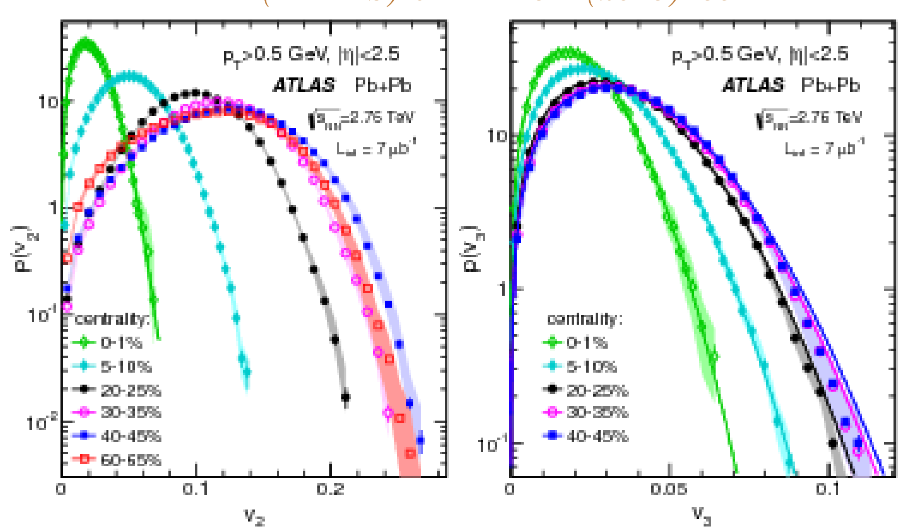

Fit data without assuming any specific model of initial state!

Fluctuations in p-Pb system and $v_2\{4\}$

- Cumulants of $v_2\{4\}$ and cumulants of $\varepsilon_2\{4\}$ in p-Pb:

$$\varepsilon_2\{4\} = \left[2\langle\varepsilon_2^2\rangle^2 - \langle\varepsilon_2^4\rangle\right]^{1/4} \Rightarrow$$

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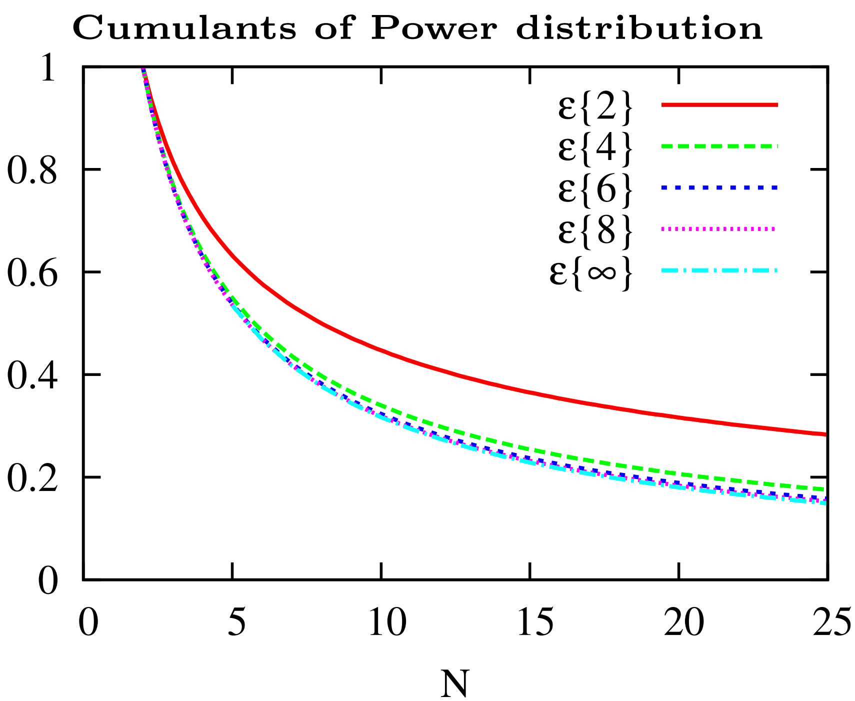

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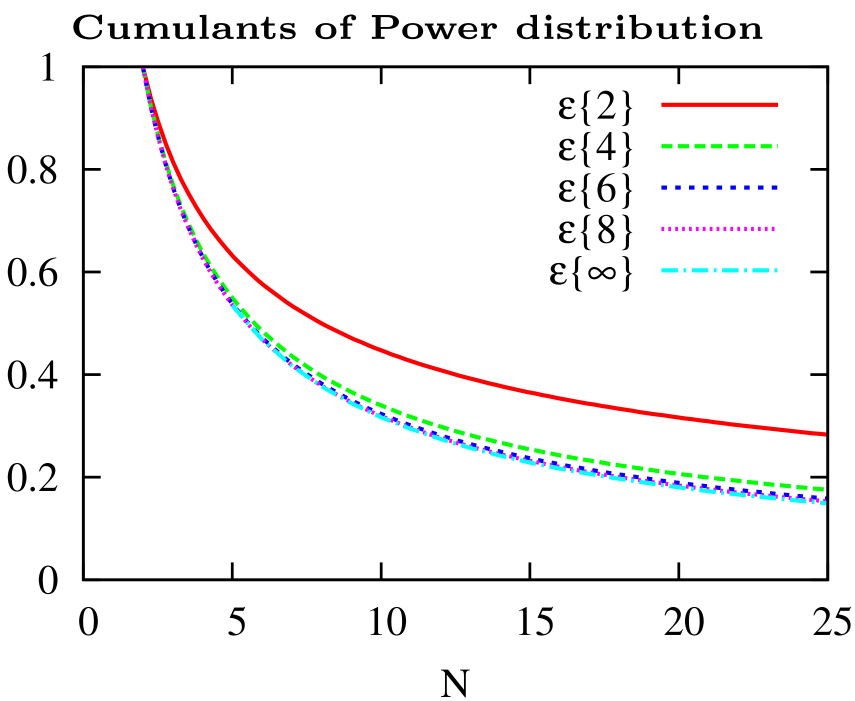

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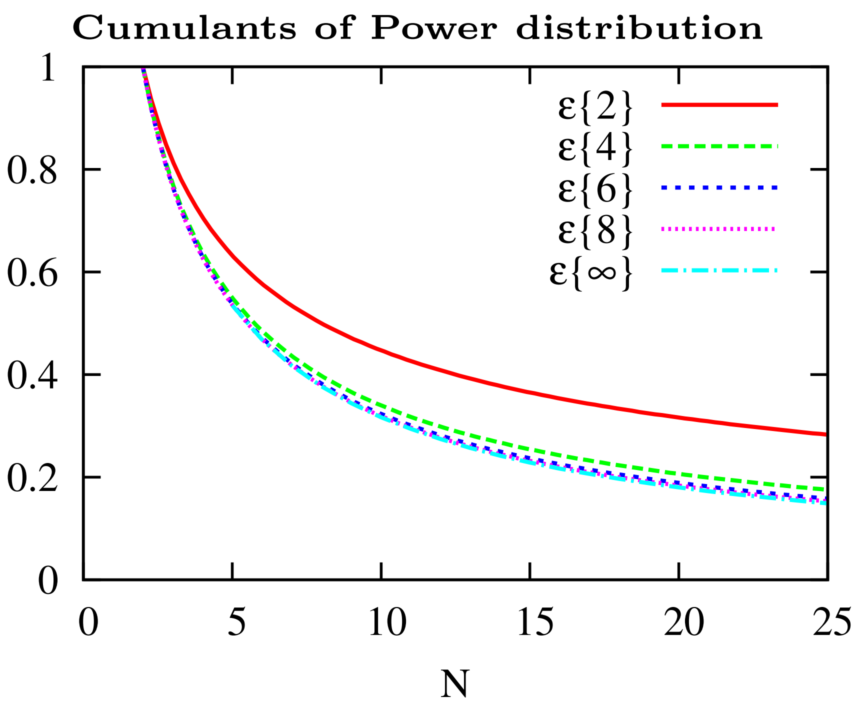

