Multipole solutions of hydrodynamics
arXiv:1405.3877

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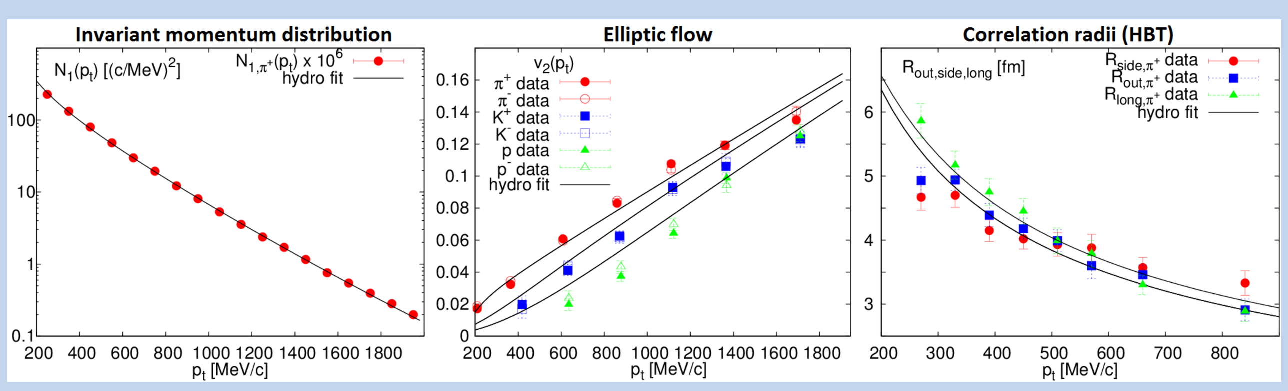
Hydrodynamics

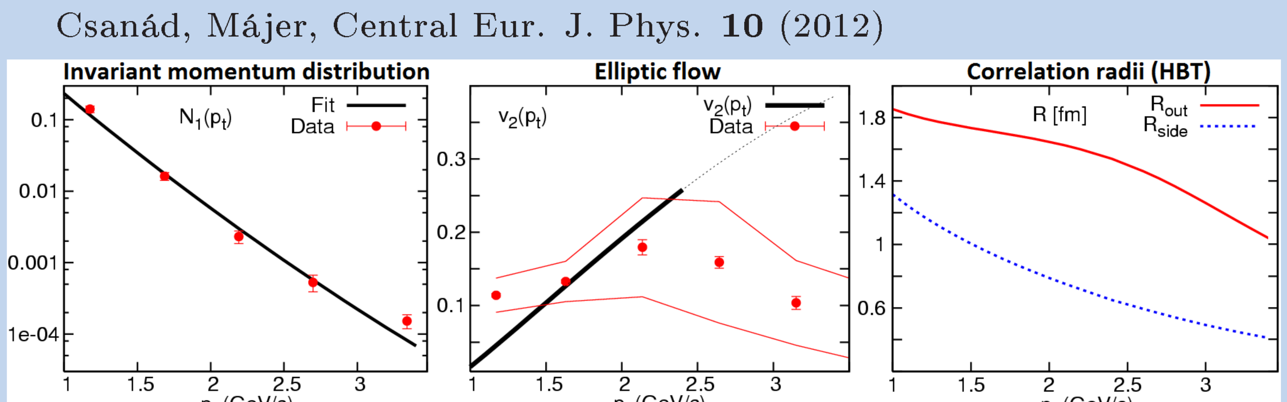
- Collective dynamics observed at RHIC
  PHENIX, Nucl. Phys. A **757**, 184 (2005)
- **Exact, analytic solutions**: important to determine initial and final state
- Famous 1+1D solutions: Landau, Hwa, Bjorken
- Many new 1+1D solutions, few 1+3D, with spherical/axial/ellipsoidal symmetry
- Energy-mom. tensor in perfect fluid:
$$T^{\mu\nu} = (\varepsilon + p)u^\mu u^\nu - pg^{\mu\nu}$$
- Continuity & e-m. conservation
$$\partial_\mu(nu^\mu) = 0 \text{ and } \partial_\nu T^{\mu\nu} = 0$$
- EoS: $\varepsilon = \kappa p$, $\kappa = c_{\text{sound}}^{-2}$, if const.
- Temperature equation via $p = nT$
$$T\partial_\mu u^\mu + \kappa u^\mu \partial_\mu T = 0$$
- Without a conserved charge: use entropy density σ
$$\varepsilon + p = T\sigma \;\Rightarrow\; d\varepsilon = Td\sigma \text{ and } dp = \sigma dT$$
- Continuity equation for σ:
$$\partial_\nu(\sigma u^\nu) = 0,$$
- If $\kappa = $ const.: exactly the same eqs.
- Solutions valid for $\{u^\mu, n, T\}$ and/or $\{u^\mu, \sigma, T\}$

