



Theory of Jet Quenching in Ultra-Relativistic Nuclear Collisions

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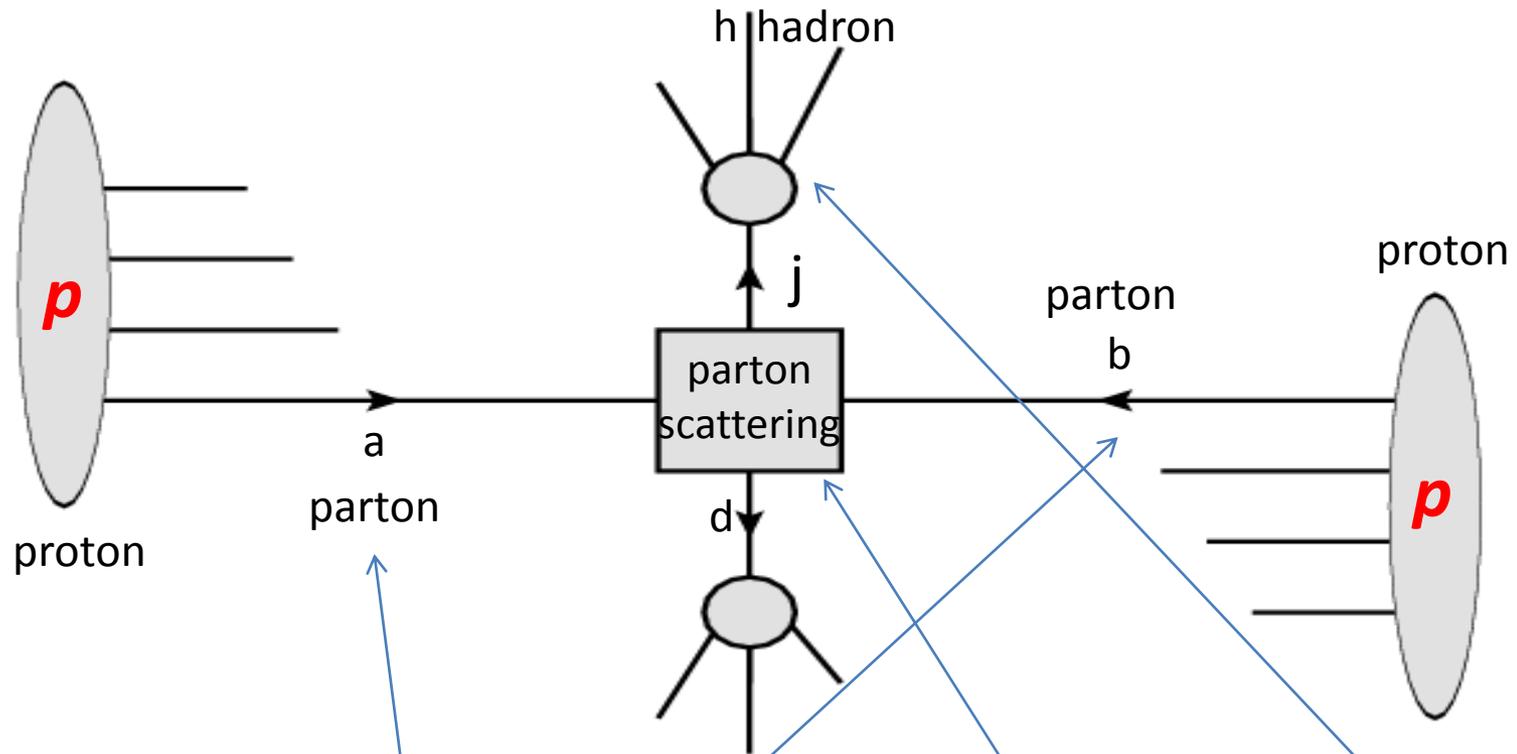
Quark Matter 2014 (May 19-24)
Darmstadt, Germany



Outline

- **Jet energy loss**
 - General picture, formalisms
 - RHIC and the LHC hadron R_{AA} (JET collaboration, etc.)
 - NLO (renormalization of q^{hat})
- **Full jet**
 - Full jet evolution and energy loss, multiple gluon emission
 - Jet substructure, Mont-Carlo models
- **Medium response**
 - Jet energy loss/deposition/redistribution
- **Lattice & AdS/CFT**
- **Heavy flavors** (Andrea Beraudo, Friday morning)
- **Summary**

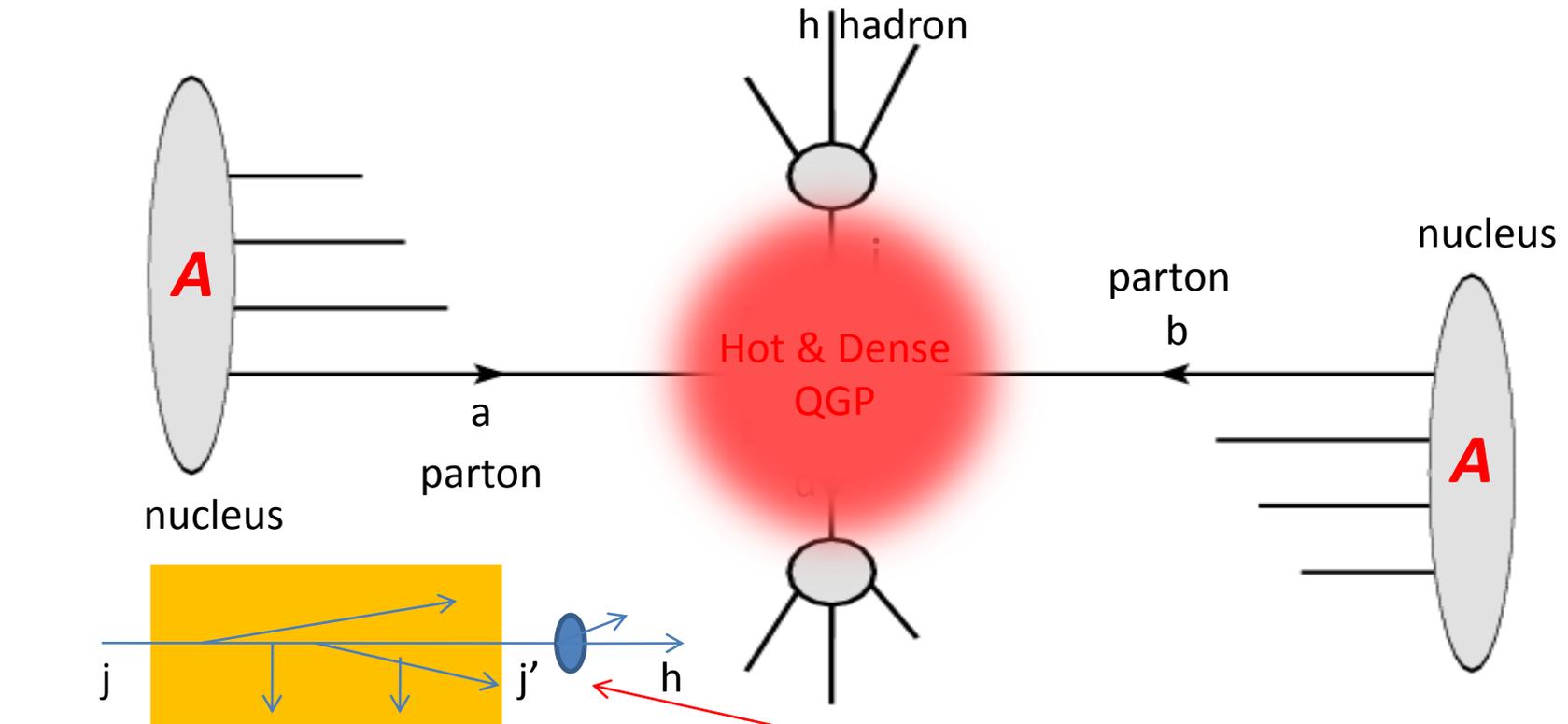
General framework of jet quenching study



$$d\sigma_h = \sum_{abjd} f_{a/A} \otimes f_{b/B} \otimes d\sigma_{ab \rightarrow jd} \otimes D_{h/j}$$

pQCD factorization: Large- p_T processes may be factorized into **long-distance** pieces in terms of **PDF & FF**, and **short-distance** parts describing **hard interactions** of partons.

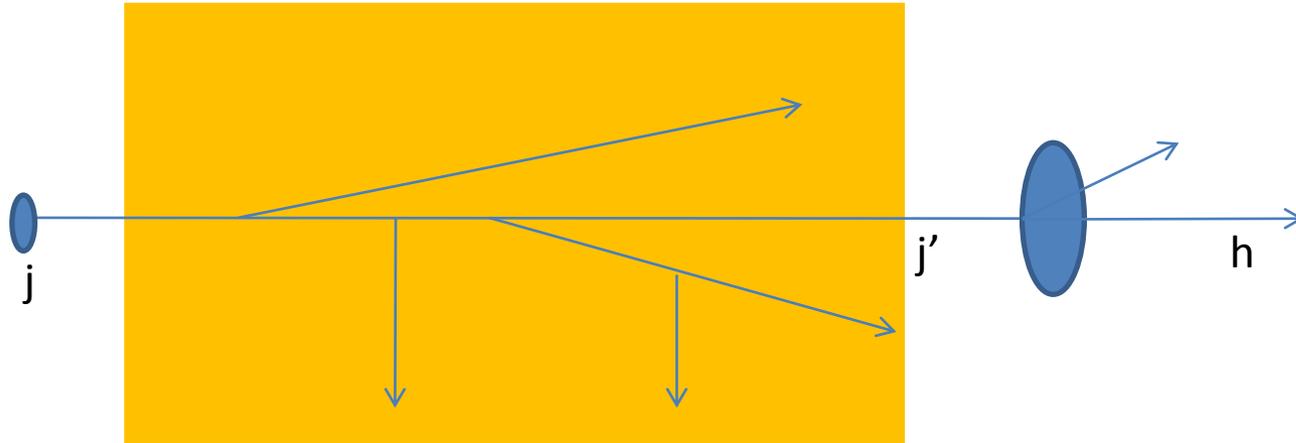
General framework of jet quenching study



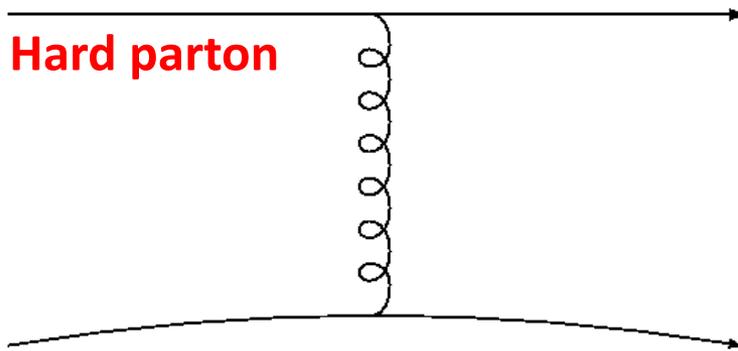
$$d\tilde{\sigma}_h = \sum_{abjd} f_{a/A} \otimes f_{b/B} \otimes d\sigma_{ab \rightarrow jd} \otimes \tilde{D}_{h/j}$$

$$d\tilde{\sigma}_h = \sum_{abjj'd} f_a \otimes f_b \otimes d\sigma_{ab \rightarrow jd} \otimes P_{j \rightarrow j'} \otimes D_{h/j'}$$

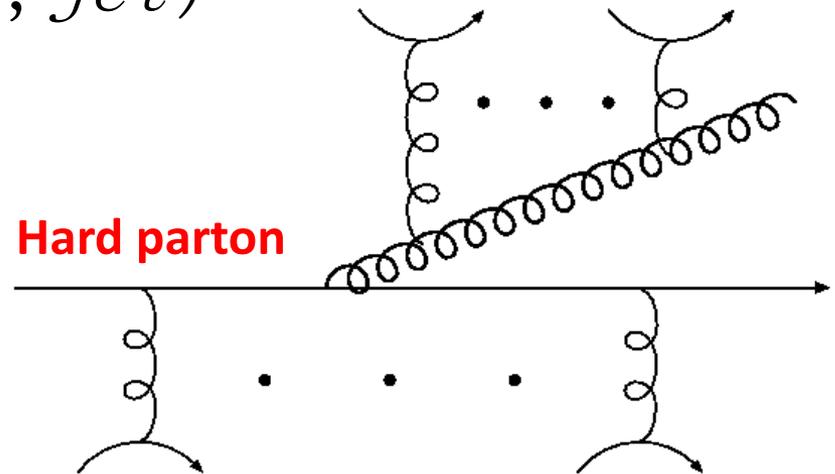
Jet evolution and energy loss in QGP



$$P_{j \rightarrow j'}(\text{medium, jet})$$



Elastic (collisional)



Inelastic (radiative)

Rad. E-loss

- **Single gluon emission**

- **Multiple soft scatterings (BDMPS-Z, ASW, AMY)**
- **Few hard scatterings (DGLV, HT)**
- **Recent developments:**
 - **AMY: finite L** (Caron-Huot, Gale 2010)
 - **GLV: finite dynamical medium** (Djordjevic, Heinz, 2008)
 - **DGLV: non-zero magnetic mass** (Djordjevic, Djordjevic, 2012)
 - **Higher Twist (HT): multiple scatterings** (Majumder 2012)

BDMPS-Z: Baier-Dokshitzer-Mueller-Peigne-Schiff-Zakharov
ASW: Amesto-Salgado-Wiedemann
AMY: Arnold-Moore-Yaffe
DGLV: Djordjevic-Gyulassy-Levai-Vitev
HT: Wang-Guo-Majumder

$$\frac{dN_g}{dxdk_{\perp}^2 dt}(T, E, \dots) = ?$$

- **Mutiple gluon emission**

- **Poisson convolution (BDMPS/ASW/DGLV)**

$$P(\Delta E) = \sum_{n=0}^{\infty} \frac{e^{-\langle N_g \rangle}}{n!} \left[\prod_{i=1}^n \int d\omega \frac{dI(\omega)}{d\omega} \right] \delta \left(\Delta E - \sum_{i=1}^n \omega_i \right)$$

- **Rate equation (AMY)**

$$\frac{df(p, t)}{dt} = \int dk f(p + k, t) \frac{d\Gamma(p + k, k)}{dkdt} - \int dk f(p, t) \frac{d\Gamma(p, k)}{dkdt}$$

- **DGLAP-like evolution equation (HT)**

$$\frac{\partial \tilde{D}(z, Q^2)}{\partial \ln Q^2} = \frac{\alpha_s}{2\pi} \int \frac{dy}{y} P(y) \int d\zeta^- K(\zeta^-, q^-, y, Q^2) \tilde{D}\left(\frac{z}{y}, Q^2\right)$$

