Bulk properties and hydrodynamics: Observables and concepts

Pasi Huovinen

J. W. Goethe Universität & Frankfurt Institute for Advanced Studies

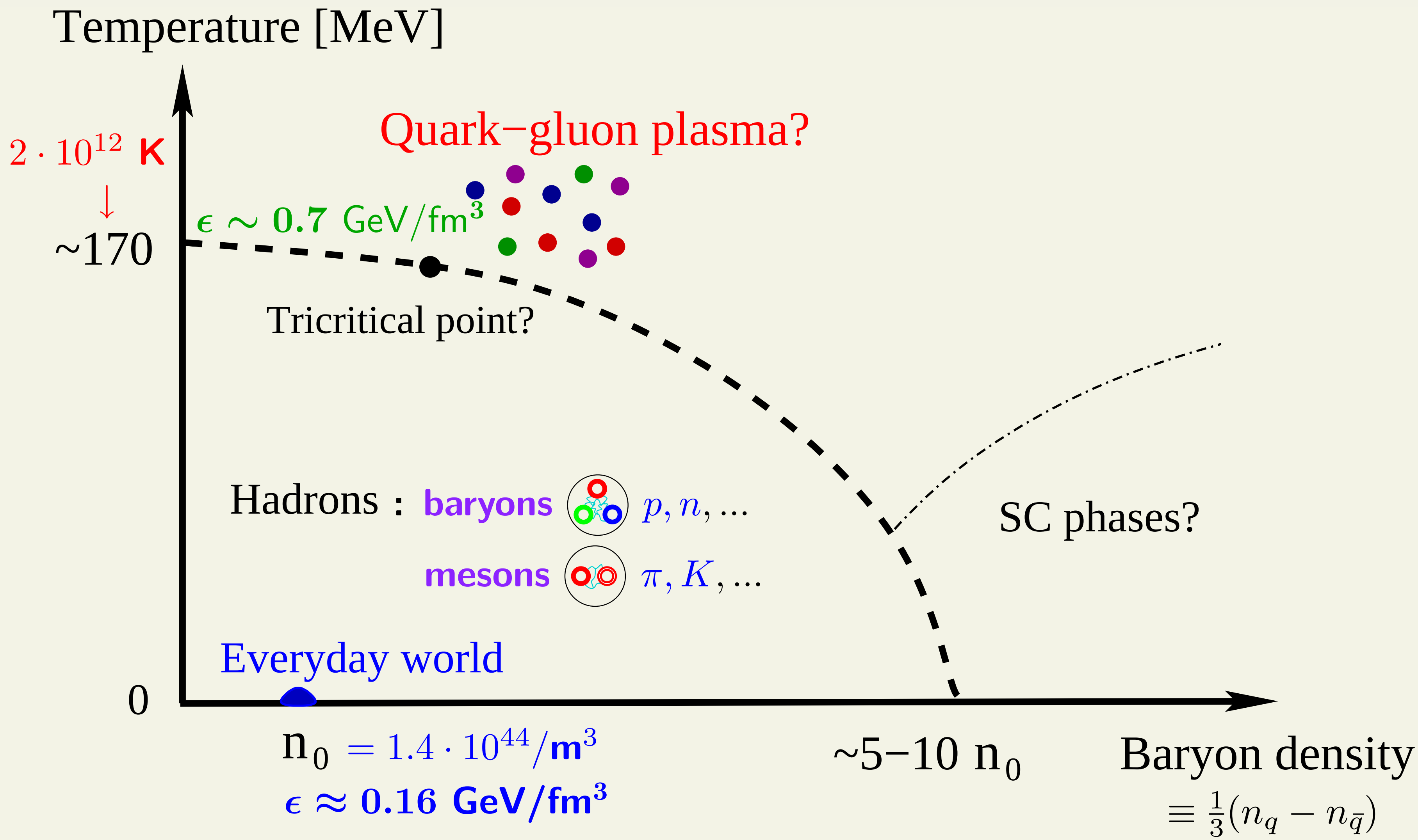
Student lecture, Quark Matter 2014

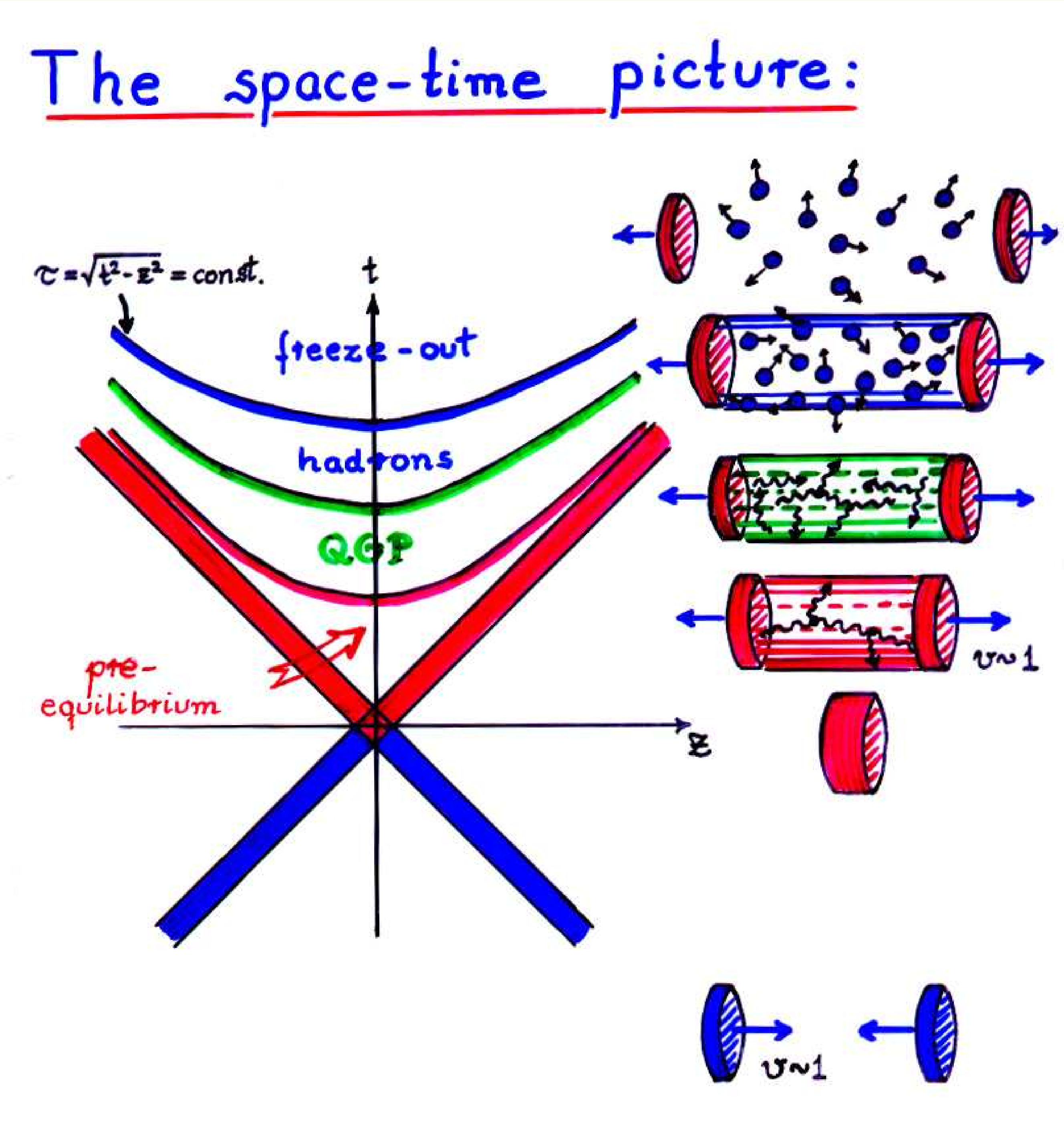
May 18, 2014, GSI

What happens when you compress nuclear matter to very high temperatures and densities?

– **Can we create strongly interacting matter?**

Nuclear phase diagram





Transient matter

- **lifetime**
  $t \sim 10\,\text{fm/c}$
  $\sim 10^{-23}$ **seconds**
- **small size**
  $r \sim 10\,\text{fm}$
  $\sim 10^{-14}\,\text{m}$
- **rapid expansion**

Multiplicity @ LHC
~ 15000

©Dirk H. Rischke

Conservation laws

Conservation of energy and momentum:

$$\partial_\mu T^{\mu\nu}(x) = 0$$

Conservation of charge:

$$\partial_\mu N^\mu(x) = 0$$

Local conservation of particle number and energy-momentum

$\Longleftrightarrow$ **Hydrodynamical equations of motion!**

This can be generalized to multicomponent systems and systems with several conserved charges:

$$\partial_\mu N_i^\mu = 0,$$

$i =$ **baryon number, strangeness, charge. . .**

Conservation of energy and momentum:

$$\partial_\mu T^{\mu\nu}(x) = 0$$

Conservation of charge:

$$\partial_\mu N^{\mu}(x) = 0$$

Consider only baryon number conservation, $i = B$**.**

$\Rightarrow$ **5 equations contain 14 unknowns!**

$\Rightarrow$ **The system of equations does not close.**

$\Rightarrow$ **Provide 9 additional equations or**
Eliminate 9 unknowns.

