

Renormalization of the jet-quenching parameter

Yacine Mehtar-Tani
IPhT Saclay



XXIV QUARK MATTER
DARMSTADT 2014



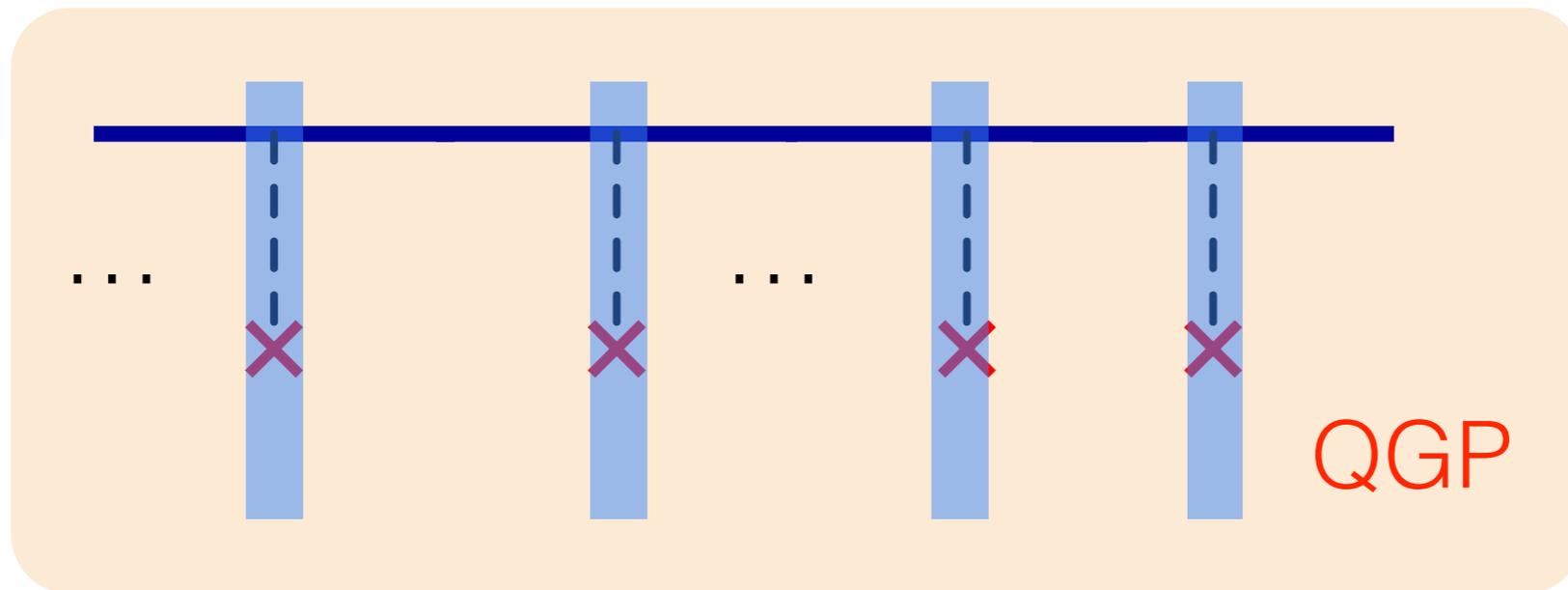
OUTLINE

- The **jet-quenching parameter** in pt-broadening and radiative energy loss in a QGP
- **Radiative corrections** to pt-broadening and radiative energy loss (universality)
- **Renormalization of the jet-quenching parameter**

Based on: J. -P. Blaizot and YMT, arXiv:1403.2323 [hep-ph]

pt-broadening in a plasma

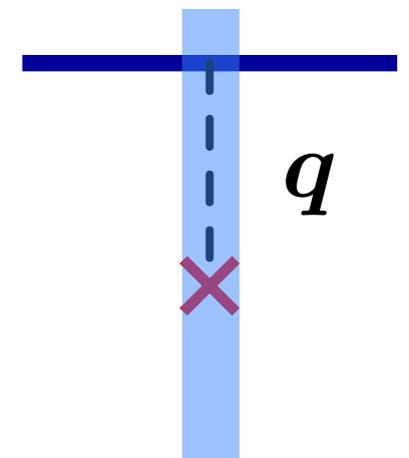
Diffusion approximation: Independent multiple scatterings