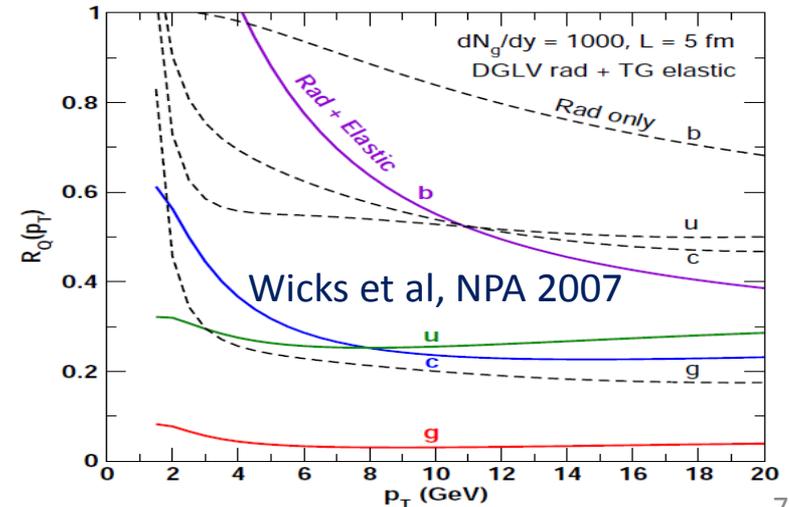
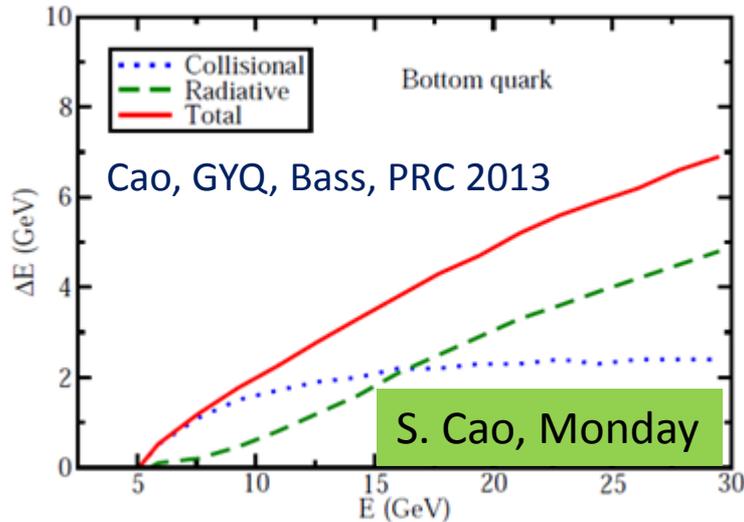
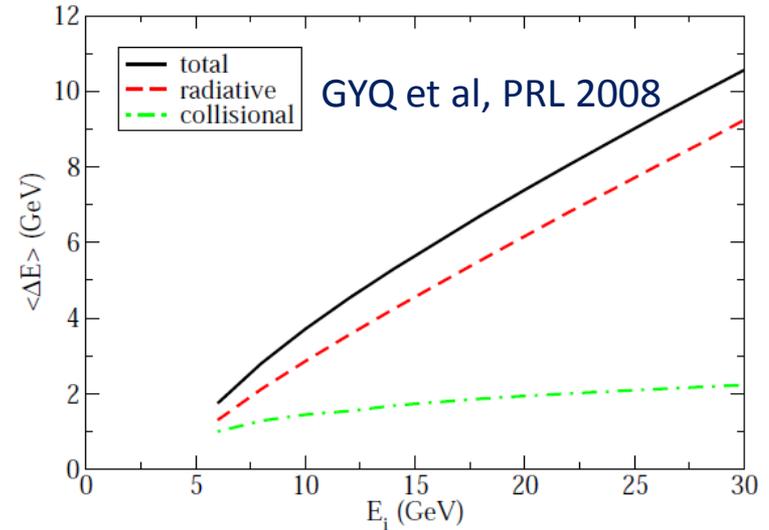
Coll. E-loss

- **First studied by Bjorken:**

- Bjorken 1982; Bratten, Thoma 1991; Thoma, Gyulassy, 1991; Mustafa, Thoma 2005; Peigne, Peshier, 2006; Djordjevic (GLV), 2006; Wicks et al (DGLV), 2007; GYQ et al (AMY), 2008...

- **Main findings:**

- dE/E small compared to rad. for large E
- Non-negligible in R_{AA} calculation (especially for heavy flavor)
- Important when studying full jet energy loss and medium response (see later)



Jet quenching phenomenology

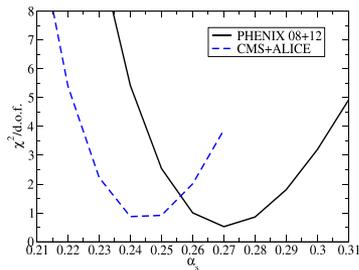
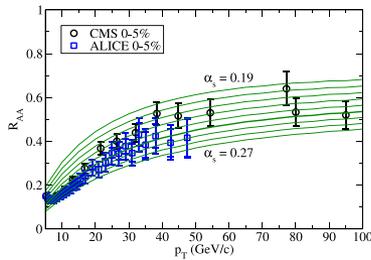
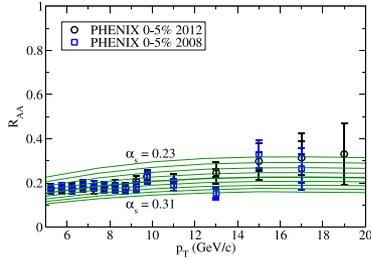
- Achieve better understanding of jet-medium interaction and extract various transport properties of QGP
- **Perform systematic study of jet quenching observables and compare to existing experimental data**



- **Build a general framework for numerical implementation of different jet quenching approaches**
 - Realistic medium evolution constrained by bulk observables (spectra, flow)
 - Hadronization of both jets and hydro (fragmentation and recombination)
- **Deliver a Monte-Carlo package**
 - Hydrodynamics + jet transport /evolution + hadronization
- **A first step: viscous hydro + semi-analytical jet quenching calculations from a few groups** (McGill-AMY, Martini-AMY, HT-BW, HT-M, GLV-CUJET, may include more in the future)

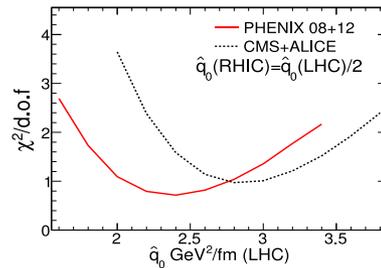
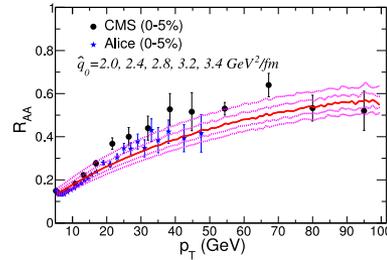
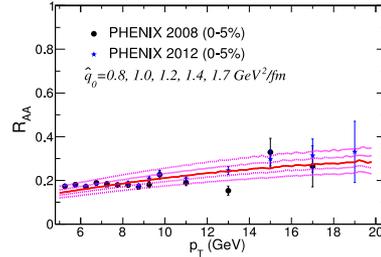
Jet quenching @ RHIC & LHC

McGill-AMY



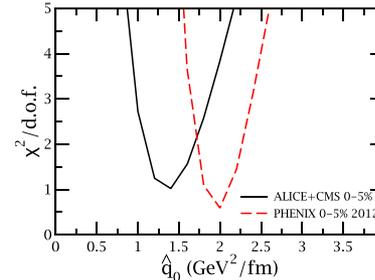
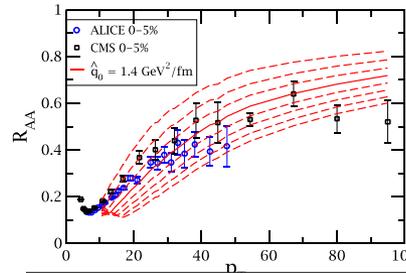
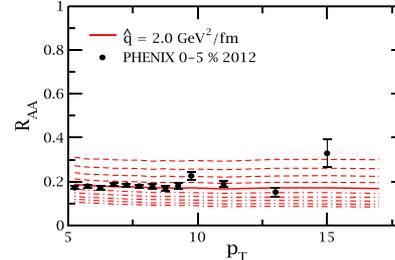
GYQ, et al, PRL 2008

HT-BW



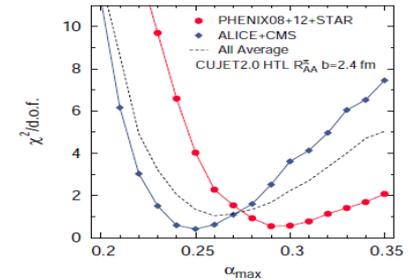
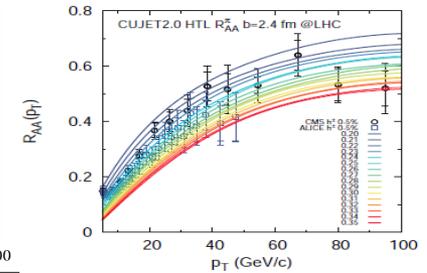
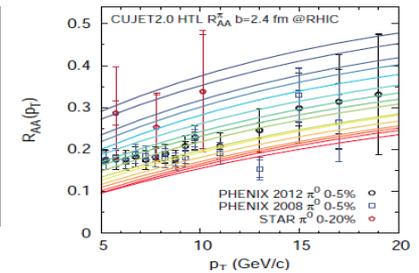
Chen, Hirano, Wang,
Wang, Zhang, PRC 2011

HT-M



Majumder, Chun,
PRL 2012

CUJET

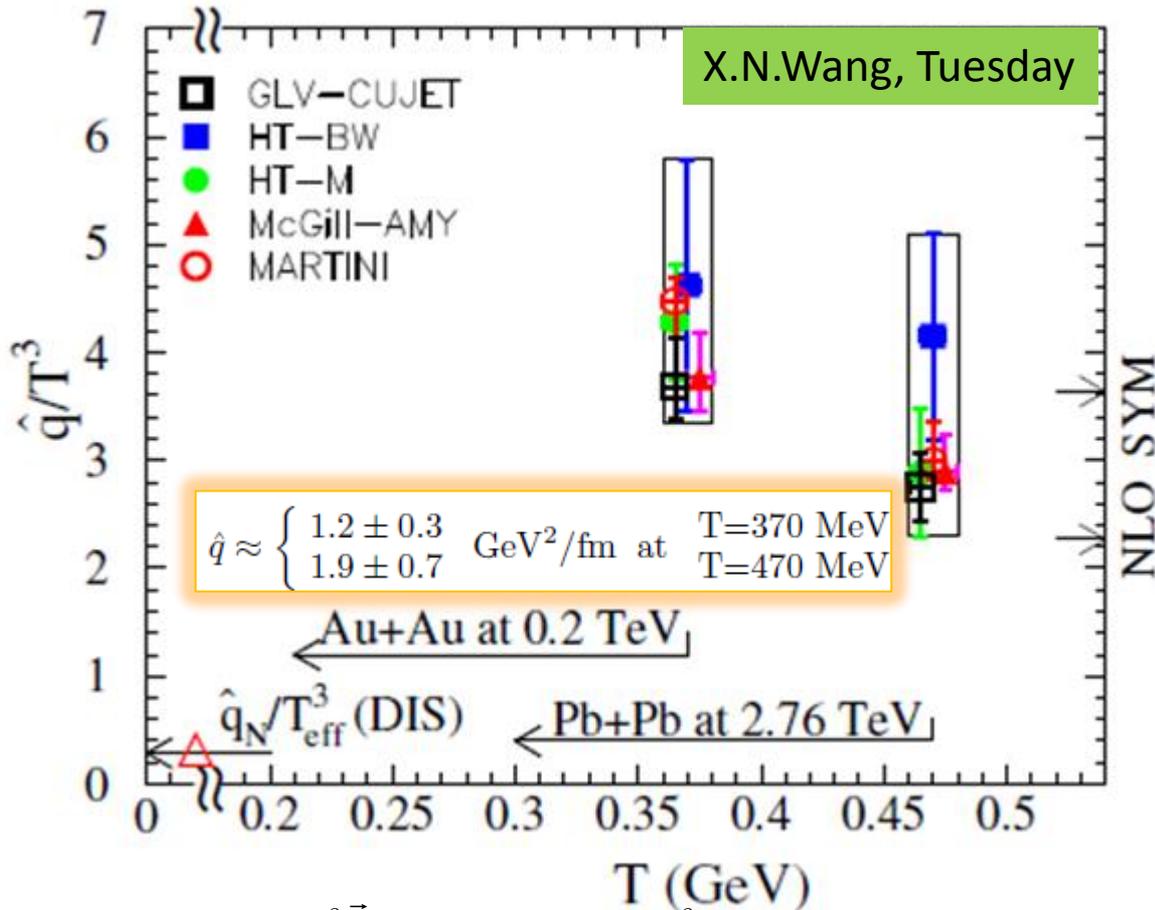


Xu, Buzzatti, Gyulassy,
arXiv: 1402.2956



, arXiv:1312.5003 [nucl-th]

Extracting jet quenching parameter



McGill-AMY:

GYQ, Ruppert, Gale, Jeon, Moore, Mustafa, PRL 2008

HT-BW:

Chen, Hirano, Wang, Wang, Zhang, PRC 2011

HT-M:

Majumder, Chun, PRL 2012

GLV-CUJET:

Xu, Buzzatti, Gyulassy, arXiv: 1402.2956

MARTINI-AMY:

Schenke, Gale, Jeon, PRC 2009

NLO SYM:

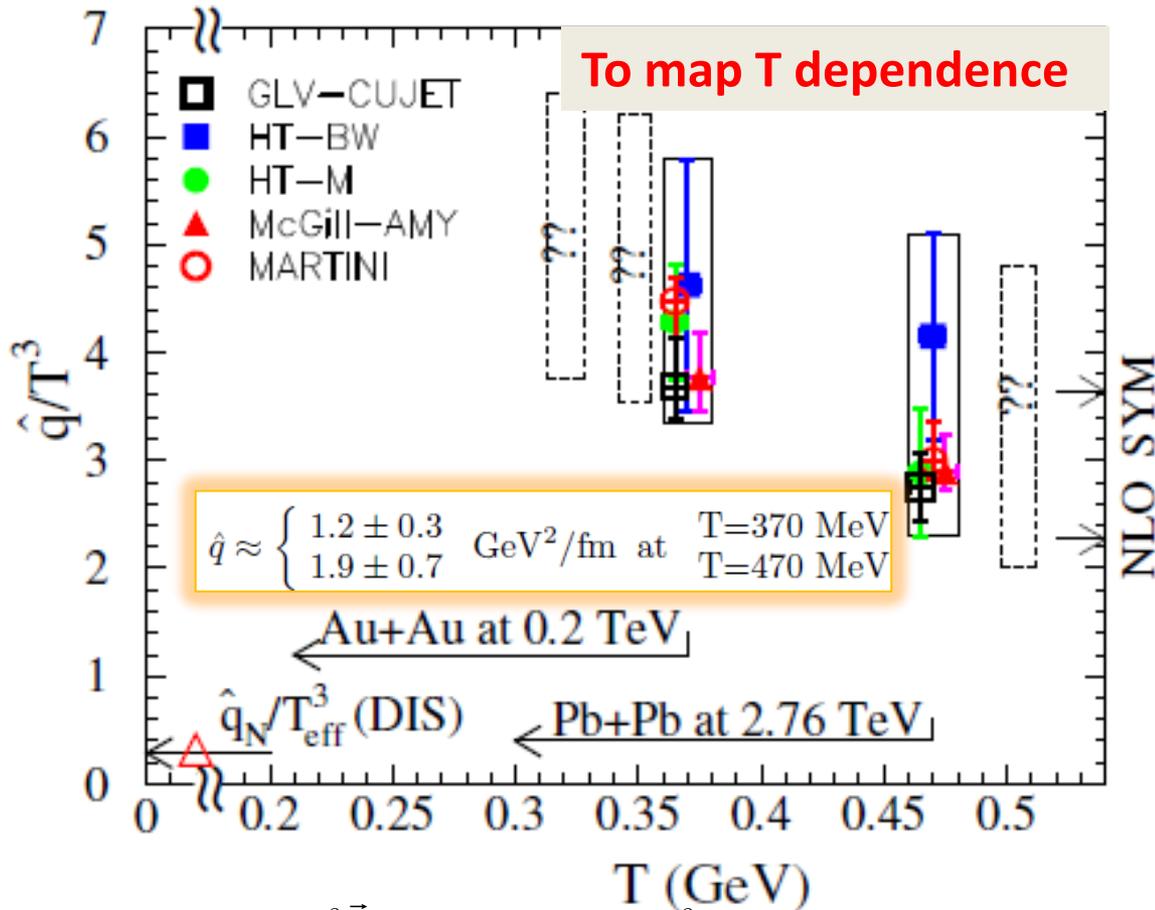
Zhang, Hou, Ren, JHEP 2013

$$\hat{q} = \frac{1}{L} \int \frac{d^2 \vec{k}_\perp}{(2\pi)^2} k_\perp^2 P(\vec{k}_\perp, L) \approx \frac{4\pi^2 \alpha_s C_R}{N_c^2 - 1} \int dy^- \langle F^{\mu+}(0) F_\mu^+(y^-) \rangle$$



