Statistical function instability under strong color magnetic field in 2PI approach

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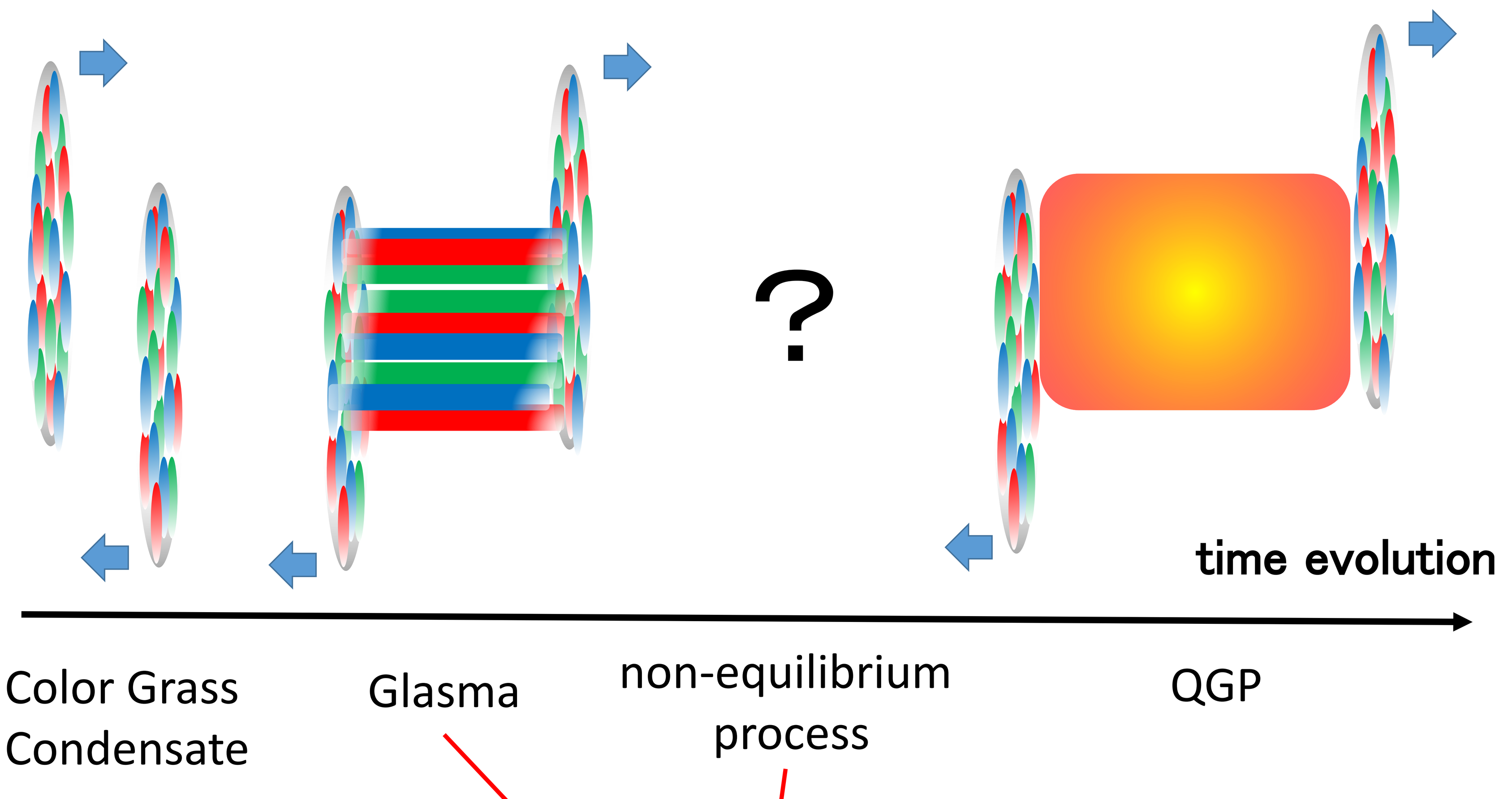
Abstract Particle production under the time dependent classical gluon fields is considered to be important in thermalization of relativistic heavy ion collisions. We calculate the time evolution of the statistical function which contains important features of particle distribution under the homogeneous, but time dependent color electromagnetic fields in the linear regime. Our results suggest that parametric resonance causes exponential growth of the statistical function in both lower and higher momentum regions.

