Top Quark Theory Overview (Pair Production at NNLO in QCD)

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SM @ LHC 2013, Freiburg, 9th – 12th April 2013

Status experiment (TOP2012)

TeVatron (CDF & D0 combined)

$\sigma\,(p\bar{p} \to t\bar{t})$ at $\sqrt{s} = 1.96$ TeV, assuming $m_t = 172.5$ GeV/c^2

$7.65 \pm 0.20\,(stat) \pm 0.29\,(syst) \pm 0.22\,(lumi)$ pb

$= 7.65 \pm 0.42$ pb (rel. 5.5%)

G. Petrillo

LHC @ 7 TeV (CMS di-lepton)

Combined $161.9 \pm 2.5^{+5.1}_{-5.0} \pm 3.6$

J. Andrea

(5%)

LHC @ 7 TeV (ATLAS & CMS combined)

$$\sigma_{t\bar{t}} = 173.3 \pm 2.3(\mathrm{stat.}) \pm 9.8(\mathrm{syst.})\ \mathrm{pb}$$

5.8 %

J. Andrea

LHC @ 8 TeV (CMS combined)

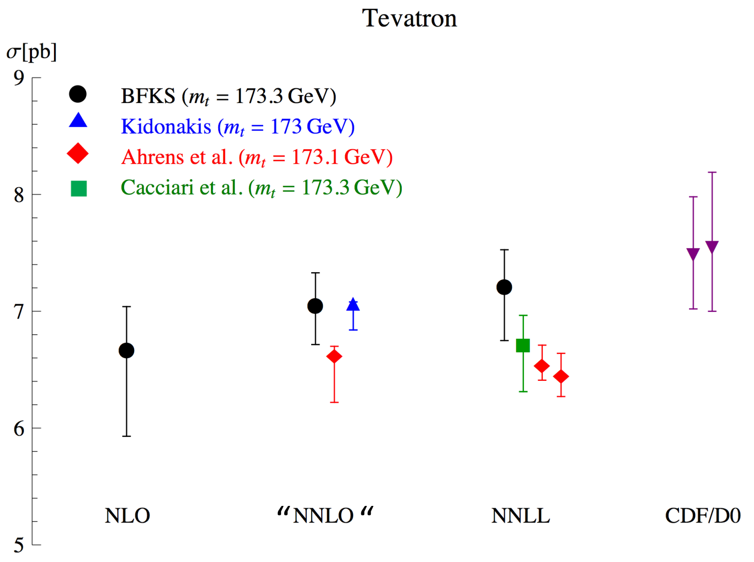
$$\sigma_{t\bar{t}} = 227 \pm 3\ (\mathrm{stat.}) \pm 11\ (\mathrm{syst.}) \pm 10\ (\mathrm{lumi})\ \mathrm{pb.}$$

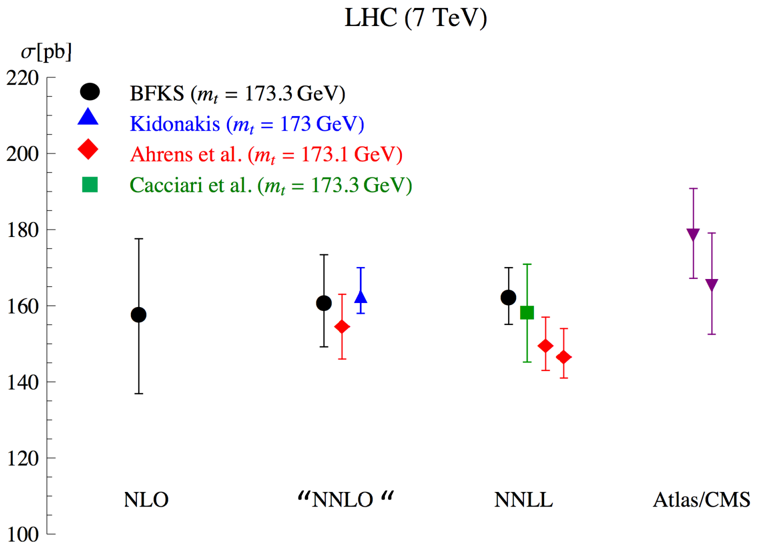
6.7 %

J. Andrea

Status theory (Outdated)

Just a year ago…





Beneke, Falgari, Klein, Schwinn, December 2011

Status theory (Current)

Before NNLO:

Beneke, Falgari, Klein, Schwinn `09-`11
Ahrens, Ferroglia, Neubert, Pecjak, Yang `10-`11
Kidonakis `04-`11
Aliev, Lacker, Langenfeld, Moch, Uwer, Wiedermann ’10
Cacciari, MC, Mangano, Mitov, Nason ‘11

NNLO:

Bärnreuther, MC, Mitov, Phys. Rev. Lett., April ‘12
MC, Mitov, JHEP, July ‘12
MC, Mitov, JHEP, October ‘12
MC, Fiedler, Mitov, submitted to Phys. Rev. Lett., March ’13

Publicly available software:

- **HATHOR**
  Aliev, Lacker, Langenfeld, Moch, Uwer, Wiedemann `10
  NLO + approximations for NNLO
- **Top++**
  Czakon, Mitov `11
  NNLO + NNLL soft gluon resummation in Mellin-space
- **TOPIXS**
  Beneke, Falgari, Klein, Piclum, Schwinn, Ubiali, Yan `12
  NLO + approximations for NNLO + NNLL soft and Coulomb resummation in x-space

Total cross section

- Factorization theorem

$$\sigma_{h_1h_2}(s,m_t)=\sum_{ij}\int dx_1dx_2\phi_{i/h_1}(x_1,\mu_F)\phi_{j/h_2}(x_2,\mu_F)\hat{\sigma}_{ij}(x_1x_2s,m_t,\alpha_s(\mu_R),\mu_R,\ \mu_F)$$

σ_{h_1,h_2}	hadronic cross section
$h_{1,2}$	hadrons
s	square of collider energy
m_t	top quark mass

