Lattice QCD study on quark mass dependence of quarkonium properties at finite temperature

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Abstract

We report our lattice QCD study on quarkonium properties at finite temperature. We perform numerical simulations on large and fine isotropic lattices by using quenched gauge field configurations. To investigate differences between charmonium and bottomonium states, we vary the quark masses in the range between the charm and bottom masses. Spatial and temporal meson correlation functions are computed at temperatures in a range from about $0.7T_c$ to $1.4T_c$. We discuss the change and dissociation of various quarkonium states in the QGP by showing the temperature and quark mass dependence of the quarkonium correlation functions as well as some preliminary results of related physical quantities: the screening masse, the quark number susceptibility and the heavy quark diffusion constant.

1. Introduction

- Suppression of yields of the quarkonium such as J/Ψ in relativistic heavy ion collision experiments at RHIC and LHC is an important signal of QGP formation [1].
- Sequential Υ suppression reported by the CMS collaboration in LHC [2] inspires theoretical interest of not only the charmonium but also the bottomonium in-medium behavior.
- Theoretical understanding of the quarkonium behavior at finite temperature plays important role to explain experimental data.
- The meson spectral function can tell information about dissociation of bound states as well as transport properties, e.g. the heavy quark diffusion constant.
- In this study we investigate the quarkonium properties at finite temperature by first principle lattice QCD calculations.
- We perform numerical simulations with several quark masses to understand the difference between the charmonium and bottomonium.

2. Simulation setup

- Standard Wilson gauge & O(a)-improved Wilson fermion actions
- In quenched QCD
- Gauge coupling: β = 7.544

→ lattice spacing: a = 0.0127 fm (a^{-1} = 15.5 GeV)

- On $144^3 \times N_\tau$ isotropic lattices (See **Table 1**.)
- With 6 quark masses (See **Table 2**.)

Table 1

N_τ	T/T_c	# confs.
72	0.71	246
48	1.07	462
42	1.22	660
36	1.42	288

$T = 1/N_\tau a$

$T_c \approx 303$ MeV

Table 2

κ	$m_0 a$	$m_{\overline{MS}}(m)$ [GeV]	m_V [GeV]
0.13236	0.031135	0.92168	3.092(4)
0.13220	0.035707	1.0047	3.274(4)
0.13180	0.047186	1.2082	3.734(4)
0.13100	0.070353	1.6045	4.641(3)
0.12950	0.11456	2.3095	6.282(3)
0.12641	0.20894	3.6302	9.450(3)

$m_0 a = \frac{1}{2\kappa} - \frac{1}{2\kappa_c}$: bare quark mass κ_c: critical κ value m_V : vector meson mass

$m_{\overline{MS}}(m)$: quark mass renormalized by $\overline{\mathrm{MS}}$ scheme at μ=m

3. Meson correlation and spectral functions

- Euclidian meson correlation function

$$G_H(\tau) \equiv \int d^3\mathbf{x} \langle J_H(\tau,\mathbf{x}) J_H(0,\mathbf{0})\rangle = \int_0^\infty \frac{d\omega}{2\pi} \boxed{\rho_H(\omega)} K(\tau,\omega)$$

Meson spectral function

having all information about in-medium quarkonium properties

Table 3

Channel	Γ_H	J^{PC}	$c\bar{c}$	$b\bar{b}$
PS	γ_5	0^{-+}	η_c	η_b
V	γ_μ	1^{--}	J/ψ	Υ
S	$\mathbf{1}$	0^{++}	χ_{c0}	χ_{b0}
AV	$\gamma_5\gamma_\mu$	1^{++}	χ_{c1}	χ_{b1}

$$J_H(\tau,\mathbf{x}) = \bar{\psi}(\tau,\mathbf{x})\Gamma_H\psi(\tau,\mathbf{x})$$

$$K(\tau,\omega) \equiv \frac{\cosh(\omega(\tau - 1/2T))}{\sinh(\omega/2T)}$$

4. Screening mass

- Spatial meson correlation function

$$G_H(z) \equiv \int dxdy \int_0^{1/T} d\tau \langle J_H(\tau,\mathbf{x}) J(0,\mathbf{0})\rangle \xrightarrow[z>>1/T]{} e^{-\boxed{M_{\mathrm{scr}}} z}$$

Screening mass

- If there is a lowest lying bound state: $M_{\mathrm{scr}} = M$: bound state mass
- High T limit (free quark case) : $M_{\mathrm{scr}} = 2\sqrt{(\pi T)^2 + m_q^2}$ m_q : quark mass