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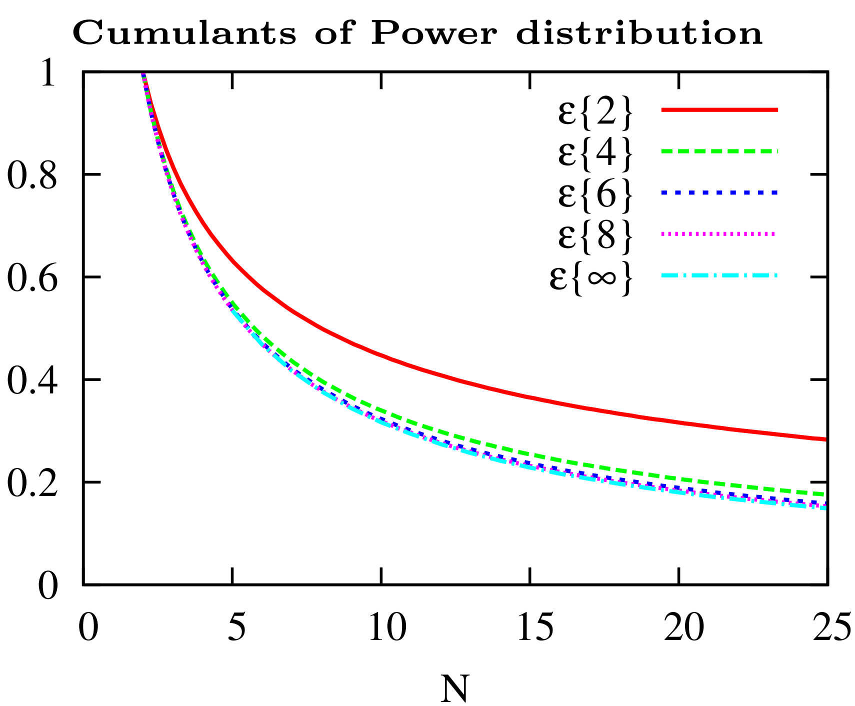

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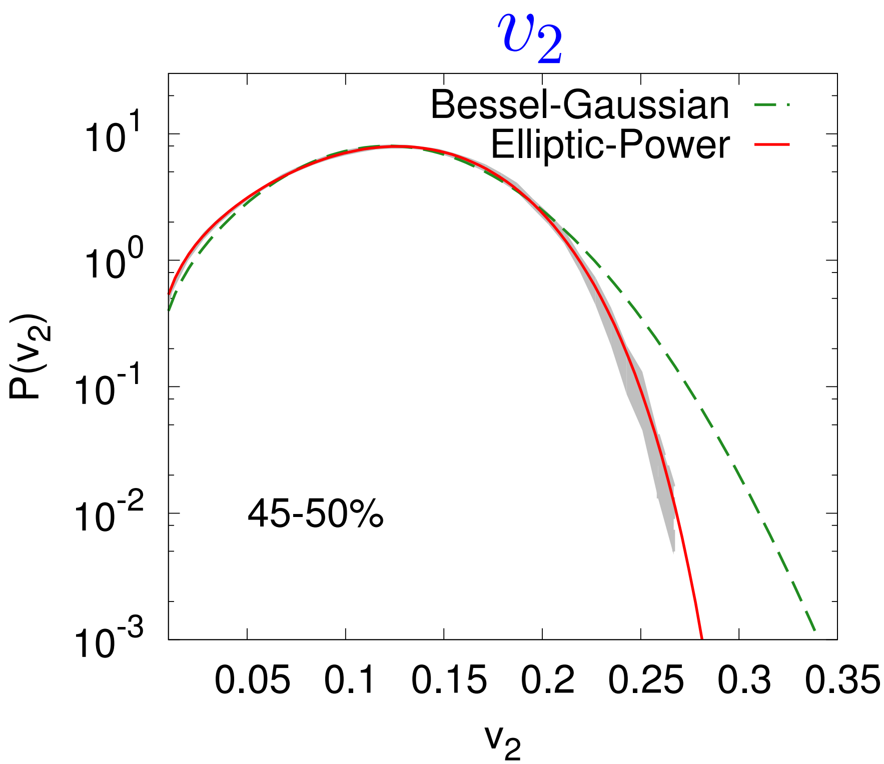
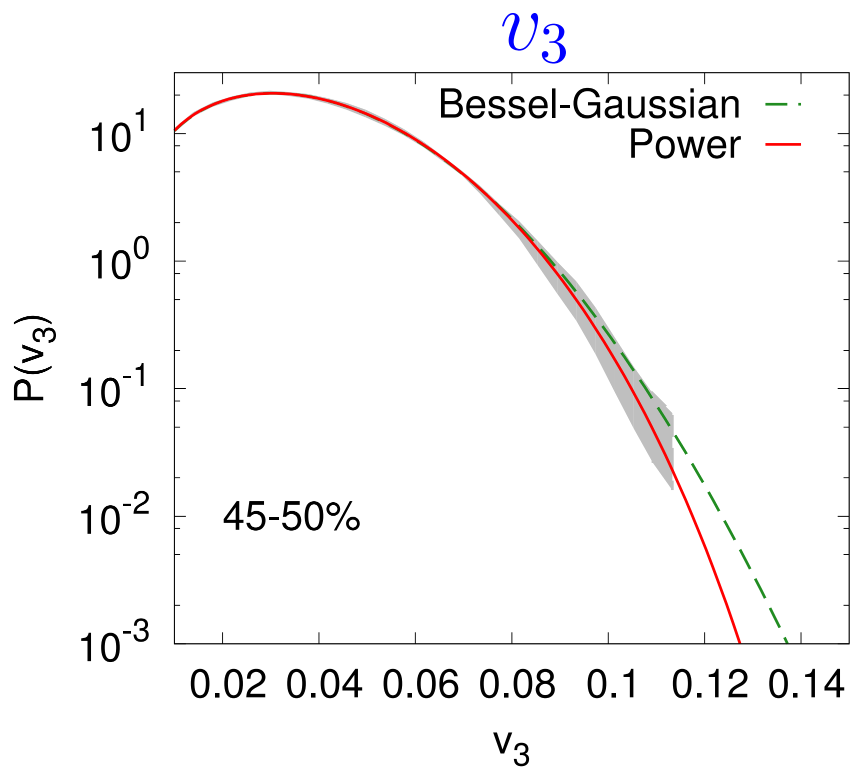
Collective expansion (hydro.) of LHC p-Pb