, arXiv:1312.5003 [nucl-th]

Extracting jet quenching parameter



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GYQ, Ruppert, Gale, Jeon, Moore, Mustafa, PRL 2008

HT-BW:

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, arXiv:1312.5003 [nucl-th]

Renormalization of q^{hat}

- **Radiative correction to transverse momentum broadening $\langle p_{\perp}^2 \rangle$** (Wu, JHEP 2011; Liou, Mueller, Wu, NPA 2013)

$$\langle p_{\perp}^2 \rangle_{rad} = \hat{q}_0 L \left[C_2 \frac{\alpha_s N_c}{\pi} \ln^2 \left(\frac{L}{\tau_0} \right) + C_1 \frac{\alpha_s N_c}{\pi} \ln \left(\frac{L}{\tau_0} \right) + C_0 \right]$$

$$\tau_0 \approx 1 / T \ll L$$

- **The double-logarithmic corrections may be absorbed into a redefinition of jet quenching parameter q^{hat}** (Iancu, arXiv:1403.1996; Blaizot, Mehtar-Tani, arXiv:1403.2323)

$$\hat{q}(\tau) = \hat{q}_0 \left[1 + \frac{\bar{\alpha}}{2} \ln \left(\frac{\tau}{\tau_0} \right) \right]$$

Y. Mehtar-Tani, Wednesday

$$\frac{\partial \hat{q}(\tau, Q^2)}{\partial \ln \tau} = \int_{\hat{q}\tau}^{Q^2} \frac{dq^2}{q^2} \bar{\alpha} \hat{q}(\tau, q^2)$$

- **For large media, anomalous length dependence of q^{hat} and mean energy loss**

$$\hat{q}(L) \propto L^{\gamma}, \quad \langle p_{\perp}^2 \rangle \propto \hat{q}_0 L^{1+\gamma}, \quad \langle \Delta E \rangle \propto \hat{q}_0 L^{2+\gamma}$$

$$\gamma = 2\sqrt{\bar{\alpha}} = 2\sqrt{\alpha_s N_c / \pi}$$

Renormalization of q^{hat}

- Calculate NLO QCD corrections to transverse momentum broadening in hadron production in SIDIS

$$\frac{d\langle k_{\perp}^2 \sigma \rangle_{NLO}}{dz} = \sigma_0 T_{qg}(x, 0, 0, \mu_f^2) \otimes H_{NLO}(x, x_B, Q^2, \mu_f^2) \otimes D(z, \mu_f^2)$$

- Collinear divergence is factorized into the redefinition of PDF & twist-4 quark-gluon correlation function

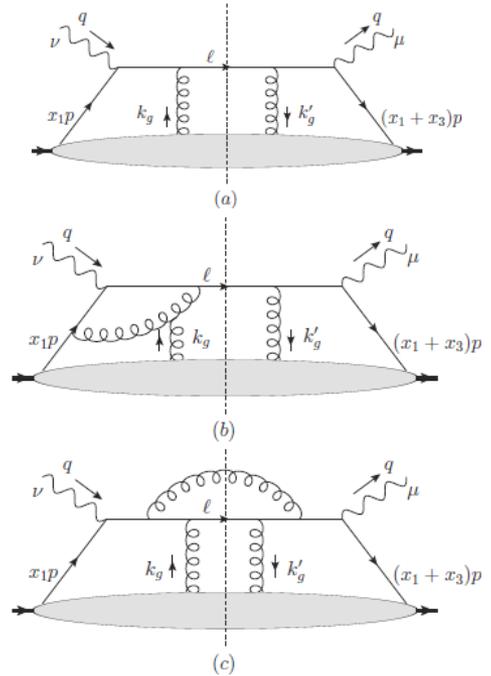
$$\frac{\partial}{\partial \ln \mu_f^2} T_{qg}(x_B, 0, 0, \mu_f^2) = \frac{\alpha_s}{2\pi} \int_{x_B}^1 \frac{dx}{x} \left[\mathcal{P}_{qg \rightarrow qg} \otimes T_{qg} + P_{qg}(\hat{x}) T_{gg}(x, 0, 0, \mu_f^2) \right]$$

- Neglecting the momentum and spatial correlations of two nucleons

$$T_{qg}(x_B, 0, 0, \mu_f^2) \approx \frac{N_c}{4\pi^2 \alpha_s} f_{q/A}(x_B, \mu_f^2) \int dy^- \hat{q}(\mu_f^2, y^-)$$

- The scale dependence of q^{hat}

$$\frac{\partial \hat{q}(\mu^2)}{\partial \ln \mu^2} = \frac{\alpha_s}{2\pi} C_A \ln\left(\frac{1}{x_B}\right) \hat{q}(\mu^2) \quad \hat{q}(\mu^2) = \hat{q}(\mu_0^2) \exp\left[\frac{\alpha_s}{2\pi} C_A \ln\left(\frac{1}{x_B}\right) \ln\left(\frac{\mu^2}{\mu_0^2}\right)\right]$$



H. Xing, Wednesday

Some other studies

- **Systematic comparison to RHIC & the LHC data**

- **CUJET2.0:** (Xu, Buzzatti, Gyulassy, arXiv: 1402.2956)
- **DGLV:** (Djordjevic, Djordjevic , arXiv:1307.4098)
- **dE/dx model:** (Betz, Gyulassy, arXiv:1404.6378)

$$\frac{dE}{dx} = \frac{dP}{d\tau}(\vec{x}_0, \phi, \tau) = -\kappa(T)P^a(\tau)\tau^z T^c \zeta_q$$

M. Djordjevic, Monday

B. Betz, Tuesday

T. Renk, Wednesday

L. Apolinario, Monday

- **Converging on initial states using bulk, jets & EM probes**

- **Improve radiative E-loss formalism beyond eikonal approximation (finite energy)**

(Apolinario, Armesto, Milhanoa, Salgadoa, arXiv:1404.7079)

- **Non-eikonal effects (large angle rescattering) largely absent above (10–15) GeV parton kinematics** (Abir, PRD 2013)

- **Jet quenching (R_{AA}) from SCET** (Kang, Lashof-Regas, Ovanesyan, Saad, Vitev)

- **Higer order 1->3 splitting in SCET** (Fickinger, Ovanesyan, Vitev, JHEP 2013)

- **Radiative E-loss due to soft rescatterings proportional to E** (Peigne, Arleo, Kolevatov, arXiv:1402.1671, Liou, Mueller, PRD 2014)

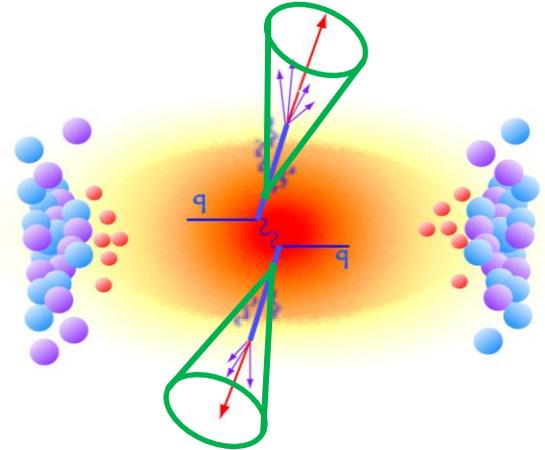
- **Viscosity effect on collisional parton energy loss** (Jiang, Hou, Li, arXiv:1405.0083, Elias, Peralta-Ramos, Calzetta. arXiv:1404.7790)

- **q^{hat} enhancement at T_c** (Li, Liao, Huang, arXiv:1401.2035, Renk, arXiv:1402.5798)

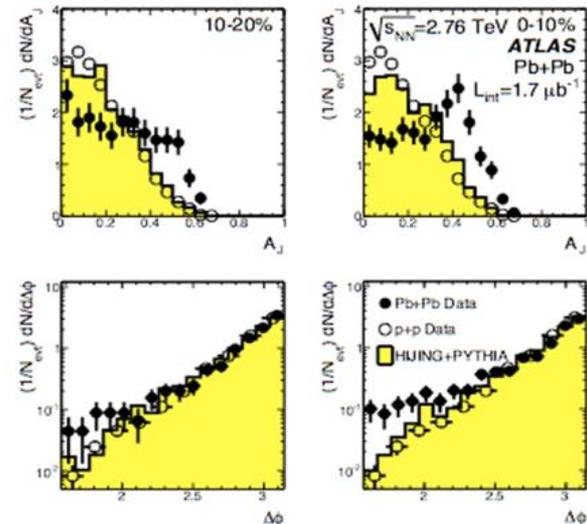
- ...

Full jet

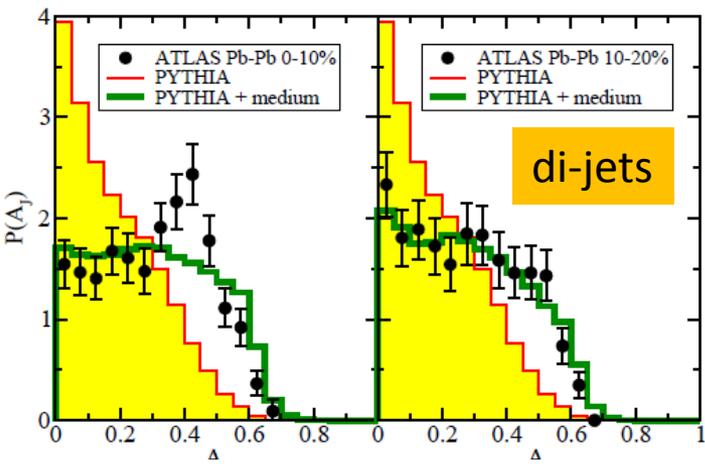
- **Recombining hadron/parton fragments, hoping to get the original parton energy/momentum**
- **Full jets might be more discriminative with sub-leading fragments**
- **Significant contribution from background in AA collisions; need reliable tools to disentangle jets from background**
- **LHC first full jet measurements in AA collisions showed strong modification of dijet energy imbalance distribution, while angular distribution largely unchanged**