Known ellipsoidal solution

- First 3D relativistic solution
  Csörgő *et al.*, Heavy Ion Phys. **A21**, 73 (2004)
$$u^\mu = \frac{x^\mu}{\tau}, \tau = \sqrt{x_\mu x^\mu},\ n = n_f \frac{\tau_0^3}{\tau^3}\nu(s)$$
$$T = T_f \left(\frac{\tau_0}{\tau}\right)^{\frac{3}{\kappa}} \frac{1}{\nu(s)},\quad s = \frac{r_x^2}{X^2} + \frac{r_y^2}{Y^2} + \frac{r_z^2}{Z^2}$$
- $\nu(s)$ arbitrary function of scaling variable s
- X, Y, Z: axes of expanding ellipsoid
- Non-accelerating, ie. $u^\nu \partial_\nu u^\mu = 0$.
- Hadron, γ, lepton source calculable
- Spectra, flow, HBT calculable
- **Describes hadron data**
  Csanád, Vargyas, Eur. Phys. J. A **44**, 473 (2010)



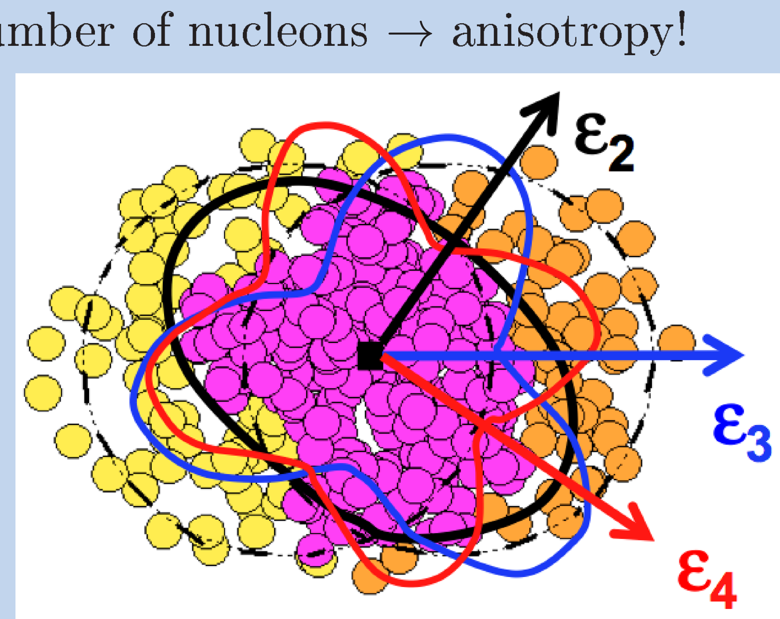
- **Describes photon data**
  Csanád, Májer, Central Eur. J. Phys. **10** (2012)



- Even compatible with thermal dileptons
  Csanád, Krizsán, Central Eur.J.Phys. **12** (2014)

Higher order anisotropies?

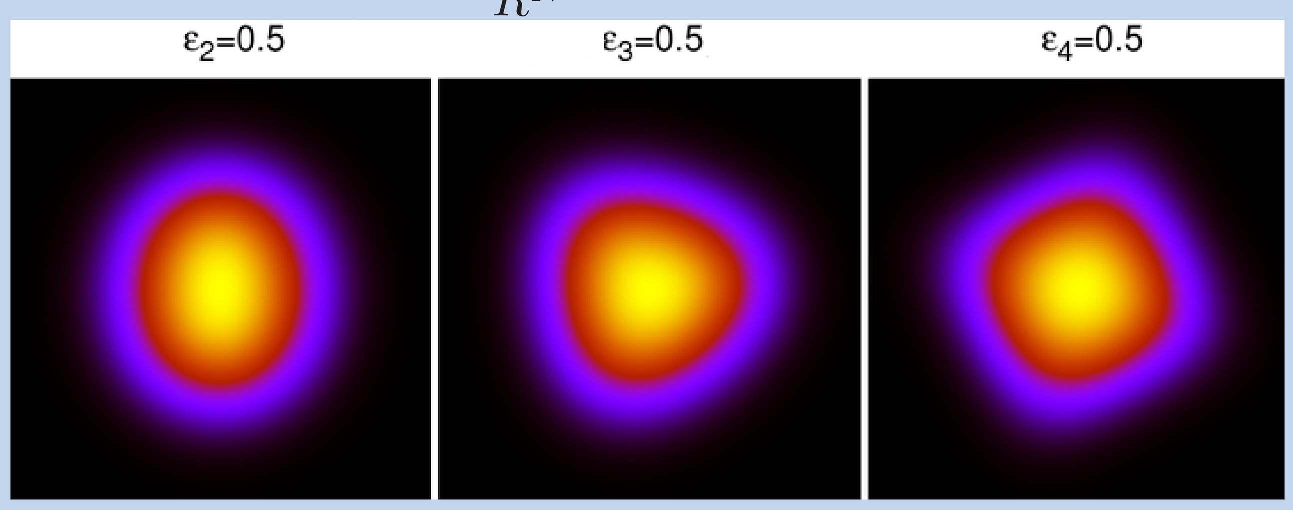
- Finite number of nucleons → anisotropy!