Fit ATLAS EbyE v_n distribution

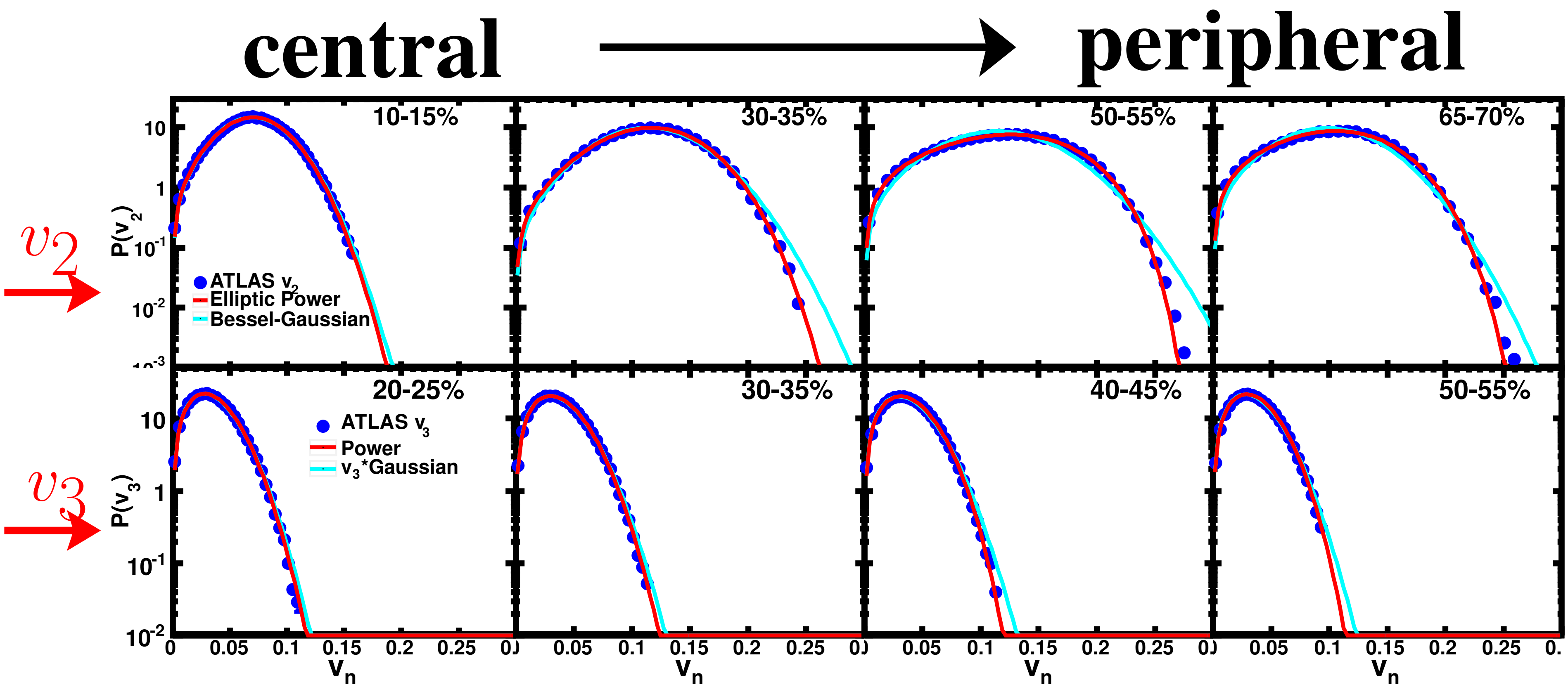
- Rescaled Elliptic Power (or Power) parameterization: ATLAS v_2 and v_3 at 45-50%.





- Fits much better than Bessel-Gaussian.
- Deviations of Bessel-Gaussian fixed at tails are mostly due to $\varepsilon_n < 1$.
- $\kappa_n \Rightarrow$ flow resp. $\varepsilon_0 \Rightarrow$ average RP eccentricity $\alpha \Rightarrow$ magnitude of fluctuations