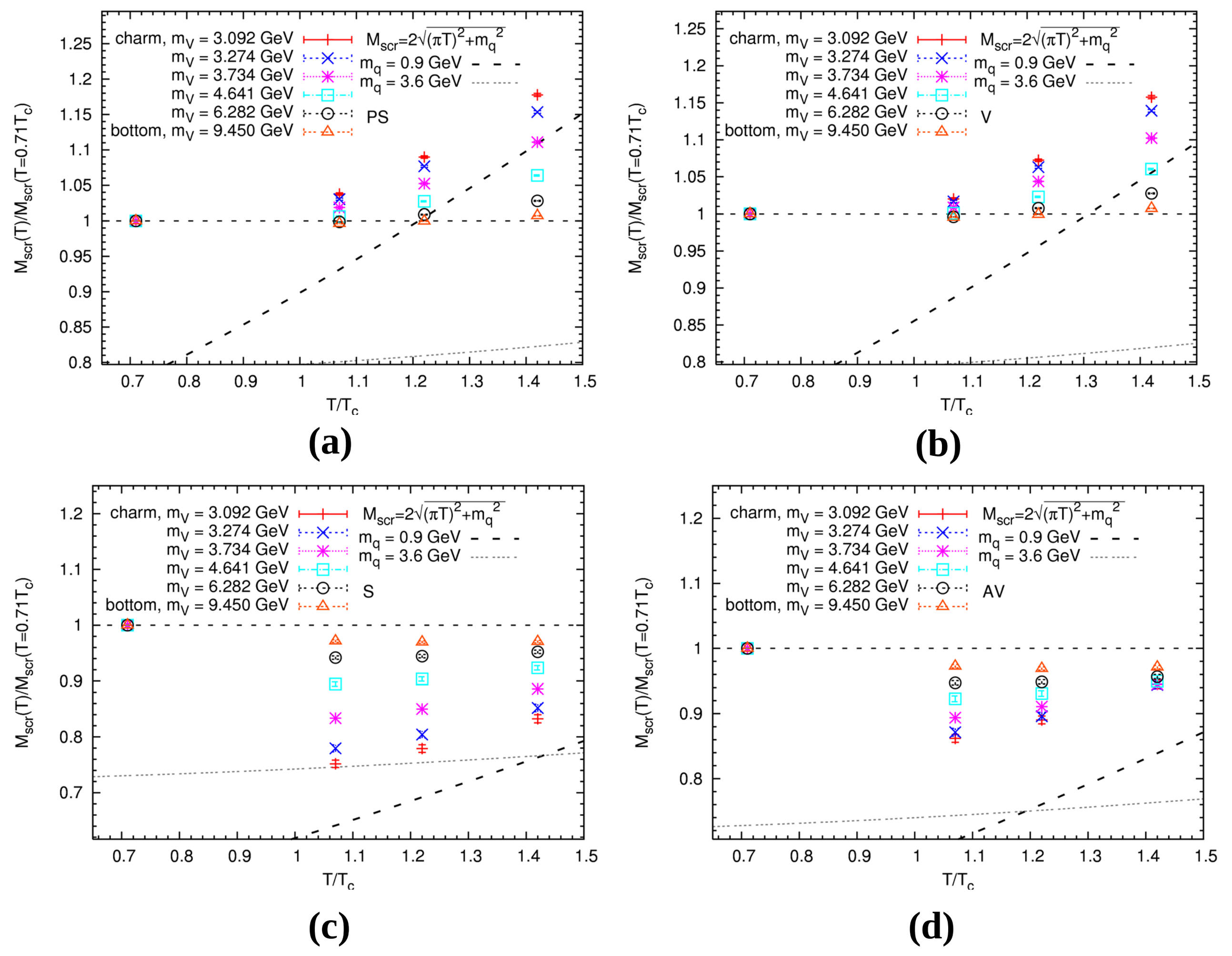


Fig. 1: Temperature and quark mass dependence of the screening masses M_{scr} for PS (a), V (b), S (c) and AV (d) channels. Values in the free quark case are also shown by dashed and dotted curves with the $\overline{\mathrm{MS}}$ quark masses for the charm and bottom, respectively. M_{scr} monotonically increases as temperature increases for the S-wave channels while data for the P-wave channels decrease at T_c then increase. Heavier quark data have smaller temperature dependence.