correlation length \ll mean-free-path \ll L

Local interaction: elastic cross-section

$$\frac{d\sigma_{\text{el}}}{d^2\mathbf{q}} \simeq \frac{g^4}{q^4} \quad (\text{for } q_{\perp} \gg m_D)$$



pt-broadening in a plasma

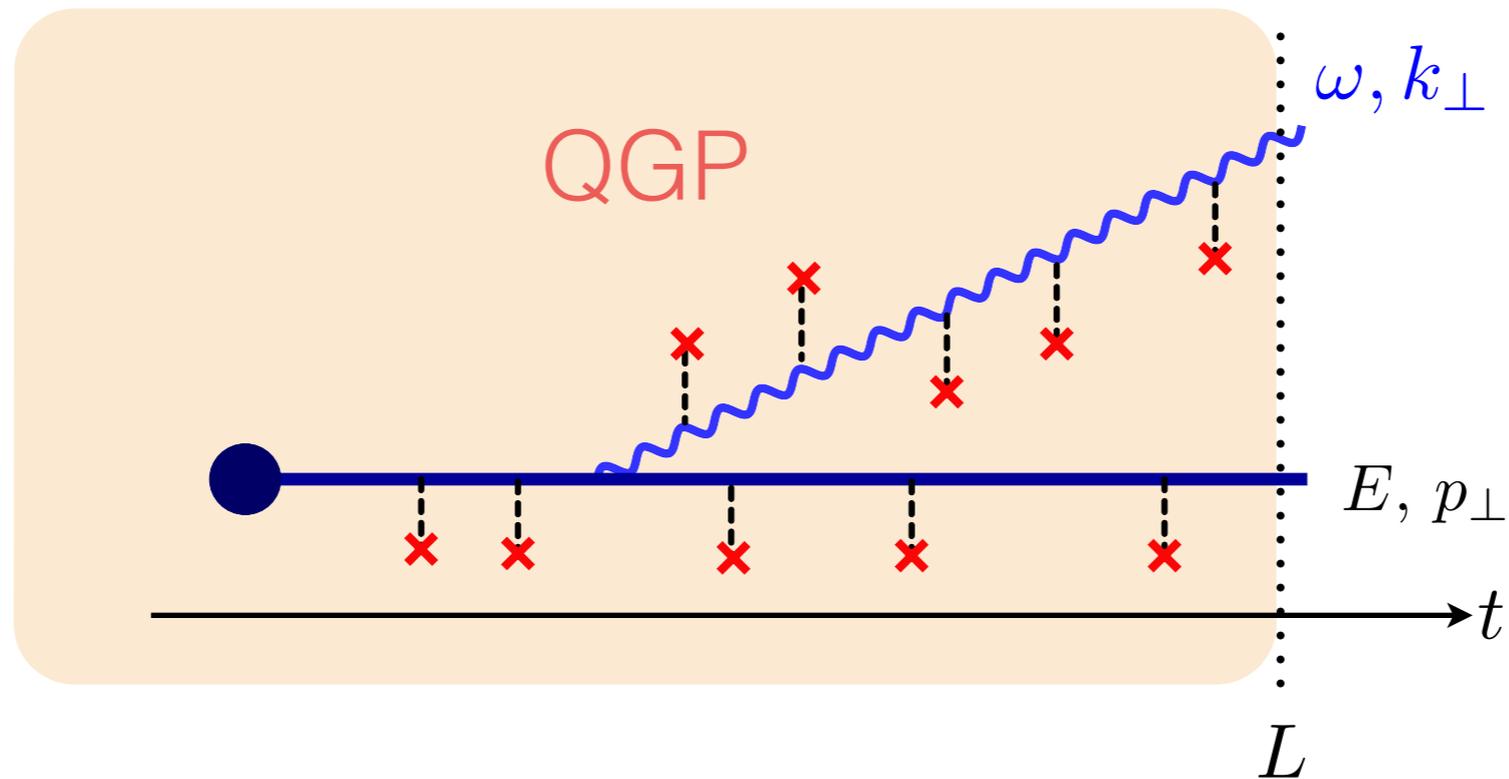
The quenching parameter is defined as a (local) diffusion coefficient

$$\hat{q} \equiv \frac{d\langle q_{\perp}^2 \rangle}{dt} = n \int_{\mathbf{q}} \mathbf{q}^2 \frac{d\sigma_{\text{el}}}{d^2\mathbf{q}} \sim \alpha_s^2 C_R n \ln \frac{k^2}{m_D^2}$$