$\phi_{i/h}$	PDF for parton i in hadron h
$\hat{\sigma}_{ij}$	partonic cross section
μ_R	renormalization scale
μ_F	factorization scale

- Scale dependence at fixed order of perturbation theory can be derived from Renormalization Group invariance

- The minimal object to calculate: $\hat{\sigma}_{ij}(\beta)$

$$\hat{\sigma}_{ij}(\hat{s},m_t,\alpha_s(m_t),m_t,m_t)=\frac{\alpha_s^2(m_t)}{m_t^2}\hat{\sigma}_{ij}(\beta)\ ,\quad \beta=\sqrt{1-\frac{4m_t^2}{\hat{s}}}\ ,\quad \hat{s}=x_1x_2s$$

β heavy quark velocity , $\hat{s}$ partonic energy squared

$$\hat{\sigma}_{ij}(\beta)=\hat{\sigma}_{ij}^{(0)}(\beta)+\alpha_s(m_t)\hat{\sigma}_{ij}^{(1)}(\beta)+\alpha_s^2(m_t)\hat{\sigma}_{ij}^{(2)}(\beta)+\ldots$$

Fluxes

- The hadronic cross section can be obtained by integration with fluxes

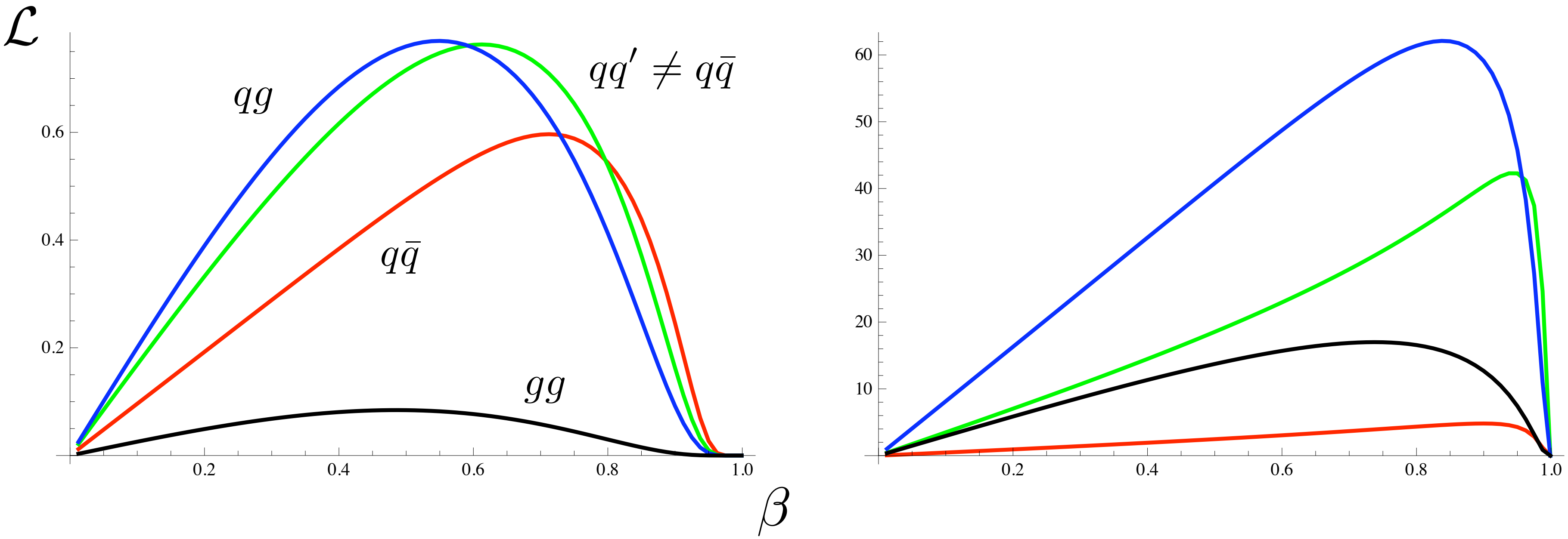
$$\sigma = \frac{\alpha_s^2}{m_t^2} \sum_{ij} \int_0^{\beta_{\max}} \mathcal{L}_{ij}(\beta)\hat{\sigma}(\beta)$$

