The QCD Equation of State

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Quark-Gluon Plasma

- Transition to the high-temperature phase of QCD, Quark-Gluon Plasma (QGP) is a crossover at $T_c \sim 154$ MeV[1]
- The deconfinement of the degrees of freedom with quantum numbers of quarks and gluons is manifested in the QGP equation of state, i.e. $p(T)$ or $\varepsilon(T)$
- The heavy-ion experiments at RHIC explore temperatures in the region of 300 MeV and at LHC around 400 MeV, with almost zero net baryon density at high center-of-mass energies
- We calculate the QCD equation of state in the $130 - 400$ MeV range at zero baryon chemical potential, $\mu_B = 0$
- Collisions at lower energies (e.g. RHIC Beam Energy Scan, future experiments at FAIR) produce systems with $\mu_B > 0$ and the equation state at $O(\mu_B^4)$ is also being studied with the Taylor expansion method (talk by Prasad Hegde, Tuesday)

[1] Aoki et al. [BW] (2010), Bazavov et al. [HotQCD] (2012)

Lattice QCD

- Quantum field theory on a discrete (Euclidean) space-time lattice, $N_s^3 \times N_\tau$, $T = 1/(aN_\tau)$
- Evaluate path integrals stochastically (importance sampling)

$$
\begin{aligned}
Z &= \int DUD\bar{\psi}D\psi \exp\{-S\}, \quad S = S_g + S_f \\
\langle \mathcal{O} \rangle &= \frac{1}{Z} \int DUD\bar{\psi}D\psi \mathcal{O} \exp\{-S\}
\end{aligned}
$$

- We use Highly Improved Staggered Quarks (HISQ)[2] and the tree-level Symanzik-improved gauge action, hence HISQ/tree
- The physics is recovered in the continuum limit ($a \to 0$, or $N_\tau \to \infty$ in the finite-temperature geometry)
- The trace anomaly or interaction measure:

$$
\varepsilon - 3p = -\frac{T}{V}\frac{d \ln Z}{d \ln a} \quad \Rightarrow \quad \frac{p}{T^4} - \frac{p}{T_0^4} = \int_{T_0}^{T} dT' \frac{\varepsilon - 3p}{T'^5}
$$

[2]Follana et al. [HPQCD] (2007)

HISQ/tree – numerical setup

- Calculation of the interaction measure requires subtraction of UV divergences (take difference of zero- and finite-temperature quantities evaluated at the same values of the gauge coupling):

$$\begin{aligned}\frac{\varepsilon - 3p}{T^4} &= R_\beta [\langle S_g \rangle_0 - \langle S_g \rangle_T] \\ &- R_\beta R_m [2 m_l (\langle \bar{l} l \rangle_0 - \langle \bar{l} l \rangle_T) + m_s (\langle \bar{s} s \rangle_0 - \langle \bar{s} s \rangle_T)]\end{aligned}$$

$$R_\beta(\beta) = -a \frac{d\beta}{da}, \quad R_m(\beta) = \frac{1}{m} \frac{dm}{d\beta}, \quad \beta = \frac{10}{g^2}$$

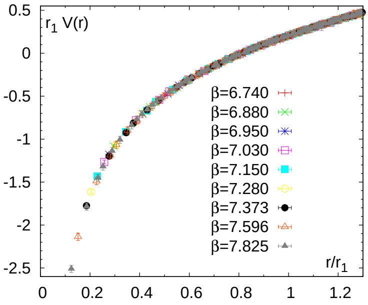
- Line of constant physics $m_l = m_s/20$ (physical $m_l = m_s/27$), $m_\pi = 160$ MeV
- Statistics (in molecular dynamics time units):

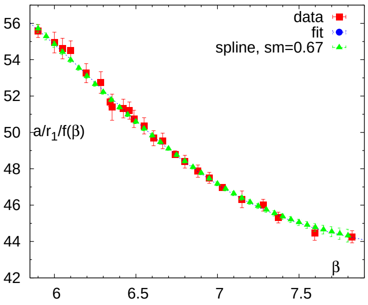
$T > 0$		$T = 0$	
$24^3 \times 6$	30-40K	$24^3 \times 32$	5-20K
$32^3 \times 8$	30-100K	32^4, $32^3 \times 64$	10-30K
$40^3 \times 10$	100-200K	48^4	5-14K
$48^3 \times 12$	50-100K	$48^3 \times 64$	8-12K
		64^4	8K