Ideal fluid approximation:

$$N^\mu = nu^\mu$$
$$T^{\mu\nu} = (\epsilon + P)u^\mu u^\nu - Pg^{\mu\mu}$$

- Particles in local thermodynamical equilibrium,
- Now N^μ and $T^{\mu\nu}$ contain 6 unknowns, ϵ, P, n and u^μ, but there are still only 5 equations!
- In thermodynamical equilibrium ϵ, P and n are not independent! They are specified by two variables, T and μ.
- The equation of state (EoS), $P(T, \mu)$ closes the system of hydrodynamic equations and makes it uniquely solvable (given initial conditions).
- EoS usually given by lattice QCD calculations and hadron resonance gas model — see lectures by Ratti and Kalweit

Dissipative hydrodynamics

General case in Landau frame

$$N^\mu = nu^\mu + \nu^\mu$$
$$T^{\mu\nu} = \epsilon u^\mu u^\nu - (P + \Pi)\Delta^{\mu\nu} + \pi^{\mu\nu}$$

(where $\Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu$)
Need 9 additional equations to determine

Π: bulk pressure
$\pi^{\mu\nu}$: shear stress tensor
ν^μ: charge flow

Usually only shear is included, bulk sometimes, charge/heat flow not so far

In the following only system with no charge/baryon current and with shear only is discussed

relativistic Navier-Stokes

dissipative currents small corrections linear in gradients

$$\pi^{\mu\nu} = 2\eta\nabla^{\langle\mu}u^{\nu\rangle}$$

η **shear viscosity coefficient**

- **resulting equations of motion acausal and unstable!**

Causal viscous hydro

bulk pressure Π, shear stress $\pi^{\mu\nu}$ heat flow q^μ treated as independent dynamical quantities that relax to their Navier-Stokes value on time scales $\tau_\Pi(e,n)$, $\tau_\pi(e,n)$, $\tau_q(e,n)$

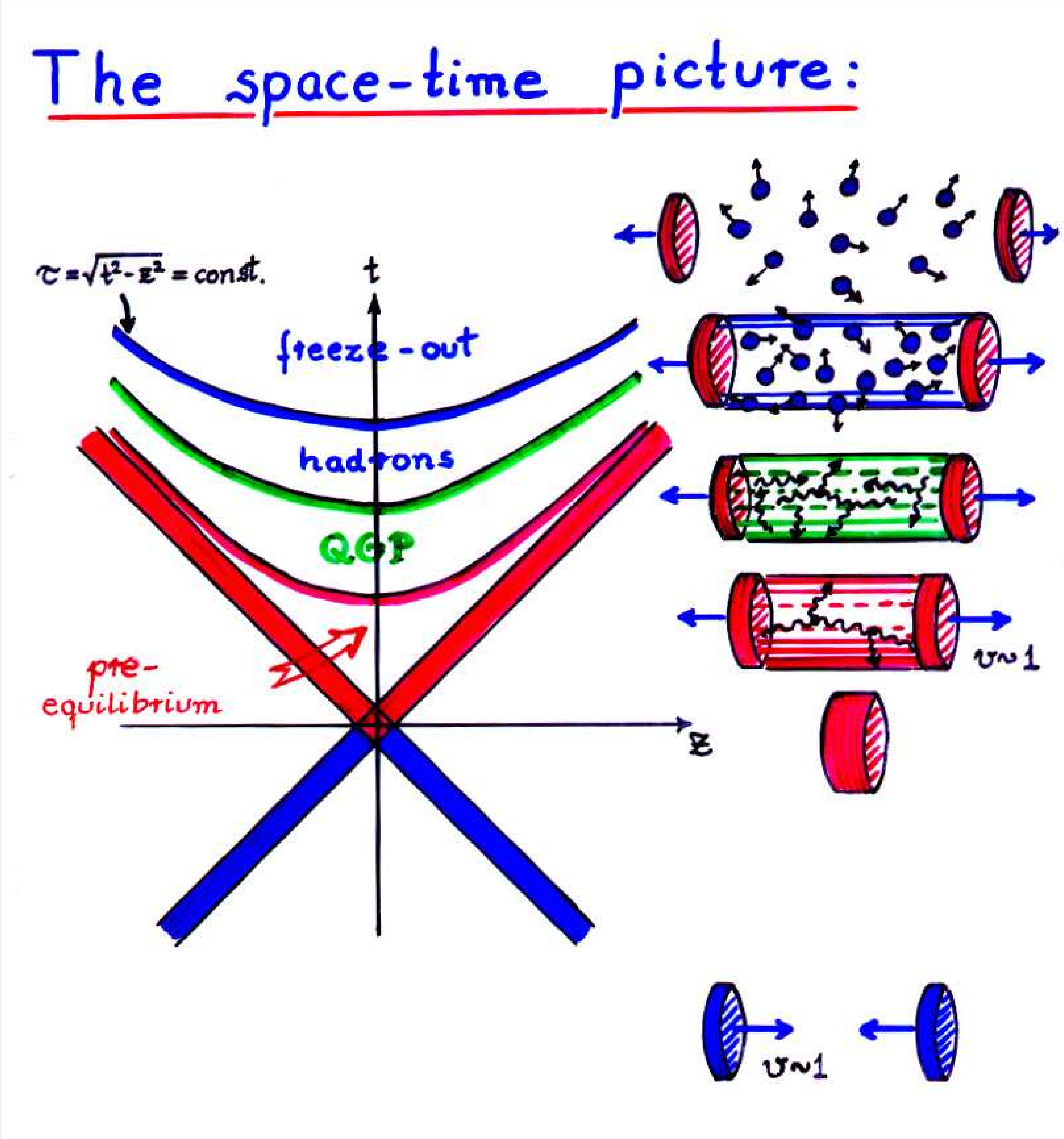
Müller, Israel & Stewart...

Israel & Stewart evolution equation for shear

$$D\pi^{\mu\nu} = -\frac{1}{\tau_\pi}\left(\pi^{\mu\nu} - 2\eta\nabla^{\langle\mu}u^{\nu\rangle}\right) - (\pi^{\lambda\mu}u^\nu + \pi^{\lambda\nu}u^\mu)Du_\lambda - \frac{1}{2}\pi^{\mu\nu}\nabla_\lambda u^\lambda + \cdots$$

leads to causal and stable equations of motion

One more parameter: relaxation time τ_π


The space-time picture:
τ = √(t² - z²) = const.
t
freeze-out
hadrons
QGP
pre-
equilibrium
z
v~1
v~1

Usefulness of hydro?

• Initial state:	unknown	}	
• Equation of state:	unknown	}	$\Rightarrow$ **Predictive power?**
• Transport coefficients:	unknown	}	
• Freeze-out:	unknown	}	

Usefulness of hydro?

- Initial state: unknown
- Equation of state: want to study
- Transport coefficients: want to study
- Freeze-out: unknown

$\Big\}$ $\Rightarrow$ **Predictive power?**

$\Longrightarrow$ **Need More Constraints!**

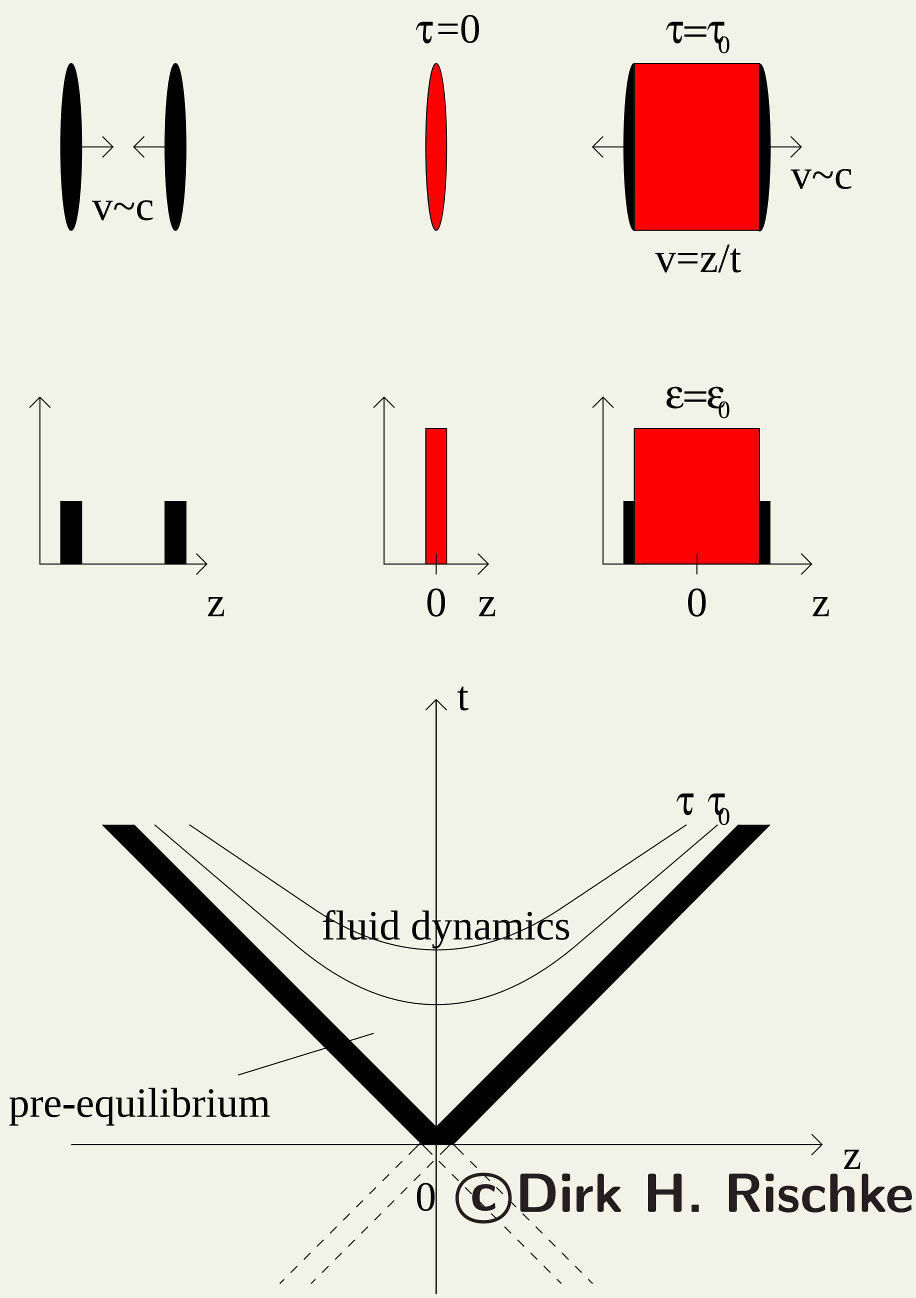
"Hydrodynamical method"

1. Use **another model** to fix unknowns (and **add new assumptions...**)
   - **initial:** color glass condensate or pQCD+saturation
   - **initial and/or final:** hadronic cascade
   - **EoS:** lattice QCD

2. Use data to fix parameters:

Principle		Example @ RHIC
• use one set of data	$\Longleftrightarrow$	$\left.\dfrac{\mathrm{d}N}{\mathrm{d}y\, p_T\, \mathrm{d}p_T}\right\|_{b=0}$ and $\dfrac{\mathrm{d}N}{\mathrm{d}y}(b)$
• fix parameters to fit it	$\Longleftrightarrow$	$\begin{cases} \epsilon_{0,\mathrm{max}} = 29.6\,\mathbf{GeV/fm}^3 \\ \tau_0 = 0.6\,\mathbf{fm}/c \\ T_{\mathrm{fo}} = 130\,\mathbf{MeV} \end{cases}$
• predict another set of data	$\Longleftrightarrow$	HBT, photons & dileptons, **elliptic flow...**