$$\beta_{\max}^{\mathrm{TeV}} = 0.98$$

$$\beta_{\max}^{\mathrm{LHC@8TeV}} = 0.999$$

$$\beta_{\max}^{\mathrm{LHC@14TeV}} = 0.9997$$

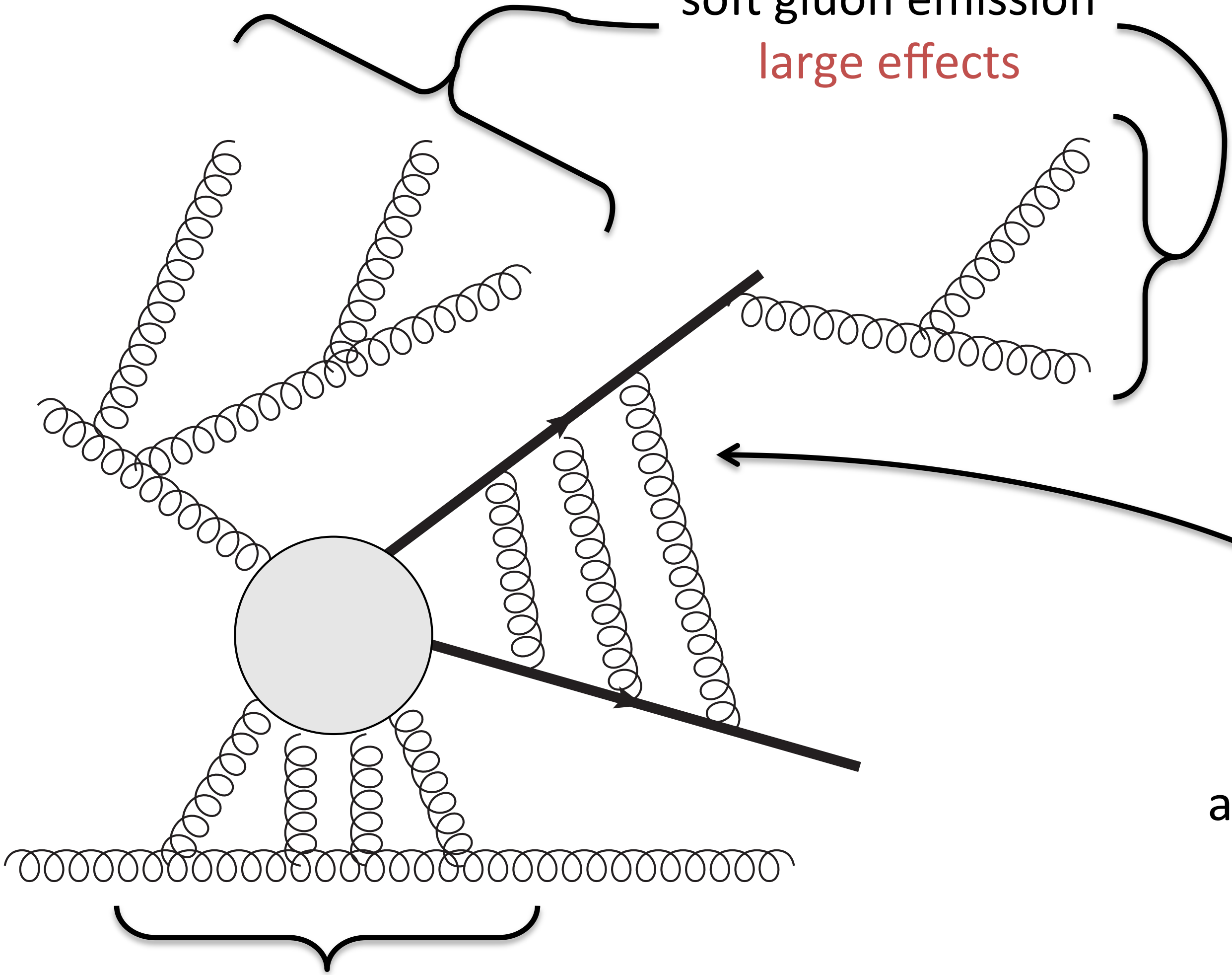
MSTW2008nnlo68cl , $\mu_F = m_t$, $m_t = 173.3$ GeV



TeVatron 1960 GeV

LHC 8 TeV

Dominant effects



soft gluon emission

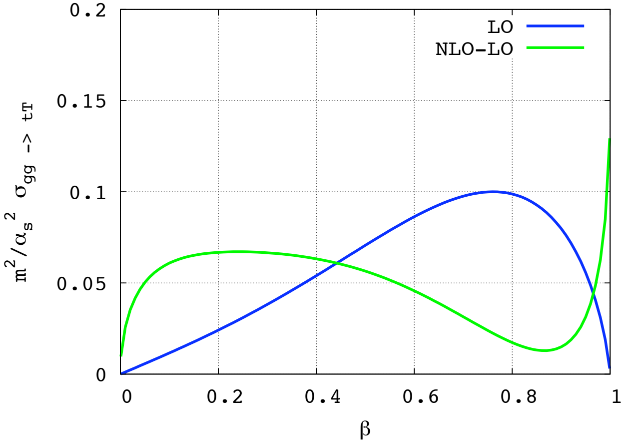
large effects

Coulomb attraction/repulsion

small effects

t-channel gluon exchange

negligible effects

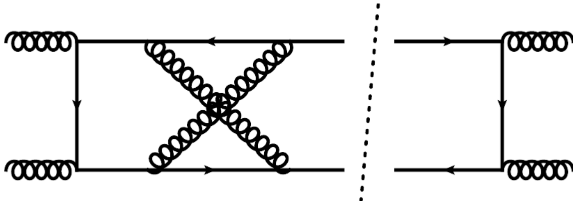


All effects can be resummed !!!

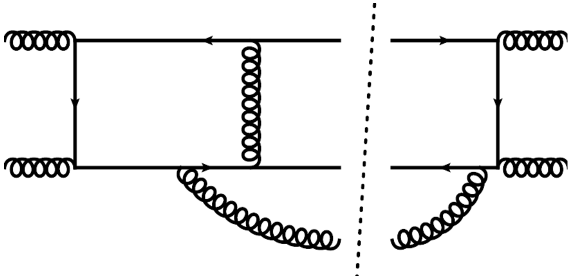
NNLO methods

There are 3 principal contributions:

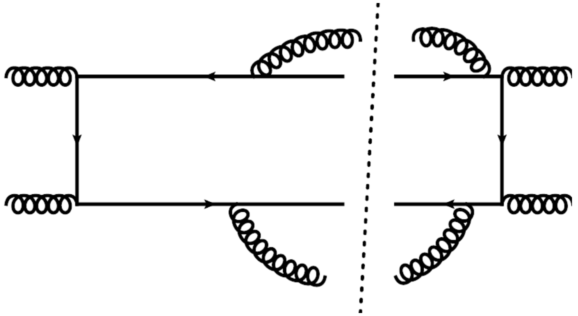
- 2-loop virtual corrections (V-V)
  MC `07 (quark annihilation)
  Bärnreuther, MC, Fiedler, in preparation (gluon fusion)



- 1-loop virtual with one extra parton (R-V)
  code by Stefan Dittmaier
  new subtraction terms: Bierenbaum, Czakon, Mitov `11



- 2 extra emitted partons at tree level (R-R)
  MC `10 `11 invention of a new subtraction scheme called STRIPPER



And 2 secondary contributions:

- Collinear subtraction for the initial state: Known, in principle. Done numerically. (the only non-differential contribution)
- One-loop squared amplitudes: Körner, Merebashvili, Rogal `07 (quark annihilation) done from scratch for gluon fusion