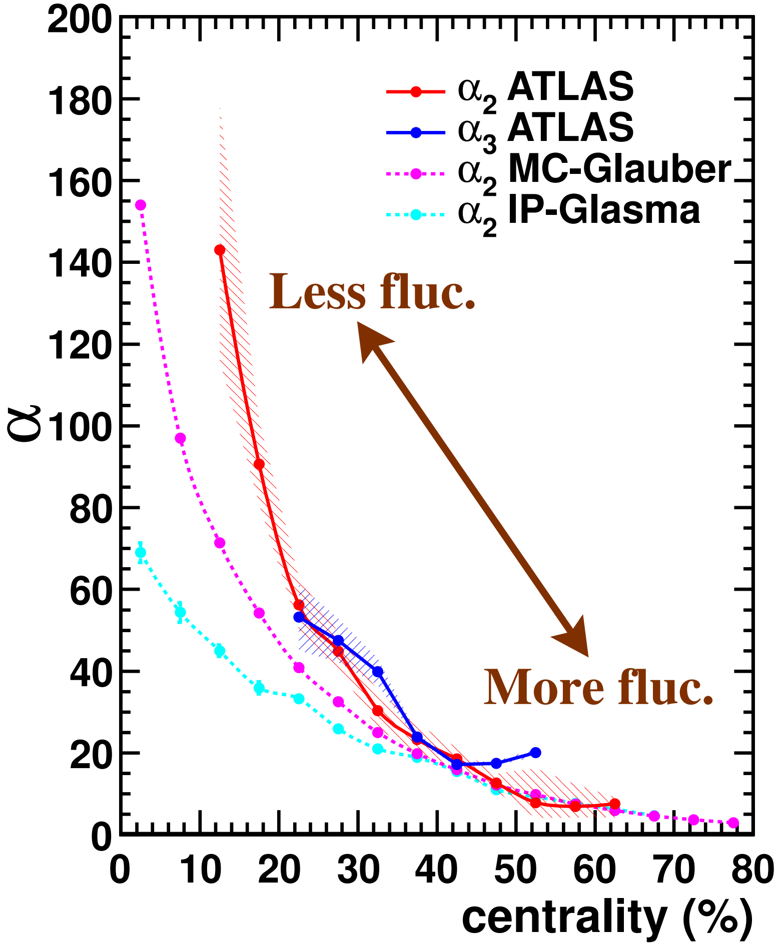
ATLAS EbyE v_n distribution



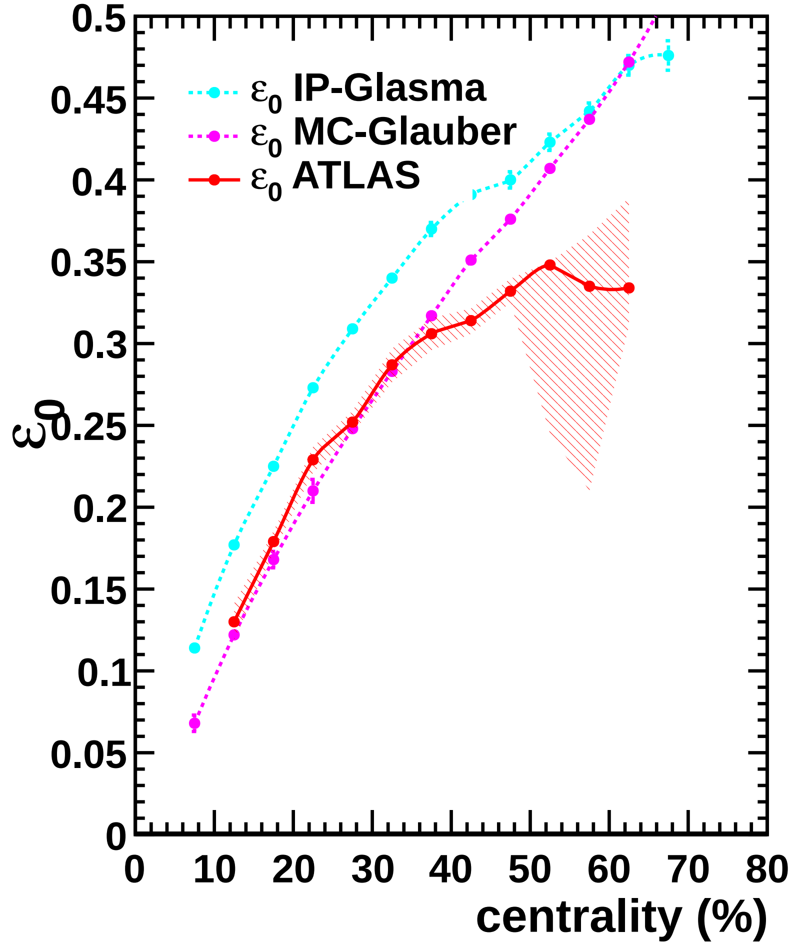
- Significant improvement with Elliptic Power and Power parameterization.
- Error of v_2 fit is dominated by systematic errors on $\sigma_v / < v >$ from ATLAS results.
- Error of v_3 fit is from statistical error of v_3 only. Systematic errors are too large.

Extract information of initial state – α and ϵ_0

Fluctuations



Average RP eccentricity



- Fluctuations become stronger for peripheral collisions.
- ε_0 grows with centrality percentage.

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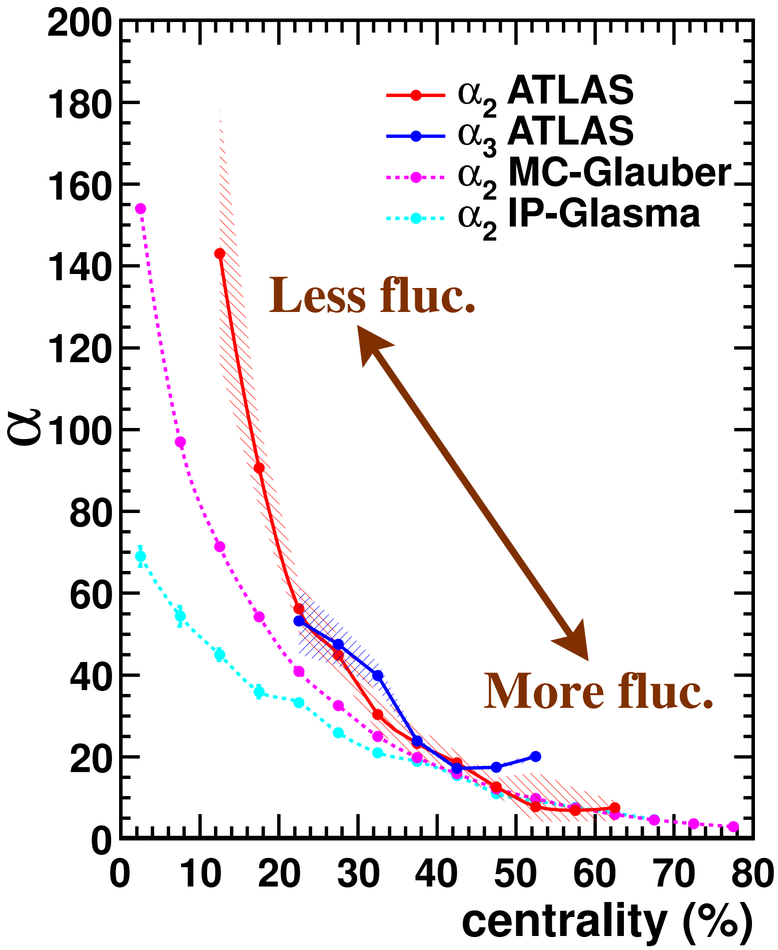

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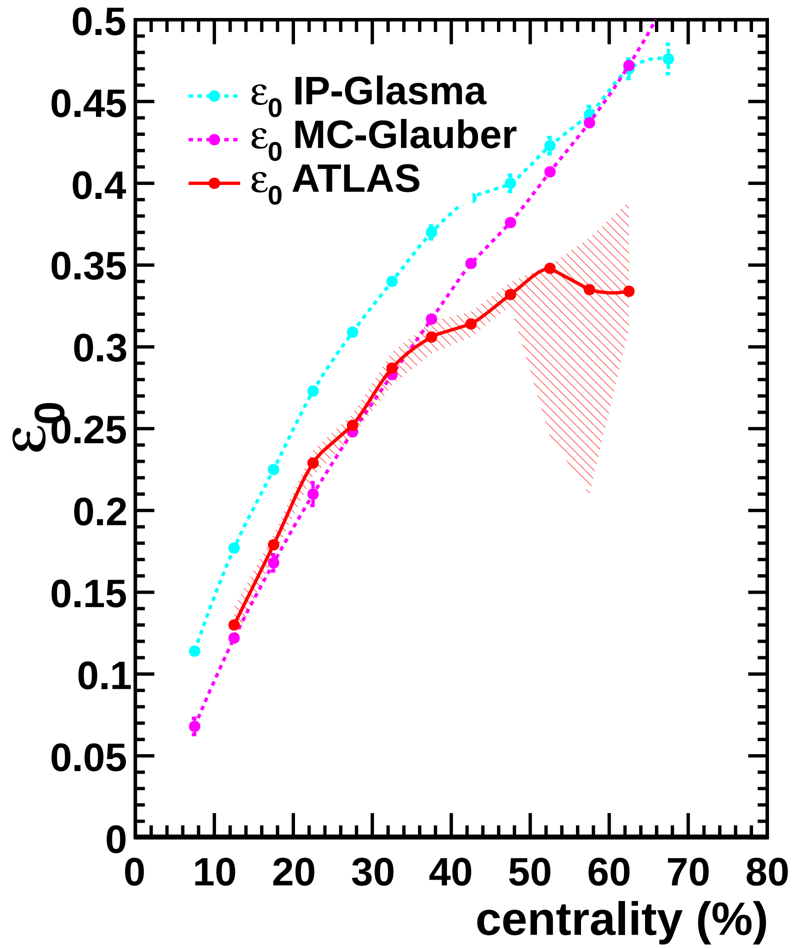