- Successfully utilized in numerical calculations
- **Exact solutions handling this?**

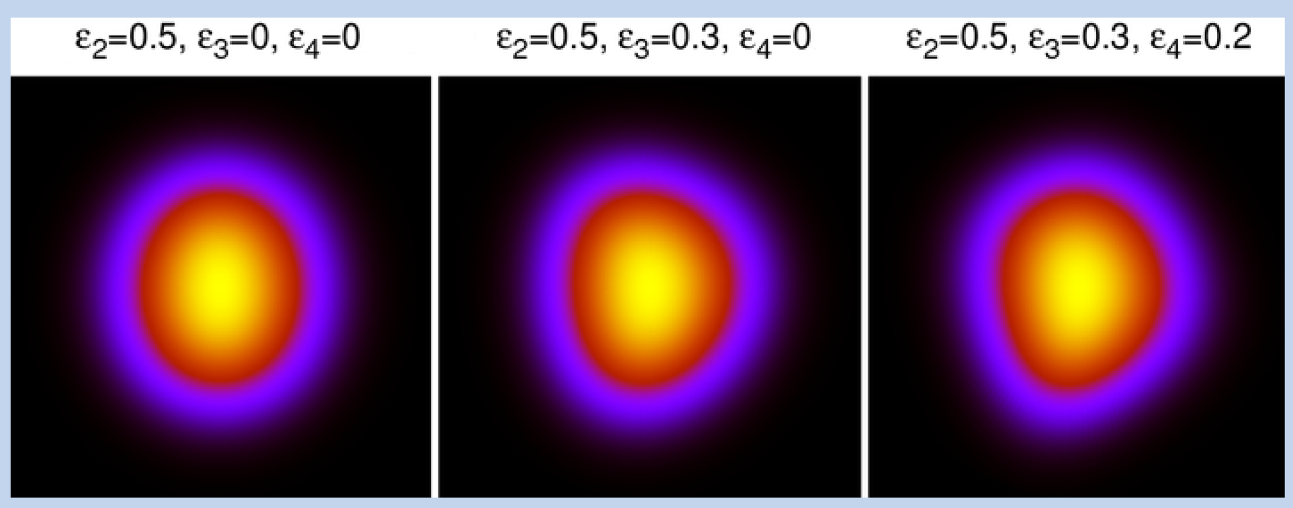
Generalization of elliptic symmetry

- **How to generalize** the ellipsoidal scaling variable of
$$s = \frac{x^2}{X^2} + \frac{y^2}{Y^2} + \frac{z^2}{Z^2}$$
- Using
$$\frac{1}{R^2} = \frac{1}{X^2} + \frac{1}{Y^2} \text{ and } \epsilon = \frac{X^2 + Y^2}{X^2 - Y^2},$$
rewrite the transverse part of s to
$$s = \frac{r^2}{R^2}\left(1 + \epsilon\cos(2\phi)\right)$$
- Generalize: ***N*-pole symmetry** in the transverse plane
$$s = \frac{r^N}{R^N}\left(1 + \epsilon_N \cos(N\phi)\right)$$



Multipole symmetries combined

- **Multiple symmetries** can be combined:
$$s = \sum_N \frac{r^N}{R^N}\left(1 + \epsilon_N \cos(N(\phi - \psi_N))\right)$$
- Aligned by Nth order reaction planes ψ_N