Additionally: divergences of two-loop amplitudes in quark annihilation: Ferroglia, Neubert, Pecjak, Yang '09

Partonic results: qQ -> tT + X

Partonic cross-section through NNLO:

$$\sigma_{ij}\left(\beta,\frac{\mu^2}{m^2}\right)=\frac{\alpha_S^2}{m^2}\left\{\sigma_{ij}^{(0)}+\alpha_S\left[\sigma_{ij}^{(1)}+L\,\sigma_{ij}^{(1,1)}\right]+\alpha_S^2\left[\sigma_{ij}^{(2)}+L\,\sigma_{ij}^{(2,1)}+L^2\sigma_{ij}^{(2,2)}\right]+\mathcal{O}(\alpha_S^3)\right\},$$

The NNLO term:

$$\sigma_{q\bar{q}}^{(2)}(\beta)=F_0(\beta)+F_1(\beta)N_L+F_2(\beta)N_L^2$$

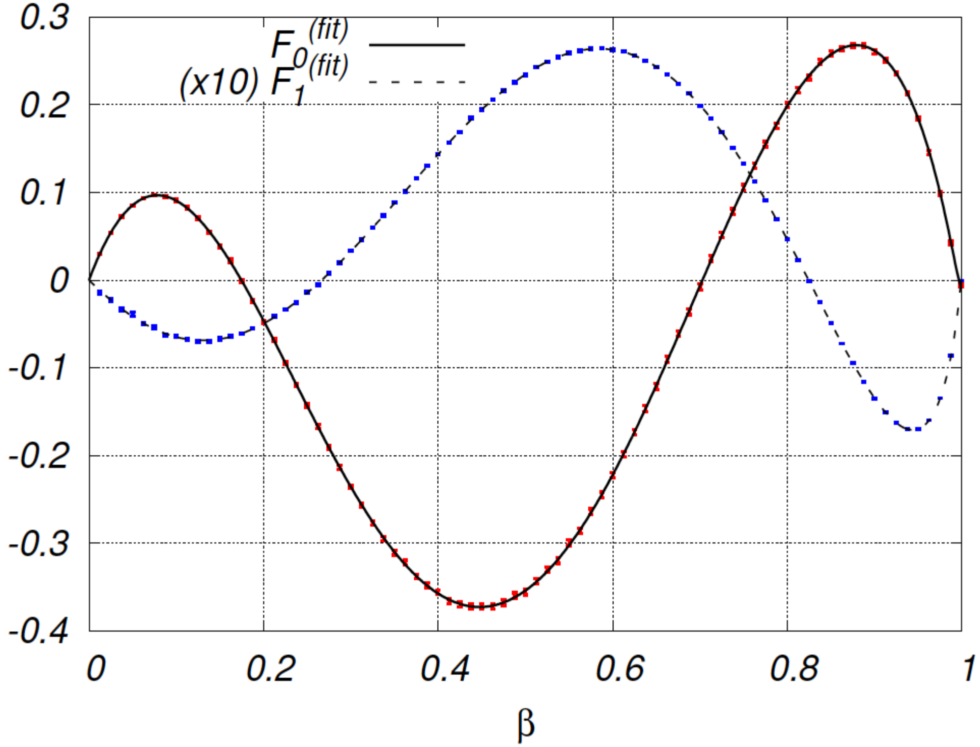
Numeric

Analytic

$$F_i\equiv F_i^{(\beta)}+F_i^{(\mathrm{fit})},i=0,1$$

The known threshold approximation