$$R = \sqrt{(\phi - \phi_J)^2 + (\eta - \eta_J)^2}$$

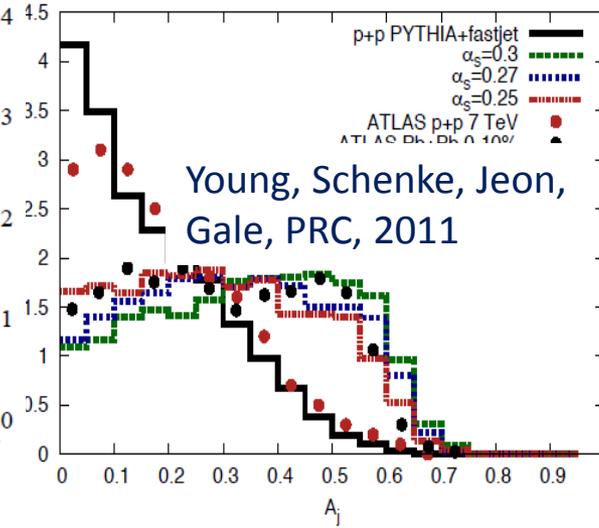


Energy imbalance of di-jets and γ -jets

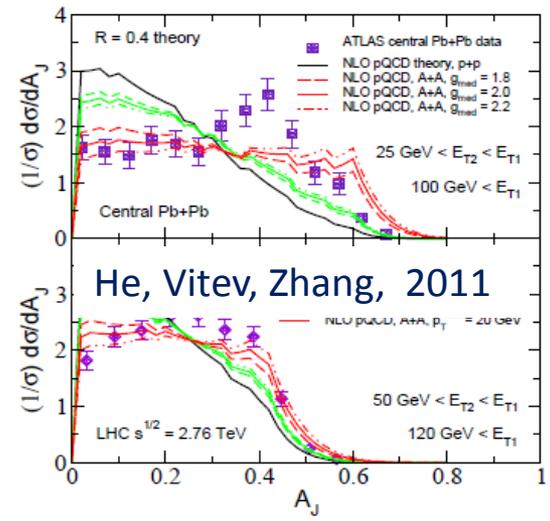


di-jets

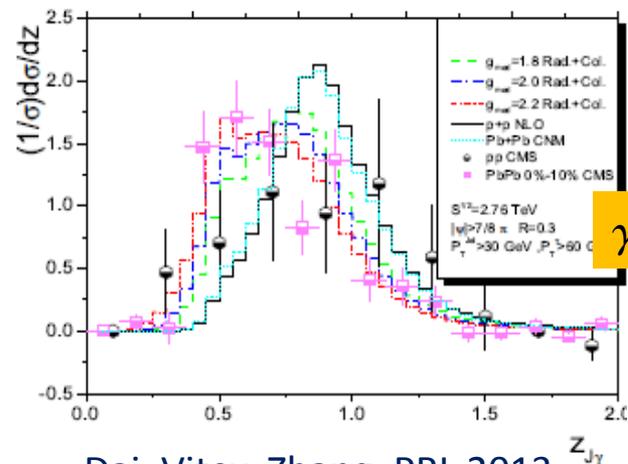
GYQ, Muller, PRL, 2011



Young, Schenke, Jeon, Gale, PRC, 2011

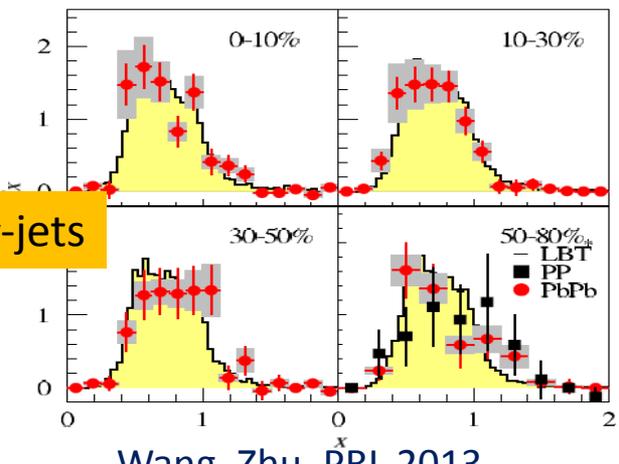


He, Vitev, Zhang, 2011



γ -jets

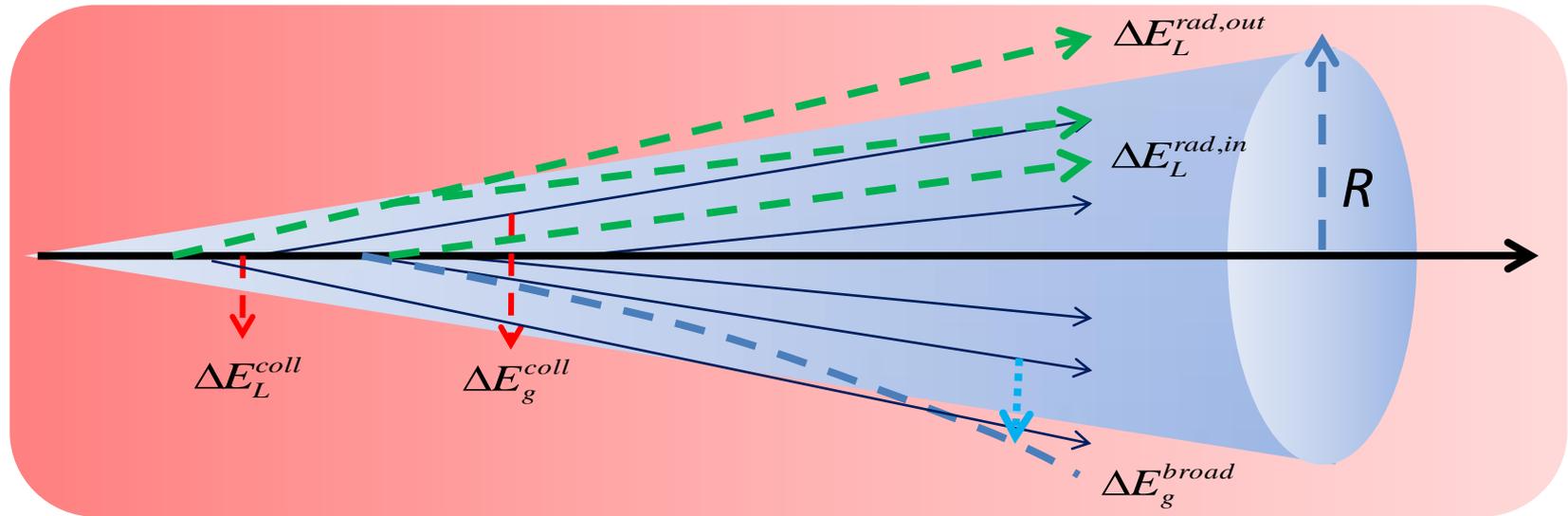
Dai, Vitev, Zhang, PRL 2013



Wang, Zhu, PRL 2013

The distribution shifting towards larger A_J (smaller x) can be explained by energy loss
 More quenching (energy loss) in more central collisions

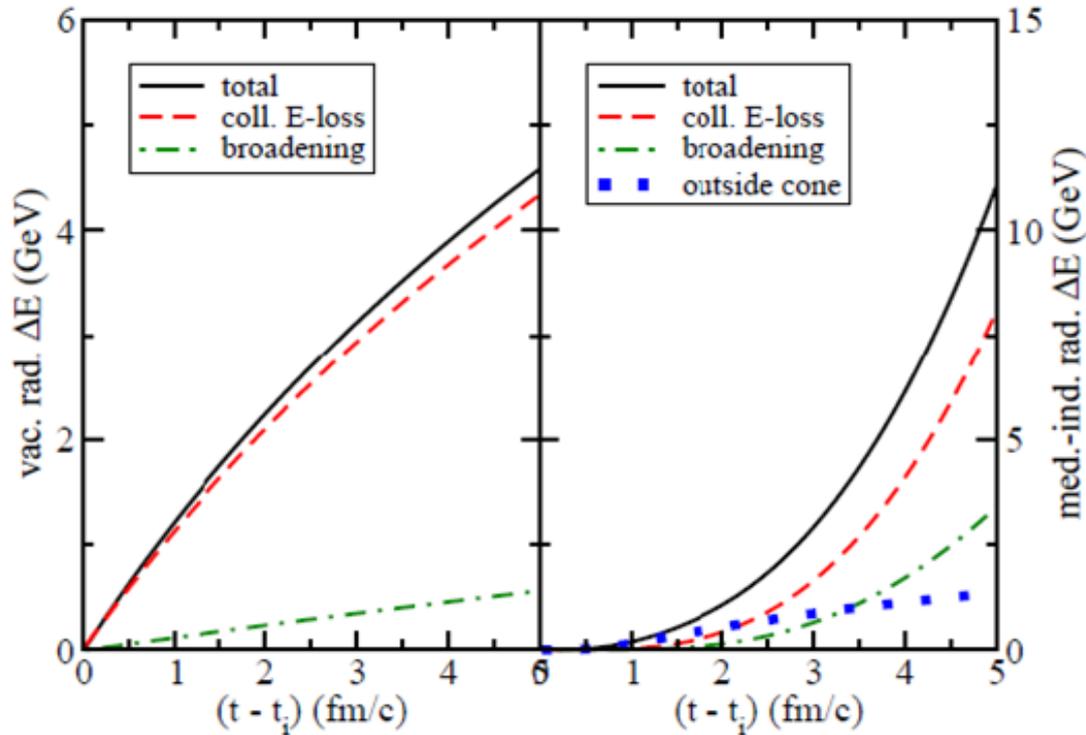
Jet shower evolution in medium



$$\frac{df_g(\omega, k_{\perp}^2, t)}{dt} = \hat{e} \frac{\partial f_g}{\partial \omega} + \frac{1}{4} \hat{q} \nabla_{k_{\perp}}^2 f_g + \frac{dN_g^{med}}{d\omega dk_{\perp}^2 dt}$$

$$\begin{aligned} E_{\text{tot}} &= E_{\text{in}} + E_{\text{lost}} \\ &= E_{\text{in}} + E_{\text{out}}(\text{radiation}) + E_{\text{out}}(\text{broadening}) + E_{\text{th}}(\text{collision}) \end{aligned}$$

Full jet energy loss



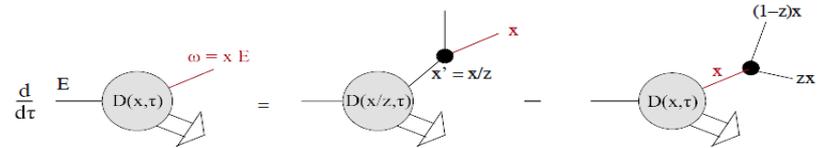
Collisional energy loss of radiated partons or medium absorption of soft radiations gives large contribution

Jet collimation mechanism (Casalderrey-Solana, Milhano, Wiedemann, JPG 2011)

$$\begin{aligned}
 E_{\text{tot}} &= E_{\text{in}} + E_{\text{lost}} \\
 &= E_{\text{in}} + E_{\text{out}}(\text{radiation}) + E_{\text{out}}(\text{broadening}) + E_{\text{th}}(\text{collision})
 \end{aligned}$$

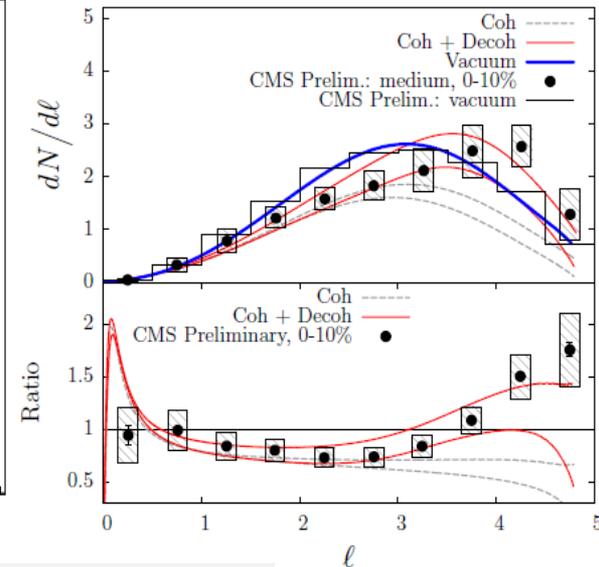
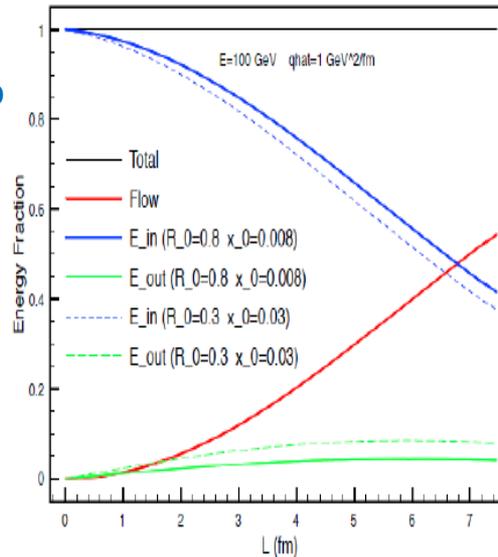
Energy flows to soft gluons

- **Color coherence is rapidly lost in medium via re-scattering, opening up the phase space for soft (large angle) emissions** (Armesto, Ma, Mehtar-Tani, Salgado, Tywoniuk, Iancu, Casalderrey-Solana, Blaizot, Dominguez, 2011, 2012)
- **Use probability description & solve rate equation to study multiple gluon emissions** (Blaizot, Iancu, Mehtar-Tani, PRL 2013; Tywoniuk, Mehtar-Tani, arXiv: 1401.8293)



K. Tywoniuk, Wednesday

- **Jet energy is rapidly degraded into many soft gluons carrying $O(T)$ energy**
- **Three different phase spaces for radiated gluons, separated by two scales x_0 & x_{th}**
- **Color decoherence manifests in the excess of soft particles**

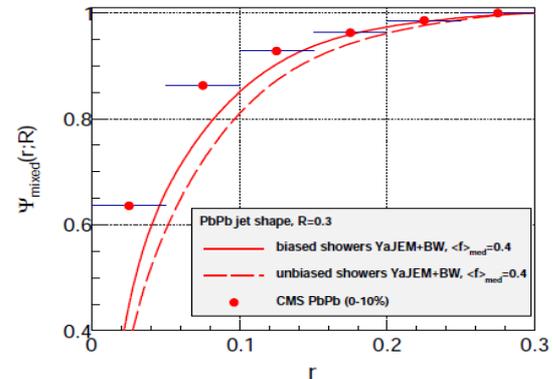
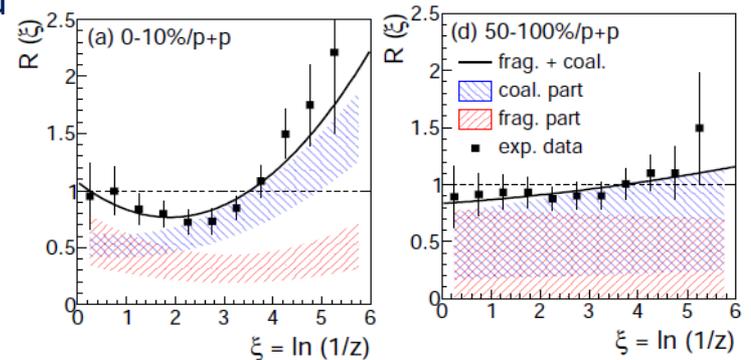
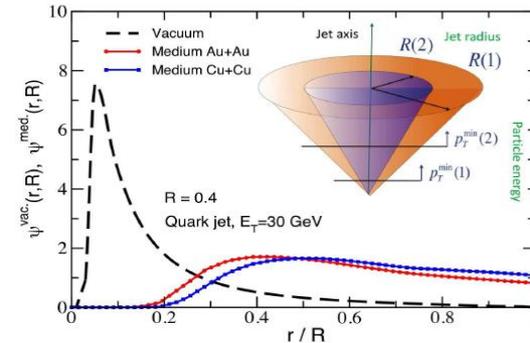


$$E_{tot} = E_{in}(x > x_0) + E_{out}(x_{th} < x < x_0) + E_{flow}(x < x_{th})$$

Jet substructure

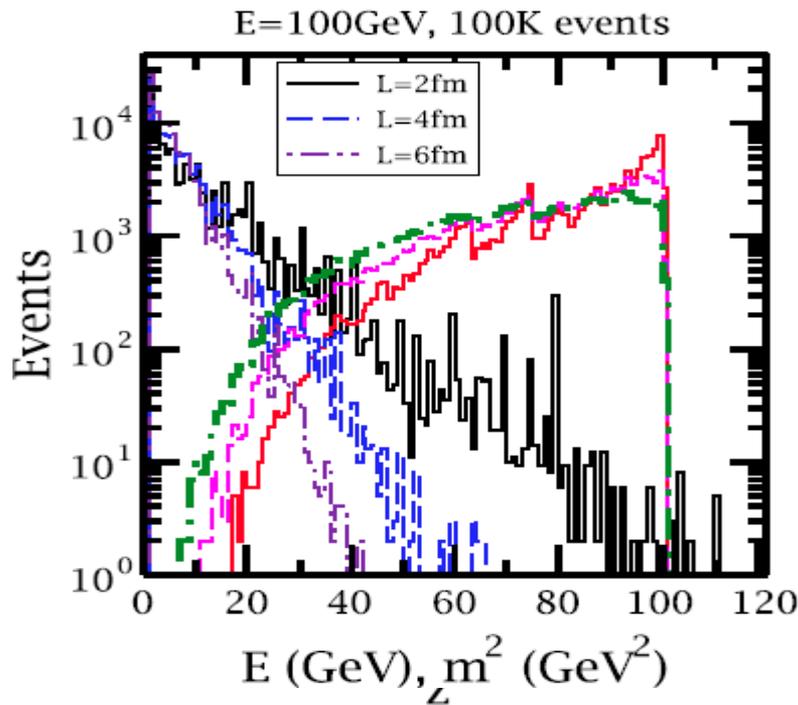
- **Broadening of jets due to medium-induced radiation** (Vitev, Zhang, Wicks, JHEP 2008, Vitev, Zhang, PRL 2010)
- **Recoiled partons (medium response) affect the jet energy loss and internal structure** (Wang, Zhu PRL 2013)
- **Competition between fragmentation and coalescence in medium-modification jet fragmentation profiles** (Ma, PRC 2013)
- **Unbiased jets and biased jets are different; data agree better with biased jets ($E_{\min} > 1\text{GeV}$) => important to include all experimental jet finding conditions** (Ramos, Renk, arXiv:1401.5283)