HISQ/tree - scale setting

- Sommer scale, $r_1 = 0.31$ fm (derived requiring that f_π is at the experimental value[3])

$$r^2 \frac{dV_{\bar{q}q}}{dr}\bigg|_{r=r_1} = 1$$

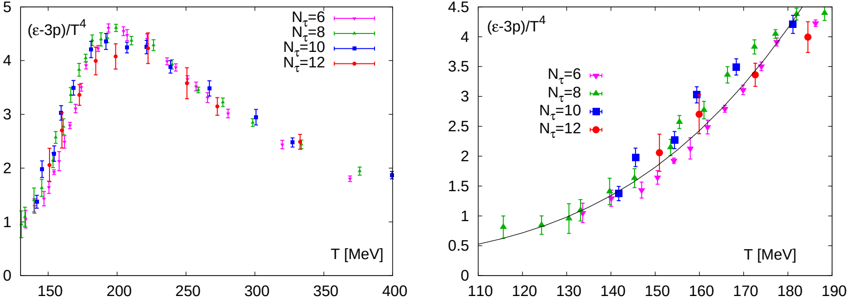




- Left: static quark anti-quark potential
- Right: lattice scale a/r_1 as function of the inverse gauge coupling β, normalized by the perturbative 2-loop result

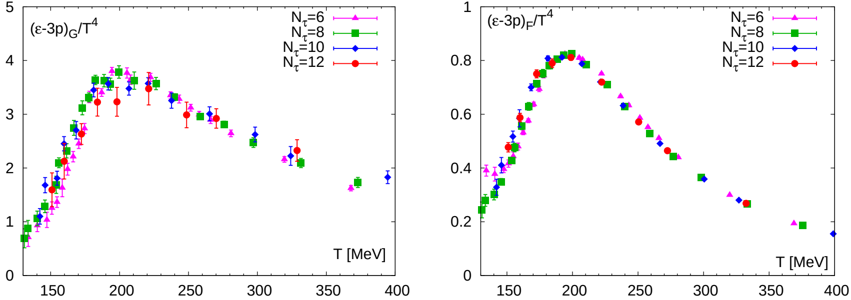
[3]MILC, PoS(Lat2010)

Interaction measure



- Cutoff effects for the HISQ/tree action at the leading order $\sim 1/N_\tau^2$
- They change with temperature: the approach to the continuum at low temperatures is from below and in the peak region from above
- $N_\tau = 6$ seems outside of the $1/N_\tau^2$ scaling regime

Interaction measure



- Left: gluon contribution

$$\left(\frac{\varepsilon - 3p}{T^4}\right)_G = R_\beta[\langle S_g \rangle_0 - \langle S_g \rangle_T]$$

- Right: (valence) quark contribution

$$\left(\frac{\varepsilon - 3p}{T^4}\right)_F = -R_\beta R_m[2m_l(\langle \bar{l}l \rangle_0 - \langle \bar{l}l \rangle_T) + m_s(\langle \bar{s}s \rangle_0 - \langle \bar{s}s \rangle_T)]$$

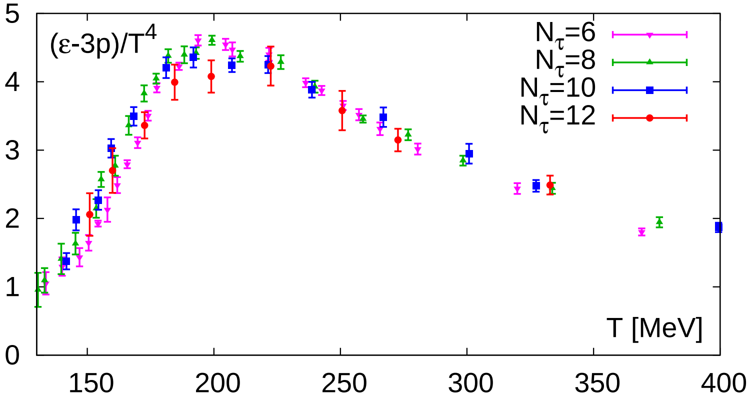
Interaction measure

- To perform the continuum extrapolation – need to interpolate the data at each N_τ
- Fit the data with cubic splines, where the knot positions are the fit parameters, determined by χ^2 minimization
- Combine spline fitting with the continuum extrapolation and use the form:

$$\frac{\varepsilon - 3p}{T^4} = A + \frac{B}{N_\tau^2} + \sum_{i=1}^{5} \left[C_i + \frac{D_i}{N_\tau^2} \right] S_i(T)$$