The probability to acquire a transverse mom. k_{\perp} after a time t is given by a Fokker-Planck equation

$$\mathcal{P}(\mathbf{k}, L) = \frac{4\pi}{\hat{q} L} \exp\left(-\frac{k^2}{\hat{q} L}\right) \quad \langle k_{\perp}^2 \rangle_{\text{typ}} \equiv \hat{q} L$$

Radiative Energy Loss



- Scatterings with the medium can induce gluon radiation
- The radiation mechanism is linked to **transverse momentum broadening**, i.e., to \hat{q}

[Baier, Dokshitzer, Mueller, Peigné, Schiff (1995-2000) Zakharov (1996)]

Radiative Energy Loss

How does it happen? After a certain number of scatterings coherence between the parent quark and gluon fluctuation is broken and the gluon is **formed** (decoherence is faster for softer gluons)

$$t_f \equiv \frac{\omega}{\langle q_{\perp}^2 \rangle} \simeq \frac{\omega}{\hat{q} t_f} \quad \longrightarrow \quad t_f = t_{\text{br}} \equiv \sqrt{\frac{\omega}{\hat{q}}}$$

maximum frequency for this mechanism $\omega_c = \frac{1}{2} \hat{q} L^2$
corresponding to $t_{\text{br}} \sim L$

[Baier, Dokshitzer, Mueller, Peigné, Schiff (1995-2000) Zakharov (1996)]

Radiative Energy Loss

Spectrum = bremsstrahlung x effective number of scatterers

The mean-energy loss dominated by « hard » but rare emissions $\omega \sim \omega_c$ (maximum coherence length $\sim L$)