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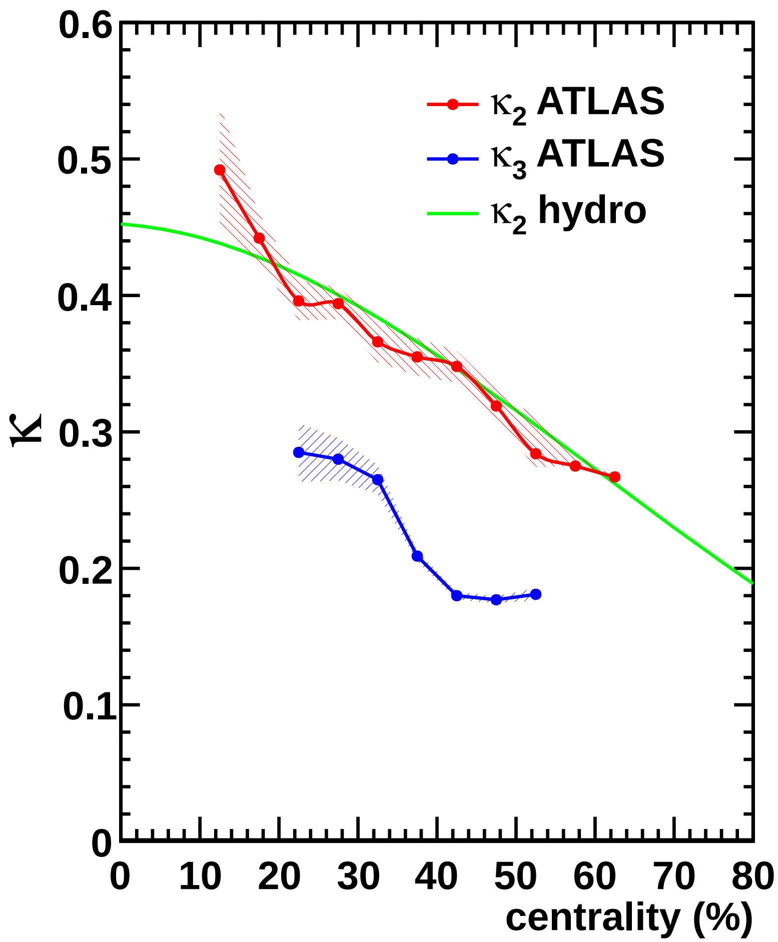
PHOBOS MC-Glauber and IP-Glasma ?

- Flow response coefficient $\kappa_n = v_n/\varepsilon_n$ vs centrality



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κ_n and extracting η/s in hydrodynamic response

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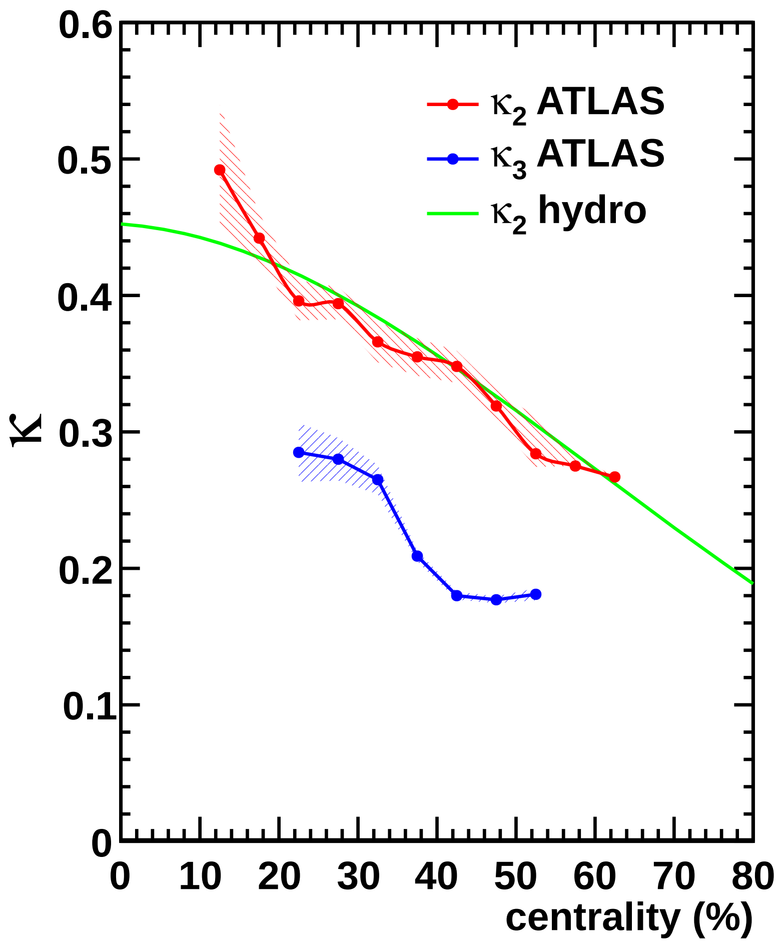

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  - Since η/s determines κ_n
- Fit by hydro.: $\delta\kappa = -\frac{\kappa^{\text{visc.}} - \kappa^{\text{ideal}}}{\eta/s}$

$$\kappa\left(\frac{\eta}{s}\right) = C_0\left[\kappa^{\text{ideal}} - \frac{\eta}{s}\delta\kappa\right],$$

$\kappa^{\text{visc.}}$ and κ^{ideal} are given by hydro.

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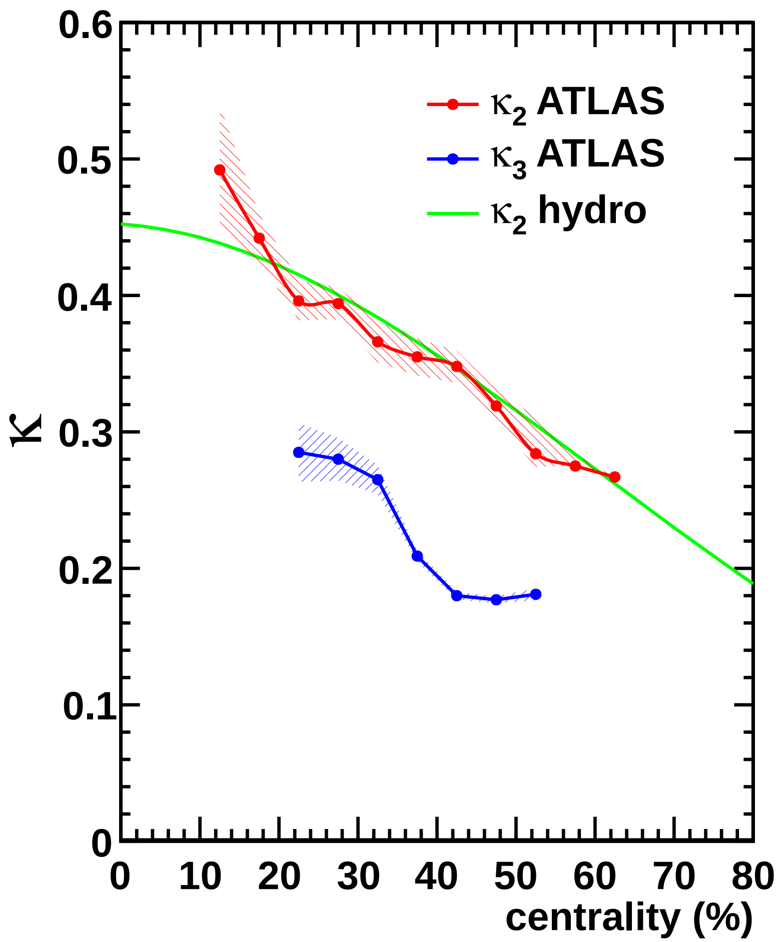

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- Since η/s determines κ_n
- Fit by hydro.: $\delta\kappa = -\frac{\kappa^{\text{visc.}} - \kappa^{\text{ideal}}}{\eta/s}$

$$\kappa\left(\frac{\eta}{s}\right) = C_0\left[\kappa^{\text{ideal}} - \frac{\eta}{s}\delta\kappa\right],$$

$\kappa^{\text{visc.}}$ and κ^{ideal} are given by hydro.

$$\text{Fit of } v_2 \quad \Rightarrow \quad \frac{\eta}{s} \sim 0.18$$

$$\Rightarrow \quad C_0 \sim 1.68$$

Our ideal hydro. needs to be rescaled to match data.