Bjorken hydrodynamics



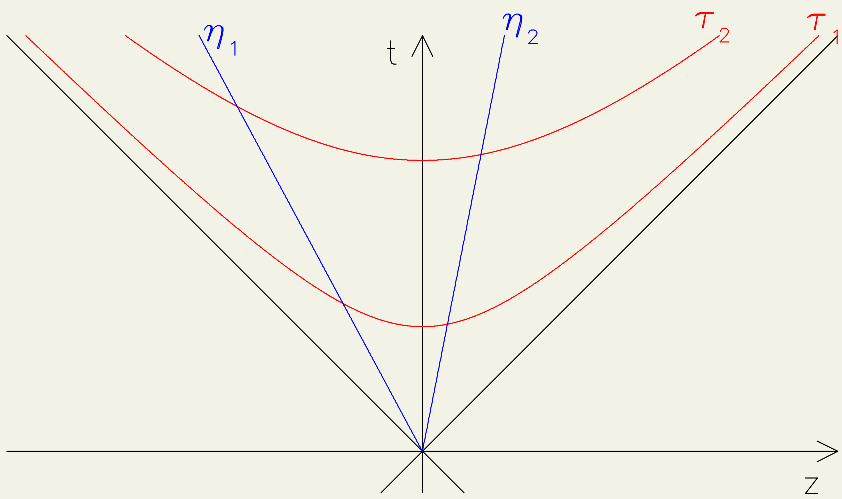
- **At very large energies, $\gamma \to \infty$ and thickness of the collision region $\to 0$**

- **Lack of longitudinal scale**
  $\Rightarrow$ **scaling flow**

$$v = \frac{z}{t}$$

- Practical coordinates to describe scaling flow expansion are
  - **Longitudinal proper time τ:**

$$\tau \equiv \sqrt{t^2 - z^2} \quad \Leftrightarrow \quad t = \tau \cosh \eta$$



  - **Space-time rapidity η_s:**

$$\eta_s = \frac{1}{2} \ln \frac{t+z}{t-z} \quad \Leftrightarrow \quad z = \tau \sinh \eta$$

- **Boost invariance:** if the initial state is independent of η_s, and flow is $v = z/t$, the system **stays independent of η_s**

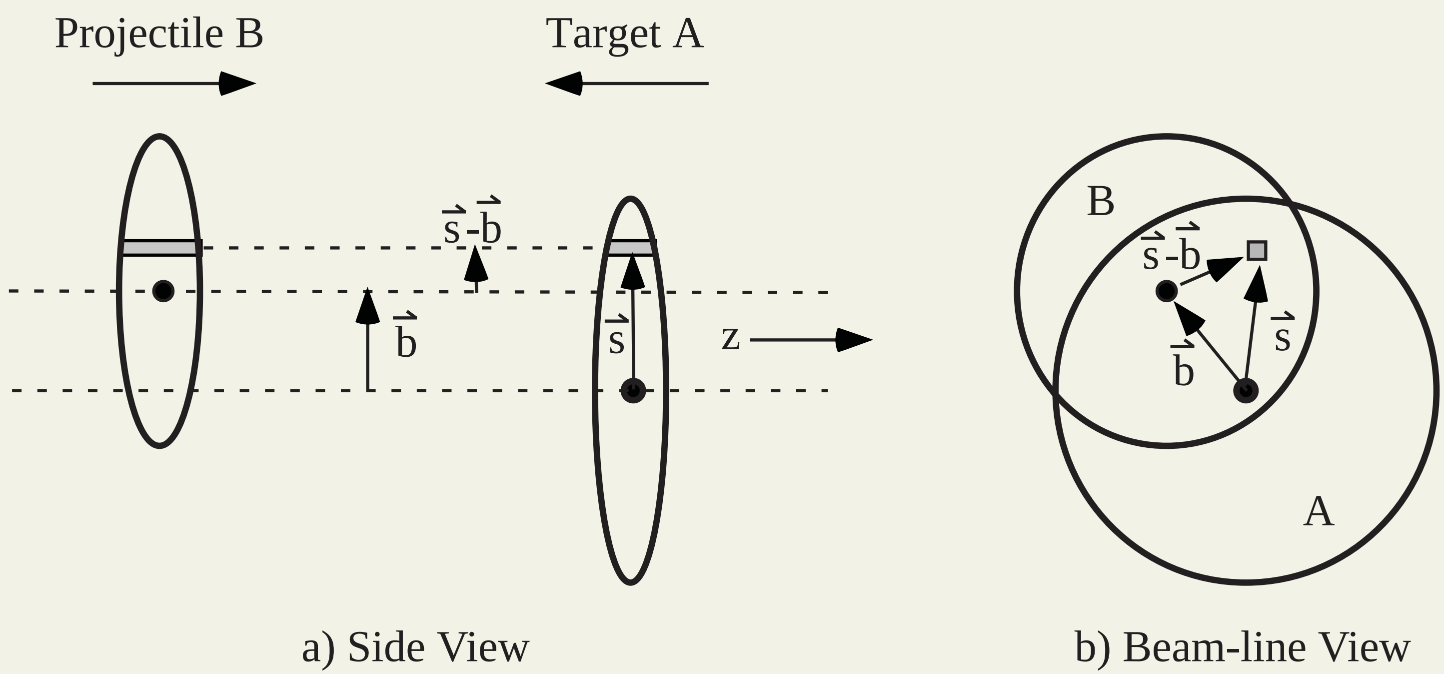
$\Rightarrow$ sufficient to solve expansion numerically in 2 dimensions

$\Rightarrow$ 2+1D hydro!

- Good approximation at LHC and highest RHIC energies

Initial density distribution

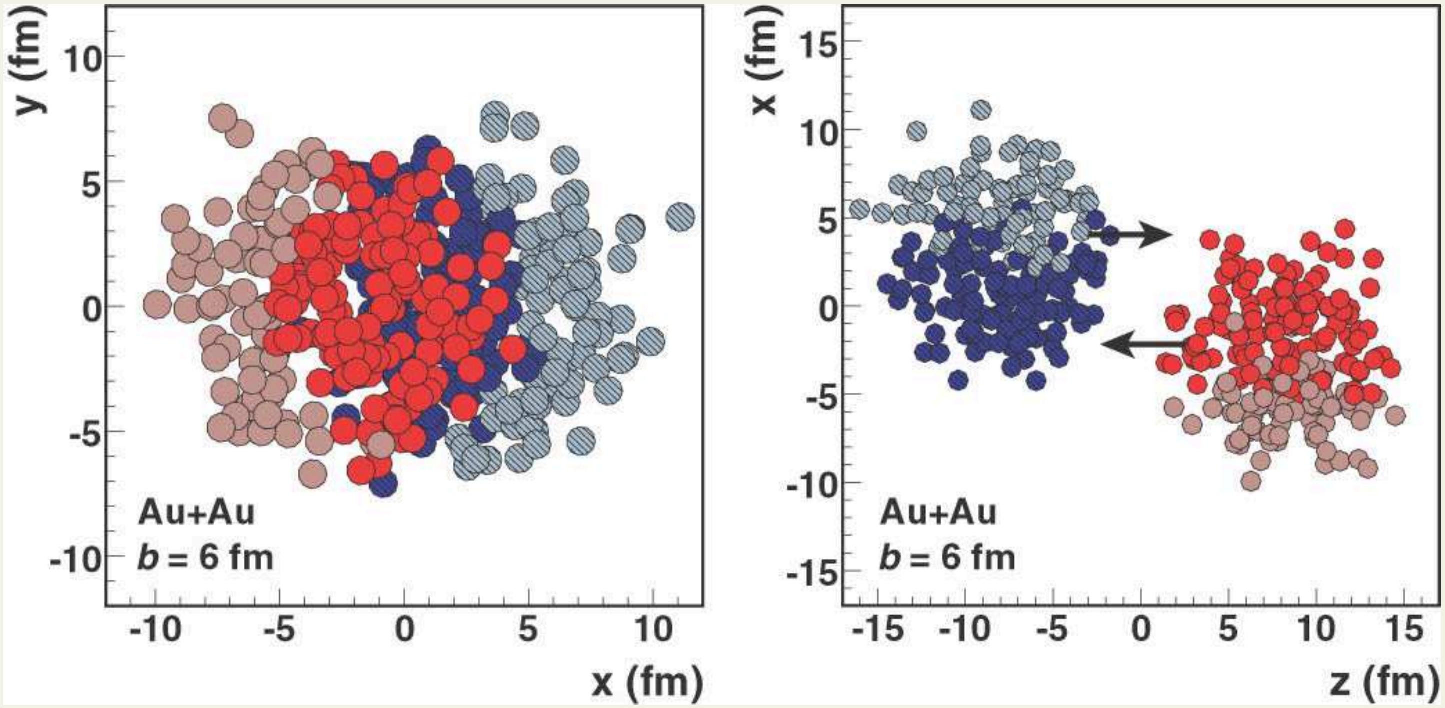
- **Nuclear geometry implies that density is not uniform**



Miller *et al.*, Ann.Rev.Nucl.Part.Sci. 57, 205 (2007)

Initial density distribution

- Nuclear geometry implies that density varies event-by-event



Miller *et al.*, Ann.Rev.Nucl.Part.Sci. 57, 205 (2007)

- evaluate average initial state, and evolve it or
- evolve many initial state $\Rightarrow$ event-by-event hydro

Models for initial conditions

- **Glauber**: geometric model determining wounded nucleons based on the inelastic nucleon-nucleon cross section (whole family of variants)
- **MC-KLN**: Color-Glass-Condensate (CGC) based model using kT -factorization
- **IP-Glasma**: CGC based model using classical Yang-Mills evolution of early-time gluon fields, including fluctuations in the particle production
- **pQCD+saturation**: calculate minijets using pQCD to get energy deposited in the collision region
- **event generators**: UrQMD (hadronic), BAMPS and AMPT (partonic) or EPOS can be used to create initial state for hydro

– so far **none of these reaches equilibrium**, but it has to be dialed in by hand