T. Luo, Wednesday



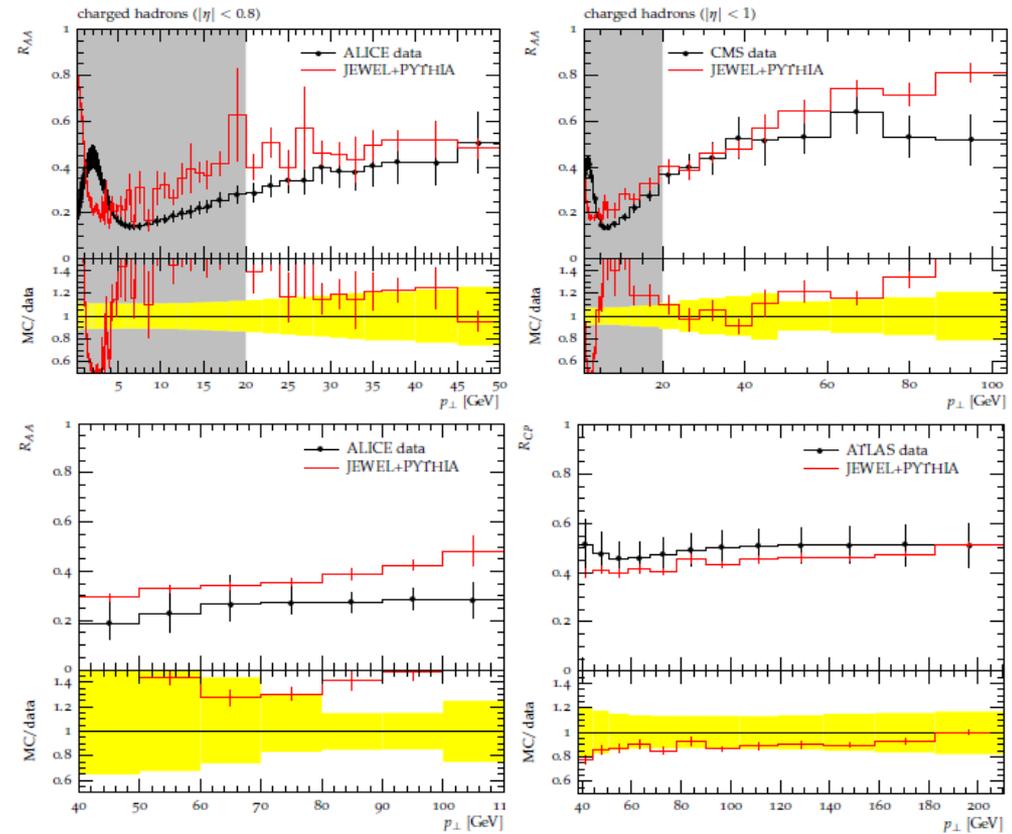
Monte-Carlo models

HIJING, Q-PYTHIA/Q-HERWIG, MARTINI, YaJEM, MATTER, JEWEL, BAMPS, AMPT, LBT, VNI/BMS ...



MATTER: in-medium jet shower MC event generator based on HT E-loss model
(Majumder, PRC 2013)

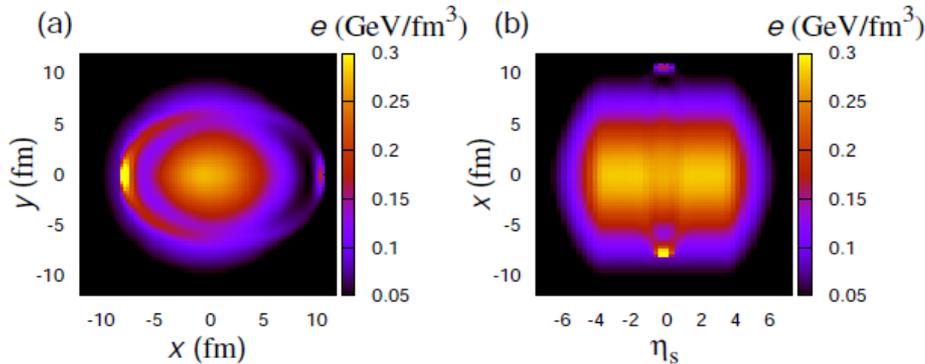
A. Majumder, Wednesday



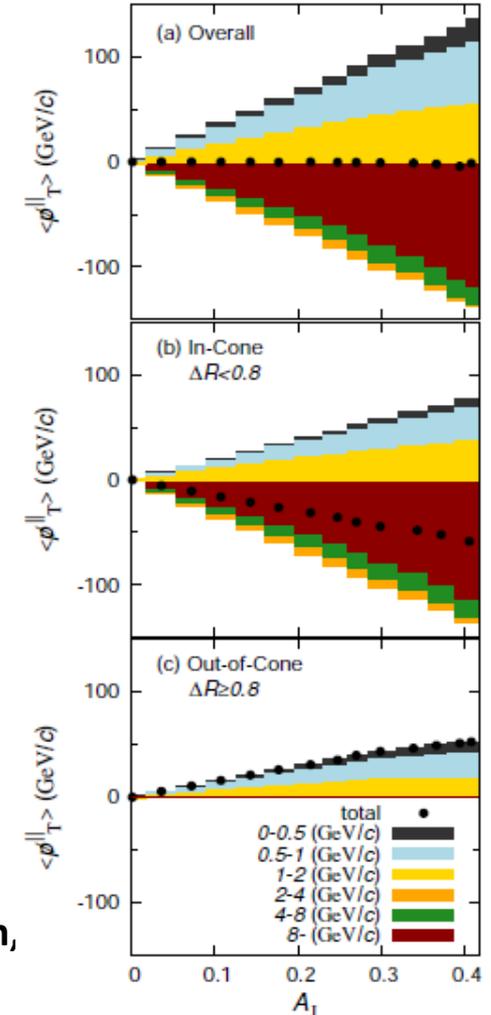
release of the **JEWEL 2.0.0 code** (Zapp, EPJC 2014)
YaJEM available online (T. Renk)

Medium response to lost energy

- Hard partons lose energy in medium; some of lost energy is deposited into medium and makes medium excitations

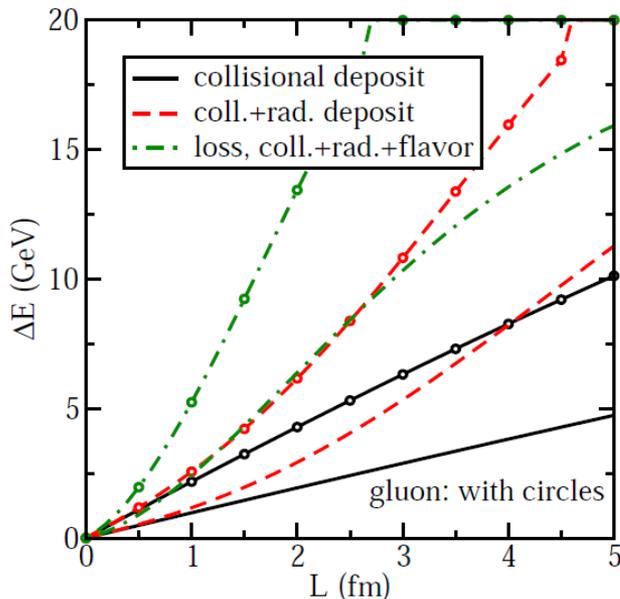
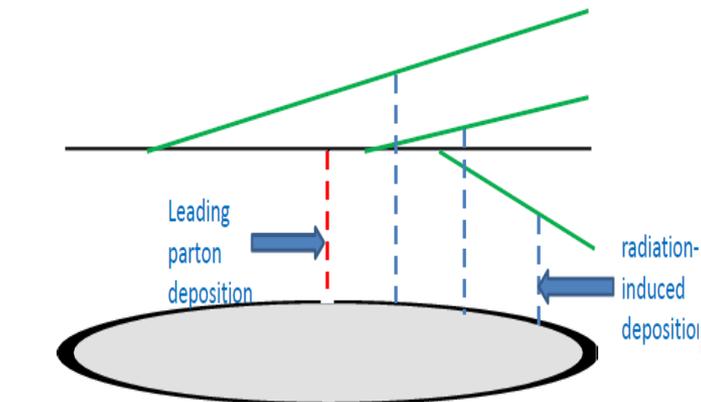


- Simulate medium response using (3+1)-D ideal hydrodynamics
- Calculate the redistribution of deposited energy through hydrodynamic evolution (Tachibana, Hirano, arXiv:1402.6469)
- Effect of deposited energy on direct, elliptic, triangular flow (Andrade, Noronha, Denicol, arXiv:1403.1789)
- SIMPLE energy deposition profiles (back-to-back 2-parton system, not 2 full jets)

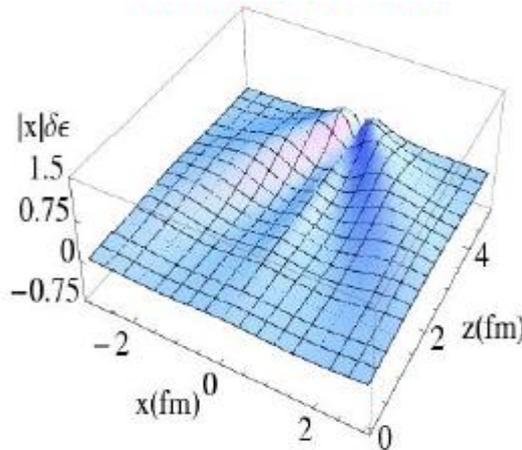


Full jet energy deposition & medium response

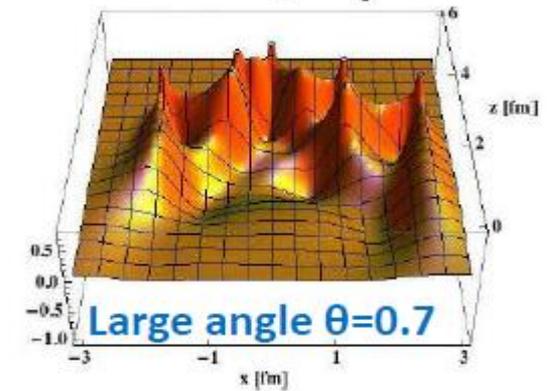
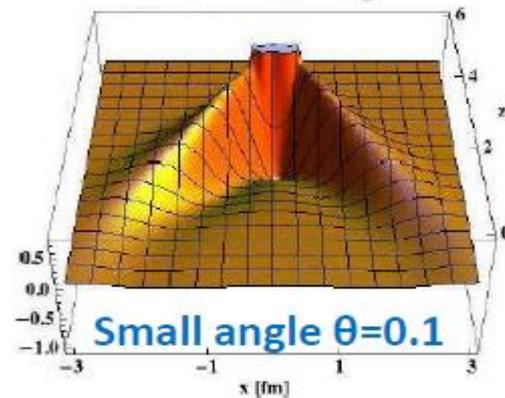
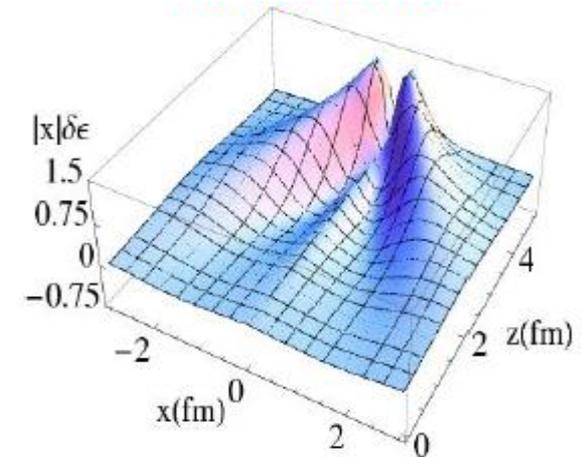
Energy deposition & medium response for jet showers are different from single partons



Single parton

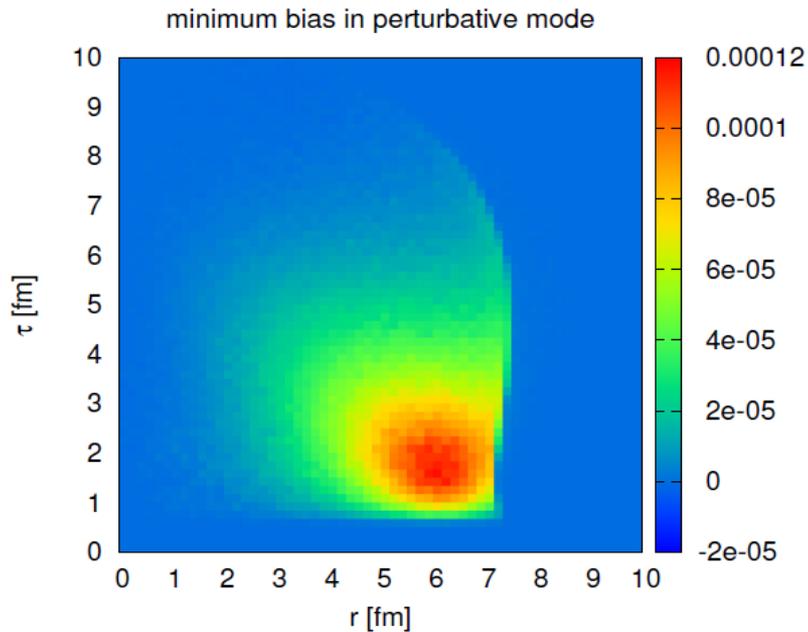


Parton shower



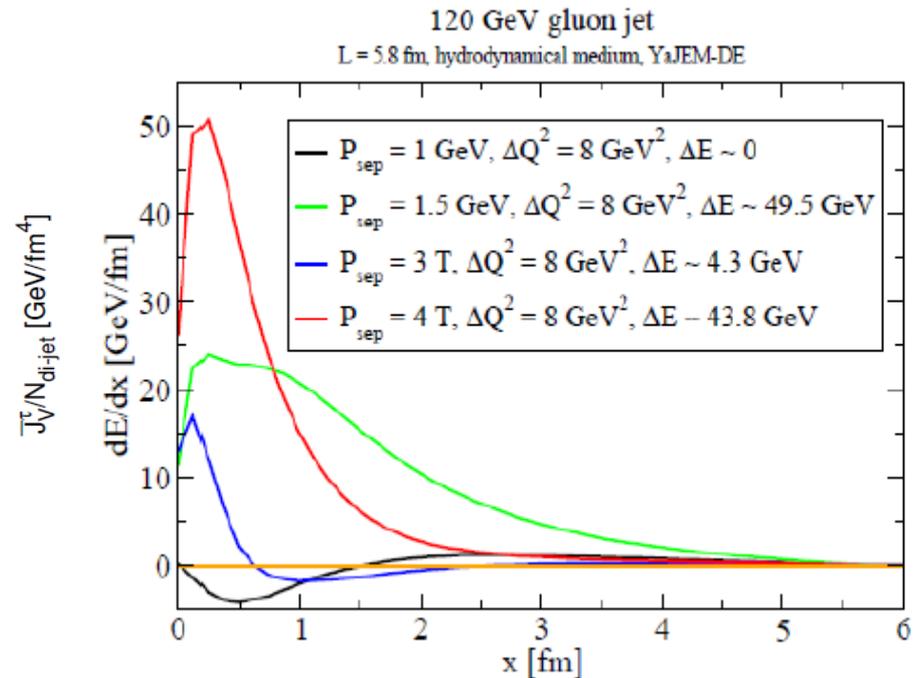
GYQ, Majumder, Song, Heinz, PRL 2009; Neufeld, Muller, PRL 2009 ; Neufeld, Vitev, PRC 2012

Full jet energy deposition & medium response



K. Zapp, Tuesday

Jet shower evolution (JEWEL) with
(1+1)-d hydro simulation



Renk, PRC 2013

Different separation scales lead to
different energy deposition profiles

How does lost energy thermalize and diffuse into medium?