Summary and conclusions

- New parameterizations of eccentricity fluctuations: Elliptic Power and Power

  1. Implement the condition $|\varepsilon_n| < 1$: large anisotropies are correctly modeled
  2. Fit all models of the initial state (Glauber, KLN, IP-Glasma, etc.)
  3. Reveals physical information of initial state: fluctuations (α) and average shape (ε_0).

- Applications

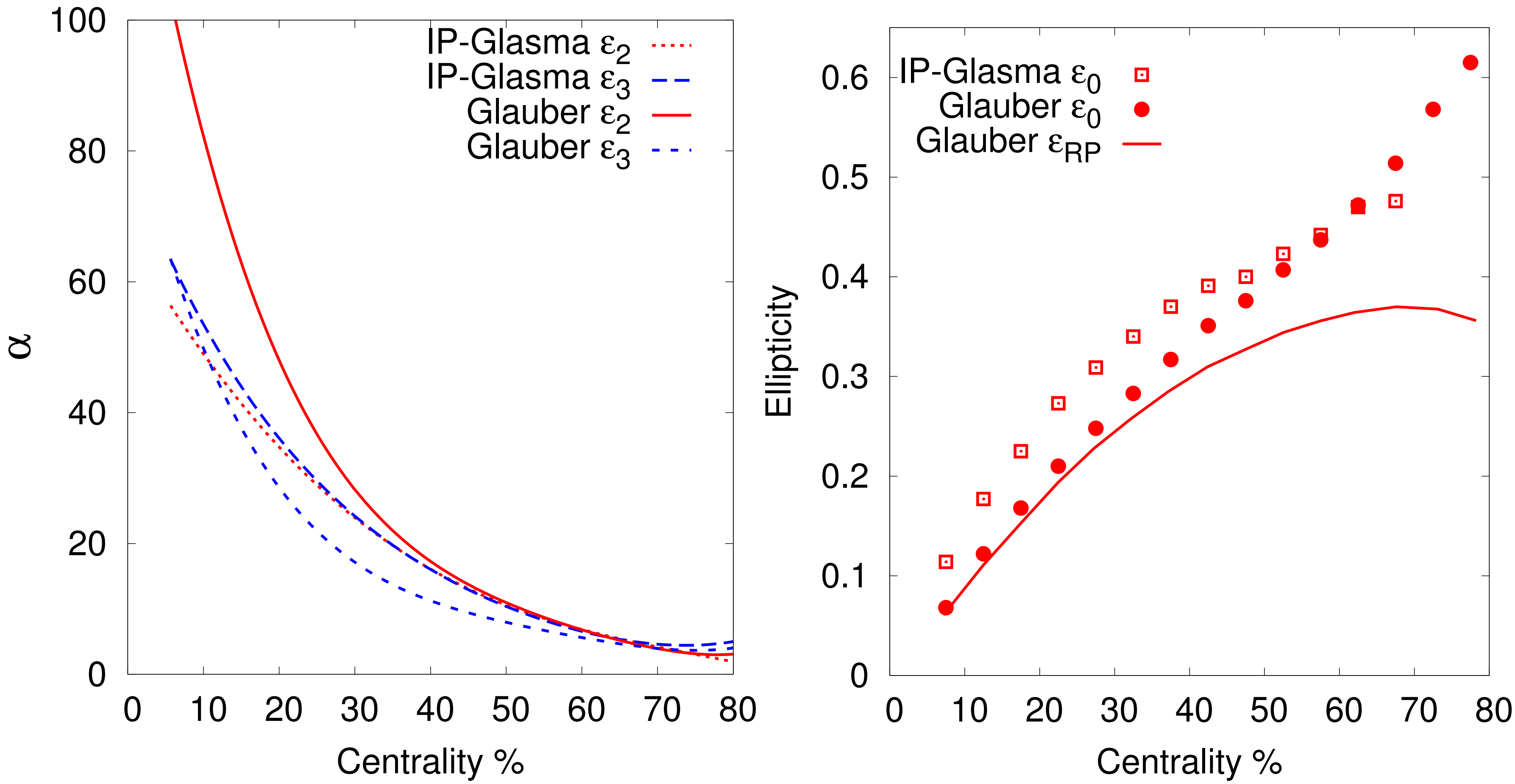
  1. Natually explains **large** $v_2\{4\}$ in p-Pb of LHC $\Rightarrow$ collective expansion of p-Pb system.
  2. Fits of ATLAS EbyE v_2 and v_3 distributions: we are able to disentangle for the first time the initial eccentricity from the hydrodynamic response **without assuming a particular model of initial conditions**.

     Info. of initial state $\Rightarrow$ Fluctuations and average eccentricity of initial state.

     Info. of the medium $\Rightarrow$ Extraction of $\eta/s \sim 0.18$.
  3. And more . . ..

Back-up slides

α and ϵ_0



Analytical expression of higher order cumulants of ε_n

- Cumulants:

$$\varepsilon_n\{4\} = \left[2\langle\varepsilon_n 2^2\rangle^2 - \langle\varepsilon_n^4\rangle\right]^{1/4}$$

$$\varepsilon_n\{6\} = \left[\frac{1}{4}\left(\langle\varepsilon_n^6\rangle - 9\langle\varepsilon_n^2\rangle\langle\varepsilon_n^4\rangle + 12\langle\varepsilon_n^2\rangle^3\right)\right]^{1/6}$$