– see lectures by Salgado and Loizides

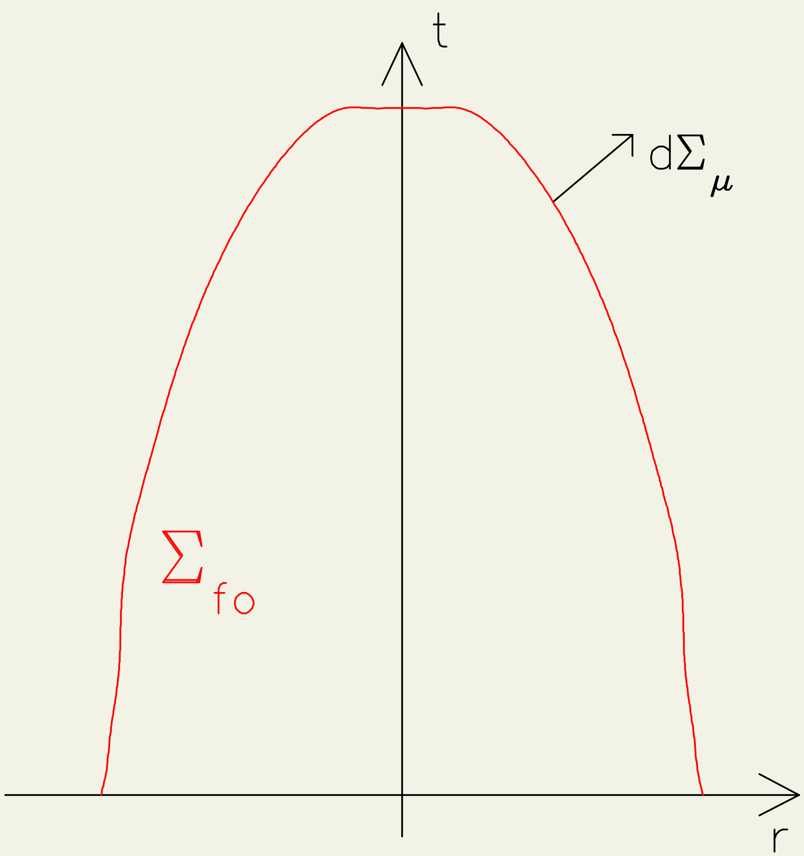
Initial conditions

Besides density distribution, one has to decide

- **Initial time τ_0: thermalization time — usually $0.2 - 1$ fm/c**
- **Initial transverse flow: often set to zero, some models provide finite transverse flow**
- **Boost-invariant or not (if not, what are the longitudinal flow and density profiles?)**
- **Initial $\pi^{\mu\nu}$: zero, Navier-Stokes value or something else?**

When to end?

- How far is hydro valid?
- How and when to convert fluid to particles?



- Kinetic equilibrium requires scattering rate $\gg$ expansion rate
- Scattering rate $\tau_{\mathrm{sc}}^{-1} \sim \sigma n \propto \sigma T^3$
- Expansion rate $\theta = \partial_\mu u^\mu$
- Fluid description breaks down when $\tau_{\mathrm{sc}}^{-1} \approx \theta$

$\rightarrow$ momentum distributions freeze-out

- $\tau_{\mathrm{sc}}^{-1} \propto T^3 \rightarrow$ rapid transition to free streaming
- Approximation: decoupling takes place on constant temperature hypersurface Σ_{fo}, at $T = T_{\mathrm{fo}}$

- Note that particle chemistry may be frozen before momentum distributions!

$\Rightarrow$ separate chemical and kinetic freeze-outs (PCE EoS)

Hybrid models

- End hydro when rescatterings still frequent
- Convert fluid to particle ensembles
- Describe evolution of particles using hadronic transport
- Advantages:
  - chemical evolution and dissipation described
  - physical decoupling
- Disadvantages:
  - all the unknowns of hadronic cascade. . .
  - where and how to switch?
- Note: The switch from fluid to cascade is **NOT** freeze-out
  $\Rightarrow$ particlization

Cooper-Frye

- **Number of particles emitted = Number of particles crossing $\Sigma_{\rm fo}$**

$$\Rightarrow \quad N = \int_{\Sigma_{\rm fo}} \mathrm{d}\Sigma_\mu \, N^\mu$$

- **Frozen-out particles do not interact anymore: kinetic theory**

$$\Rightarrow \quad N^\mu = \int \frac{\mathrm{d}^3\mathbf{p}}{E} p^\mu f(x, p\cdot u)$$

$$\Rightarrow \quad N = \int \frac{\mathrm{d}^3\mathbf{p}}{E} \int_{\Sigma_{\rm fo}} \mathrm{d}\Sigma_\mu \, p^\mu f(x, p\cdot u)$$

- **Invariant single inclusive momentum spectrum: (Cooper-Frye formula)**

$$\boxed{E\frac{\mathrm{d}N}{\mathrm{d}\mathbf{p}^3} = \int_{\Sigma_{\rm fo}} \mathrm{d}\Sigma_\mu \, p^\mu f(x, p\cdot u)}$$

Cooper and Frye, PRD 10, 186 (1974)

Blast wave

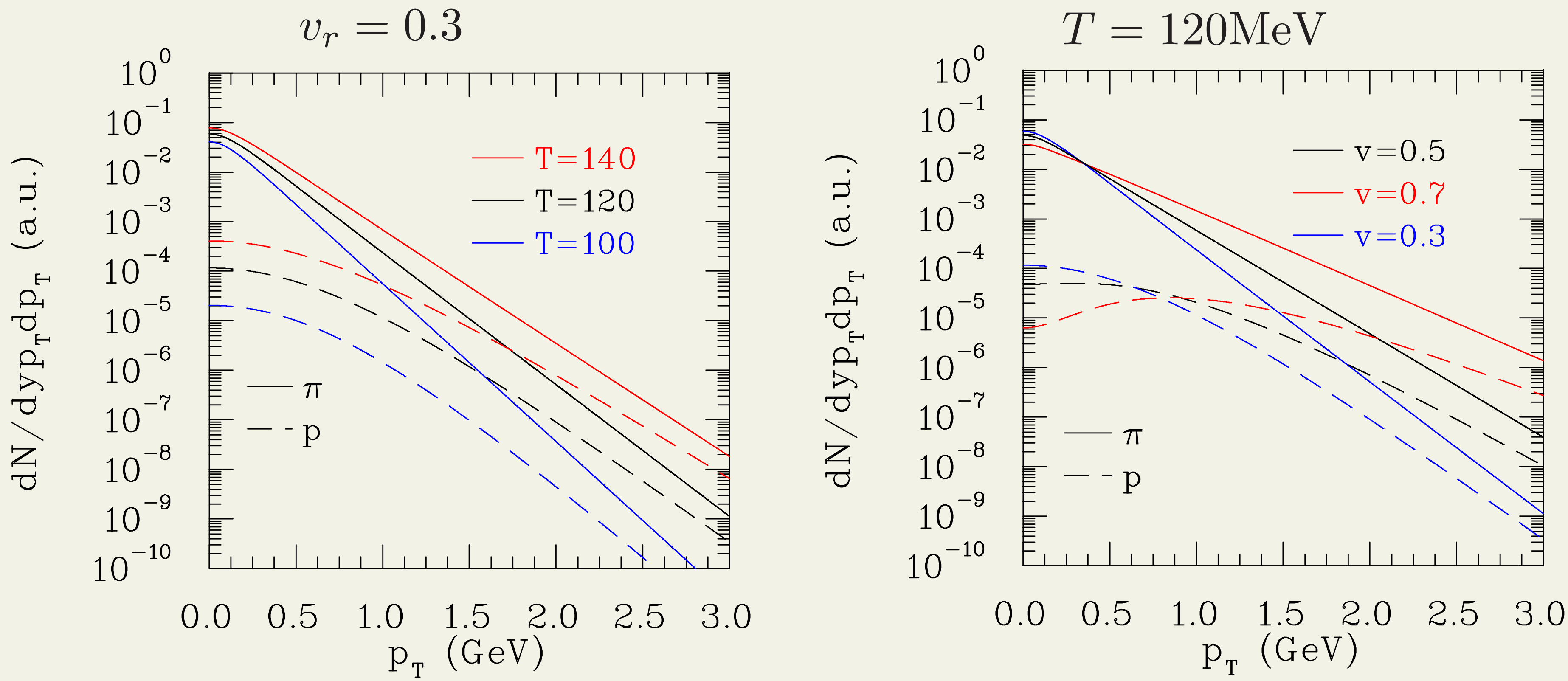
(Siemens and Rasmussen, PRL 42, 880 (1979))

- Freeze-out surface a thin cylindrical shell radius r, thickness $\mathrm{d}r$, expansion velocity v_r, decoupling time τ_{fo}, boost invariant

- Cooper-Frye for Boltzmannions

$$\frac{\mathrm{d}N}{\mathrm{d}y\, p_T\, \mathrm{d}p_T} = \frac{g}{\pi}\, \tau_{\mathrm{fo}}\, r\, m_T\, \mathrm{I}_0\left(\frac{v_r \gamma_r p_T}{T}\right) \mathrm{K}_1\left(\frac{\gamma_r m_T}{T}\right)$$

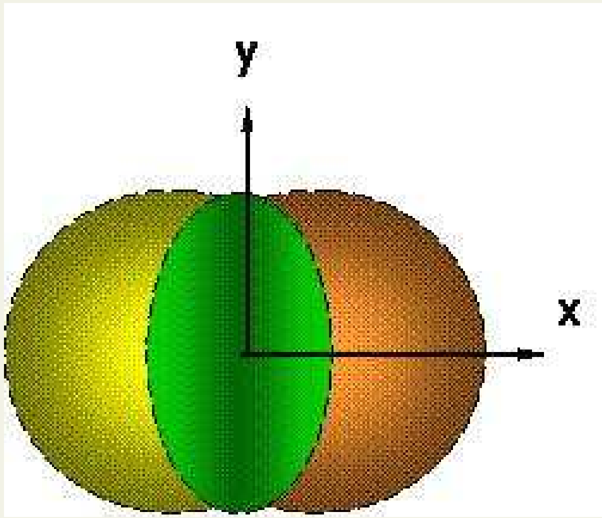
effect of temperature and flow velocity



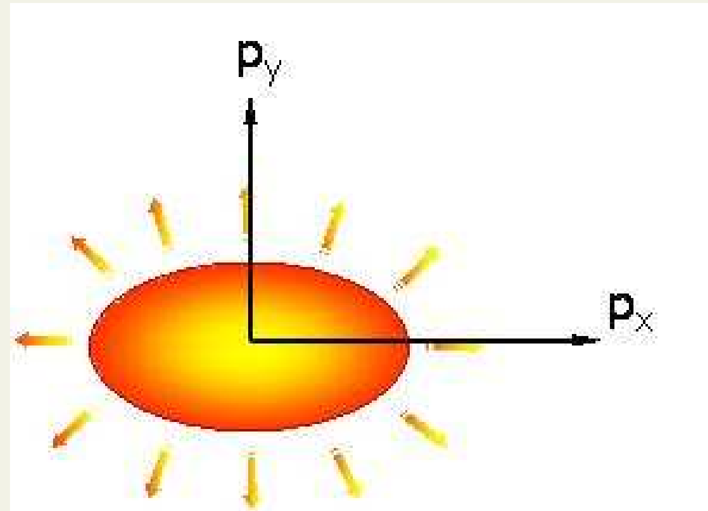
- The larger the temperature, the flatter the spectra
- The larger the velocity, the flatter the spectra ⇒ **blueshift**
- The heavier the particle, the more sensitive it is to flow (shape and slope)

