Heavy Quarks and Mesons at Finite Chemical Potential in Gauge/Gravity Duality

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Motivation

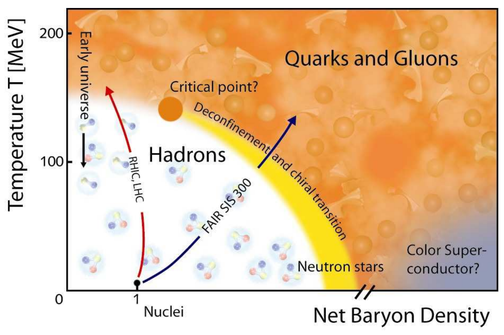

Fig. 1: Sketch of the QCD phase diagram (credits: GSI Darmstadt).

The Quark-Gluon Plasma (QGP) of QCD is experimentally studied by heavy ion collisions at RHIC and LHC (high temperatures), and especially FAIR (larger chemical potential). Heavy probes provide a unique window on the physics of the QGP.

Their theoretical description in strongly coupled plasmas is very challenging. We use gauge/gravity ('AdS/CFT') duality to study heavy probes moving in **generic strongly coupled gauge theory plasmas**, looking for *universal behavior* of screening, the free energy, and the running coupling in heavy quark–antiquark pairs, as well as the drag force on single heavy quarks. Specifically, we employ **deformations of AdS_5 spacetime** endowed with a charged black hole, which yields a simple holographic model (*i. e.*, use $(4+1)$-dim. physics to describe $(3+1)$-dim. physics) of deconfined plasma at non-zero **temperature** and **chemical potential**.

The Models

1. 'Plasma' of conformal field theory, **$\mathcal{N}=4$ SYM**:

   Strongly coupled $\mathcal{N}=4$ supersymm. Yang–Mills with $N_\mathrm{c}\to\infty$
   $\longleftrightarrow$ Classical Supergravity on AdS_5

   The AdS_5 metric is

$$\mathrm{d}s^2 = \frac{R^2}{z^2}\left(-\left(1-\frac{z^4}{z_\mathrm{h}^4}\right)\mathrm{d}t^2 + \mathrm{d}\vec{x}^2 + \frac{\mathrm{d}z^2}{1-\frac{z^4}{z_\mathrm{h}^4}}\right).$$

   - AdS length $R/\alpha'^2 = 4\pi g_\mathrm{YM}^2 N_\mathrm{c}$
   - Schwarzschild black hole horizon $z_\mathrm{h} = (\pi T)^{-1}$

2. More interesting: Non-conformal field theory from **deformations** of dual AdS_5 (*e. g.* [1]). **Non-zero chemical potential** μ from adding a $U(1)$ gauge field to the bulk, *e. g.* by putting charge on the black hole. Such a model is the **CGN model** proposed by [2]:

$$\mathrm{d}s^2 = \frac{R^2\mathrm{e}^{c^2z^2}}{z^2}\left[-h(z)\mathrm{d}t^2 + \mathrm{d}\vec{x}^2 + \frac{1}{h(z)}\mathrm{d}z^2\right]$$
$$\text{with}\quad h(z) = 1-\frac{z^4}{z_\mathrm{h}^4}\left(1+Q^2\left(1-\frac{z^2}{z_\mathrm{h}^2}\right)\right).$$

   - *Ad hoc* deformation with parameter c
   - Black hole charge Q (Reissner–Nordström black hole)

$$T = \frac{1}{\pi z_\mathrm{h}}\left(1-\frac{1}{2}Q^2\right), \qquad \mu = (\text{const})\frac{Q}{z_\mathrm{h}} \equiv \frac{Q}{z_\mathrm{h}}$$

3. More consistent: Model from (super-)gravity action [3]:

$$S = \frac{1}{2\kappa_5^2}\int \mathrm{d}^5x\sqrt{-g}\left(\mathcal{R}[g_{\mu\nu}] - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi) - \frac{f(\phi)}{4}F_{\mu\nu}F^{\mu\nu}\right)$$

   - Dynamic fields: metric $g_{\mu\nu}$, scalar field ϕ, gauge field $F_{\mu\nu} = \partial_{[\mu}A_{\mu]}$
   - Scalar–gauge coupling $f(\phi) = \cosh(12/5)\cosh(6(\phi-2)/5)$
   - Consistent deformation with parameter κ
   - **1-parameter models**: Analytic solution of EOMs with

$$g_{\mu\nu}\mathrm{d}x^\mu\mathrm{d}x^\nu = \mathrm{e}^{2A(z)}\left(-h(z)\mathrm{d}t^2 + \mathrm{d}\vec{x}^2\right) + \frac{\mathrm{e}^{2B(z)}}{h(z)}\mathrm{d}z^2,$$
$$A(z) = \log\left(\frac{R}{z}\right), \qquad \phi(z) = \sqrt{\frac{3}{2}}\kappa z^2, \qquad A_\mu\mathrm{d}x^\mu = \Phi(z)\mathrm{d}t,$$
$$\text{Boundary conditions} \longleftrightarrow (\mu, T)$$