Lattice study of jet quenching

$$\hat{q} = \frac{1}{L} \int \frac{d^2 \vec{k}_\perp}{(2\pi)^2} k_\perp^2 P(\vec{k}_\perp, L)$$

$$P(\vec{k}_\perp, L^-) = \int \frac{d^2 \vec{k}'_\perp}{(2\pi)^2} e^{-i\vec{k}'_\perp \cdot \vec{y}_\perp} P(\vec{y}_\perp, L^-)$$

$$P(\vec{y}_\perp, L^-) = \frac{1}{N_c} \text{Tr} \langle \mathcal{W}(\vec{y}_\perp, L^-) \rangle$$

$$\frac{dP(\vec{y}_\perp, L^-)}{dL} = -V(\vec{y}_\perp) P(\vec{y}_\perp, L^-)$$

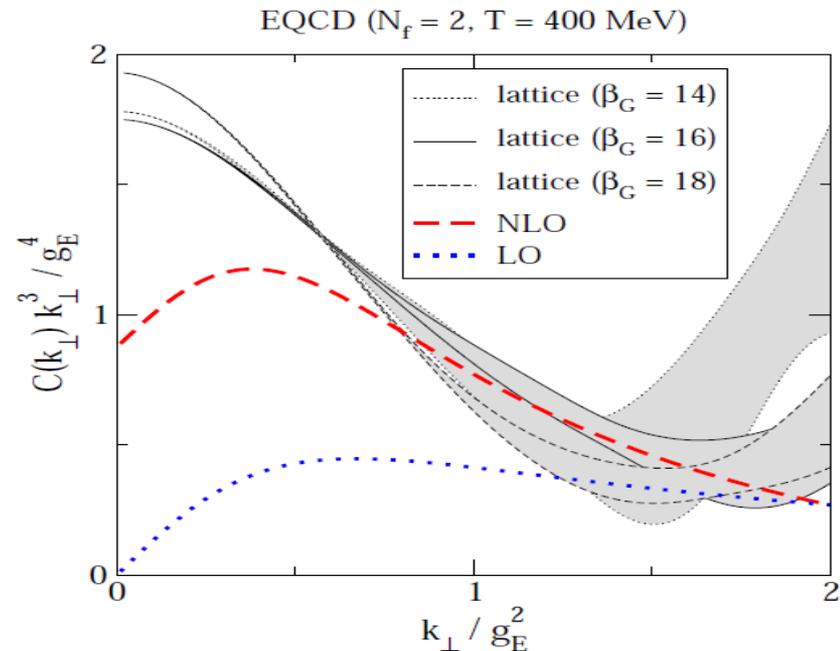
$$V(\vec{y}_\perp) = \int \frac{d^2 \vec{k}_\perp}{(2\pi)^2} (1 - e^{i\vec{k}_\perp \cdot \vec{y}_\perp}) C(\vec{k}_\perp)$$

Majumder, PRC 2013; Panero, Rummukainen, Schafer, PRL 2014; Laine, Rothkopf, JHEP 2013; arXiv:1310.2413; Caron-Huot, PRD 2009; Benzke, Brambilla, Escobedo, Vairo, JHEP 2013; D'Eramo, Lekaveckas, Liu, Rajagopal, JHEP 2013

Quenched SU(2): $\hat{q} \approx 1.3 - 3.3 \text{ GeV}^2/\text{fm}@T = 400 \text{ MeV}$

Lattice EQCD: $\hat{q} \approx 6 \text{ GeV}^2/\text{fm} \pm 20\% @ \text{RHIC}$

M. Panero, Tuesday



Discrepancy is unclear so far and needs further investigation

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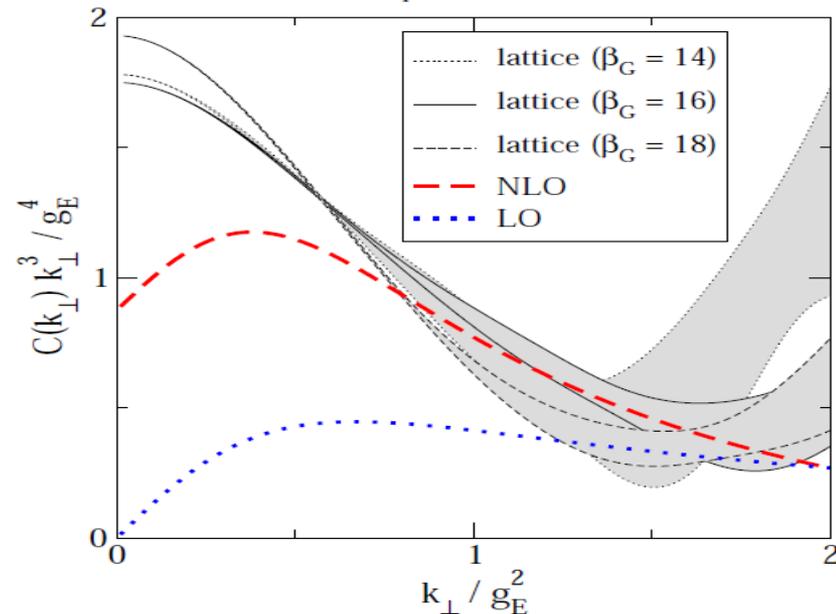
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$$\hat{q} \approx \begin{cases} 1.2 \pm 0.3 \\ 1.9 \pm 0.7 \end{cases} \text{ GeV}^2/\text{fm} \text{ at } \begin{matrix} T=370 \text{ MeV} \\ T=470 \text{ MeV} \end{matrix}$$

EQCD ($N_f = 2, T = 400 \text{ MeV}$)



Discrepancy is unclear so far and needs further investigation

AdS/CFT

- **NLO correction for finite t'Hooft coupling** (Zhang, Hou, Ren, JHEP 2013)

$$\hat{q}_{SYM}(\lambda) = \hat{q}_{SYM}^{(0)} [1 + \kappa \lambda^{-1/2} + O(\lambda^{-1})] \quad \kappa \simeq -1.97.$$

- **Better agreement with data using finite endpoint momentum shooting strings for more realistic description of an energetic quark** (Ficnar, Gubser, Gyulassy, arXiv:1311.6160)
- **Jets emerging from a slab of plasma have the same shape as vacuum jets, except less energy and broader angle** (Chesler, Rajagopal, arXiv:1402.6756)

K. Rajagopal, Wednesday

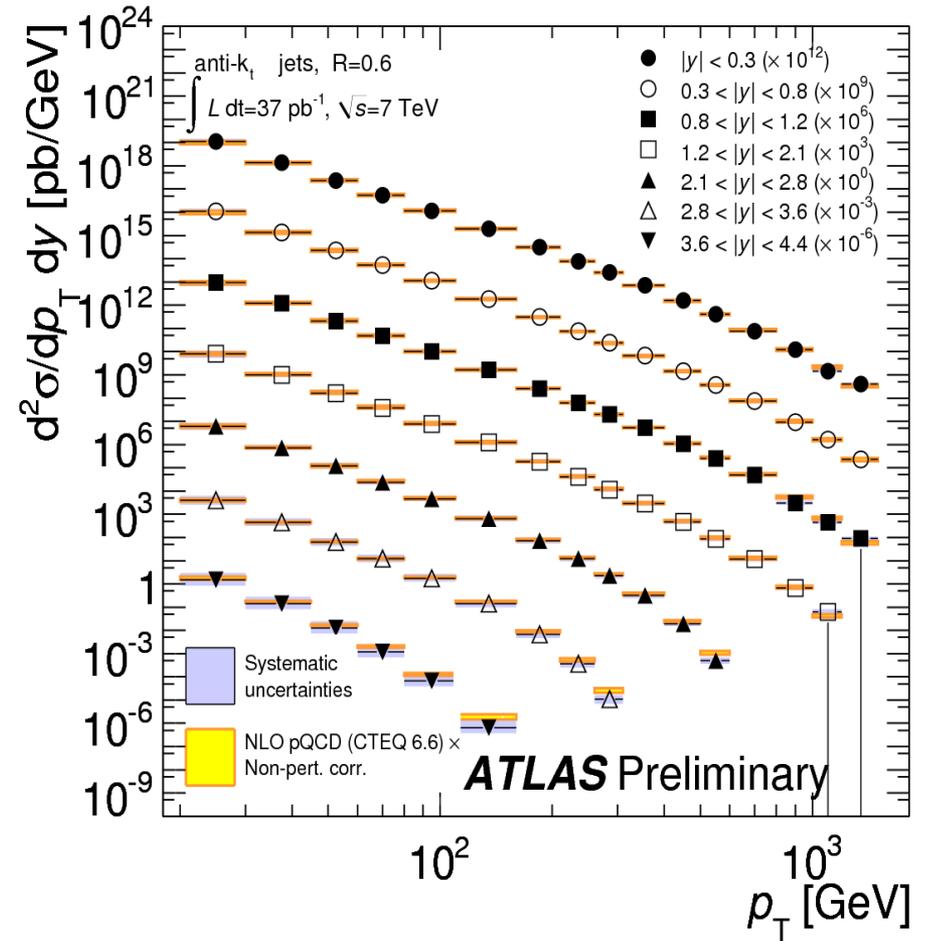
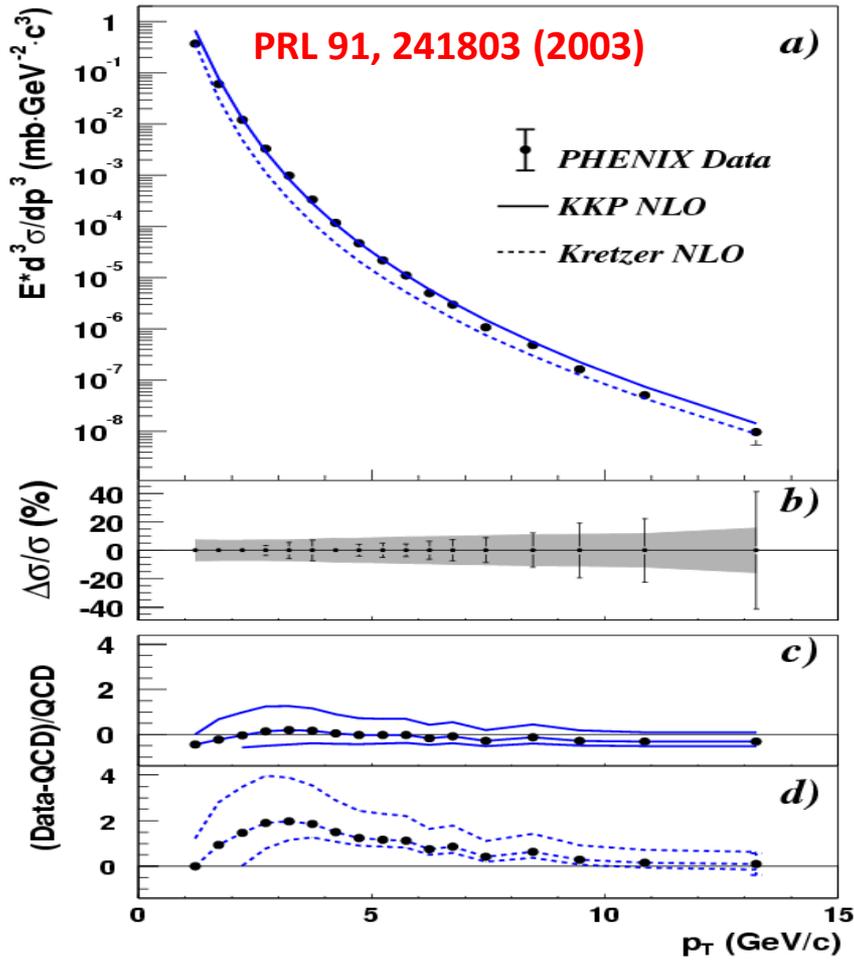
- **A hybrid strong/weak coupling model** (Casalderrey-Solana, Gulhan, Milhano, Pablos, Rajagopal, arXiv:1405.3864)

D. Pablos, Wednesday

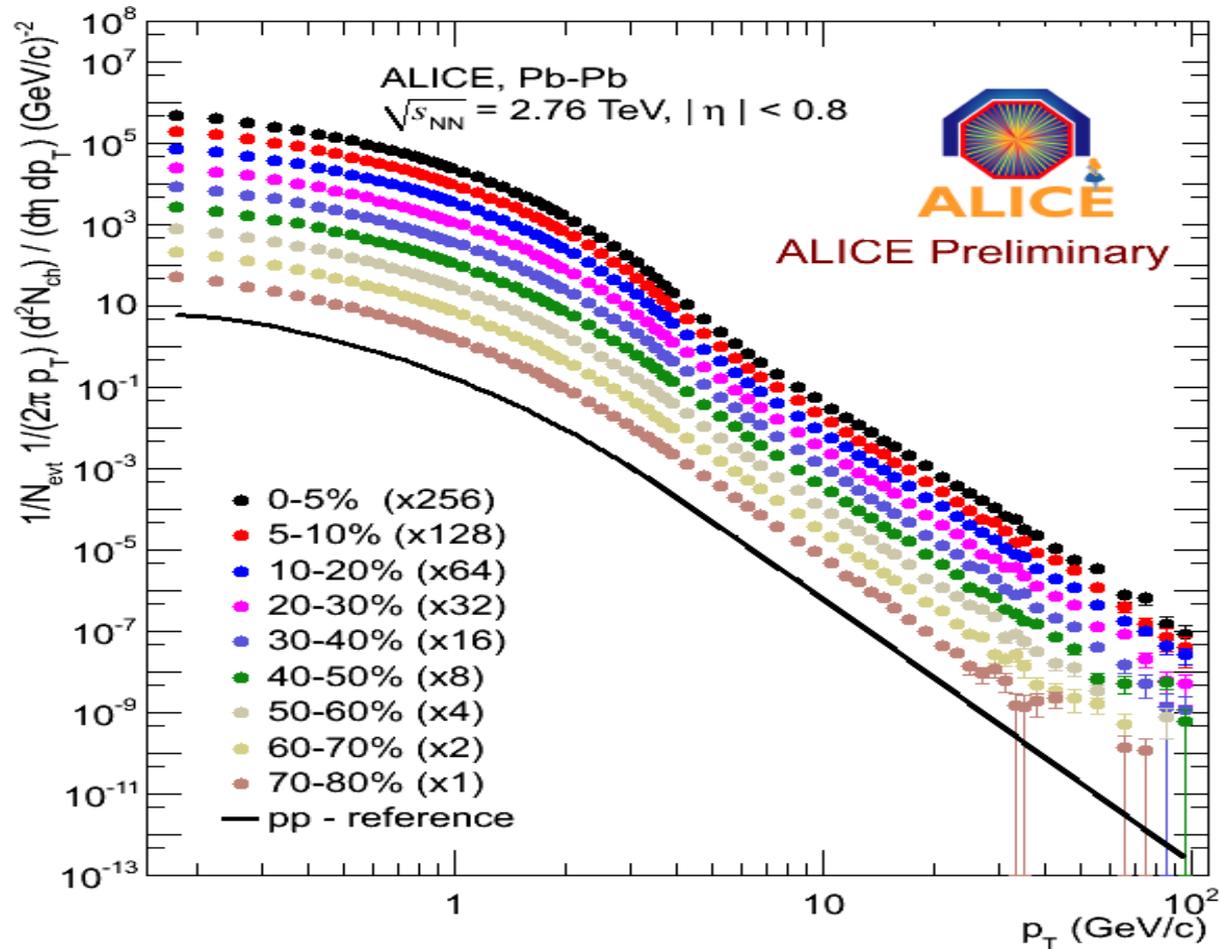
Summary

- **Significant theoretical effort and progress**
 - Systematic phenomenological studies, q^{hat} from JET Collab., Lattice & AdS/CFT
 - Progress in NLO jet energy loss, full jet evolution and modification, Monte-Carlo models, jet energy deposit/redistribution
- **Future**
 - Discrepancy in q^{hat} values
 - NLO jet energy loss
 - Full jet modification (energy loss, substructure...)
 - Realistic jet transport and medium response simultaneously (thermalization, lost energy redistribution...)
 - ...