$$\ldots = \ldots$$

- Elliptic Power $f_n = \langle(1-\varepsilon_n^2)^k\rangle$

$$\varepsilon_n\{4\} = (1-f_1)^{1/2}$$

$$\varepsilon_n\{6\} = (1-2f_2+2f_1^2-f_2)^{1/4}$$

$$\ldots = \ldots$$

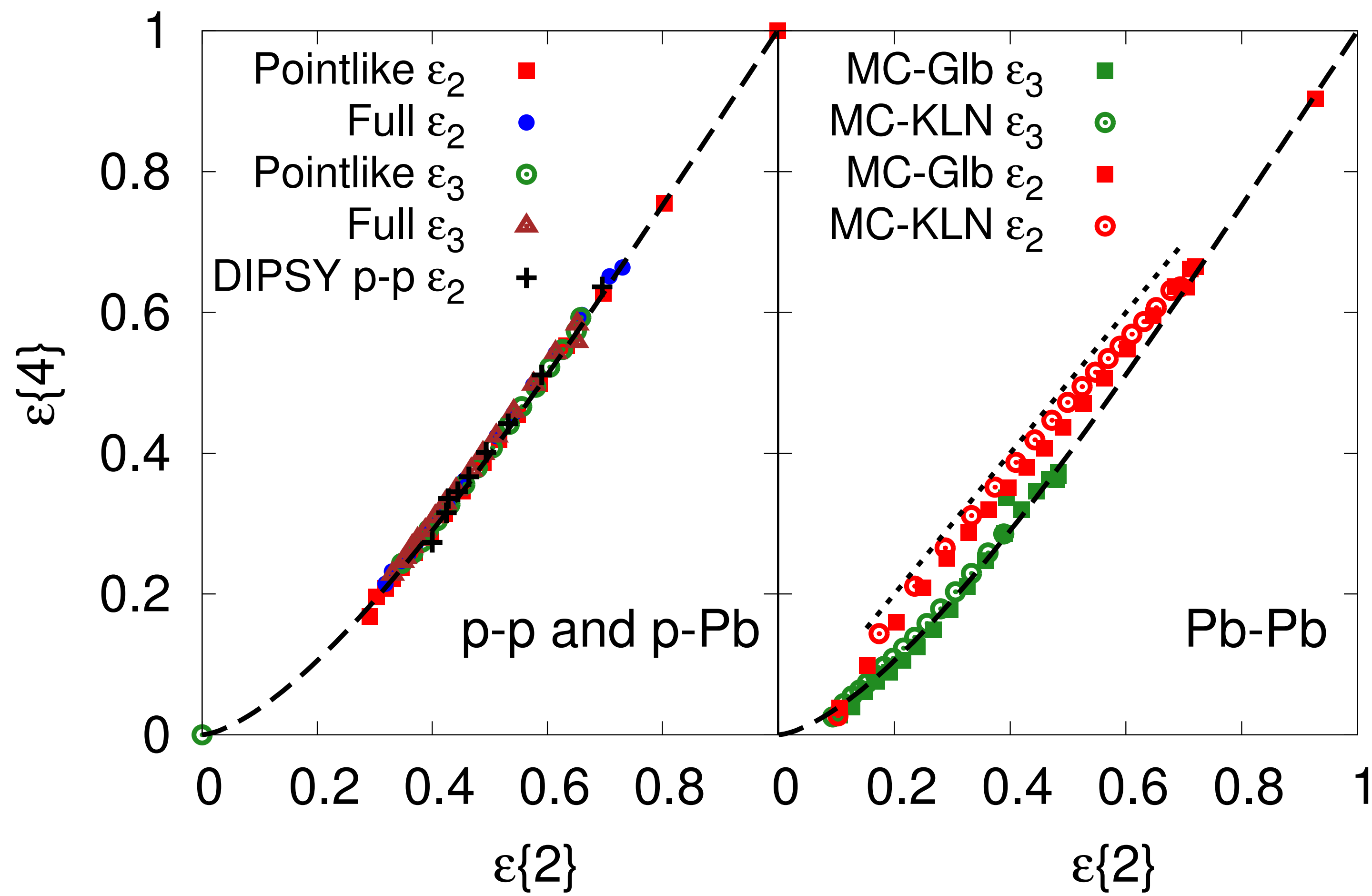
- Power

$$\varepsilon_n\{4\} = \frac{1}{\sqrt{1+\alpha}}$$

$$\varepsilon_n\{6\} = \left[\frac{2}{(1+\alpha)^2(2+\alpha)}\right]^{1/4}$$

$$\ldots = \ldots$$

Cumulants from Power distribution

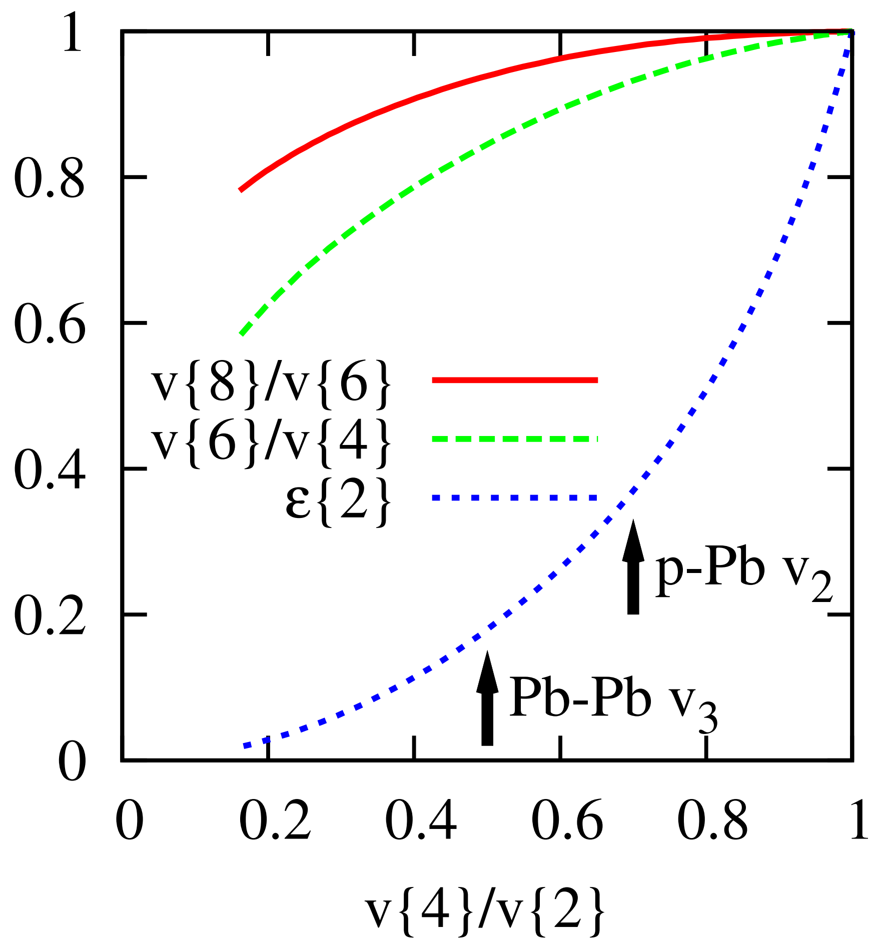


Universal relation between cumulants:

$$\varepsilon_n\{4\} = \left(\frac{2}{2+\alpha}\right)^{1/4} \varepsilon_n\{2\}$$

Cumulants from Power distribution

- Patterns of certain ratios of cumulants of v_n



- $\frac{v_2\{4\}}{v_2\{2\}} \sim 0.7 \Rightarrow \frac{v_2\{6\}}{v_2\{4\}} \sim 0.93$, and $\frac{v_2\{8\}}{v_2\{6\}} \sim 0.98$.