Elliptic flow v_2

spatial anisotropy $\rightarrow$ final azimuthal momentum anisotropy



$\Rightarrow$



- Anisotropy in coordinate space + rescattering

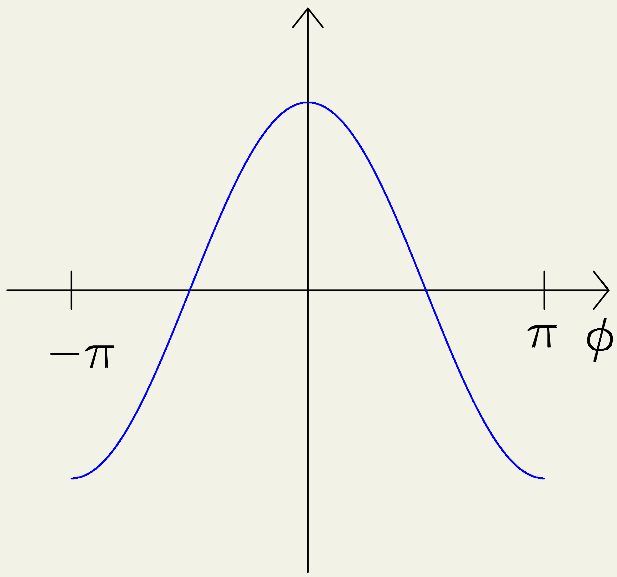
$\Rightarrow$ Anisotropy in momentum space

Elliptic flow v_2

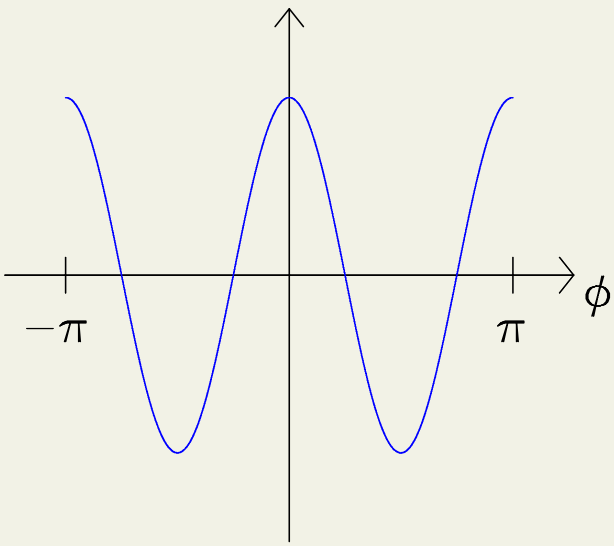
- **Fourier expansion of momentum distribution:**

$$\frac{\mathrm{d}N}{\mathrm{d}y\, p_T \mathrm{d}p_T\, \mathrm{d}\phi} = \frac{1}{2\pi}\frac{\mathrm{d}N}{\mathrm{d}y\, p_T \mathrm{d}p_T}(1 + 2v_1(y,p_T)\cos\phi + 2v_2(y,p_T)\cos 2\phi + \cdots)$$

$\mathbf{v_1}$: **Directed flow: preferred direction**

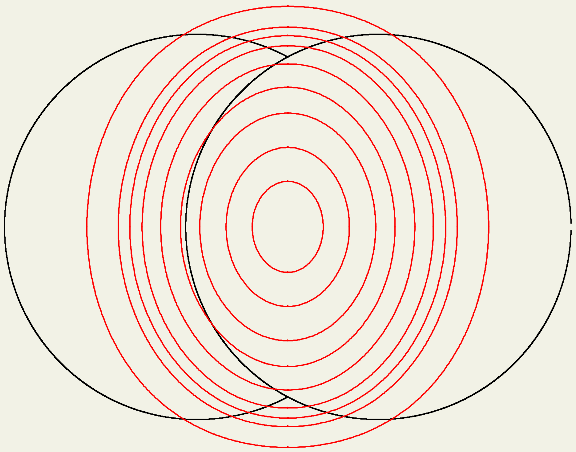


$\mathbf{v_2}$: **Elliptic flow: preferred plane**

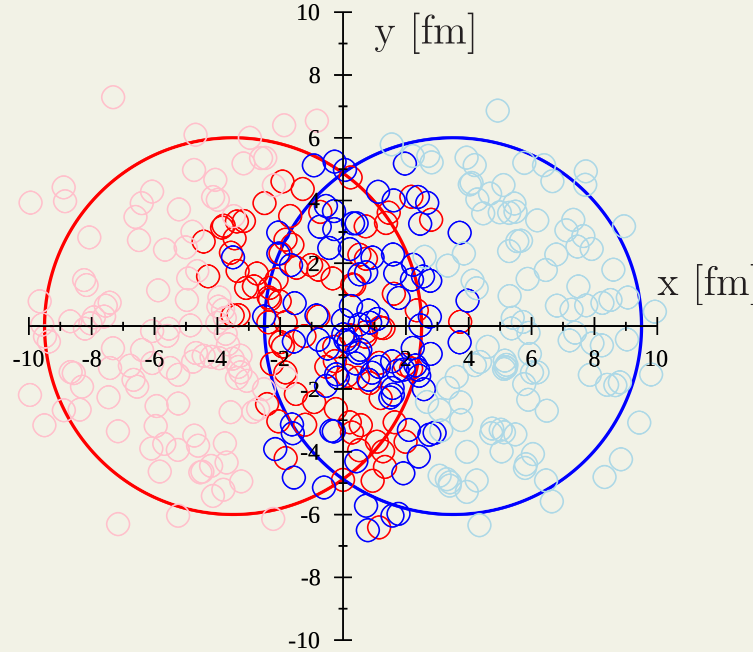


sensitive to speed of sound $c_s^2 = \partial p/\partial e$ **and shear viscosity** η

event-by-event



$\Longrightarrow$

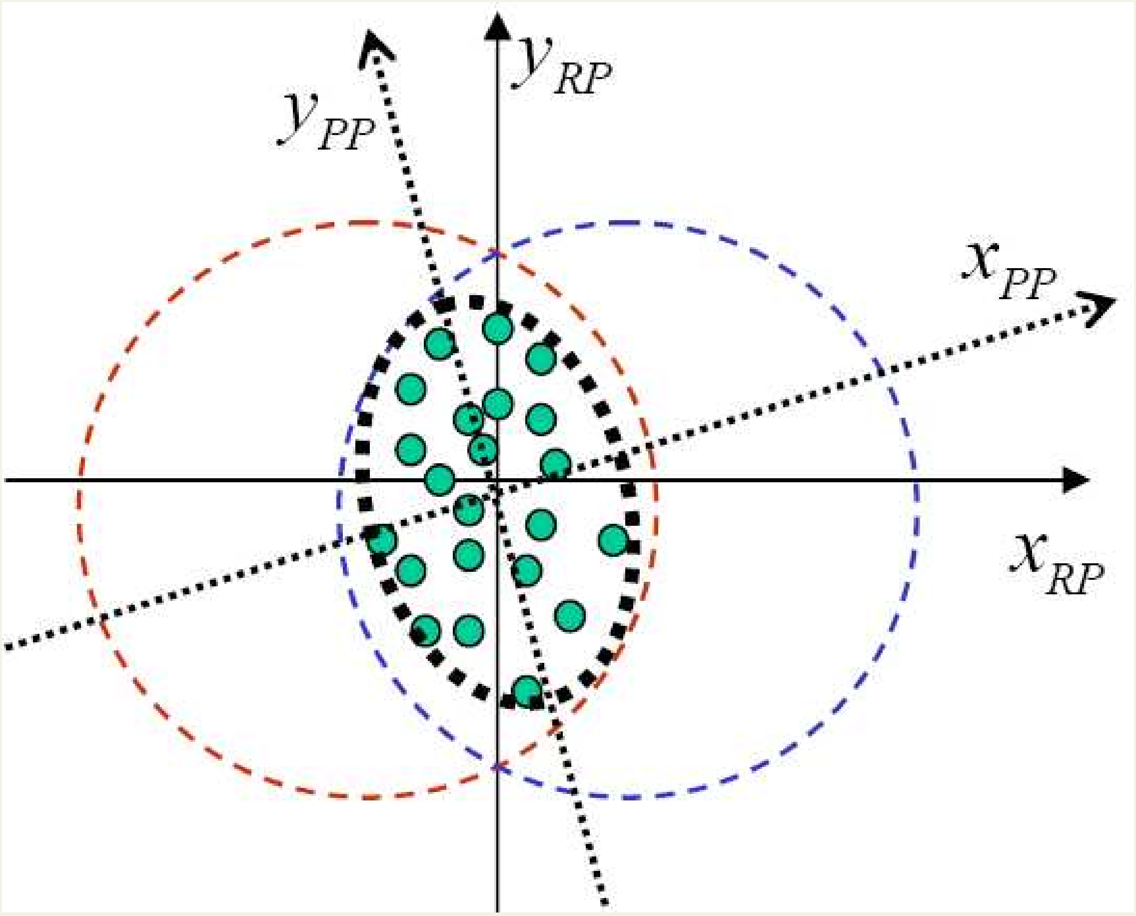


- **shape fluctuates event-by-event**
- **all coefficients v_n finite**

$$\frac{\mathrm{d}N}{\mathrm{d}y\mathrm{d}\phi} = \frac{\mathrm{d}N}{\mathrm{d}y}\left[1 + \sum_n 2v_n \cos(2(\phi - \Psi_n))\right]$$

All the planes. . .