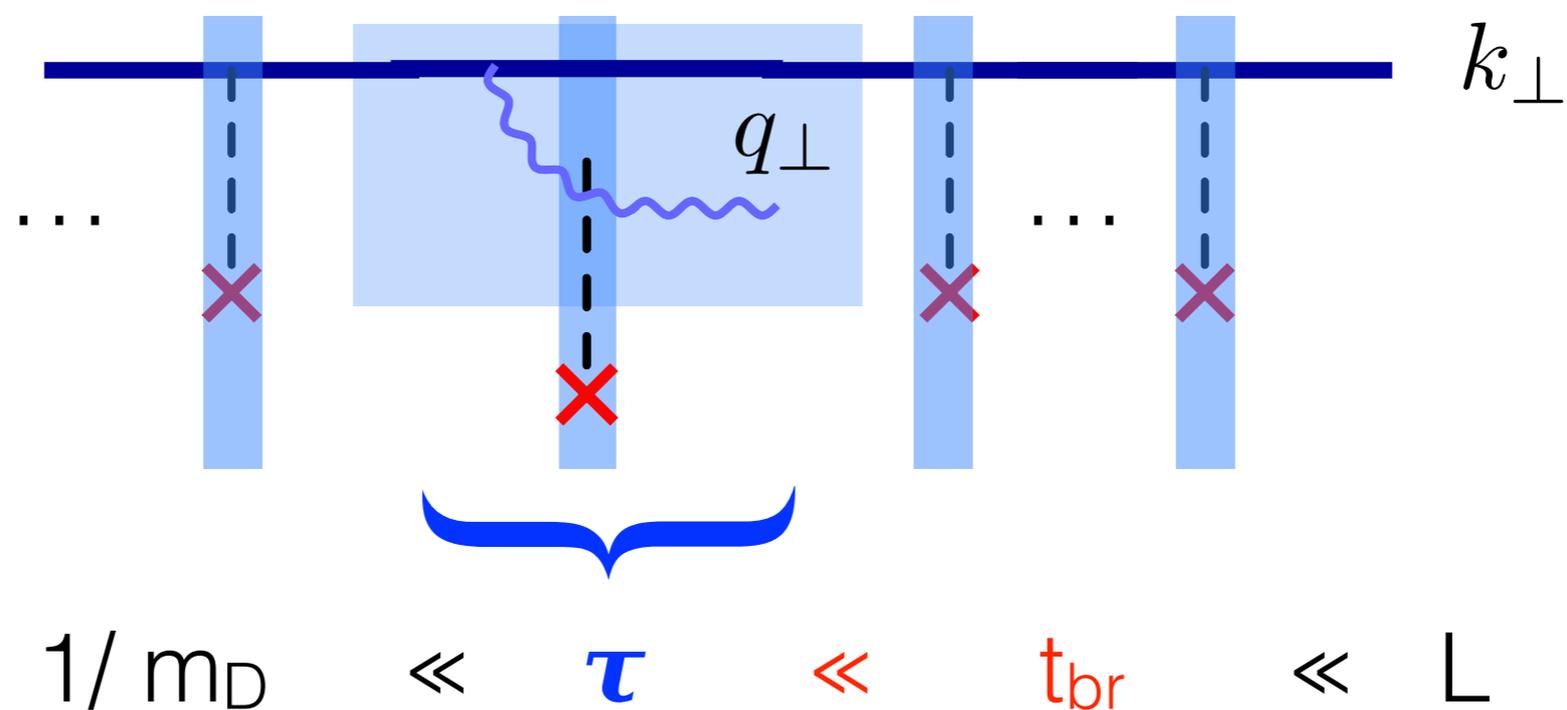
$$\Delta E = \int^{\omega_c} d\omega \omega \frac{dN}{d\omega} \simeq \alpha_s C_R \hat{q} L^2 \propto \omega_c$$

[Baier, Dokshitzer, Mueller, Peigné, Schiff (1995-2000) Zakharov (1996)]

Radiative corrections to pt-broadening

The radiative correction exhibits a logarithmic singularity when the duration of the gluon fluctuation $\tau \rightarrow 0$

The leading logarithmic contribution requires that the **time scale** of the fluctuation τ to be smaller than the radiative coherence length $t_{br} \sim (\omega/\hat{q})^{1/2}$



Radiative correction to pt-broadening

The radiative correction exhibits a logarithmic singularity when the duration of the gluon fluctuation $\tau \rightarrow 0$

- Possible interpretation of the fluctuation as local
- Double Log corrections to the jet-quenching parameter

$$\Delta \hat{q}(L, \mathbf{k}^2) \equiv \frac{\alpha_s N_c}{\pi} \int_{\tau_0}^L \frac{d\tau}{\tau} \int_{\hat{q}\tau}^{\mathbf{k}^2} \frac{d\mathbf{q}^2}{\mathbf{q}^2} \hat{q}(\tau_0)$$

DL: Single scattering : $\mathbf{q} \gg \hat{q} \tau$ (LPM suppression)

Radiative correction to pt-broadening

To logarithmic accuracy $k_{\perp}^2 \sim \hat{q}L$

$$\Delta\hat{q}(L) \simeq \frac{\alpha_s N_c}{\pi} \hat{q} \ln^2 \frac{L}{\tau_0}$$

[T. Liou, A. H. Mueller, B. Wu, (2013) J.P. Blaizot, F. Dominguez, E. Iancu, YMT (2013)]

[Single hard scattering regime: NLO in the Higher-Twist approach Z. Kang et al (2013)]

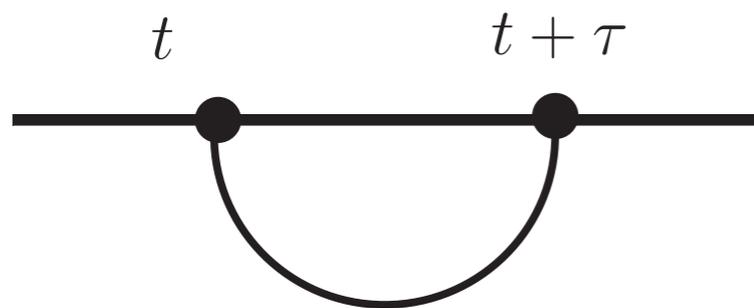
The size of the fluctuation extends up the length of the medium (non local correction)