5. Quark number susceptibility

χ_{00} : Quark number susceptibility

$$\rho_{00}^V(\omega) = 2\pi\chi_{00}\omega\delta(\omega) \;\Rightarrow\; G_{00}^V(\tau) = T\chi_{00}$$

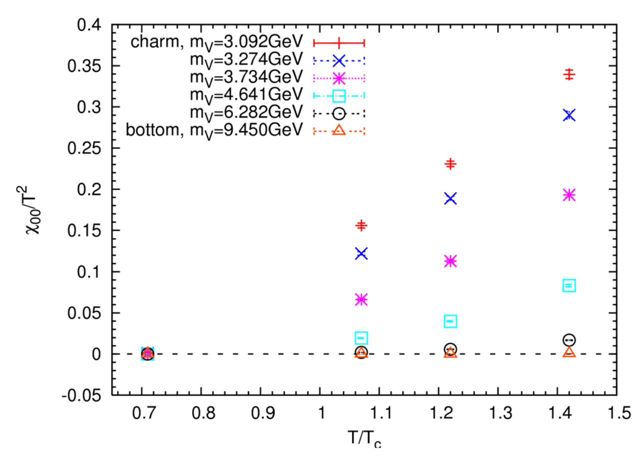
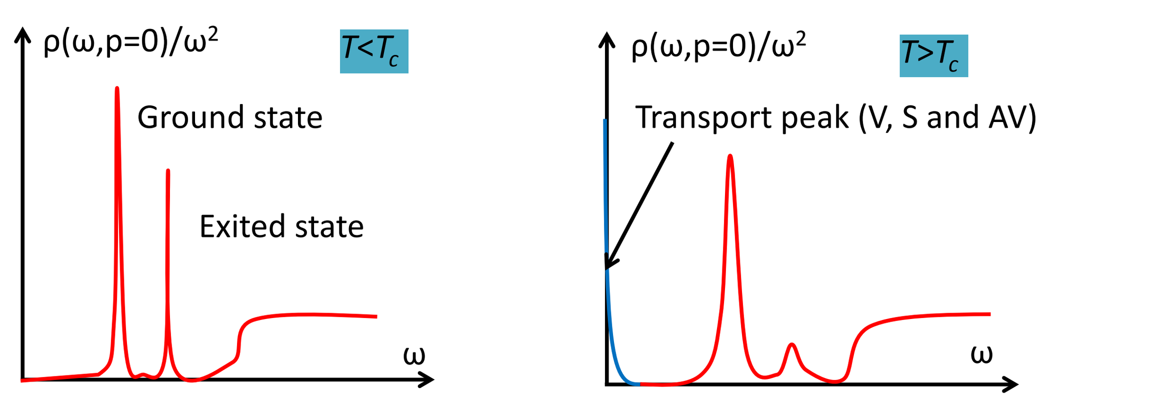


Fig. 2: Temperature dependence of the quark number susceptibility χ_{00}. Similarly to the screening mass in the S-wave case, χ_{00} increases monotonically as temperature increases and heavier quark data have smaller temperature dependence.

6. Reconstructed correlator and diffusion constant

- Expected modification of the spectral function above T_c



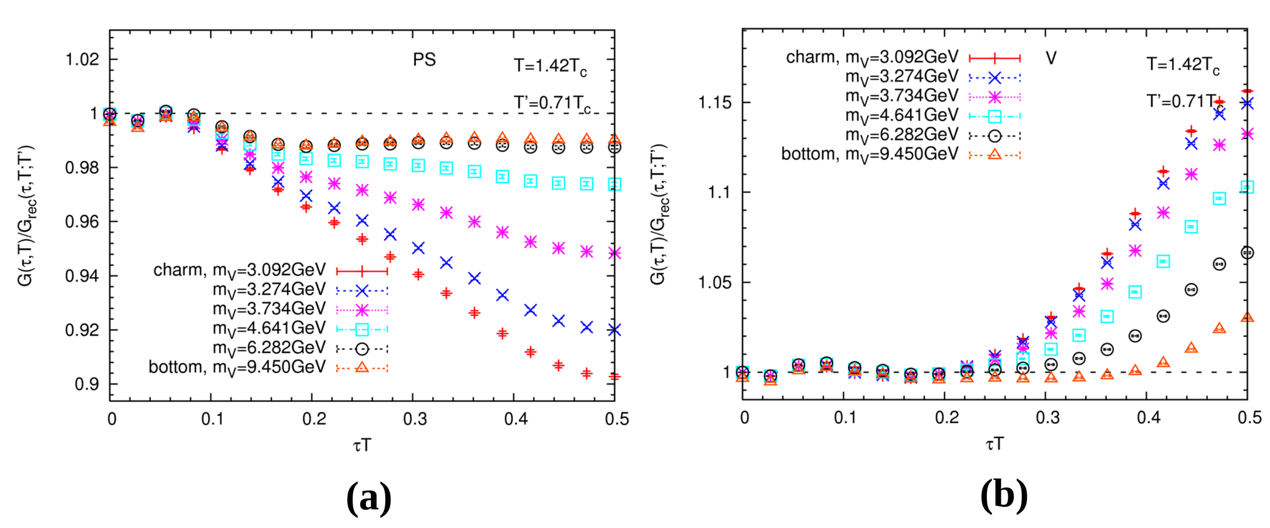
- Smearing of the bound state peaks
- Appearance of the transport peak around ω=0 (V, S and AV)