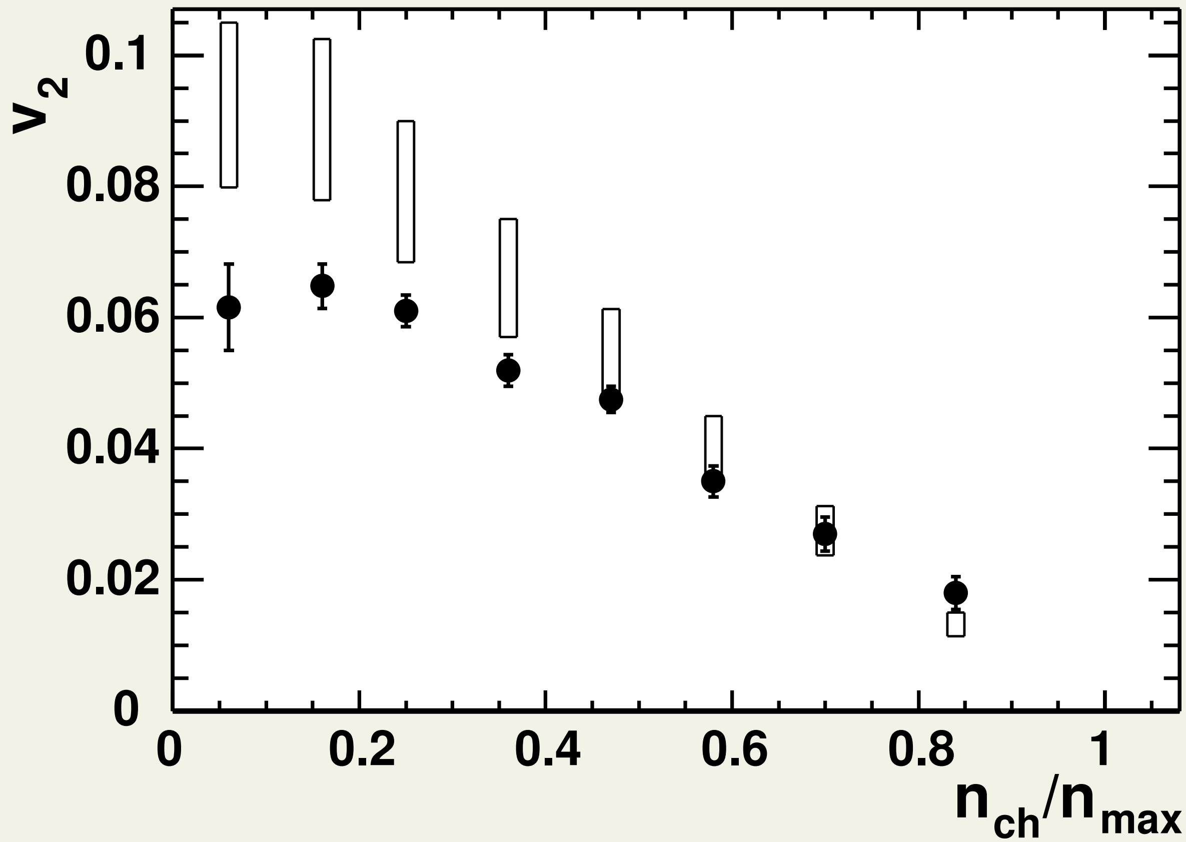
Voloshin *et al.* Phys. Lett. B 659, 537 (2008)



- X_{RP}: **Reaction plane, spanned by beam and impact parameter**
- X_{PP}: **Participant plane, maximises spatial anisotropy** ϵ_n
- Ψ_n: **Event plane, maximises anisotropy** v_n

Success of ideal hydrodynamics

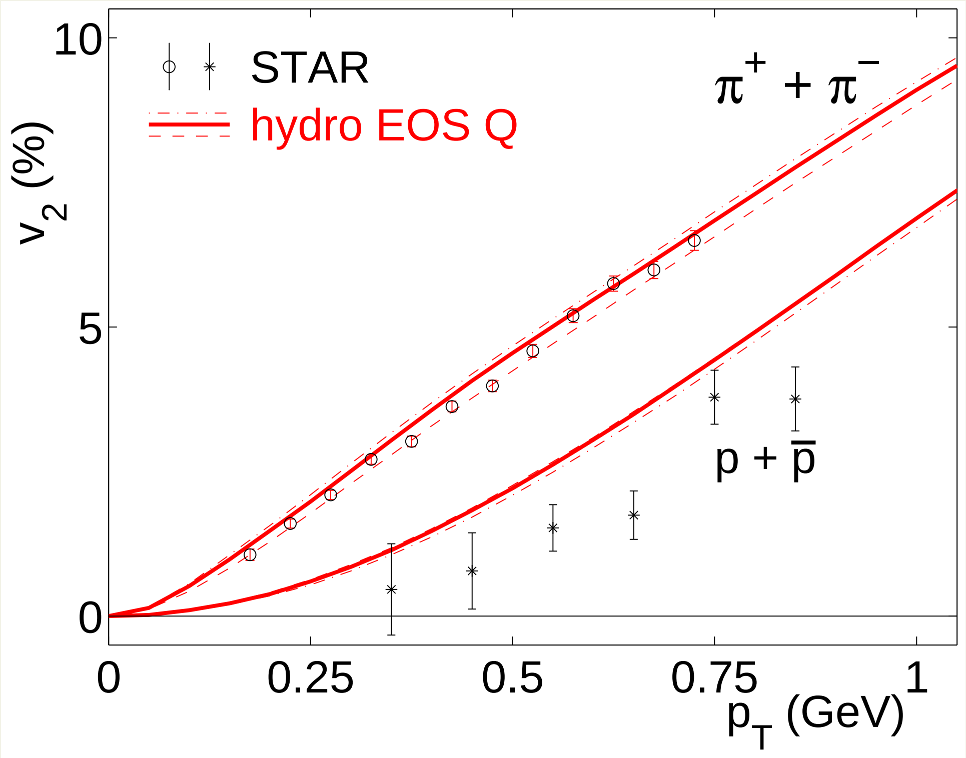
- p_T-averaged v_2 of charged hadrons:

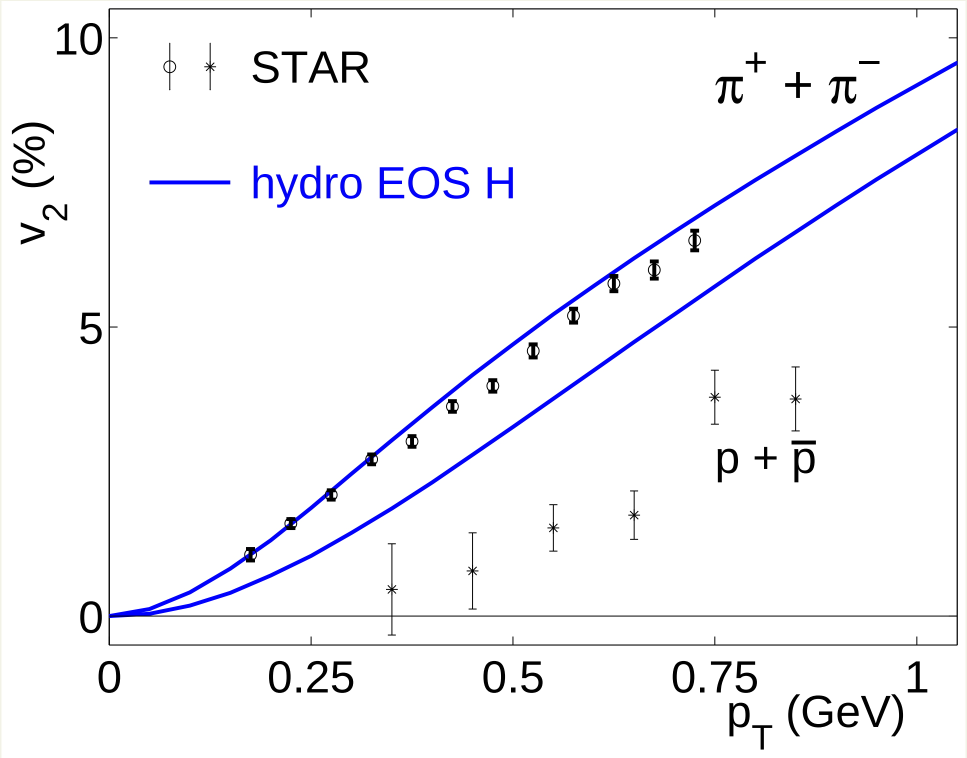


- works beautifully in central and semi-central collisions
- but why is $v_{2,\mathrm{obs}} > v_{2,hydro}$ in most central collisions? $\Rightarrow$ fluctuations!