- Basically any shape can be described!
- In three dimensions: add $\frac{z^N}{R^N}$
- No change in the longitudinal direction
- More general scaling variables possible, higher order asymmetries in 3D

New solutions

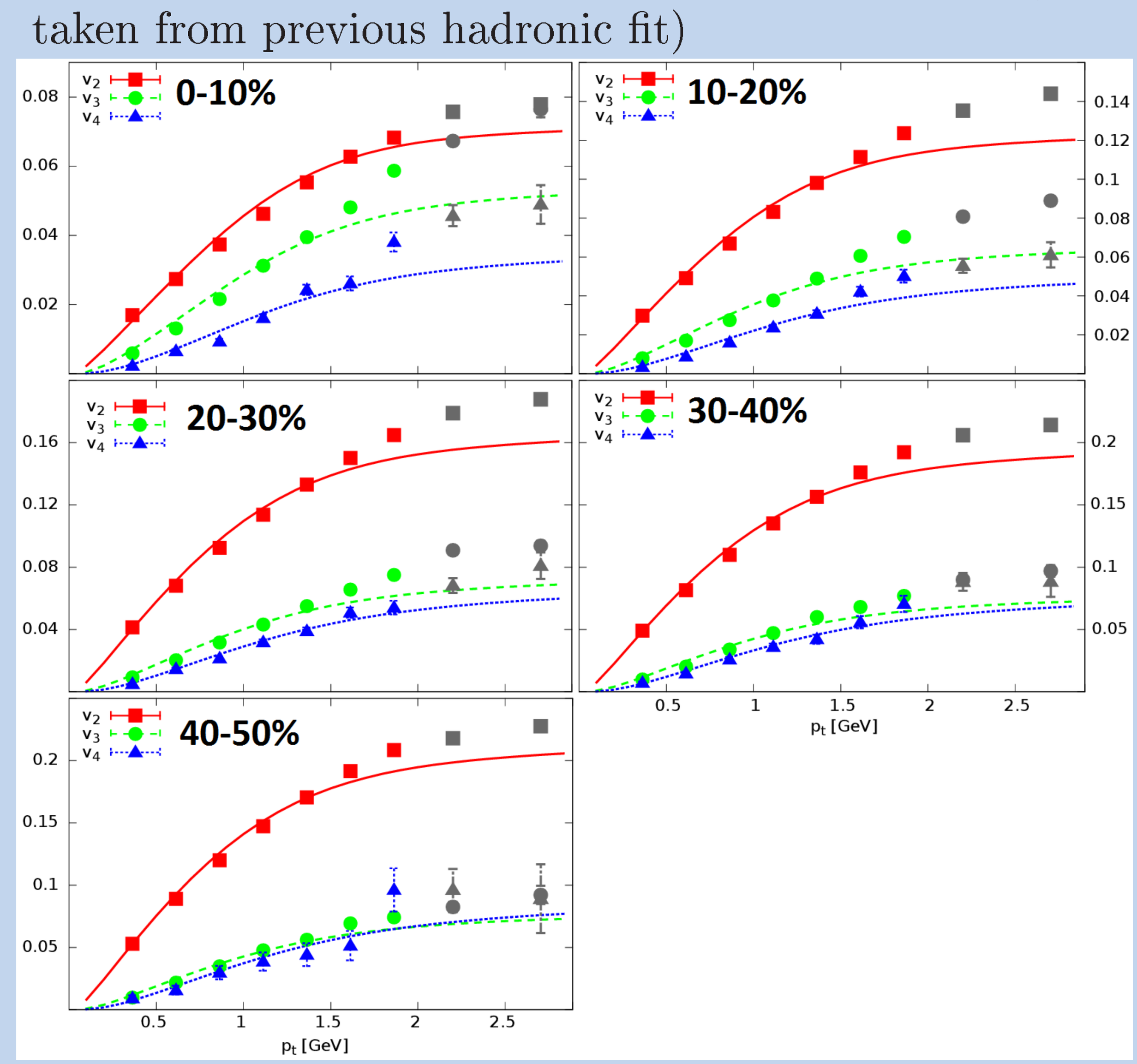
- **New solutions** with multipole symmetries
$$s = \sum_N \frac{r^N}{R^N}\left(1 + \epsilon_N \cos(N(\phi - \psi_N))\right) + \frac{z^N}{Z^N}$$
$$u^\mu = \gamma\left(1, \frac{\dot{R}}{R} r\cos\phi, \frac{\dot{R}}{R} r\sin\phi, \frac{\dot{R}}{R} z\right)$$
$$T = T_f \left(\frac{\tau_f}{\tau}\right)^{3/\kappa} \frac{1}{\nu(s)} \text{ and } n = n_f \left(\frac{\tau_f}{\tau}\right)^3 \nu(s)$$
- **Higher order harmonics**: via hadronic source