- Reconstructed correlator

$$G_{\mathrm{rec}}(\tau,T;T') \equiv \int_0^\infty \frac{d\omega}{2\pi}\rho(\omega,T')K(\tau,\omega,T)$$

$$G_{\mathrm{rec}}(\tau,T;T') = \sum_{\tau'=\tau;\Delta\tau'=N_\tau}^{N'_\tau - N_\tau + \tau} G(\tau',T')$$

a useful tool to investigate temperature dependence of the spectral function



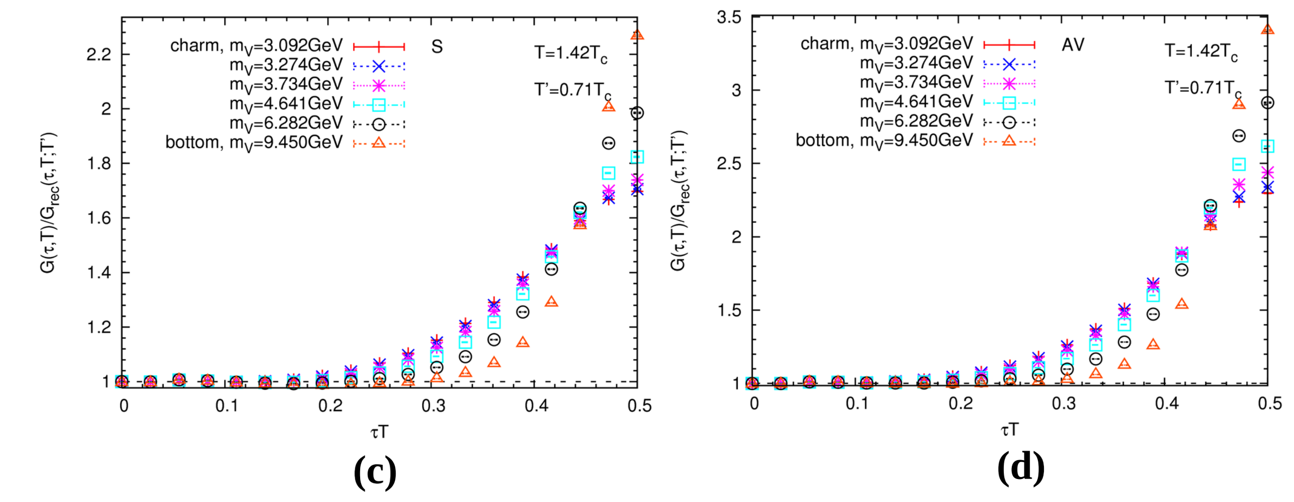


Fig. 4: Quark mass dependence of the ratio $G(\tau,T)/G_{\mathrm{rec}}(\tau,T;T')$ for PS (a), V(b), S (c) and AV(d) channels. The ratio deviates from 1 at long distance, which suggests the modification of the spectral function in the low frequency region. Especially for V, S and AV channels there is strong enhancement at the long distance, which indicates the appearance of the transport peak around zero frequency.

- Diffusion constant

$$D = \frac{1}{6\chi_{00}} \lim_{\omega\to 0} \sum_{i=1}^{3} \frac{\rho_{ii}^V(\omega,\mathbf{0})}{\omega}$$

$\rho_{ii}^V(\omega)$: spatial component of vector spectral function

related to the vector spectral function around zero frequency

- If the transport peak is most dominant part of the difference $G(1/2T,T)$–$G_{\mathrm{rec}}(1/2T,T;T')$ at $1.42T_c$, one can roughly estimate the diffusion constant.