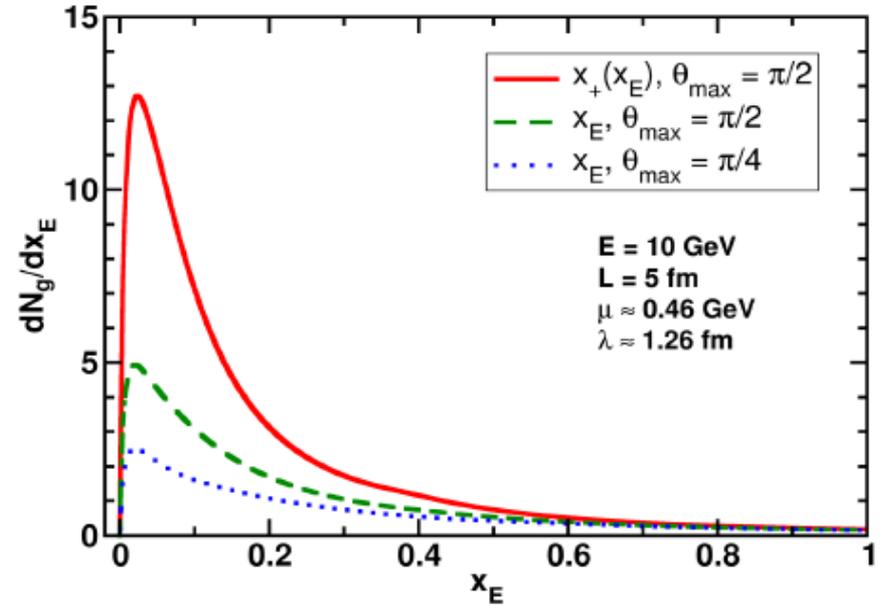
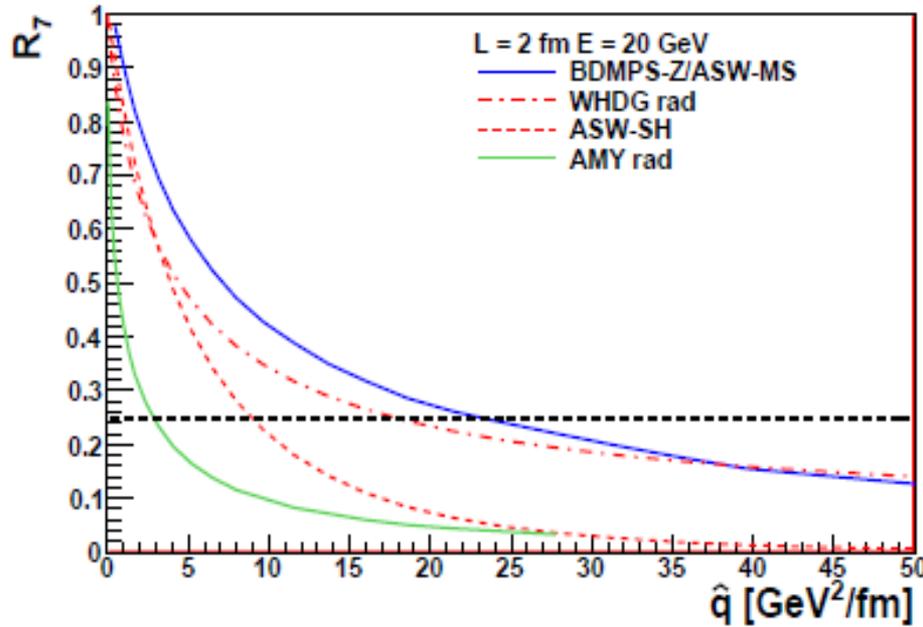
pQCD works for p-p



Soft & hard



Comparison of different models



$$R_n = \int_0^1 d\epsilon (1 - \epsilon)^{n-1} P(\epsilon)$$

$$x_+ = \frac{1}{2} x_E \left(1 + \sqrt{1 - \left(\frac{k_T}{x_E E} \right)^2} \right)$$

$$x_E = x_+ \left(1 + \left(\frac{k_T}{x_+ E^+} \right)^2 \right),$$

Single gluon emission kernel

$$dP \propto \alpha_s C_R \frac{dy}{y} \frac{dI_{\perp}^2}{I_{\perp}^2}$$

Soft and collinear divergence

BDMPS-Z formalism:

Formation time: $t_f \approx \frac{\omega}{I_{\perp}^2}$  $t_f \approx \sqrt{\frac{\omega}{\hat{q}}}$

Momentum broadening: $I_{\perp}^2 \approx \hat{q} t_f$ $\omega \frac{dN}{d\omega} \approx \alpha_s \frac{L}{\tau_f} \approx \alpha_s \sqrt{\frac{\hat{q} L^2}{\omega}}$

Emission angle: $\theta \approx \frac{l_{\perp}}{\omega} \approx \left(\frac{\hat{q}}{\omega^3} \right)^{1/4}$

No emission if $t_f > L$, i.e., $\omega > \omega_c \approx \hat{q} L^2$

Mean energy loss in a single emission: $\langle \omega \rangle \approx \alpha_s \omega_c \approx \alpha_s \hat{q} L^2$

Emission is dominated by ω close to upper limit ω_c , i.e., energy loss is dominated by hard emissions with small angles

MARTINI-AMY, McGill-AMY

- Solving the following rate equations

$$\frac{dP_g(p)}{dt} = \int_k P_{q(\bar{q})}(p+k) \frac{d\Gamma_{qg}^q(p+k, p)}{dk} + \int_k P_g(p+k) \frac{d\Gamma_{gg}^g(p+k, k)}{dk} - \int_k P_g(p) \left[\frac{d\Gamma_{q\bar{q}}^g(p, k)}{dk} + \frac{d\Gamma_{gg}^g(p, k)}{dk} \Theta(k-p/2) \right]$$

$$\frac{dP_{q(\bar{q})}(p)}{dt} = \int_k P_{q(\bar{q})}(p+k) \frac{d\Gamma_{qg}^q(p+k, k)}{dk} - \int_k P_{q(\bar{q})}(p) \frac{d\Gamma_{qg}^q(p, k)}{dk} + 2 \int_k P_g(p+k) \frac{d\Gamma_{q\bar{q}}^g(p+k, k)}{dk}$$

- The interaction between the parton and the medium is controlled by the HTL re-summed elastic collision rate

$$\frac{d\Gamma_{el}}{d^2q_{\perp}} = \frac{C_a}{(2\pi)^2} \frac{g^2 m_D^2 T}{q_{\perp}^2 (q_{\perp}^2 + m_D^2)}$$

- In the fluid rest frame, The average transverse momentum transfer squared per mean free path (\hat{q}^{hat})

$$\hat{q} = \int^{q_{\text{max}}} d^2q_{\perp} q_{\perp}^2 \frac{d\Gamma_{el}}{d^2q_{\perp}} \quad \hat{q} = C_a \alpha_s m_D^2 T \ln(1 + q_{\text{max}}^2/m_D^2)$$

CUJET

- DGLV induced gluon radiative distribution

$$x \frac{dN_{Q \rightarrow Q+g}}{dx}(\mathbf{r}, \phi) = \int d\tau \rho(\mathbf{r} + \hat{\mathbf{n}}(\phi)\tau, \tau) \int \frac{d^2 \mathbf{q}_T}{\pi} \frac{d^2 \sigma_{\text{eff}}}{d^2 \mathbf{q}_T} \int \frac{d^2 \mathbf{k}_T}{\pi} \alpha_s(k_T^2/(x(1-x)))$$

$$\times \frac{12(\mathbf{k}_T + \mathbf{q}_T)}{(\mathbf{k}_T + \mathbf{q}_T)^2 + \chi(\tau)} \cdot \left(\frac{(\mathbf{k}_T + \mathbf{q}_T)}{(\mathbf{k}_T + \mathbf{q}_T)^2 + \chi(\tau)} - \frac{\mathbf{k}_T}{k_T^2 + \chi(\tau)} \right) \left(1 - \cos \left[\frac{(\mathbf{k}_T + \mathbf{q}_T)^2 + \chi(\tau)}{2x_+ E} \tau \right] \right)$$

- The effective running differential quark-gluon cross section

$$\frac{d^2 \sigma_{\text{eff}}}{d^2 \mathbf{q}_T} = \frac{\alpha_s^2(q_T^2)}{(\mathbf{q}_T^2 + f_E^2 \mu^2(\tau))(\mathbf{q}_T^2 + f_M^2 \mu^2(\tau))}$$

- The effective jet transport coefficient in an idealized static and homogeneous thermal equilibrium medium.

$$\hat{q}(E, T; \alpha_{max}, f_E, f_M) = \rho_g(T) \int_0^{\sqrt{6ET}} dq_T^2 q_T^2 \frac{d\sigma_{qg}}{dq_T^2}$$

Higher Twist (HT-BW)

- Medium modified quark fragmentation functions

$$\tilde{D}_q^h(z_h, Q^2) = D_q^h(z_h, Q^2) + \frac{\alpha_s(Q^2)}{2\pi} \int_0^{Q^2} \frac{d\ell_T^2}{\ell_T^2} \int_{z_h}^1 \frac{dz}{z} \left[\Delta\gamma_{q \rightarrow qg}(z, \ell_T^2) D_q^h\left(\frac{z_h}{z}\right) + \Delta\gamma_{q \rightarrow gq}(z, \ell_T^2) D_g^h\left(\frac{z_h}{z}\right) \right]$$

- The jet transport parameter for a quark is related to the gluon distribution density of the medium

$$\hat{q} = \frac{4\pi C_F \alpha_s}{N_c^2 - 1} \int dy^- \langle F^{ai+}(0) F_i^{a+}(y^-) \rangle e^{i\xi p^+ y^-}$$

- The corresponding quark energy loss

$$\frac{\Delta E}{E} = \frac{2N_c \alpha_s}{\pi} \int dy^- dz d\ell_T^2 \frac{1 + (1-z)^2}{\ell_T^4} \left(1 - \frac{1-z}{2}\right) \hat{q}(E, y) \sin^2 \left[\frac{y^- \ell_T^2}{4Ez(1-z)} \right]$$

- In a dynamical evolving medium

$$\hat{q}(\tau, r) = \left[\hat{q}_0 \frac{\rho_{QGP}(\tau, r)}{\rho_{QGP}(\tau_0, 0)} (1-f) + \hat{q}_h(\tau, r) f \right] \cdot \frac{p \cdot u}{p_0}$$

- To include the contribution from hadronic contribution

$$\hat{q}_h = \frac{\hat{q}_N}{\rho_N} \left[\frac{2}{3} \sum_M \rho_M(T) + \sum_B \rho_B(T) \right]$$

Higher Twist (HT-M)

- Vacuum DGLAP evolution equation

$$\frac{\partial D_q^h(z, Q^2)}{\partial \log(Q^2)} = \frac{\alpha_S(Q^2)}{2\pi} \int_z^1 \frac{dy}{y} P_{q \rightarrow i}(y) D_i^h\left(\frac{z}{y}, Q^2\right)$$

- Medium-modified DGLAP evolution equation

$$\frac{\partial D_q^h(z, Q^2; q^-) \Big|_{\zeta_i}^{\zeta_f}}{\partial \log(Q^2)} = \frac{\alpha_S}{2\pi} \int_z^1 \frac{dy}{y} \int_{\zeta_i}^{\zeta_f} d\zeta P(y) K_{q^-, Q^2}(y, \zeta) D_q^h\left(\frac{z}{y}, Q^2; q^- y\right) \Big|_{\zeta}^{\zeta_f}$$

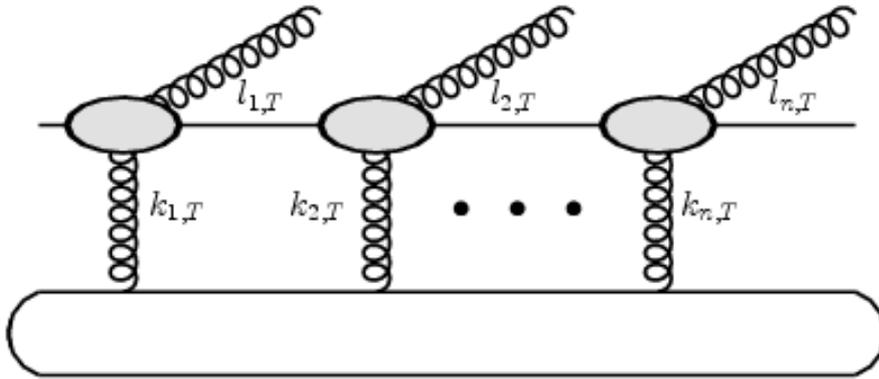
- The modification from the medium is contained in the factor

$$K_{q^-, Q^2}(y, \zeta) = \frac{[\hat{q}_A(\zeta) - (1-y)\hat{q}_A/2 + (1-y)^2\hat{q}_F]}{Q^2} \left[2 - 2 \cos\left(\frac{Q^2(\zeta - \zeta_i)}{2q^- y(1-y)}\right) \right]$$

- q^{hat} scales with the entropy density

$$\hat{q}(s) = \hat{q}_0 \frac{s}{s_0}$$

Medium-modified FF



$$\hat{q} = d(\Delta p_T)^2 / dt$$

$$\tau_F = 2q^- y(1-y) / l_{\perp}^2$$

For soft transverse kicks $k_T \ll l_T$, the medium-dependent correction to FF (splitting function):

$$\Delta \tilde{D}(z, Q^2, q^-)_{t_i}^{t_f} = \frac{\alpha_s}{2\pi} \int \frac{dl_{\perp}^2}{l_{\perp}^2} \int \frac{dy}{y} P(y) \int_{t_i}^{t_f} dt \frac{\hat{q}(t)}{l_{\perp}^2} K_m(t, q^-, y, l_{\perp}^2) D\left(\frac{z}{y}, \mu_0^2, q^- y\right)_t^{t_f}$$