- Small numerical errors
  - Agrees with limits



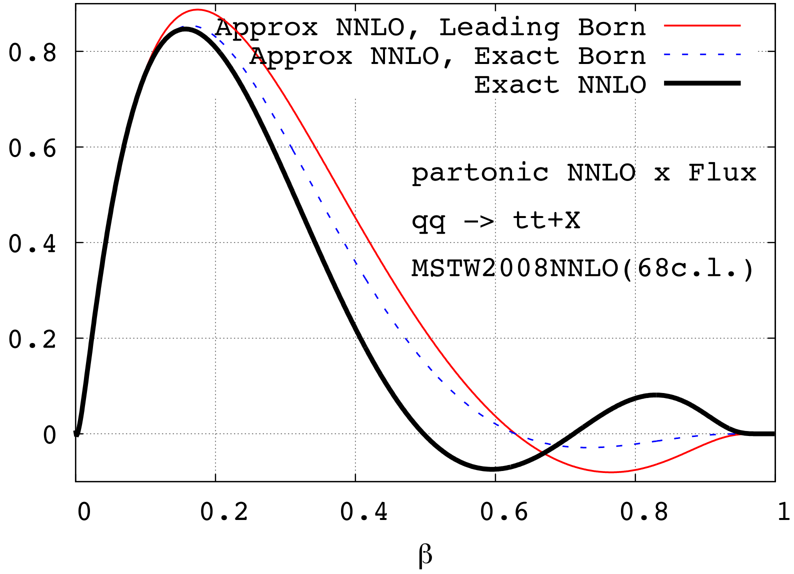
Bärnreuther, MC, Mitov '12

Beneke, MC, Falgari, Mitov, Schwinn `09

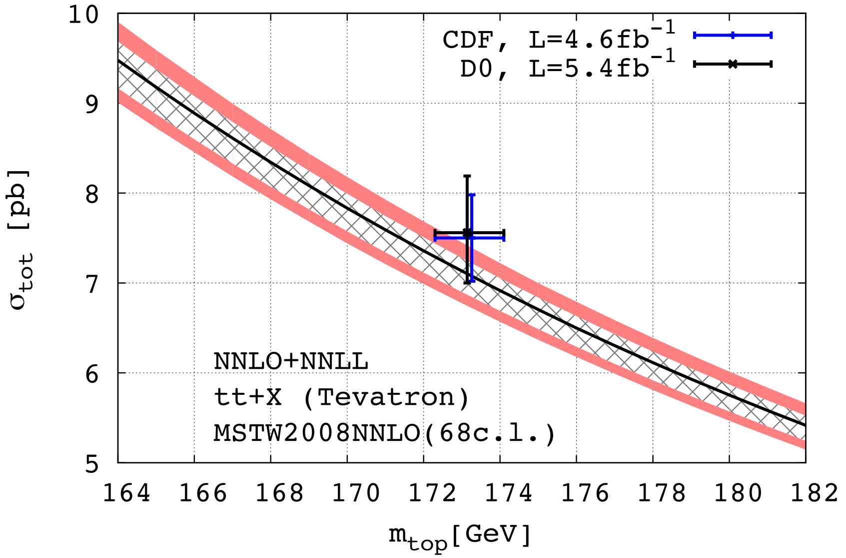
Partonic results: qQ -> tT + X

Bärnreuther, MC, Mitov `12

After inclusion of the flux at NNLO:



Comparison to data:

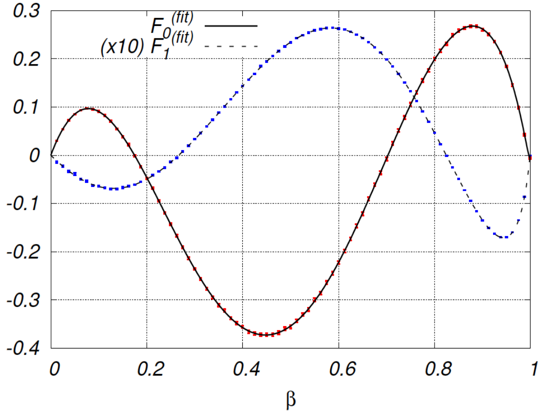


Small effect due to accidental cancellations ≈ -1 %

Partonic results: all-fermionic

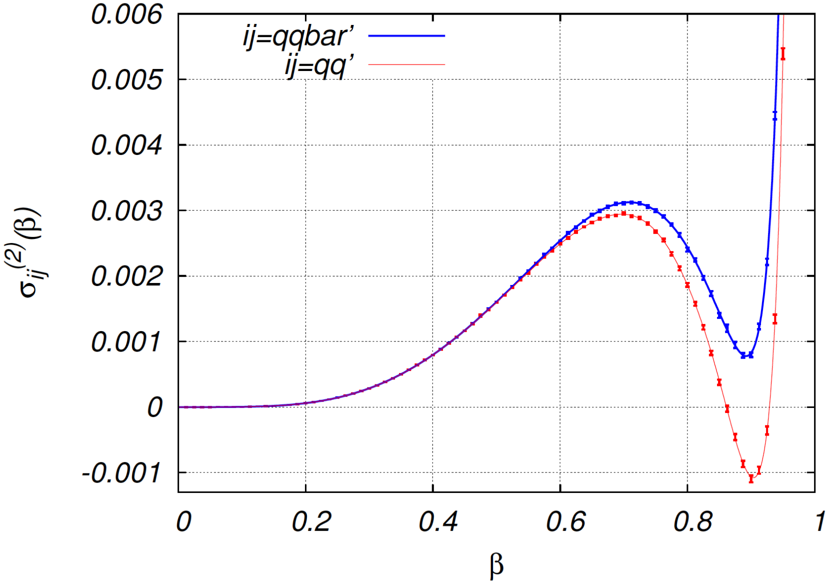
MC, Mitov '12

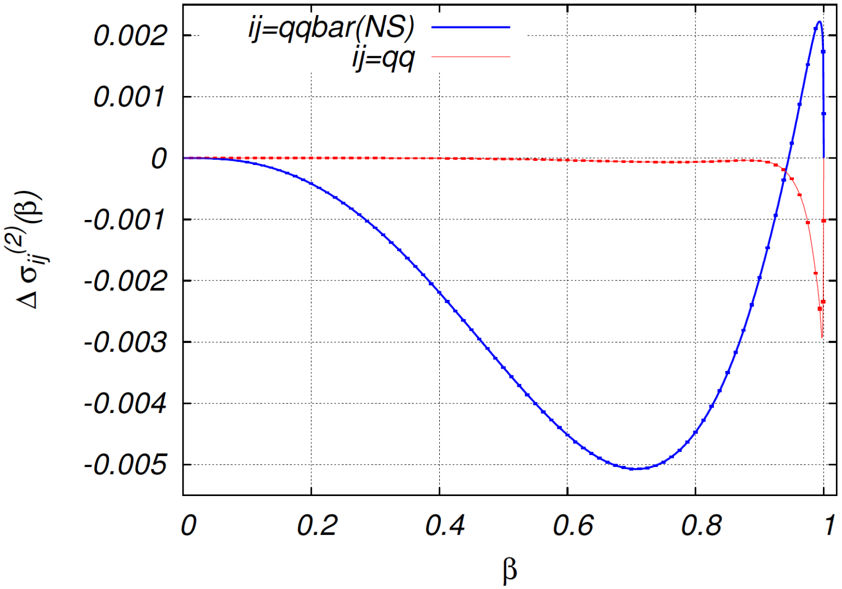
$$q\bar{q} \to t\bar{t} + q\bar{q}\big|_{\rm NS}\,,$$
$$q\bar{q}' \to t\bar{t} + q\bar{q}'\,,$$
$$qq' \to t\bar{t} + qq'\,,$$
$$qq \to t\bar{t} + qq\,.$$