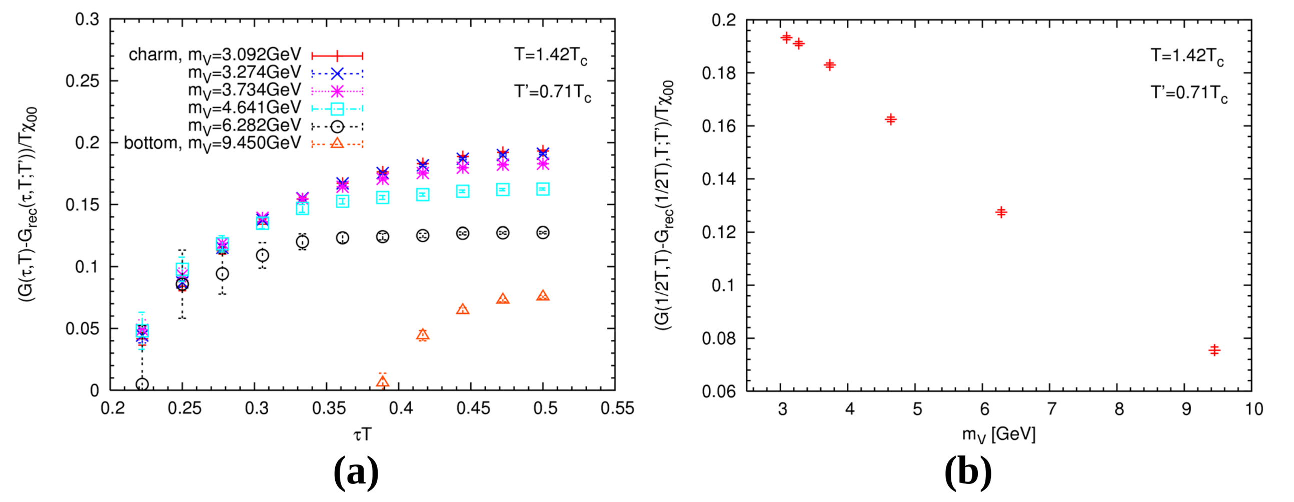


Fig. 5: The difference $G(\tau,T)$-$G_{\mathrm{rec}}(\tau,T;T')$ (a) and its value at the midpoint $\tau=1/2T$ (b). The midpoint value decreases as the quark mass increases.

Assuming an Ansatz [3],

$$\rho_{ii}^V(\omega << T) = 2\chi_{00}\frac{T}{M}\frac{\omega\eta}{\omega^2+\eta^2} \qquad \eta \equiv \frac{T}{MD} \qquad M \equiv m_q a$$

$$2\pi TD \approx \begin{cases} 1-3 \ (m_q = 1-2 \text{ GeV}) \text{ for charm,} \\ 0.3-0.8 \ (m_q = 3.5-5 \text{ GeV}) \text{ for bottom.} \end{cases}$$

7. Conclusion and outlook

- Quarkonium properties at finite temperature are studied in quenched QCD on large and fine isotropic lattices.
- To investigate the difference between the charmonium and bottomonium, quark mass is varied in the range between the charm and bottom masses.
- The screening mass and the quark number susceptibility have smaller temperature dependence for the heavier quark.
- The analysis using the reconstructed correlator suggests the thermal modification of the spectral function in the low frequency region as well as the appearance of the transport peak for V, S, AV channels.
- The difference between the measured and reconstructed correlators gives a rough estimation of the heavy quark diffusion constant $2\pi TD \approx 1-3$ for charm and $0.3-0.8$ for bottom, where further investigation is needed to check the uncertainty.
- Investigating the spectral function directly and taking the continuum limit are our future plan.

8. References

[1] T. Matsui and H. Satz, Phys. Lett. B **178**, 416 (1986).
[2] S. Chatrchyan et al. [CMS Collaboration], Phys. Rev. Lett. **109**, 222301 (2012) [arXiv:1208.2826[nucl-ex]].
[3] H. T. Ding, A. Francis, O. Kaczmarek, F. Karsch, H. Satz and W. Soeldner, Phys. Rev. D 86, 014509 (2012) [arXiv:1204.4945 [hep-lat]].