   - Two choices:
     - ➢ Treat ϕ as dilaton: add $\kappa z^2/2$ to A, B ('string frame' model)
     - ➢ Don't treat ϕ as dilaton ('Einstein frame' model)

Thermodynamics

- According to [2], the CGN model exhibits a phase transition due to the warp factor $\mathrm{e}^{c^2z^2}$ in the metric: Confining low-μ/T phase (area law in Wilson loop), deconfined high-μ/T phase (screening).
- However, at low (μ, T), considering *moving* probes, we find apparently unphysical behavior in the (*ad hoc*) CGN and similar models.
- More consistent 1-parameter models silent about the low-μ/T regime, as for such low values of (μ, T) no black hole solutions exist.

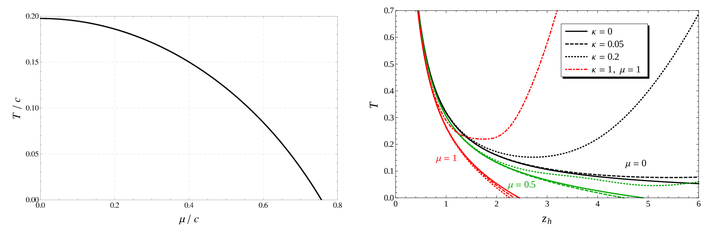

Fig. 2: **Left:** Phase diagram of the CGN model. **Right:** Temperature in the 1-parameter models as a function of horizon position z_h. At small μ, the minimal attainable temperature is $T_\mathrm{min}(\mu) > 0$, excluding parts of the (μ, T) plane.

Observables and Method

- We compute for
  - ➢ *moving* heavy quark–antiquark $(Q\bar{Q})$ pairs: **screening distance** L_s, **free energy** $E(L)$, and **running coupling** $\alpha_{Q\bar{Q}}(L)$
  - ➢ single moving heavy quark: **energy loss** via drag force $\mathrm{d}p/\mathrm{d}t$
- Extremize classical Nambu–Goto action of 5d string attached to quark(s), get Wilson loop [4] and worldsheet momentum current [5]:

$$S_\mathrm{NG} = -\frac{1}{2\pi\alpha'}\int \mathrm{d}^2\sigma\sqrt{-\det g_{ab}}\,, \qquad g_{ab} = \partial_a X^\mu \partial_b X^\nu g_{\mu\nu},$$

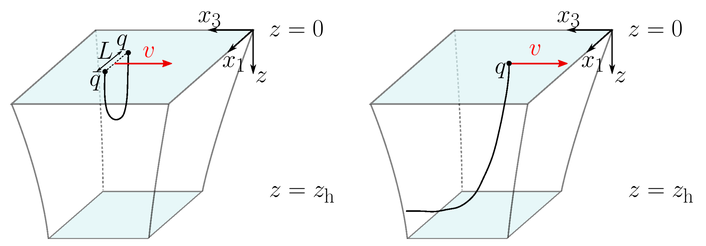

Fig. 3: **Left:** Sketch of string configuration for $Q\bar{Q}$ pair. **Right:** Sketch of trailing string configuration for single quark. Quarks are at the $z=0$ boundary.

$$L_\mathrm{s} = \max\left\{L_{\text{any string config}}\right\}, \qquad E(L) = -\frac{S_\mathrm{NG}^{Q\bar{Q}} - 2S_\mathrm{NG}^\mathrm{trail}}{\mathcal{T}},$$
$$\alpha_{Q\bar{Q}}(L) = \frac{3}{4}L^2\frac{\mathrm{d}E(L)}{\mathrm{d}L}, \qquad \frac{\mathrm{d}p}{\mathrm{d}t} = -\left.\frac{\delta S_\mathrm{NG}^\mathrm{trail}}{\delta(\partial_z X)}\right|_\text{on shell}$$

Screening Distance & Free Energy at $\mu > 0$ [6]

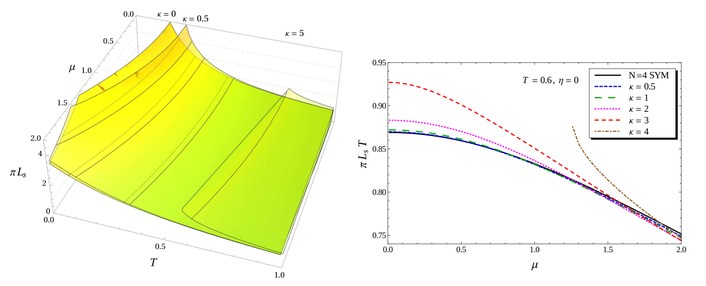

Fig. 4: Screening distance in 1-parameter models, $Q\bar{Q}$ at rest. **Left:** ϕ treated as dilaton. The value in $\mathcal{N}=4$ SYM case is a *lower bound*. (Same result in simpler CGN model.) **Right:** ϕ not treated as dilaton. The universality of the lower bound is lost at large μ.