1. Early Thermalization Problem



hydrodynamics is applicable

considerably short thermalization time $\tau_{\rm thermal} \sim 0.6 - 1.0\,{\rm fm/c}$

Heinz (2005)

field particle conversion under strong gluon fields plays an important role

- decay of background fields (e.g. preheating in inflation theory) Kofman *et. al.* (1997)
- Schwinger mechanism Kharzeev *et. al.* (2005)

2. Instabilities under Strong Fields

Background fields rapidly decay and explosive particle production occurs when the system have some **instabilities**

we assume
- homogeneous color magnetic field
- SU(2) pure Yang-Mills theory

remark : other well known instabilities
- Weibel instability Mrowczynski (1988), Romatschke, Venugopalan (2006)
- classical chaos in Yang-Mills theory Iida *et. al.* (2013)

two types of instability triggered by color magnetic fields

$$\boldsymbol{B}^a = \nabla \times \boldsymbol{A}^a - \frac{1}{2} g \epsilon^{abc} \boldsymbol{A}^b \times \boldsymbol{A}^c$$

"abelian" configuration

$$A^a_x = -\frac{1}{2} B y \delta^{a3} \quad A^a_y = \frac{1}{2} B x \delta^{a3}$$

"non-abelian" configuration

$$A^a_i = \sqrt{B(t)}\,(\delta^{a2}\delta_{ix} + \delta^{a1}\delta_{iy})$$

homogeneous magnetic field

$$B^3_z \neq 0 \qquad B^a_{x,y} = 0$$

Fujii, Itakura (2008)
Iwazaki (2009)

Nielsen-Olesen instability

Berges *et. al.* (2012)

parametric resonance

We discuss the stability under the "non-abelian" magnetic field in the linear regime.

classical EOM

$$\partial_t^2 B(t)^{1/2} + B(t)^{3/2} = 0$$

classical solution is periodic function of time

$$B(t) = B_0 {\rm cn}^2(\sqrt{B_0}t\,;1/\sqrt{2})$$

EOMs of fluctuation

$$\frac{d^2\vec{a}}{dt^2} + \Omega^2[B(t)]\vec{a} = 0$$

linear ODE with periodic coefficients (Hill's equation)

3. Floquet Theory

Floquet theory is a strong tool to analyze the stability under periodic "potential" (c.f. Bloch's theorem)

fundamental matrix solution: $\Phi(t) = \left(\vec{\phi}_1(t) \quad \ldots \quad \vec{\phi}_n(t)\right)$ — *indep. sol. of ODE with T-periodic potential*

if $\Phi(t)$ is a matrix solution, $\Phi(t+T)$ is also a matrix solution $\quad \Phi(t+T) = \Phi(t)M$ — constant and regular