Can we still talk about independent multiple scattering (2-D diffusion) ? Yes, to logarithmic accuracy

Independent multiple radiative corrections

1-dim model:

Introducing a **correction (radiative)** with a logarithmic phase space (**non-local corrections**)



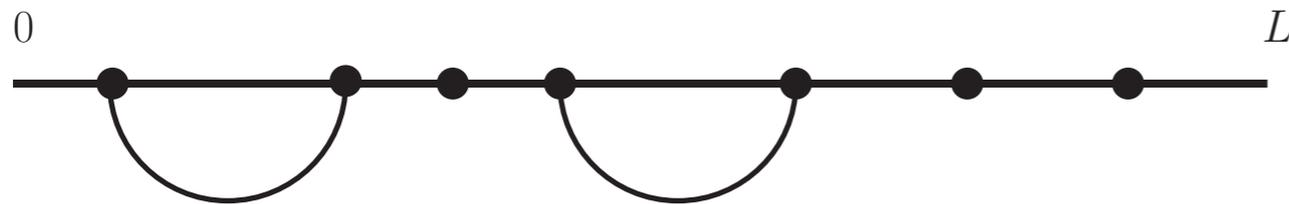
$$\equiv \sigma_0 \alpha \int_{\tau_0}^L \frac{d\tau}{\tau}$$

and consider the large medium limit such that

$$\alpha \ln \frac{L}{\tau_0} \sim 1 \quad \text{and} \quad \alpha \ll 1$$

Independent multiple radiative corrections

To logarithmic accuracy, only disconnected graphs contribute



The survival probability reads

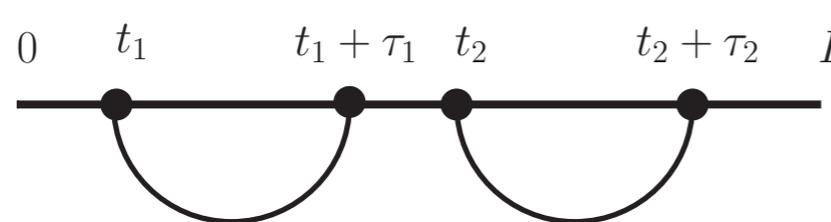
$$S(L) \rightarrow \sum_n \frac{1}{n!} \left| \begin{array}{c} 0 \quad L \\ \text{---} \bullet \text{---} \\ \text{---} \bullet \text{---} \end{array} \right. + \left. \begin{array}{c} 0 \quad L \\ \text{---} \bullet \text{---} \bullet \text{---} \\ \text{---} \bullet \text{---} \end{array} \right|_n$$

$$\sigma_0 \rightarrow \sigma(L) = \sigma_0 \left(1 + \alpha \ln \frac{L}{\tau_0} \right)$$

Therefore, independent scattering approximation is still valid even if radiative corrections are non-local

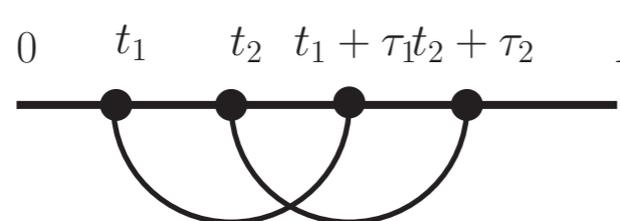
Independent multiple radiative corrections

the factorized contribution (disconnected graph)



$$\approx \frac{1}{2!} \left(\sigma_0 \alpha L \ln \frac{L}{\tau_0} \right)^2$$

connected graph



$$\approx \frac{\pi^2}{12} (\sigma_0 \alpha L)^2 \quad (\text{suppressed})$$

Independent multiple radiative corrections

Hence, radiative corrections to the broadening probability can be reabsorbed in a redefinition of the jet-quenching parameter