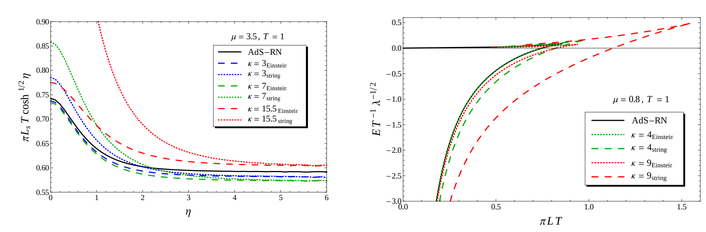

Fig. 5: Screening distance and free energy in 1-parameter models, $Q\bar{Q}$ moving. **Left:** Screening distance vs. $Q\bar{Q}$ rapidity η. At finite η and large μ, in both models $L_\mathrm{s}^{\mathcal{N}=4\ \mathrm{SYM}}$ is no longer a lower bound. **Right:** Free energy is very robust in 'Einstein frame'.

Running Coupling

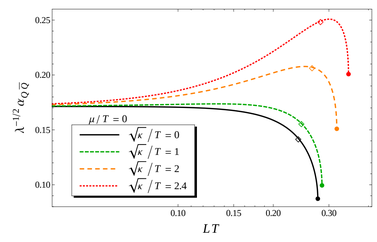

Fig. 6: Running coupling in undeformed $\mathcal{N}=4$ SYM and 'string frame' model.

- Running coupling drops at thermal scale $L_\mathrm{th} \sim 1/T$; find: scale L_th only weakly influenced by μ
- Coupling becomes constant in the UV: restoration of conformality
- Find **universal rising** above UV value in all non-conformal models
- Surprisingly, scale L_max of maximum of $\alpha_{Q\bar{Q}}$ is $L_\mathrm{max} \propto 1/T^2$

Commonalities & Differences in the Models

- **Lower bound on L_s:**
  - ➢ In CGN and the 'string frame' model, $L_\mathrm{s}^{\mathcal{N}=4\ \mathrm{SYM}}$ is a lower bound under all deformations, also at $\mu > 0$ (extends [8]).
  - ➢ This no longer holds in 'Einstein frame' model and for moving $Q\bar{Q}$.
  - ➢ But then, the screening distance and the free energy are quite robust under deformations.
- **Ultrarelativistic scaling of L_s:** At large η, the scaling [7] is same for all models also for $\mu > 0$: $L_\mathrm{s} \propto \cosh^{-1/2}(\eta)$.
- **Large-μ/T scaling of L_s** quite simple and same in all models:

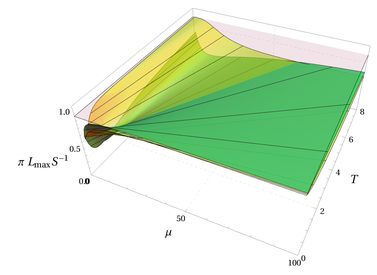

Fig. 7: Asymptotic μ/T-scaling of screening distance in all models under consideration. The function S for the light green surface is given below.

$$L_\mathrm{max} = g\left(\frac{\mu}{T}\right)\frac{0.869}{\pi}\frac{R^{x(\varphi)}}{T^{1-x(\varphi)}\mu^{x(\varphi)}} = g\left(\frac{\mu}{T}\right)\frac{0.869}{\pi}S$$
$$\text{with } x(\varphi) = \left(\frac{2\arctan\left(\frac{\mu}{T}\right)}{\pi}\right)^n, \quad n \approx 7, \quad R \approx 8, \quad 0.6 \lesssim g\left(\frac{\mu}{T}\right) \leq 1.$$

- **Drag force:** In non-conformal models, at non-zero deformation $\mathrm{d}p/\mathrm{d}t \not\propto v$, unlike in conformal $\mathcal{N}=4$ SYM:

$$\frac{\mathrm{d}p}{\mathrm{d}t}\{\text{CGN, Str, Einst}\} = -\frac{\sqrt{\lambda}}{2\pi}\frac{\{\mathrm{e}^{c^2z_\mathrm{WS}^2}, \mathrm{e}^{\kappa z_\mathrm{WS}^2}, 1\}}{z_\mathrm{WS}^2}v\,, \quad \text{but } z_\mathrm{WS} = z_\mathrm{WS}(v).$$

Outlook

- Thermodynamics: Compute and analyze ρ conjugate to μ
  - ➢ Show thermodynamic (in-)consistency of CGN model
- Go beyond simple scalar ansatz $\phi \propto z^2$; analyze phase transition
- Use larger class of models, *e. g.* 'more realistic' 2-parameter models
- Different implementation of chemical potential?

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