Relaxation-time approximation and relativistic viscous hydrodynamics from kinetic theory

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Relativistic viscous hydrodynamics and of BE in RTA

- Fluid dynamical equation of motion : $\partial_\mu T^{\mu\nu} = 0$, where

$$T^{\mu\nu} = \int dp\, p^\mu p^\nu\, f(x,p) = \epsilon u^\mu u^\nu - P\Delta^{\mu\nu} + \pi^{\mu\nu}, \qquad dp = \frac{gd^3p}{(2\pi)^3 p^0}.$$

- For a system close to local thermodynamic equilibrium, $f = f_0 + \delta f$,

$$\pi^{\mu\nu} = \Delta^{\mu\nu}_{\alpha\beta} \int dp\, p^\alpha p^\beta\, \delta f, \qquad \dot{\pi}^{\langle\mu\nu\rangle} = \Delta^{\mu\nu}_{\alpha\beta} \int dp\, p^\alpha p^\beta\, \delta \dot{f}.$$

[Denicol, Koide, Rischke, PRL **105**, 162501]

- Relativistic Boltzmann equation in RTA [Anderson, Witting, Physica **74**, 466]

$$p^\mu \partial_\mu f = -\left(u\cdot p\right) \frac{\delta f}{\tau_R} \quad \Rightarrow \quad f = f_0 - \frac{\tau_R}{(u\cdot p)} p^\mu \partial_\mu f.$$

- Iterative sol. [Romatschke, PRD **85**, 065012; AJ, PRC **87**, 051901(R), **88**, 021903(R)]

$$\delta f^{(1)} = -\frac{\tau_R}{u\cdot p} p^\mu \partial_\mu f_0, \qquad \delta f^{(2)} = \frac{\tau_R}{u\cdot p} p^\mu p^\nu \partial_\mu \left(\frac{\tau_R}{u\cdot p} \partial_\nu f_0\right).$$

Evolution equation for shear stress tensor

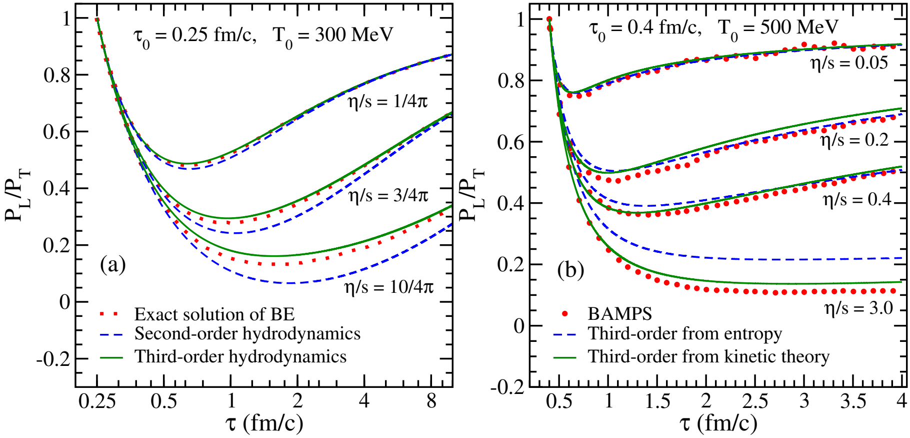
- For $\delta f = \delta f^{(1)}$: $\pi^{\mu\nu} = 2\tau_\pi\beta_\pi\sigma^{\mu\nu}$, where $\tau_\pi = \tau_R, \quad \beta_\pi = 4P/5$.
- Similarly for $\delta f = \delta f^{(1)} + \delta f^{(2)}$, we obtain second- and third-order evolution equation for shear stress tensor [AJ, PRC **88**, 021903(R)]:

$$\dot{\pi}^{\langle\mu\nu\rangle} = -\frac{\pi^{\mu\nu}}{\tau_\pi} + 2\beta_\pi\sigma^{\mu\nu} + 2\pi_\gamma^{\langle\mu}\omega^{\nu\rangle\gamma} - \frac{10}{7}\pi_\gamma^{\langle\mu}\sigma^{\nu\rangle\gamma} - \frac{4}{3}\pi^{\mu\nu}\theta - \frac{1}{3\beta_\pi}\pi_\gamma^{\langle\mu}\pi^{\nu\rangle\gamma}\theta$$
$$- \frac{38}{245\beta_\pi}\pi^{\mu\nu}\pi^{\rho\gamma}\sigma_{\rho\gamma} - \frac{22}{49\beta_\pi}\pi^{\rho\langle\mu}\pi^{\nu\rangle\gamma}\sigma_{\rho\gamma} + \frac{25}{7\beta_\pi}\pi^{\rho\langle\mu}\omega^{\nu\rangle\gamma}\pi_{\rho\gamma}$$
$$- \frac{24}{35}\nabla^{\langle\mu}\left(\pi^{\nu\rangle\gamma}\dot{u}_\gamma\tau_\pi\right) + \frac{12}{7}\nabla_\gamma\left(\tau_\pi\dot{u}^{\langle\mu}\pi^{\nu\rangle\gamma}\right) + \frac{6}{7}\nabla_\gamma\left(\tau_\pi\dot{u}^\gamma\pi^{\langle\mu\nu\rangle}\right)$$
$$- \frac{2}{7}\nabla_\gamma\left(\tau_\pi\nabla^{\langle\mu}\pi^{\nu\rangle\gamma}\right) - \frac{1}{7}\nabla_\gamma\left(\tau_\pi\nabla^\gamma\pi^{\langle\mu\nu\rangle}\right) + \frac{4}{35}\nabla^{\langle\mu}\left(\tau_\pi\nabla_\gamma\pi^{\nu\rangle\gamma}\right)$$
$$- \frac{2}{7}\tau_\pi\omega^{\rho\langle\mu}\omega^{\nu\rangle\gamma}\pi_{\rho\gamma} - \frac{2}{7}\tau_\pi\pi^{\rho\langle\mu}\omega^{\nu\rangle\gamma}\omega_{\rho\gamma} - \frac{10}{63}\tau_\pi\pi^{\mu\nu}\theta^2 + \frac{26}{21}\tau_\pi\pi_\gamma^{\langle\mu}\omega^{\nu\rangle\gamma}\theta$$

- Invoking 2^{nd} law of thermodynamics [El, Xu, Greiner, PRC **81**, 041901(R)]

$$\dot{\pi}^{\langle\mu\nu\rangle} = -\frac{\pi^{\mu\nu}}{\tau'_\pi} + 2\beta'_\pi\sigma^{\mu\nu} - \frac{4}{3}\pi^{\mu\nu}\theta + \frac{5}{36\beta'_\pi}\pi^{\mu\nu}\pi^{\rho\gamma}\sigma_{\rho\gamma} - \frac{16}{9\beta'_\pi}\pi_\gamma^{\langle\mu}\pi^{\nu\rangle\gamma}\theta, \quad \beta'_\pi = \frac{2P}{3}$$

Comparison with exact solution of BE and BAMPS



- Derived a novel third-order evolution equation for shear pressure.
- Iterative solution of BE was used instead of Grad's approximation.
- Evolution equation for shear tensor derived directly from definition.
- Good agreement with exact solution of BE as well as BAMPS.