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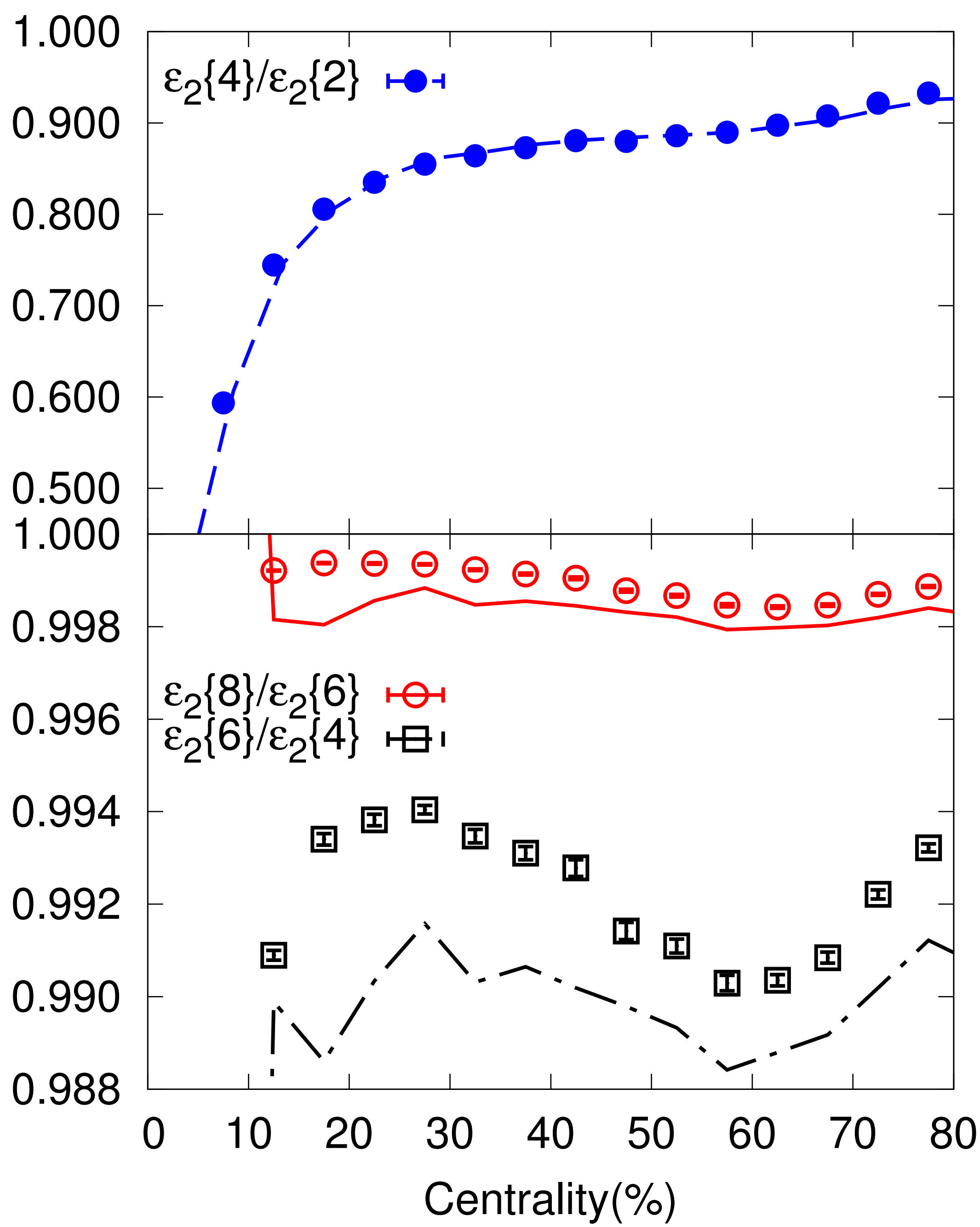
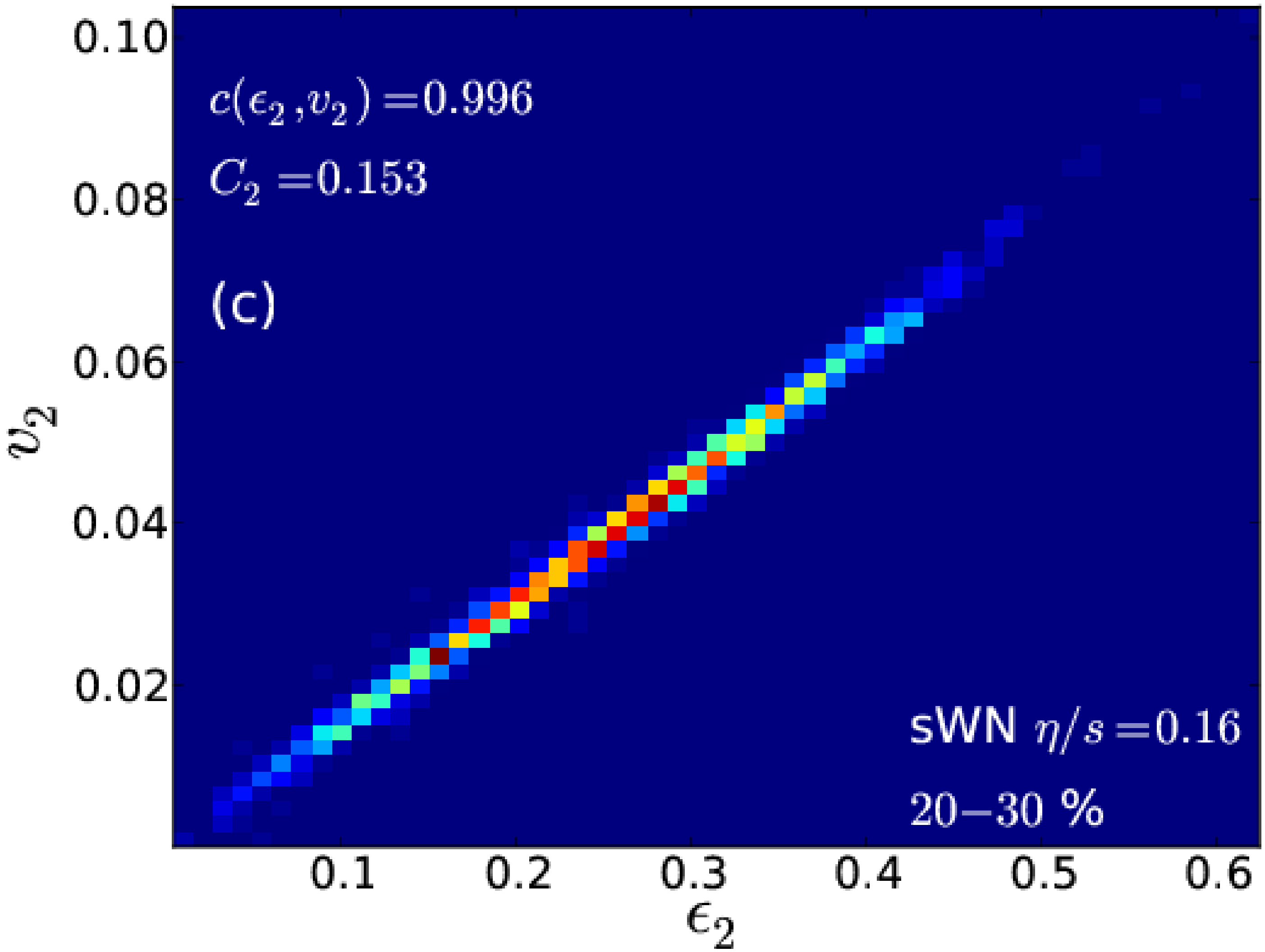


EbyE dissipative hydro. with shear viscosity = 0.16.



H.Niemi et al.,Phys.Rev. C87 (2013) 054901

Goodness of fit