For l_T ordered emissions, one may obtain DGLAP evolution equation for MMFF

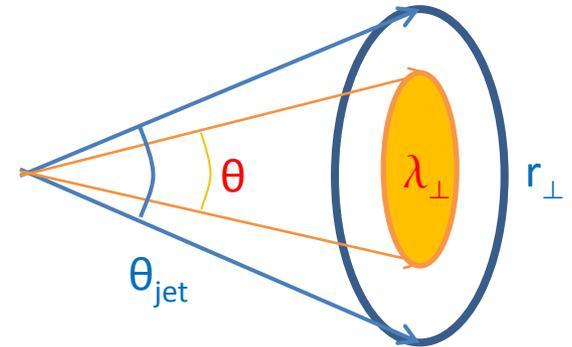
$$\frac{\partial \tilde{D}(z, Q^2, q^-)_{t_i}^{t_f}}{\partial \ln Q^2} = \frac{\alpha_s}{2\pi} \int \frac{dy}{y} P(y) \int_{t_i}^{t_f} dt \frac{\hat{q}(t)}{Q^2} K_m(t, q^-, y, Q^2) D\left(\frac{z}{y}, Q^2, q^- y\right)_t^{t_f}$$

Color coherence & de-coherence

- **BDMPS-Z: emission dominated by hard emissions with small angles**

$$\tau_f \approx \omega / k_{\perp}^2 \approx \sqrt{\omega / \hat{q}} \quad \langle \omega \rangle \approx \alpha_s \omega_c \approx \alpha_s \hat{q} L^2$$

- **E.g. qq^{bar} emitting system has a transverse size r_⊥**
- **Glucos emitted at angle θ have wavelength λ_⊥**



- **Vacuum:**

- **In-cone (λ_⊥ < r_⊥): independent emission by each quark**
- **Out-of-cone (λ_⊥ > r_⊥): coherent emission by qq^{bar} system**

- **Medium:**

- **If medium color field varies over r_⊥, then q and q^{bar} lose their color coherence**
- **Jet-medium interaction destroys the color coherence**
- **Phase space for soft radiation at large angles opens up**
- **De-coherence parameter**

$$\Delta_{med} = 1 - \exp\left(-\frac{1}{12} \frac{r_{\perp}^2}{\lambda_{\perp}^2}\right) = 1 - \exp\left(-\frac{1}{12} \theta_{jet}^2 \hat{q} L^3\right)$$

- **Massive antenna: a heavy-quark gluon antenna loses coherence faster than a light-quark gluon antenna (lose more energy loss)**
(Calvo, Moldes, Salgado, arXiv:1403.4892)

$$r_{\perp} = \theta_{jet} L$$

$$\lambda_{\perp} = 1 / k_{\perp} = 1 / \theta \omega = \sqrt{\hat{q} L}$$

$$\tau_d = (\hat{q} \theta_{jet}^2)^{-1/3}$$

Armesto, Ma, Mehtar-Tani, Salgado, Tywoniuk, 2011; Mehtar-Tani, Salgado, Tywoniuk, 2011; Iancu, Casalderrey-Solana 2011; Blaizot, Dominguez, Iancu, Mehtar-Tani, 2012

Energy flow due to soft emissions

- Color coherence is rapidly lost via re-scattering in medium
- Color de-coherence opens up the phase space for soft-large-angle emissions
- Probability description:

$$\frac{d\tilde{P}}{dxdt} \approx \alpha_s P(x) \frac{1}{t_f} \approx \alpha_s P(x) \left(\frac{\hat{q}}{xE} \right)^{1/2}$$

- Jet energy is converted into many soft gluons carrying energy of $O(T)$
- Evolution (rate) equation:

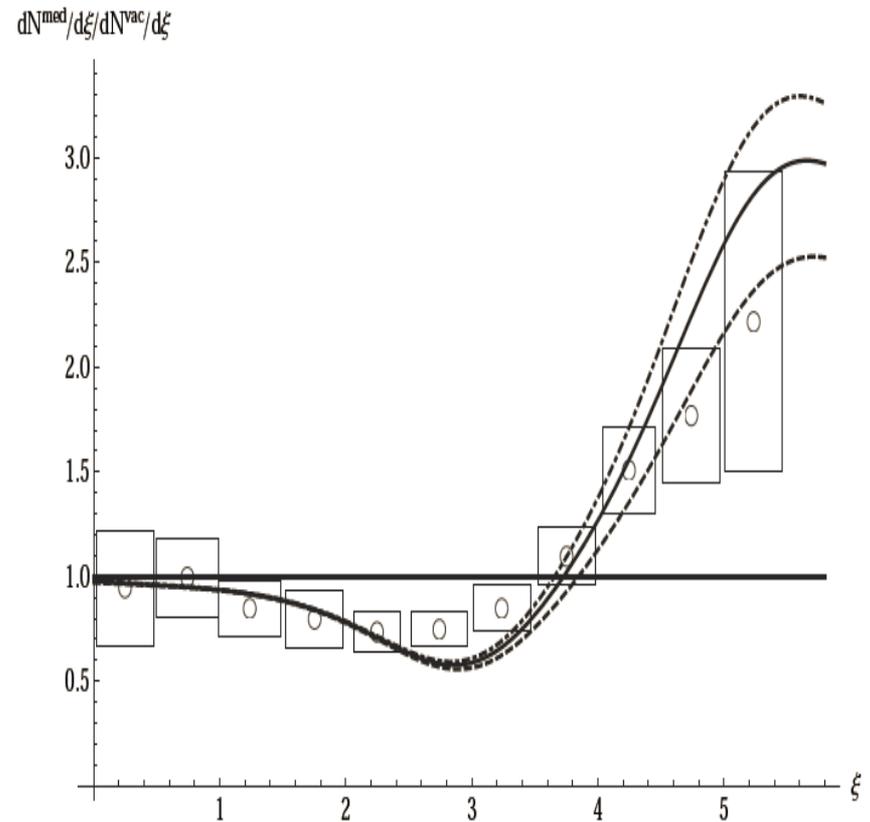
$$\frac{\partial D(x, t)}{\partial t} = \int dz \left[2\mathcal{K} \left(z, \frac{x}{z} \right) D \left(\frac{x}{z}, t \right) - \mathcal{K} (z, x) D(x, t) \right]$$

$$D(x, t) \equiv x \frac{dN}{dx} \quad t = \alpha_s \sqrt{\frac{\hat{q}}{E}} L \quad \mathcal{K}(z, x) \equiv \frac{1}{\sqrt{x}} \frac{P_{gg}(z)}{\sqrt{z(1-z)}}$$

Blaizot, Iancu, Mehtar-Tani, PRL 2013

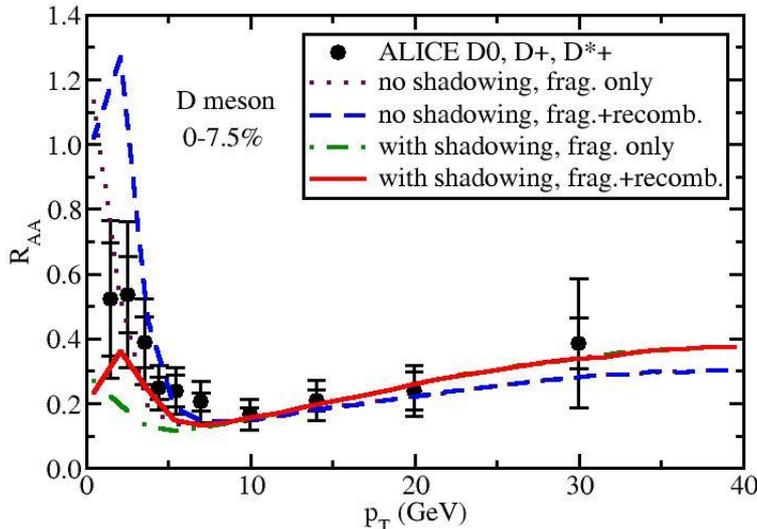
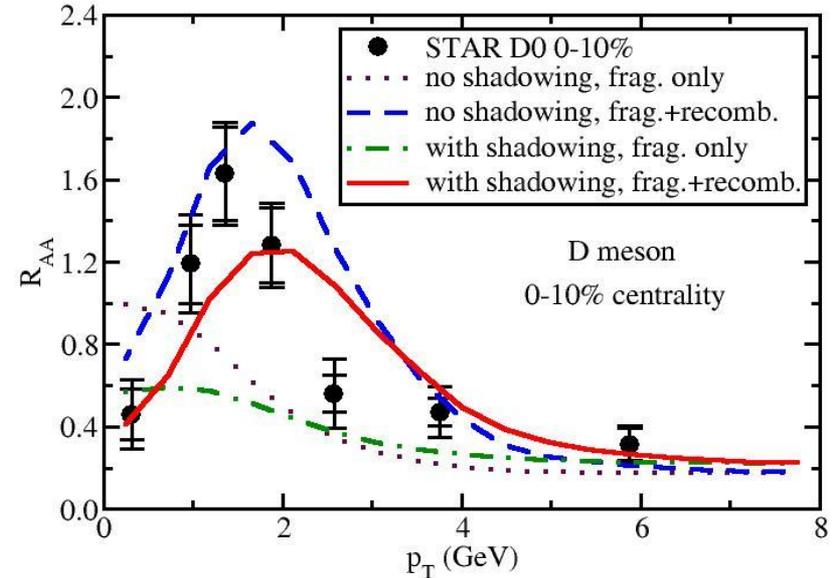
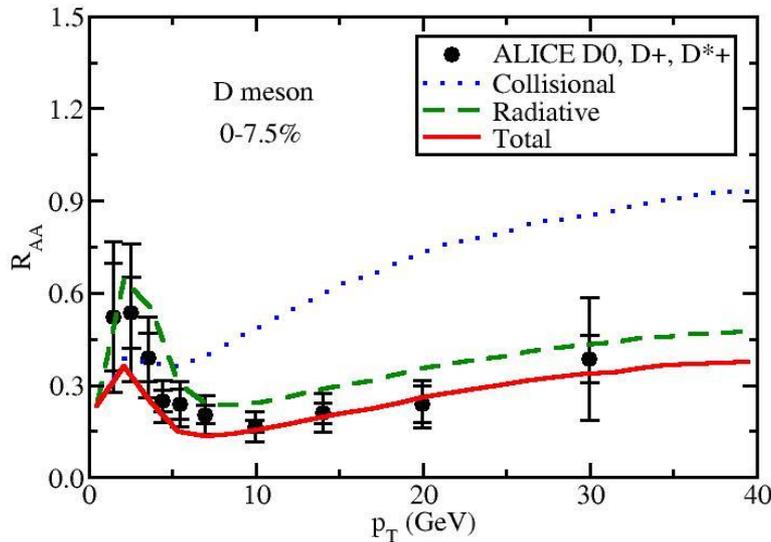
Jet energy loss and fragmentation

- Massless QED in 1 + 1 dimensions used to study the dynamical modification of jet fragmentation in the medium
- The suppression of the in-medium fragmentation function is due to the partial screening of jet color charges by co-moving medium-induced gluon
- The energy does not transfer outside the jet cone, but transfer from the high energy jet to low energy hadrons
- Below a certain experimental cutoff, soft hadrons are not counted as a part of the jet, leading to an effective energy loss



Kharzeev, Loshaj, PRD 2013

Nuclear modification of D mesons

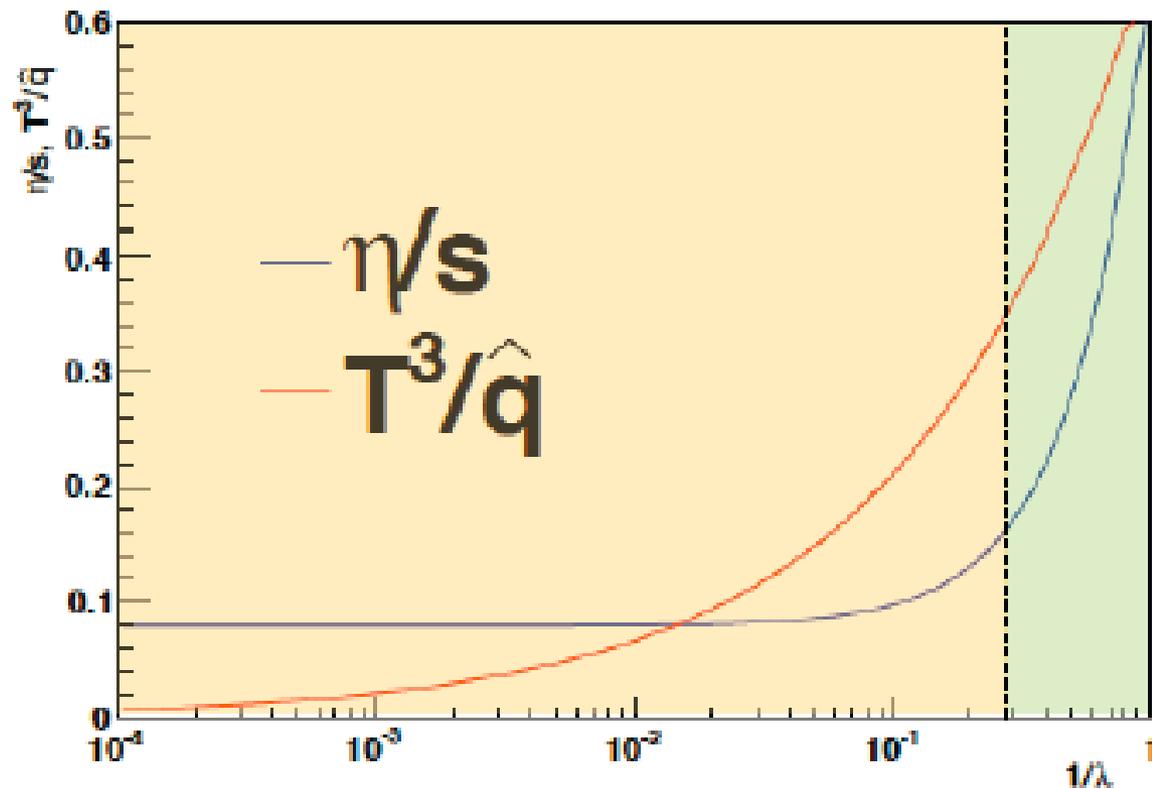


Collisional dominates low p_T , radiative dominates high p_T .

Shadowing effect reduces R_{AA} significantly at low p_T .

Recombination mechanism raises R_{AA} at intermediate p_T , and has more significant contribution to R_{AA} at RHIC energy

Weak v.s. strong coupling



At weak coupling:

$$\frac{T^3}{\hat{q}} \approx \# \frac{\eta}{s}$$

At strong coupling:

$$\frac{T^3}{\hat{q}} \ll \# \frac{\eta}{s}$$

Majumder, Muller, Wang,
PRL 2007