$$\hat{q}_0 \rightarrow \hat{q}(L) \equiv \hat{q}_0 \left(1 + \frac{\bar{\alpha}}{2} \ln^2 \frac{L}{\tau_0} \right)$$

leaving the diffusion picture intact

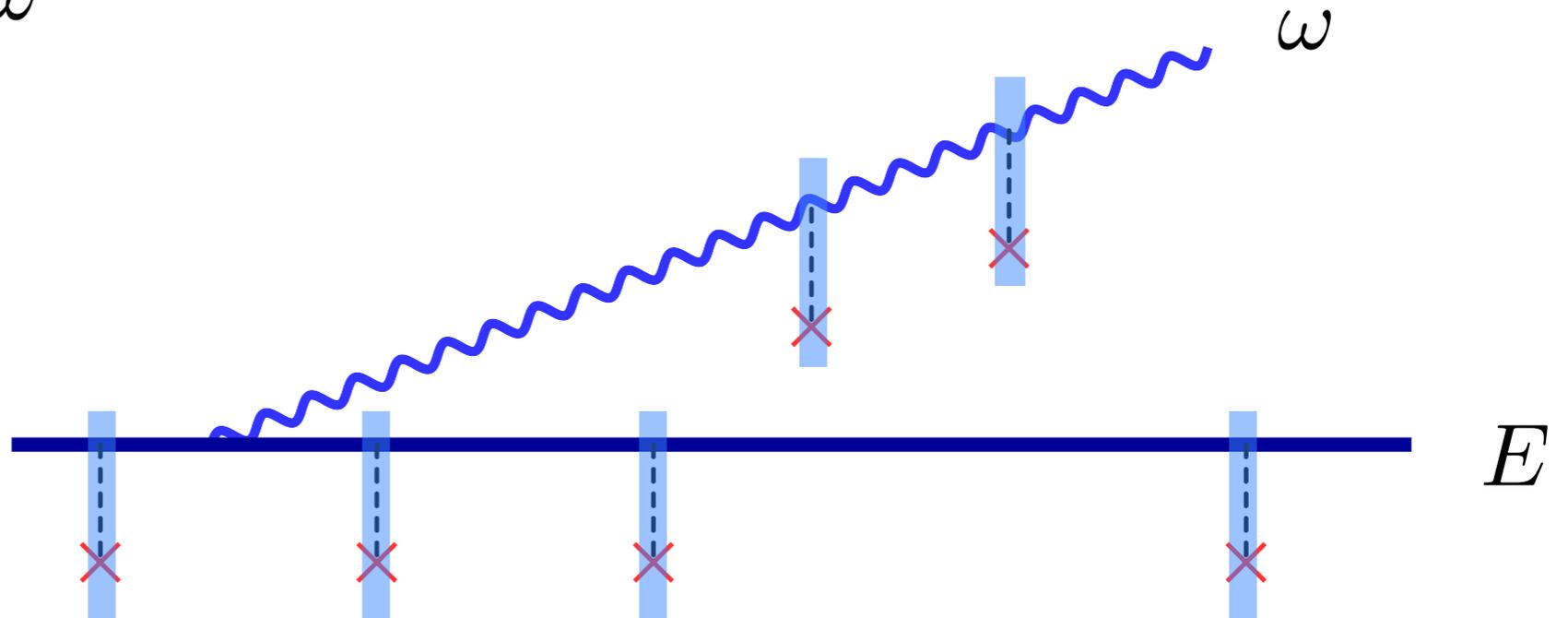
$$\mathcal{P}(\mathbf{k}, L) = \frac{4\pi}{\hat{q}(L) L} \exp \left(-\frac{\mathbf{k}^2}{\hat{q}(L) L} \right)$$

Radiative corrections to Energy loss

What about the radiative corrections to the **medium-induced gluons spectrum** which is also function of the **quenching parameter**?