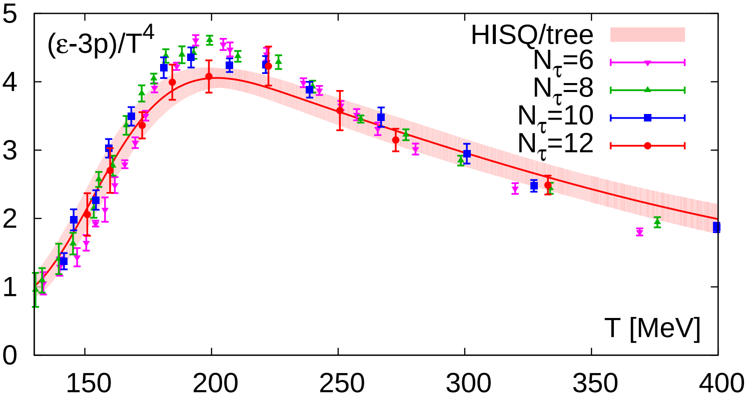
  to simultaneously fit the data at $N_\tau = 8$, 10 and 12
- $S_i(T)$ are B-splines
- Add the Hadron Resonance Gas model value for the interaction measure and its slope at $T = 130$ MeV as a constraint
- The fit uses two internal knots and has overall 37 degrees of freedom, giving $\chi^2/dof = 0.838$
- Final errors include bootstrap errors on the fit and 2% errors on the scale determination, added linearly

Interaction measure



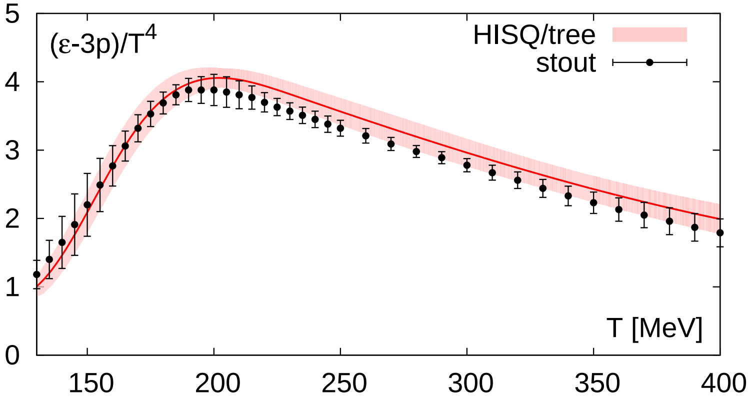
HISQ/tree data at $N_\tau = 6$, 8, 10 and 12

Interaction measure



HISQ/tree data and the continuum extrapolation

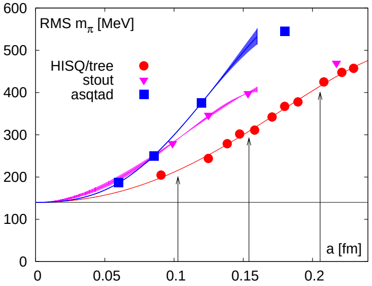
Interaction measure

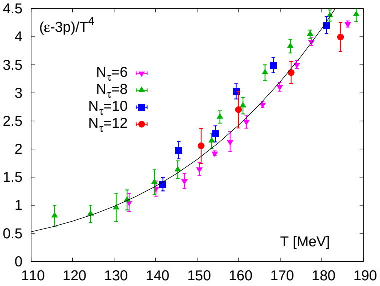


HISQ/tree and stout (Borsanyi et al. [BW] (2014)) continuum extrapolation

Interaction measure at low temperature

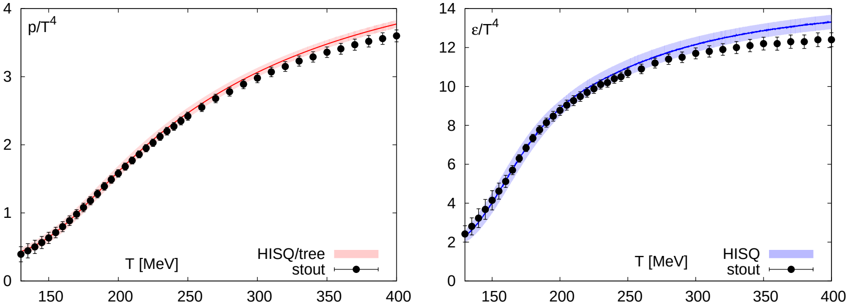
- Taste symmetry breaking makes the average pion mass heavier, the problem is worse on coarser lattices





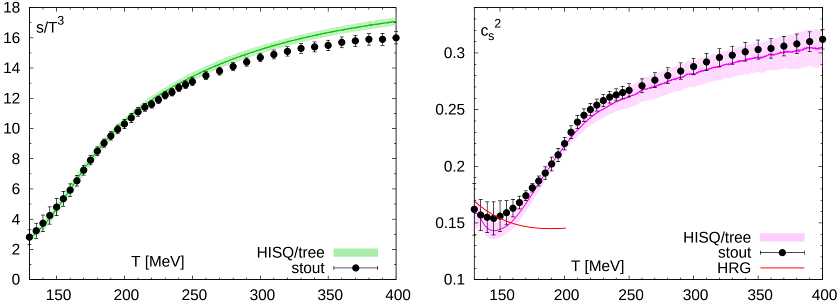
- Left: root-mean-squared pion mass as function of lattice spacing
- Right: the interaction measure at low temperature