the matrix solution becomes as follows

$$\Phi(t) = F(t)\exp\left((\log M)\frac{t}{T}\right)$$

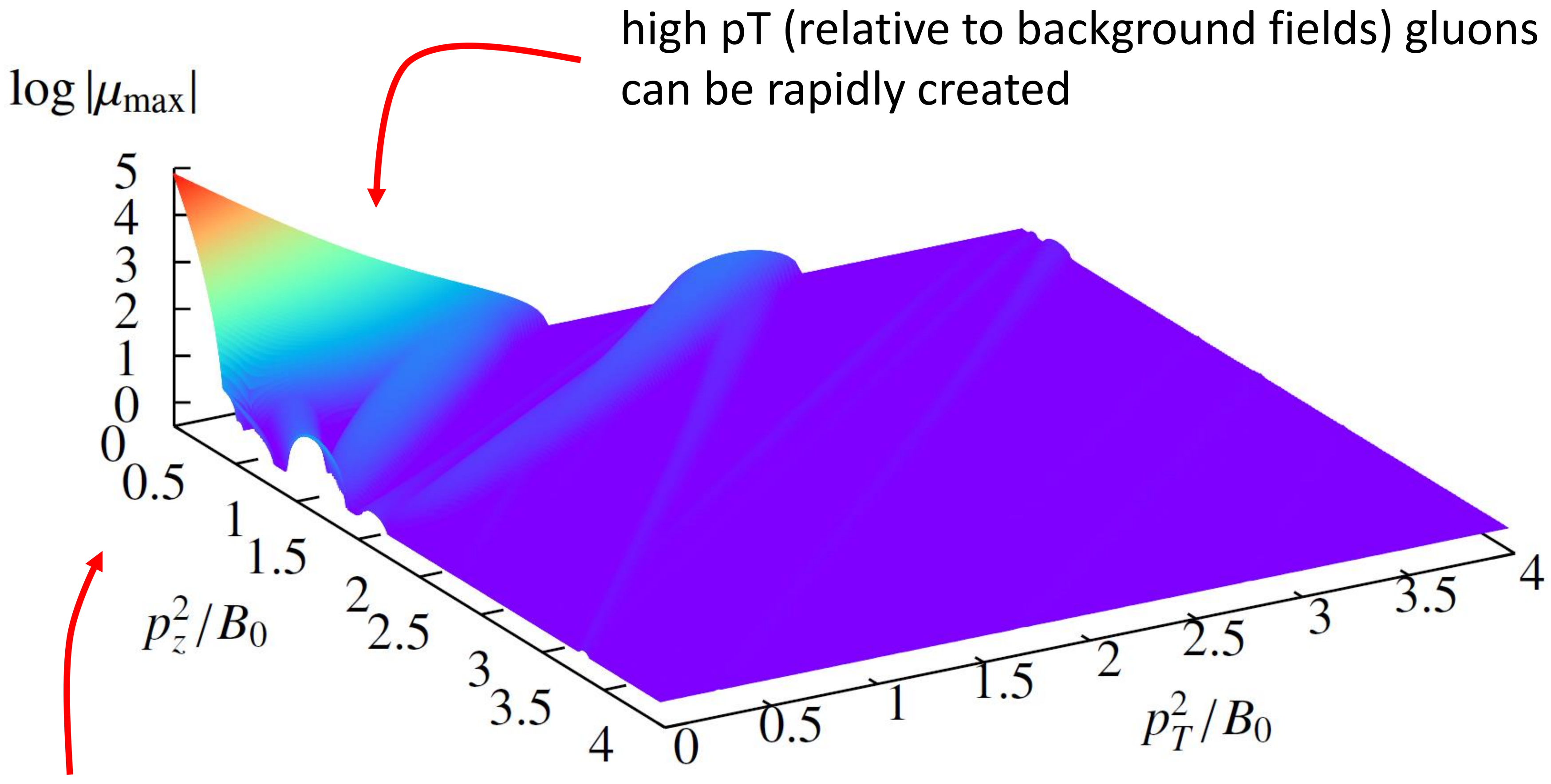
$$F(t+T) = F(t)$$

characteristic multipliers μ_i *:* eigenvalues of M

$|\mu| > 1$ the solution is unstable (exponentially diverge)

$|\mu| \leq 1$ the solution is stable

4. Instability Band



high pT (relative to background fields) gluons can be rapidly created

large growth rate around the zero momentum region (similar to Nielsen-Olesen instability)

maximum characteristic multipliers vs momentum

In the linear regime, the statistical function (effective particle number) also satisfy Hill's equation

$$\mathcal{F}(t,t;\boldsymbol{p}) = \frac{1}{\omega_{\boldsymbol{p}}}\left(n(t)_{\boldsymbol{p}} + \frac{1}{2}\right)$$

$$\partial_t^2 \mathcal{F}(t,t',\boldsymbol{p}) = -\Omega^2[B(t)]\mathcal{F}(t,t',\boldsymbol{p})$$

instability ≀ rapid particle creation

SUMMARY

- ➢ We calculate the instability band of gluon under the classical color magnetic fields.
- ➢ The momentum dependence of growth rate has also been decided.
- ➢ Parametric resonance occurs
  - ➢ lower momentum modes are very unstable (similar to Nielsen-Olesen instability)
  - ➢ relatively high pT gluons are unstable

- ◆ How about more realistic cases? back reaction, expanding geometry, spatial inhomogeneity of background fields (e.g MV model)
- ◆ closed descriptions of field + particle system are needed (2PI formalism)