Bärnreuther, MC, Mitov '12

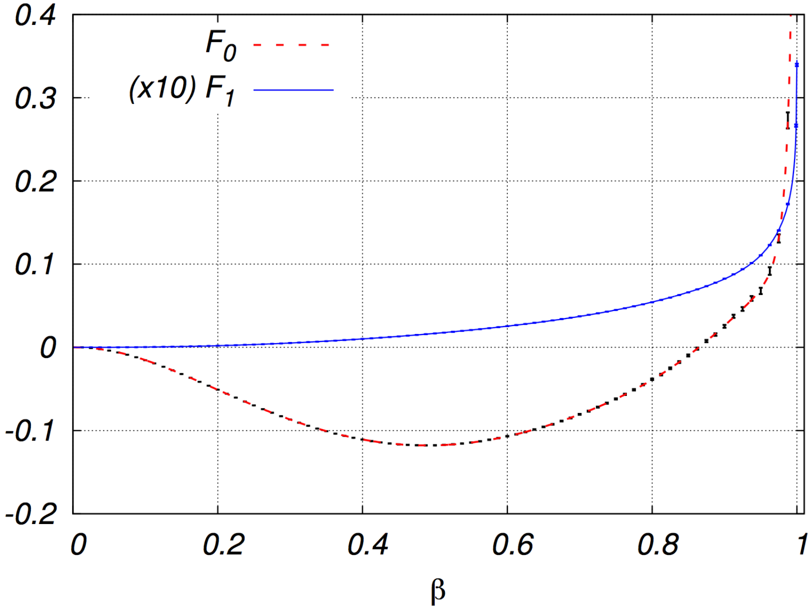
These partonic cross-sections are very small.
Compare to the ones involving gluons!

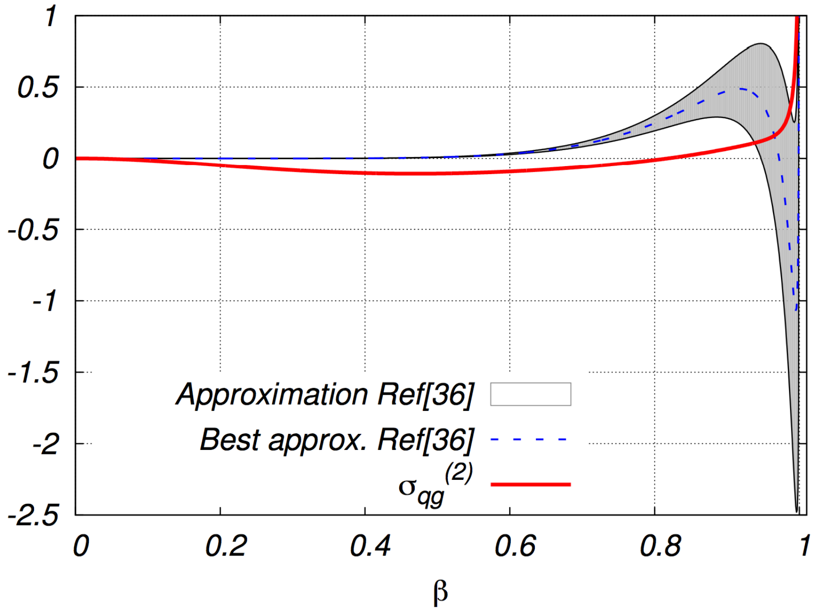




Partonic results: gq -> tT + X

MC, Mitov `12





-0.8 % effect on the cross section at the TeVatron

-1.1 % effect at the LHC @ 8 TeV

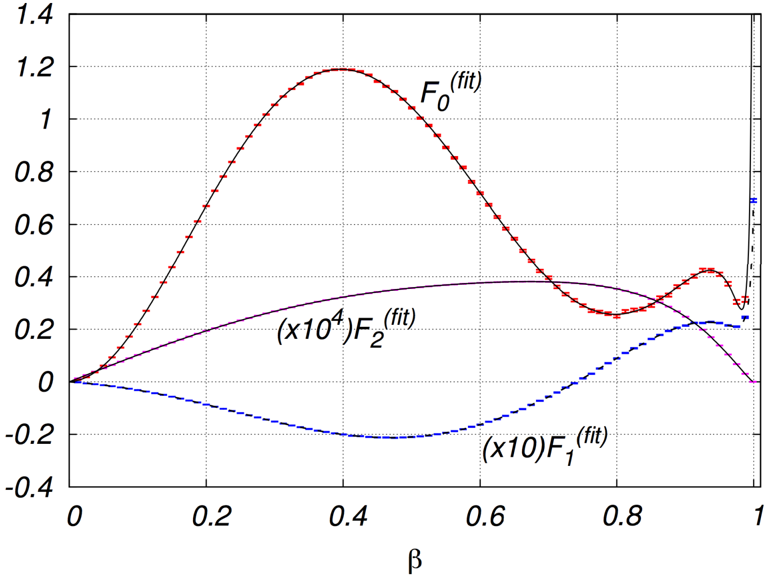
Comparison
With HATHOR

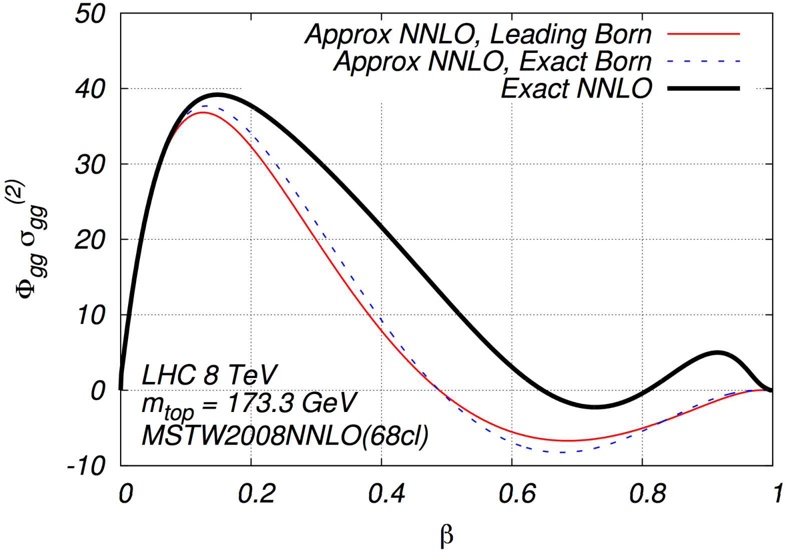
		Tevatron	LHC 7 TeV	LHC 8 TeV	LHC 14 TeV
I_1	Due to $\sigma_{qg}^{(1)}$ [pb]	-0.068	-0.88	-0.48	9.01
I_2	Due to $\sigma_{qg}^{(2)}$ [pb]	-0.057	-1.82	-2.25	-4.07
I_3	$\sigma_{qg}^{(2)}$(Hathor; $(A+B)/2$) [pb]	0.040	5.78	8.11	27.36
I_4	$(I_3 - I_2)/\sigma_{\text{tot}}$ [%]	1.4	4.9	4.7	3.7