Pressure and the energy density



- Left: pressure, calculated as an integral of the interaction measure, starting from the Hadron Resonance Gas model value at $T = 130$ MeV
- Right: the energy density, calculated as a linear combination of pressure and the interaction measure
- The errors are from bootstrap analysis

Entropy and the velocity of sound



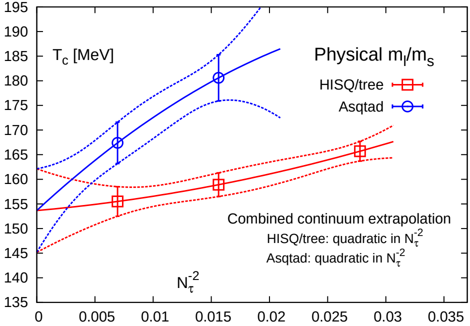
- Left: the entropy density $s/T^3 = (p+\varepsilon)/T^4$
- Right: the velocity of sound, $c_s^2 = dp/d\varepsilon$
- The errors are from bootstrap analysis

Conclusion

- We calculate the interaction measure on the lattice at several values of the cutoff $1/N_\tau = 1/6$, $1/8$, $1/10$ and $1/12$ in the temperature range $T = 130 - 400$ MeV
- The HISQ/tree action has reduced (compared to other staggered fermion schemes) cutoff effects at low temperature
- The leading cutoff effects are $\sim 1/N_\tau^2$ and depend on the temperature, the data at $N_\tau = 6$ is not in the scaling regime
- Our analysis performs fitting of the data at fixed cutoff and the continuum extrapolation in $1/N_\tau^2$ simultaneously
- The HISQ/tree continuum extrapolation for the interaction measure $(\varepsilon - 3p)/T^4$ agrees with the stout result within errors
- With the integral method we calculate the pressure, choosing the Hadron Resonance Gas value at $T = 130$ MeV as reference point
- We also calculate the energy and entropy density and the velocity of sound, the latter acquires minimum at $T \sim 146$ MeV

Extra

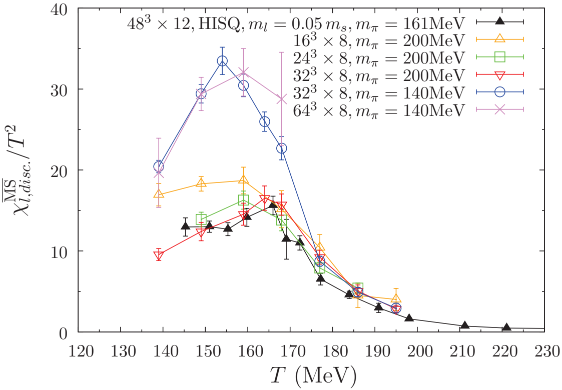
T_c determination, staggered



- Combined extrapolation using asqtad and HISQ data sets
- The final result for the chiral transition temperature at the physical quark masses in the continuum limit

$$T_c = 154 \pm 9 \text{ MeV}$$

T_c determination, domain-wall



- Disconnected chiral susceptibility
- The crossover temperature with DWF: $T_c = 155(1)(8)$ MeV (1402.5175), confirms staggered result

HISQ/tree - scale setting

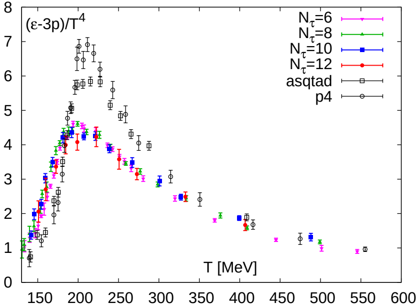
Fit a/r_1 data with the Ansatz:

$$\frac{a}{r_1} = \frac{c_0 f(\beta) + c_2(10/\beta) f^3(\beta)}{1 + d_2(10/\beta) f^2(\beta)},$$

$$f(\beta) = (b_0(10/\beta))^{-b_1/(2b_0^2)} \exp(-\beta/(20 b_0))$$

$$c_0 = 43.1281 \pm 0.2868$$
$$c_2 = 343236 \pm 41191$$
$$d_2 = 5513.84 \pm 754.821$$

Interaction measure



Current HISQ/tree and previous HotQCD result (2009) with the asqtad and p4 action at $m_l = m_s/10$ LCP