Success of ideal hydrodynamics

Kolb, Heinz, Huovinen et al ('01) minbias Au+Au at RHIC



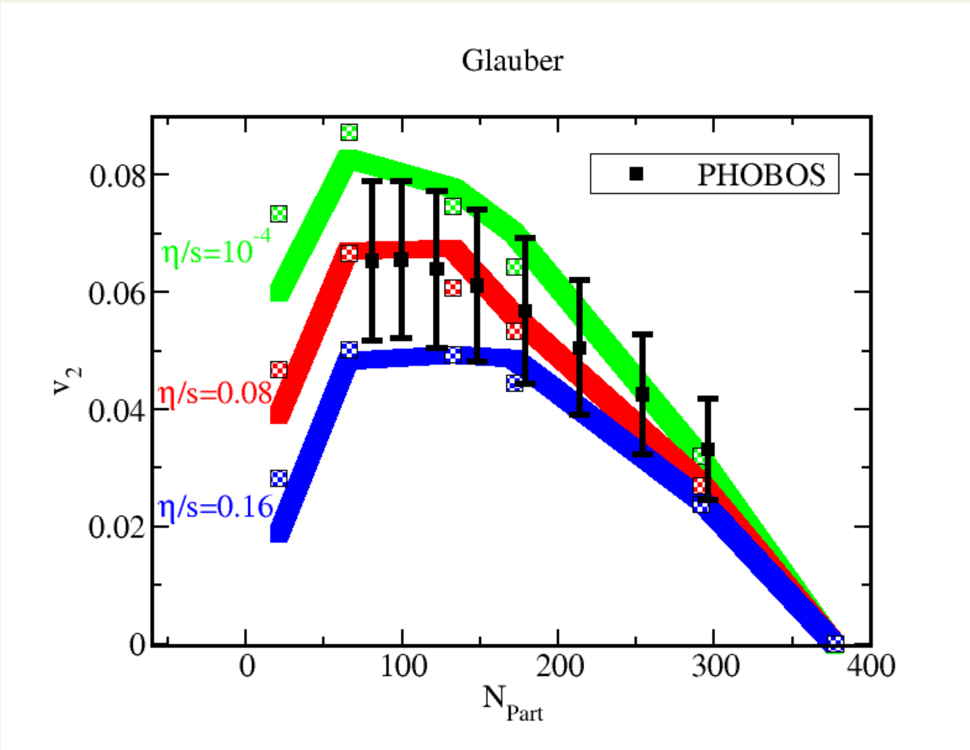


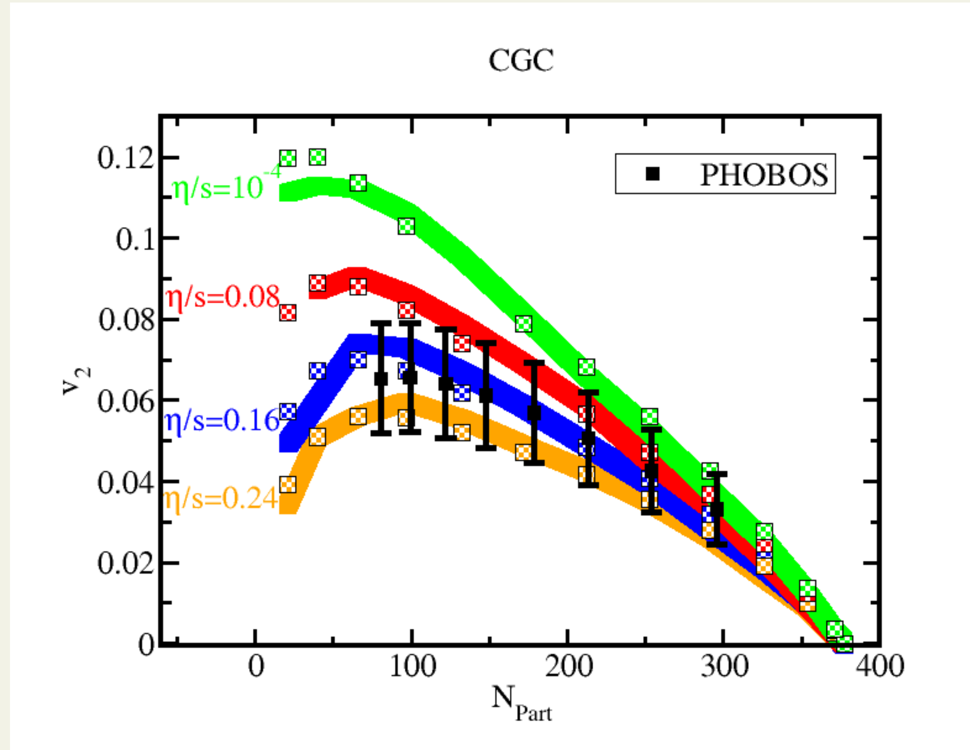
not perfect agreement but plasma EoS favored

ideal fluid? — so how ideal is plasma actually. . . ?

η/s from comparison with observed v_2

- **Luzum & Romatschke, Phys.Rev.C78:034915,2008**

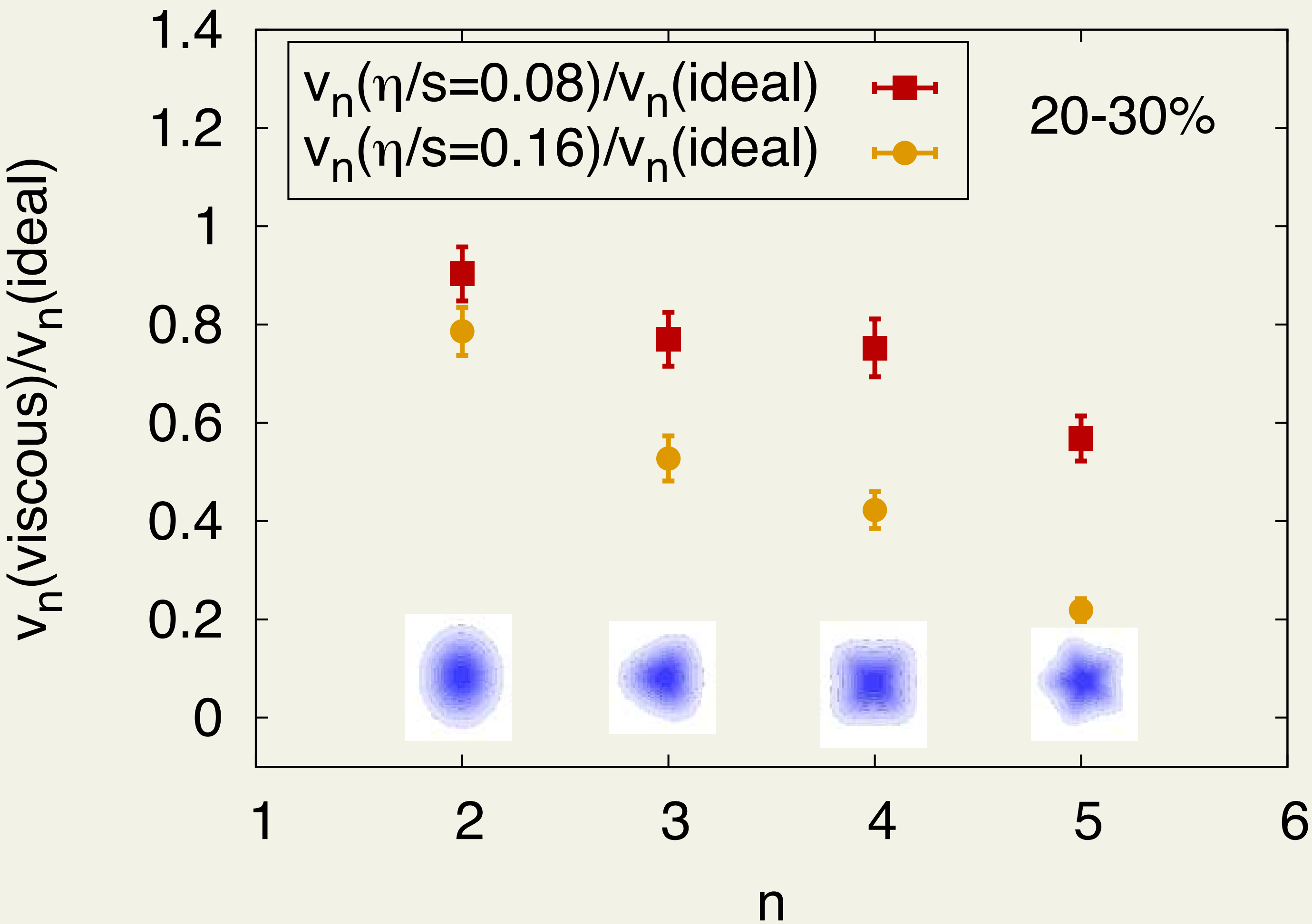




- $\eta/s = 0.08$ **or** $\eta/s = 0.16$ **depending on initialization**
- **consensus:** $1 < 4\pi\frac{\eta}{s} < 5$

Sensitivity to η/s

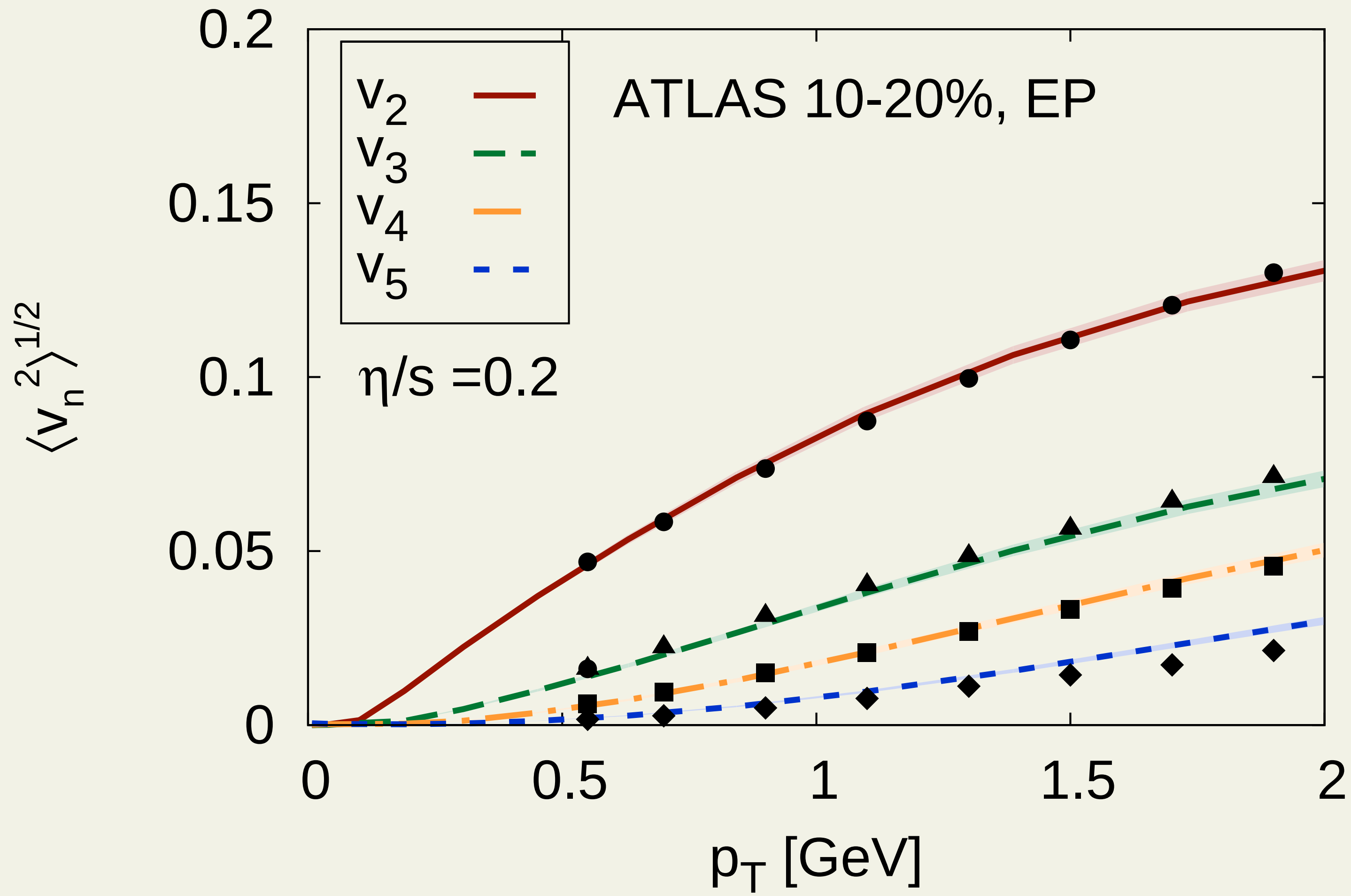
Schenke *et al.* Phys.Rev.C85:024901,2012