Partonic results: gg -> tT + X

MC, Fiedler, Mitov `13

$$\sigma_{gg}^{(2)}(\beta) = F_0(\beta) + F_1(\beta)N_L + F_2(\beta)N_L^2$$



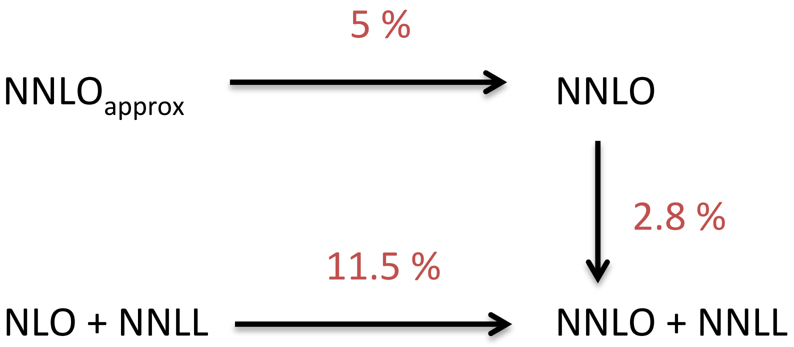


Top++ MC, Mitov

Version 1.4

Version 2.0

Precision of
the gluon channel



5 %

$\text{NNLO}_{\text{approx}} \longrightarrow \text{NNLO}$

2.8 %

11.5 %

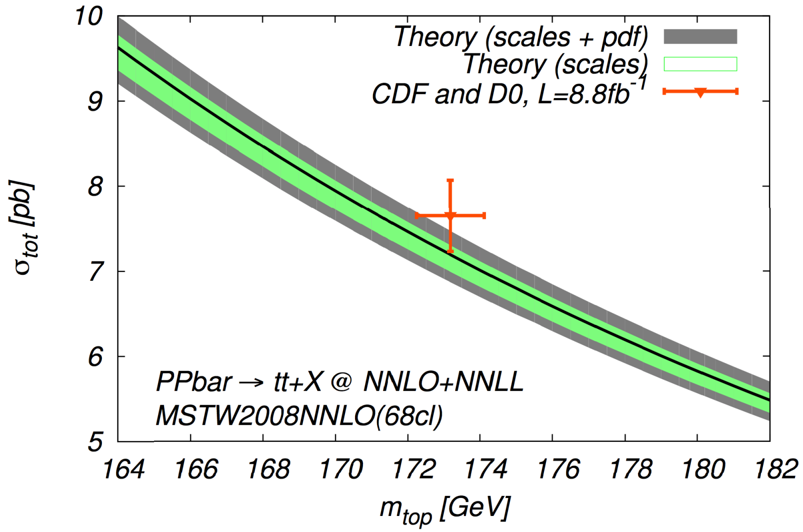
NLO + NNLL → NNLO + NNLL

Predictions for hadron colliders

MC, Fiedler, Mitov `13

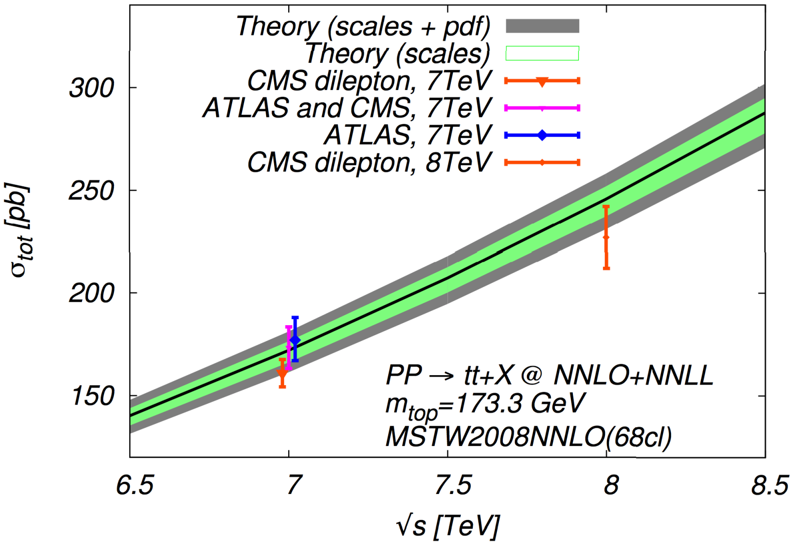
NNLO + NNLL

Collider	$\sigma_{\rm tot}$ [pb]	scales [pb]	pdf [pb]
Tevatron	7.164	$+0.110(1.5\%)$ $-0.200(2.8\%)$	$+0.169(2.4\%)$ $-0.122(1.7\%)$
LHC 7 TeV	172.0	$+4.4(2.6\%)$ $-5.8(3.4\%)$	$+4.7(2.7\%)$ $-4.8(2.8\%)$
LHC 8 TeV	245.8	$+6.2(2.5\%)$ $-8.4(3.4\%)$	$+6.2(2.5\%)$ $-6.4(2.6\%)$
LHC 14 TeV	953.6	$+22.7(2.4\%)$ $-33.9(3.6\%)$	$+16.2(1.7\%)$ $-17.8(1.9\%)$