Observables

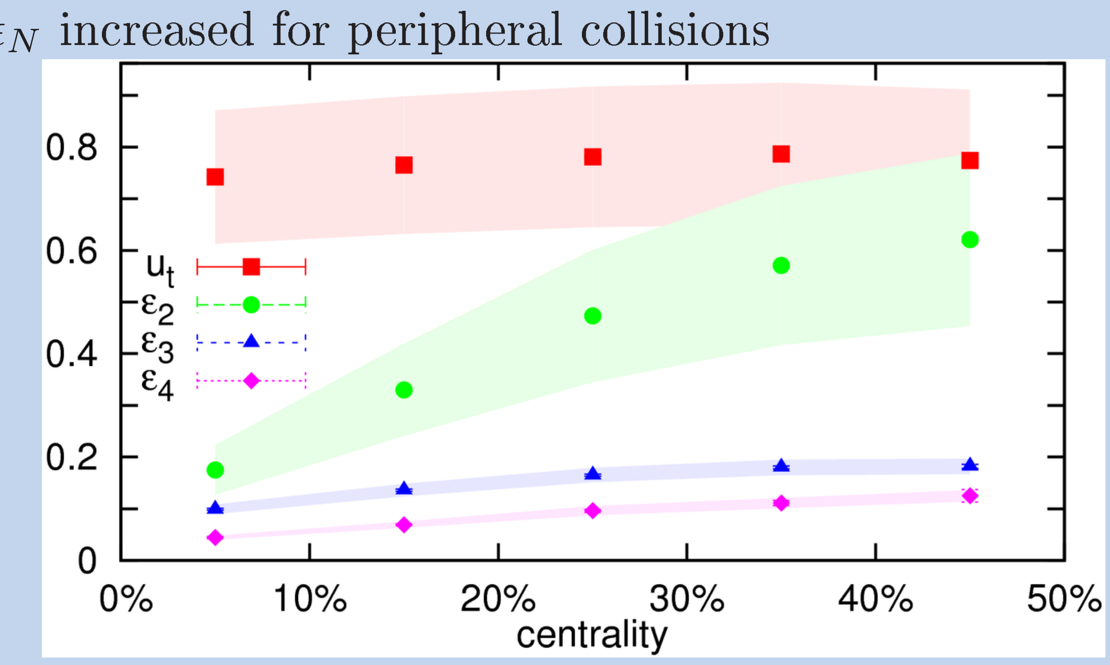
- Hadronic source: Maxwell-Jüttner distribution:
$$S(x,p)d^4x = n(x)\exp\left[-\frac{p_\mu u^\mu(x)}{T(x)}\right]\delta(\tau - \tau_f)\frac{p_\mu u^\mu}{u^0}d^4x.$$
- **Transverse momentum spectrum and flow**:
$$N(p) = \int S(x,p)d^4x \text{ and } N(p_t) = \frac{1}{2\pi}\int_0^{2\pi} N(p)|_{p_z=0}\, d\alpha$$
$$v_n(p_t) = \frac{\frac{1}{2\pi}\int_0^{2\pi} N(p)|_{p_z=0}\cos(n\alpha)d\alpha}{N(p_t)} = \langle\cos(n\alpha)\rangle$$
- Choose **Gaussian temperature profile**, i.e. $\nu(s) = e^{bs}$

Comparison to PHENIX data

- PHENIX measured v_2, v_3 and v_4
  Phys. Rev. Lett. **107** (2011) 252301
- Hadron fits with a similar, but ellipsoidal model done in
  Csanád, Vargyas, Eur. Phys. J. A **44**, 473 (2010)
- Fitted parameters: ϵ_N and transverse flow u_t (others taken from previous hadronic fit)



- **Successful fit**
- See details in arXiv:1405.3877
- Results not sensitive to temperature gradient b
- Good fit possible for $b \in [0.05, 0.20]$
- Fixing b: systematic error on the other parameters
- u_t does not vary strongly
- ϵ_N increased for peripheral collisions



Parameter dependence of v_n's

- Strong dependence on u_t (note shape change) and b
- Weaker dependence on T_f
- No dependence on τ_f (not shown here)
- Practically linear dependence on ϵ_n (not shown here)