Effective Field Theory vs. Anomalous Couplings

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Motivation

- July 4, 2012: New particle found! (Compatible with SM Higgs)

Besides that, no sign of new physics!

➡ **Next steps:**

- Try to measure Higgs couplings as precise as possible.
  (look for deviations from SM Higgs)
- Continue search for new physics.
  But what to look for ? SUSY? Extra dimensions ? Little Higgs ? ...

General approach:
Try to parametrize deviations from SM in a model-independent way

Effective Field Theory

Which features should an extension of the SM have ?

- Extension should satisfy S-Matrix axioms (unitary, analyticity).
- Symmetries of the SM should be respected:
  - Lorentz invariance
  - $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge symmetry
- Should be possible to recover SM in an appropriate limit.
- Extension should be general enough to capture any physics beyond SM, but also give some guidance where to look for new physics.
- Should be possible to calculate radiative corrections to SM interactions in the extended theory.
- Should be possible to calculate radiative corrections in the new interactions in the extended theory

Effective Field Theory

This features can be realized by an effective quantum field theory [Weinberg '79]

Construction:

- Add additional operators to the SM Lagrangian. (SM contains operators up to dim 4 → coupling constants dimensionless)

$$\mathcal{L}_{eff} = \mathcal{L}_{SM} + \sum_{n=1}^{\infty} \sum_i \frac{c_i^{(n)}}{\Lambda^n} \mathcal{O}_i^{(n+4)}$$

- Operators are constructed by defining particle content, e.g: $\mathcal{O}_{WW} = \phi^\dagger \hat{W}^{\mu\nu} \hat{W}_{\mu\nu} \phi$
- Coupling constants inverse powers of mass, mass scale Λ
- Operators are suppressed if accessible energy low compared to mass scale.

➡ Lower dimensional operators more important.
(relevant: dim 6, for complete list, see [Buchmueller, Wyler '86])

Effective Field Theory

Important properties of the effective theory:

- Low energy effective theory ($s < \Lambda^2$).
- Λ : Scale of new physics.
- Fields defined in operator: Particles present at low energies.
- Recover Standard Model in the limit $\Lambda \to \infty$.

EXAMPLE: Fermi theory of weak interaction / Z'

Z' boson: $\mathcal{O} = \frac{c}{M_{Z'}^2}\bar{\psi}\psi\bar{\psi}\psi$

At low energies: Four fermions present, $\Lambda = M_{Z'}$

Effective Field Theory

BUT THIS ALSO MEANS:

- Low energy effective theory only valid below scale Λ.
- If $s \approx \Lambda^2$: Higher dimensional operators are no longer suppressed

  → Approach no longer useful.
- For Z' example: If $s \approx M_{Z'}^2$:

  Effective theory has to be replaced by a theory which contains Z' as low energy content.
- Effects of higher dim. operators grow with energy

  → Violation of unitarity!

Anomalous couplings

Anomalous couplings usually used for electroweak interactions [Gaemers,Gounaris '79]

- Lagrangian approach:

$$\begin{aligned}\mathcal{L} &= ig_{WWV}\Big(g_1^V(W^+_{\mu\nu}W^{-\mu} - W^{+\mu}W^-_{\mu\nu})V^\nu + \kappa_V W^+_\mu W^-_\nu V^{\mu\nu} + \frac{\lambda_V}{M_W^2}W^{\nu+}_\mu W^{-\rho}_\nu V^\mu_\rho \\ &+ig_4^V W^+_\mu W^-_\nu(\partial^\mu V^\nu + \partial^\nu V^\mu) - ig_5^V\epsilon^{\mu\nu\rho\sigma}(W^+_\mu\partial_\rho W^-_\nu - \partial_\rho W^+_\mu W^-_\nu)V_\sigma \\ &+\tilde{\kappa}_V W^+_\mu W^-_\nu \tilde{V}^{\mu\nu} + \frac{\tilde{\lambda}_V}{m_W^2}W^{\nu+}_\mu W^{-\rho}_\nu \tilde{V}^\mu_\rho\Big)\end{aligned}$$

$$V=\gamma, Z;\; W^\pm_{\mu\nu} = \partial_\mu W^\pm_\nu - \partial_\nu W^\pm_\mu,\; V_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu$$

Note:

- Not necessarily gauge invariant, need to impose

$$g_1^\gamma = 1 \qquad g_4^\gamma = g_5^\gamma = 0$$

- Additional terms can be generated by adding derivatives ∂_μ

  → Would get another factors M_W^{-1} , as this is the only scale in the theory.

Anomalous couplings

- Vertex function approach:

$$\begin{aligned}\Gamma_V^{\alpha\beta\mu} \;=\;& f_1^V (q-\bar{q})^\mu g^{\alpha\beta} - \frac{f_2^V}{M_W^2}(q-\bar{q})^\mu P^\alpha P^\beta + f_3^V (P^\alpha g^{\mu\beta} - P^\beta g^{\mu\alpha}) \\ & + i f_4^V (P^\alpha g^{\mu\beta} + P^\beta g^{\mu\alpha}) + i f_5^V \epsilon^{\mu\alpha\beta\rho}(q-\bar{q})_\rho \\ & - f_6^V \epsilon^{\mu\alpha\beta\rho} P_\rho - \frac{f_7^V}{m_W^2}(q-\bar{q})^\mu \epsilon^{\alpha\beta\rho\sigma} P_\rho (q-\bar{q})_\sigma\end{aligned}$$