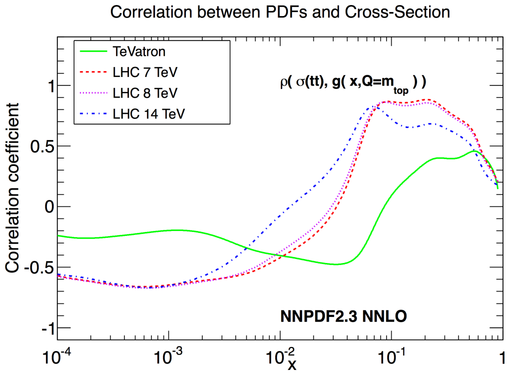
NNLO

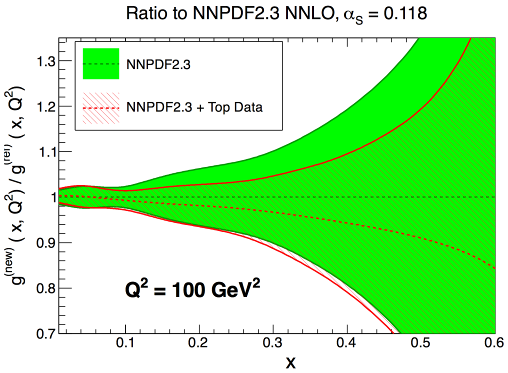
Collider	$\sigma_{\rm tot}$ [pb]	scales [pb]	pdf [pb]
Tevatron	7.009	$+0.259(3.7\%)$ $-0.374(5.3\%)$	$+0.169(2.4\%)$ $-0.121(1.7\%)$
LHC 7 TeV	167.0	$+6.7(4.0\%)$ $-10.7(6.4\%)$	$+4.6(2.8\%)$ $-4.7(2.8\%)$
LHC 8 TeV	239.1	$+9.2(3.9\%)$ $-14.8(6.2\%)$	$+6.1(2.5\%)$ $-6.2(2.6\%)$
LHC 14 TeV	933.0	$+31.8(3.4\%)$ $-51.0(5.5\%)$	$+16.1(1.7\%)$ $-17.6(1.9\%)$

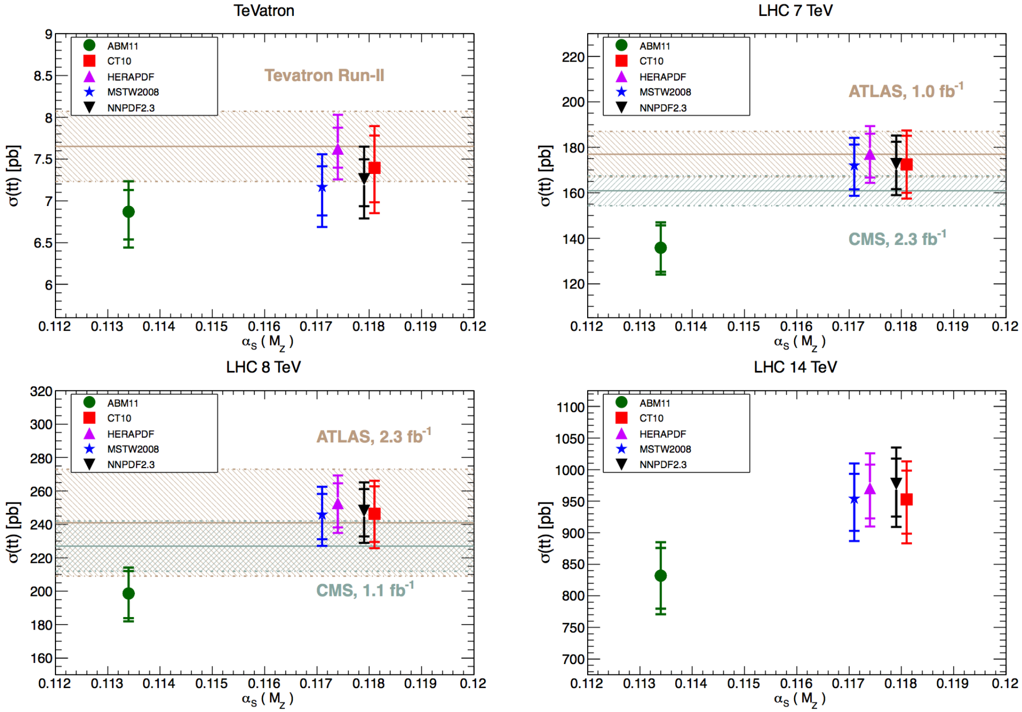


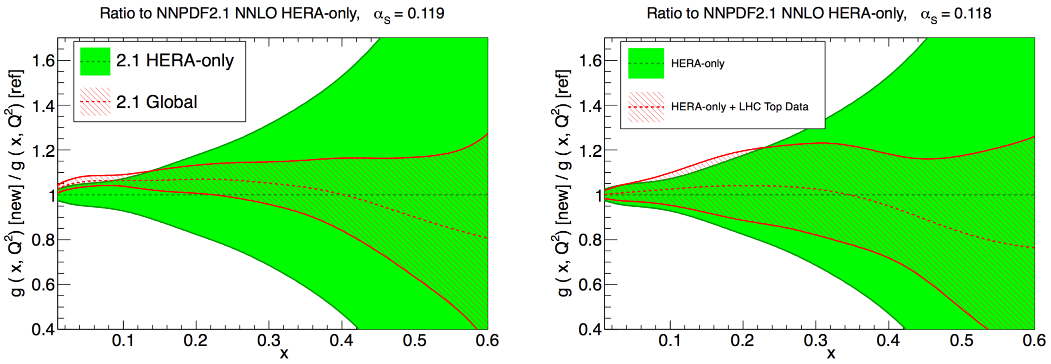
Application to PDF studies

MC, Mangano, Mitov, Rojo `13









Outlook

Next project: calculation of the Forward-Backward asymmetry

Current technical status: description of on-shell top quark pair production in a fully differential Monte Carlo

What is possible without new concepts?
NNLO including decays in the Narrow Width Approximation