- **higher coefficients are suppressed more by dissipation**

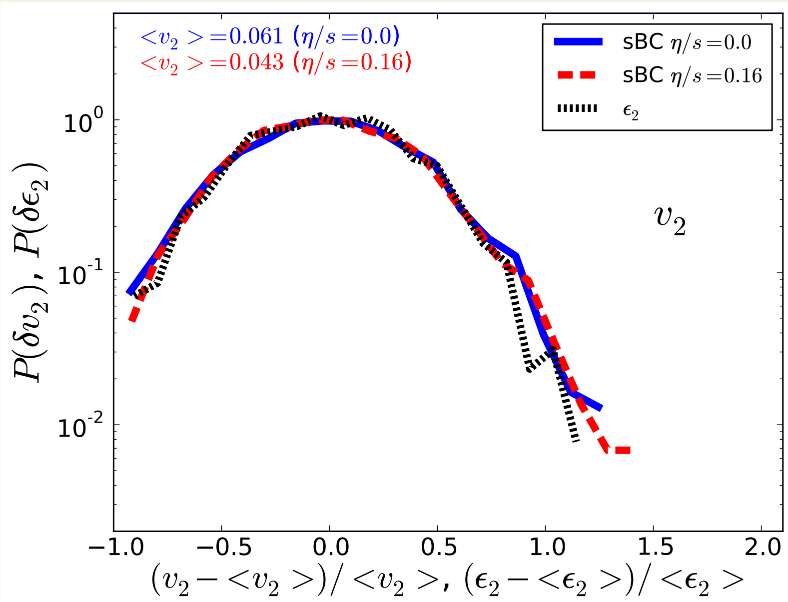
η/s from v_n

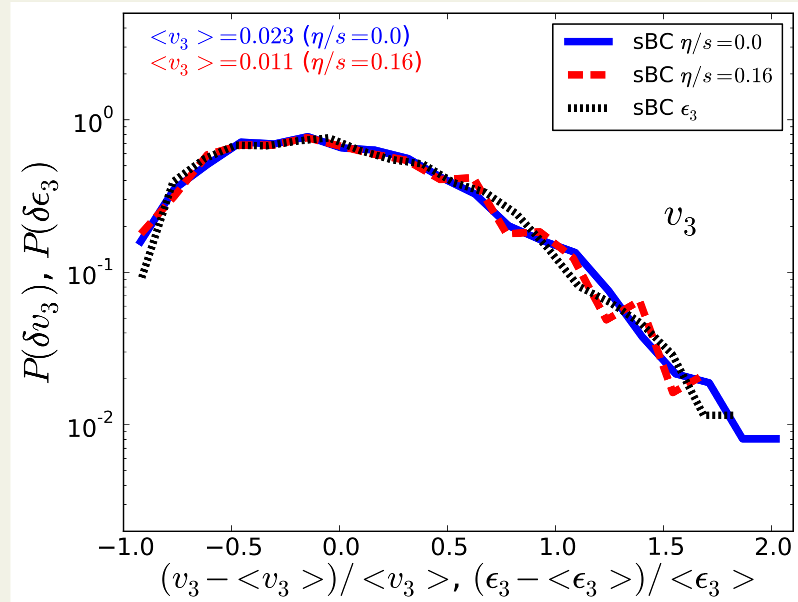
Gale *et al.* Phys.Rev.Lett. 110, 012302 (2013)

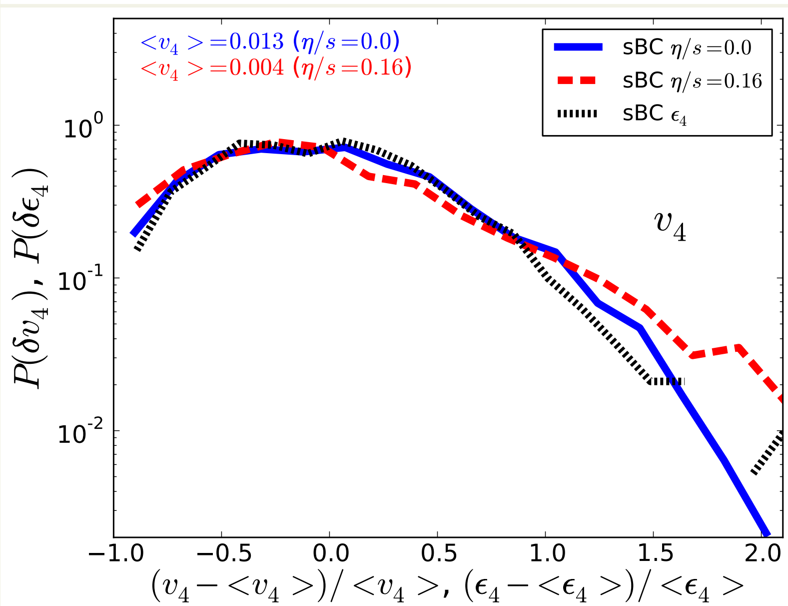


- IP-Glasma initialization
- looks promising!

Distributions of v_n event-by-event





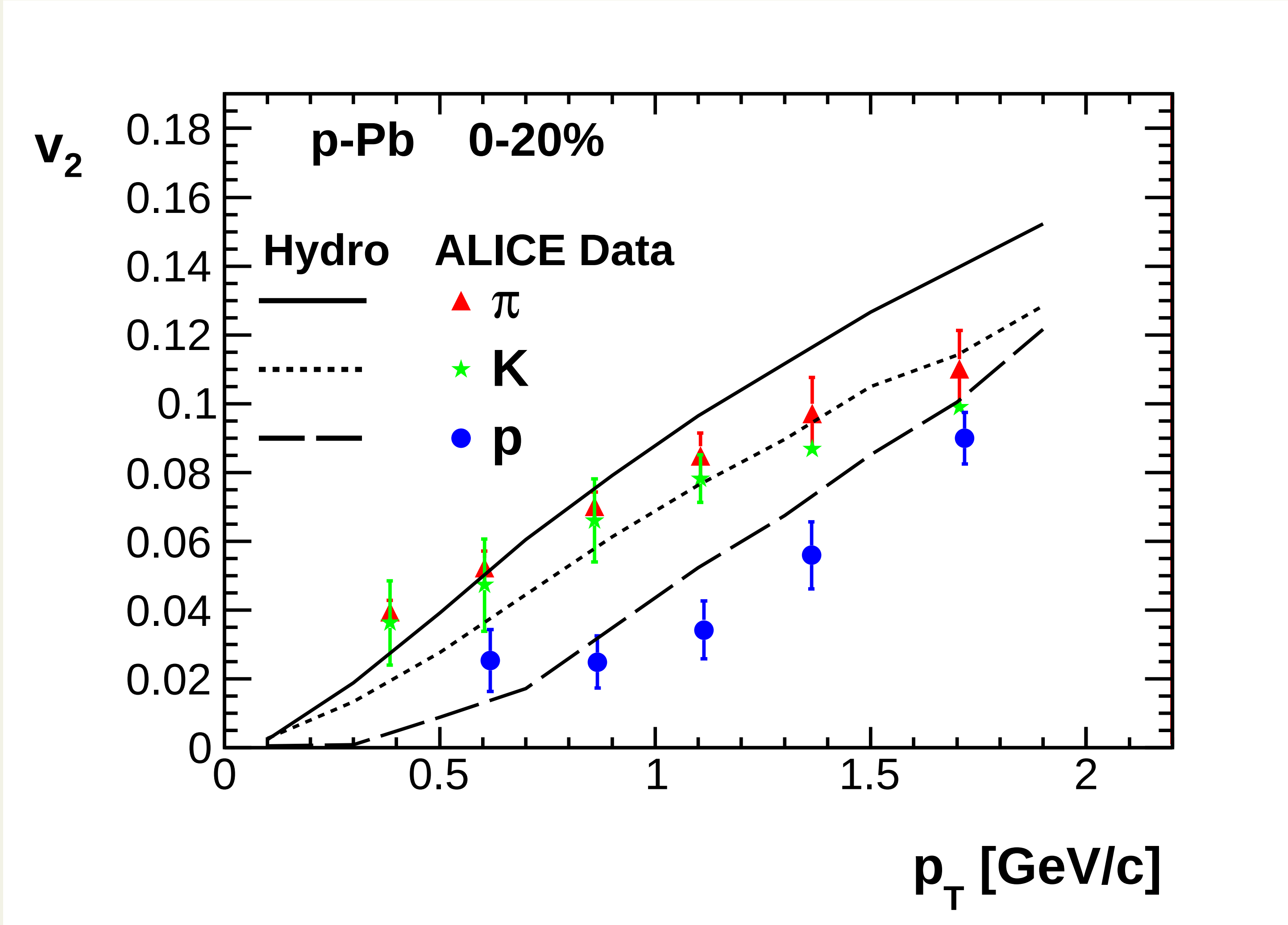


Niemi *et al.* Phys.Rev.C87, 054901 (2013)

- $\delta v_n \approx \delta\epsilon_n$ **independent of η/s**
- **measurement of initial state?**

Flow in small systems?

- at LHC, even $p + Pb$ collisions seem to show collective behaviour



Bozek *et al.* Phys.Rev.Lett. 111, 172303 (2013)

- expect to hear much about this!

Summary

- Hydrodynamics is a useful tool to model collision dynamics
  - approximation at its best
  - but it can reproduce (a lot of) the data
- we have observed hydrodynamical behaviour at RHIC and LHC
  - and will observe more!
- There are many variants of the model
  - initialization
  - 2+1D vs. 3+1D
  - pure hydro vs. hybrid
  - etc.
- We aim to understand transport properties of QGP and test the initial state models using hydro