$P, q, \bar{q}$ four momenta of V, W^-, W^+

- Momentum space analogue of Lagrangian approach.
- Coefficients f_i^V are form factors that depend on P^2, but functional form $f_i^V(P^2)$ is arbitrary.
- W-boson charge and gauge invariance require $f_1^\gamma(0) = 1, \quad f_4^\gamma, f_5^\gamma \sim P^2$
- Sometimes vertex- and Lagrangian approach are mixed by treating $g^V, \kappa_V, \Lambda_V, \cdots$ as form factors (but Lagrangian is in position space)

Transitions between effective field theory and anomalous coupling

Start with the operators contributing to trilinear couplings [Hagiwara,Ishihara,Szalapski,Zeppenfeld '92]

$$\mathcal{O}_{WWW} = \mathrm{Tr}[W_{\mu\nu}W^{\nu\rho}W_\rho^\mu] \qquad \mathcal{O}_{\tilde{W}WW} = \mathrm{Tr}[\tilde{W}_{\mu\nu}W^{\nu\rho}W_\rho^\mu]$$

$$\mathcal{O}_W = (D_\mu\Phi)^\dagger W^{\mu\nu}(D_\nu\Phi) \qquad \mathcal{O}_{\tilde{W}} = (D_\mu\Phi)^\dagger \tilde{W}^{\mu\nu}(D_\nu\Phi)$$

$$\mathcal{O}_B = (D_\mu\Phi)^\dagger B^{\mu\nu}(D_\nu\Phi)$$

Relations between approaches can be derived:

$$g_1^Z = 1 + c_W \frac{m_Z^2}{2\Lambda^2}$$

$$\kappa_\gamma = 1 + (c_W + c_B)\frac{m_W^2}{2\Lambda^2}$$

$$\kappa_Z = 1 + (c_W - c_B \tan^2\theta_W)\frac{m_W^2}{2\Lambda^2}$$

$$\lambda_\gamma = \lambda_Z = c_{WWW}\frac{3g^2 m_W^2}{2\Lambda^2}$$

$$g_4^V = g_5^V = 0$$

$$\tilde{\kappa}_\gamma = c_{\tilde{W}}\frac{m_W^2}{2\Lambda^2}$$

$$\tilde{\kappa}_Z = -c_{\tilde{W}}\tan^2\theta_W \frac{m_W^2}{2\Lambda^2}$$

$$\tilde{\lambda}_\gamma = \tilde{\lambda}_Z = c_{\tilde{W}WW}\frac{3g^2 m_W^2}{2\Lambda^2}$$

$$f_1^\gamma = 1 + c_{WWW}\frac{3g^2P^2}{4\Lambda^2}$$

$$f_1^Z = 1 + c_W\frac{m_Z^2}{\Lambda^2} - c_{WWW}\frac{3g^2P^2}{4\Lambda^2}$$

$$f_2^\gamma = f_2^Z = c_{WWW}\frac{3g^2m_W^2}{2\Lambda^2}$$

$$f_3^\gamma = 2 + (c_B + c_W)\frac{m_W^2}{2\Lambda^2} + c_{WWW}\frac{3g^2m_W^2}{2\Lambda^2}$$

$$f_3^Z = 2 + \left(c_W(1+\cos^2\theta_W) - c_B\sin^2\theta_W\right)\frac{m_Z^2}{2\Lambda^2} + c_{WWW}\frac{3g^2m_W^2}{2\Lambda^2}$$

$$f_4^V = f_5^V = 0$$

$$f_6^\gamma = +c_{\tilde{W}}\frac{m_W^2}{2\Lambda^2} - c_{\tilde{W}WW}\frac{3g^2m_W^2}{2\Lambda^2}$$

$$f_6^Z = -c_{\tilde{W}}\tan^2\theta_W\frac{m_W^2}{2\Lambda^2} - c_{\tilde{W}WW}\frac{3g^2m_W^2}{2\Lambda^2}$$

$$f_7^\gamma = f_7^Z = -c_{\tilde{W}WW}\frac{3g^2m_W^2}{4\Lambda^2}$$

Effective Theory vs. Anomalous Couplings

Relations between different couplings:

$$\Delta g_1^Z = \Delta\kappa_Z + \tan^2\theta_W \Delta\kappa_\gamma \qquad \Delta g_1^Z = g_1^Z - 1,\ \Delta\kappa_{\gamma,Z} = \kappa_{\gamma,Z} - 1$$

$$0 = \tilde{\kappa}_Z + \tan^2\theta_W \tilde{\kappa}_\gamma$$

➡ Effective field theory approach leads to simplifications (due to restriction to dim 6).

➡ Transition between the approaches work only if coupling constants are not modified to form factors !

Unitarity

Effective Field theory:

- Dimension 6 operators yield amplitudes $\sim \frac{s}{\Lambda^2}$.
- Will eventually violate unitarity → But then the theory is useless (low energy effective theory), should not be applied anymore!

Anomalous Couplings:

- Yields amplitudes $\sim \frac{s}{M_W^2}$.
- Leads to unitarity violation → Use vertex function and choose form factors such that they decrease at high energies (also done with lagrangian, hard to justify).

➡ Valid up to arbitrary energies.

IMPORTANT:

- Measurement will NEVER violate unitarity!
- Effective theory that fits data will not violate unitarity → No need for form factors!

Summary

- Term “Anomalous couplings” used for different approaches, not a unique name.
- Transition between EFT approach and anomalous couplings approach as long as there are no form factors and coupling constants are constant.
- EFT provides a better interpretation, in particular regarding applicability.
- EFT provides cleaner approach, separating more important contributions from less important ones.
- Form factors are introduced ad hoc to preserve unitarity, unnecessary in the EFT approach.