The BDMPS spectrum (soft gluon radiation $\omega \ll E$)

$$\omega \frac{dN}{d\omega} \sim \bar{\alpha} \sqrt{\frac{\hat{q}}{\omega}} L$$

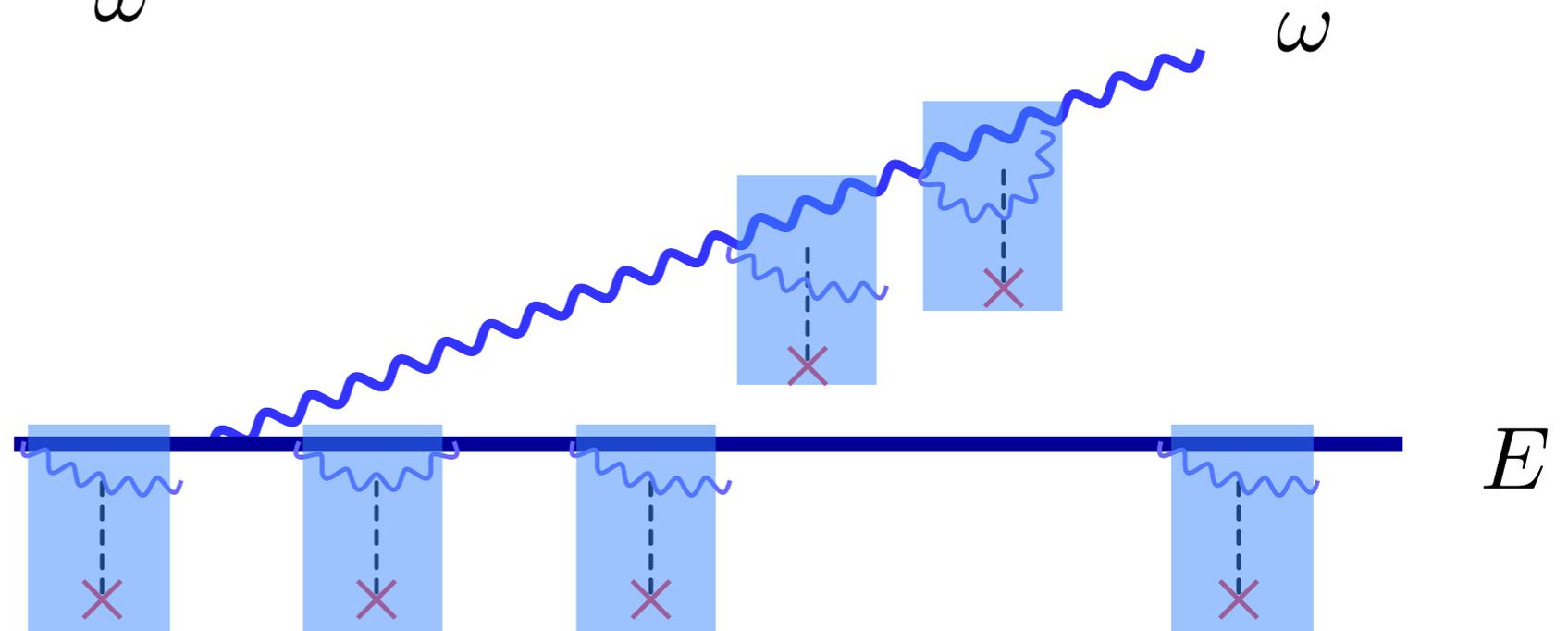


Radiative corrections to Energy loss

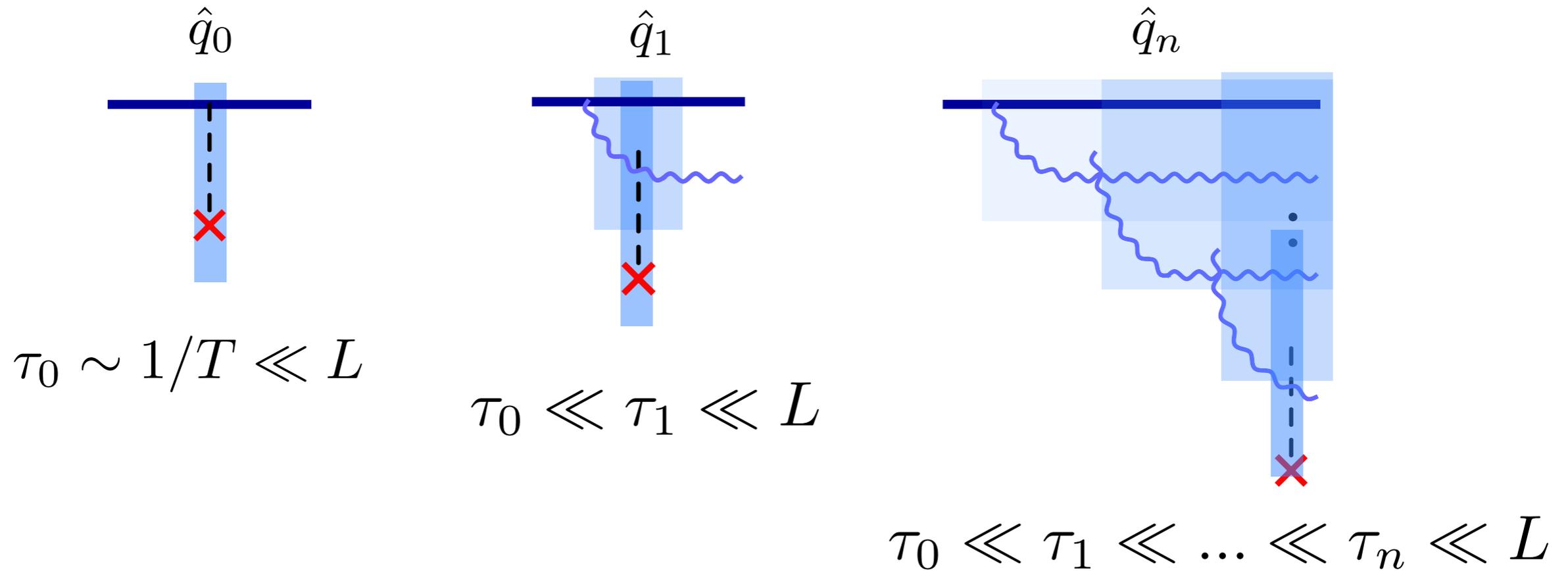
The Double logarithmic divergence can be absorbed in a **redefinition of \hat{q}**

The BDMPS spectrum (soft gluon radiation $\omega \ll E$)

$$\omega \frac{dN}{d\omega} \sim \bar{\alpha} \sqrt{\frac{\hat{q} + \Delta\hat{q}}{\omega}} L$$



The first α_s correction is enhanced by a **double log (DL)** which is resummed with **strong ordering in formation time** (or energy) and **transverse mom.** of **overlapping** successive gluon emissions



Renormalization Group Equation for the quenching parameter

$$\frac{\partial \hat{q}(\tau, Q^2)}{\partial \ln(\tau/\tau_0)} = \int_{\hat{q}\tau}^{Q^2} \bar{\alpha}(\mathbf{q}) \frac{d\mathbf{q}^2}{q^2} \hat{q}(\tau, \mathbf{q}^2) \quad \text{with initial condition} \quad \hat{q}(\tau_0) \equiv \hat{q}_0$$

Universality of the double logs

As a consequence to the renormalization of the quenching parameter, the DL's not only enhance the **pt-broadening** but also the **radiative energy loss** expectation:

For large media (asymptotic behavior)

$$\langle k_{\perp}^2 \rangle \propto L^{1+\gamma}$$

anomalous dimension

$$\Delta E \propto L^{2+\gamma}$$

$$\gamma = \sqrt{\frac{4\alpha_s N_c}{\pi}}$$

To be compared to N=4 SYM (strong-coupling) estimate $\Delta E \sim L^3$

[Gubser et al, Hatta et al, Chesler et Yaffe (2008)]

SUMMARY

- We have shown that to Double-Log accuracy radiative corrections can be reabsorbed in a **renormalization of the jet-quenching parameter** without altering the classical picture of **independent scatterings**
- For large media the renormalized quenching parameter increases compared to the standard perturbative estimate and exhibits an anomalous scaling \Rightarrow **consequences for phenomenological studies that aim to probe the nature of the QGP in HIC**