

Top mass theory overview

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Introduction

Classical mechanics

- Mass is defined as product of density and volume of matter
 - classical concept
- *The quantity of matter is that which arises jointly from its density and magnitude. A body twice as dense in double the space is quadruple in quantity. This quantity I designate by the name of body or of mass.*

Newton

Atomic theory

- Mass is conserved Lavoisier
- Mass of body is sum of mass of its constituents
 $M(X) = N_A m_a(X)$ Avogadro

Special relativity

- Equivalence principle
 $E = mc^2$ Einstein

PHILOSOPHIÆ NATURALIS PRINCIPIA MATHEMATICA.

DEFINITIONES.

DEFINITIO I.

Quantitas materiæ est mensura ejusdem orta ex illius densitate et magnitudine conjunctim.

ÆR densitate duplicata, in spatio etiam duplicato, fit quadruplus; in triplicato sextuplus. Idem intellige de nive & pulveribus per compressionem vel liquefactionem condensatis. Et par est ratio corporum omnium, quæ per causas quascunque diversimode condensantur. Medii interea, si quod fuerit, interstitia partium libere pervadentis, hic nullam rationem habeo. Hanc autem quantitatem sub nomine corporis vel massæ in sequentibus passim intelligo. Innotescit ea per corporis cujusque pondus: Nam ponderi proportionalem esse reperi per experimenta pendulorum accuratissime instituta, uti posthac docebitur.

DEFINITIO II.

Quantitas motus est mensura ejusdem orta ex velocitate et quantitate materiæ conjunctim.

Motus totius est summa motuum in partibus singulis; ideoque in corpore duplo majore, æquali cum velocitate, duplus est, & dupla cum velocitate quadruplus.

Standard Model

- Higgs boson gives mass to matter fields via Higgs-Yukawa coupling
 - large top quark mass m_t

QCD

- Classical part of QCD Lagrangian

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu}^a F_b^{\mu\nu} + \sum_{\text{flavors}} \bar{q}_i (i\not{D} - m_q)_{ij} q_j$$

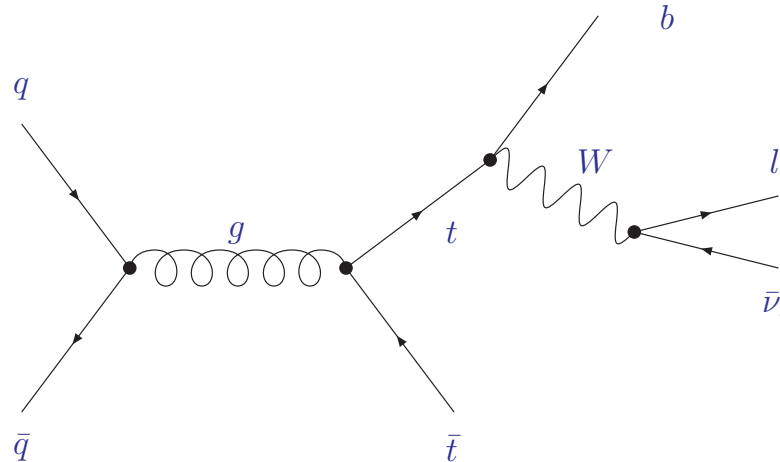
- field strength tensor $F_{\mu\nu}^a$ and matter fields q_i, \bar{q}_j
- covariant derivative $D_{\mu,ij} = \partial_\mu \delta_{ij} + ig_s (t_a)_{ij} A_\mu^a$
- Formal parameters of the theory (no observables)
 - strong coupling $\alpha_s = g_s^2 / (4\pi)$
 - quark masses m_q

Challenge

- Suitable observables for measurements of α_s, m_q, \dots
 - comparison of theory predictions and experimental data

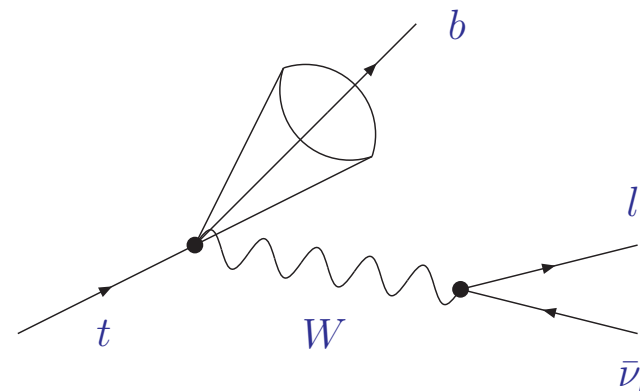
Top-quark pair-production

- Born process ($q\bar{q}$ -channel) with leptonic decay $t \rightarrow b l \bar{\nu}_l$



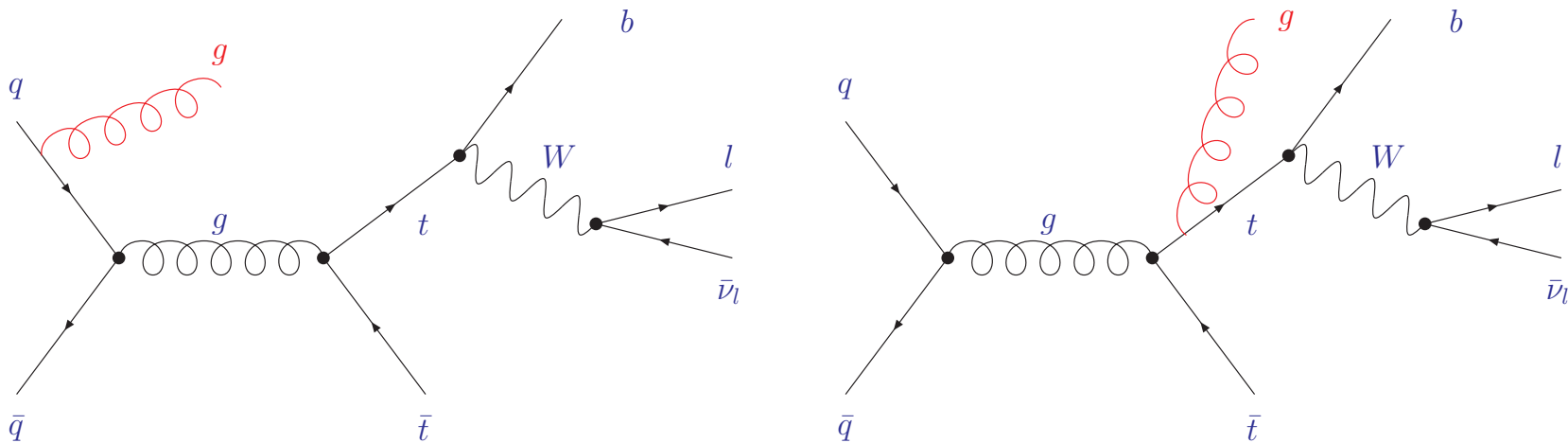
Top-quark mass

- Current methods based on reconstructed physics objects
 - jets, identified charged leptons, missing transverse energy
 - $m_t^2 = (p_{W\text{-boson}} + p_{b\text{-jet}})^2$



Radiative corrections

- Real corrections (examples): gluon emission
 - phase space integration \rightarrow infrared divergences (soft/collinear singularities)

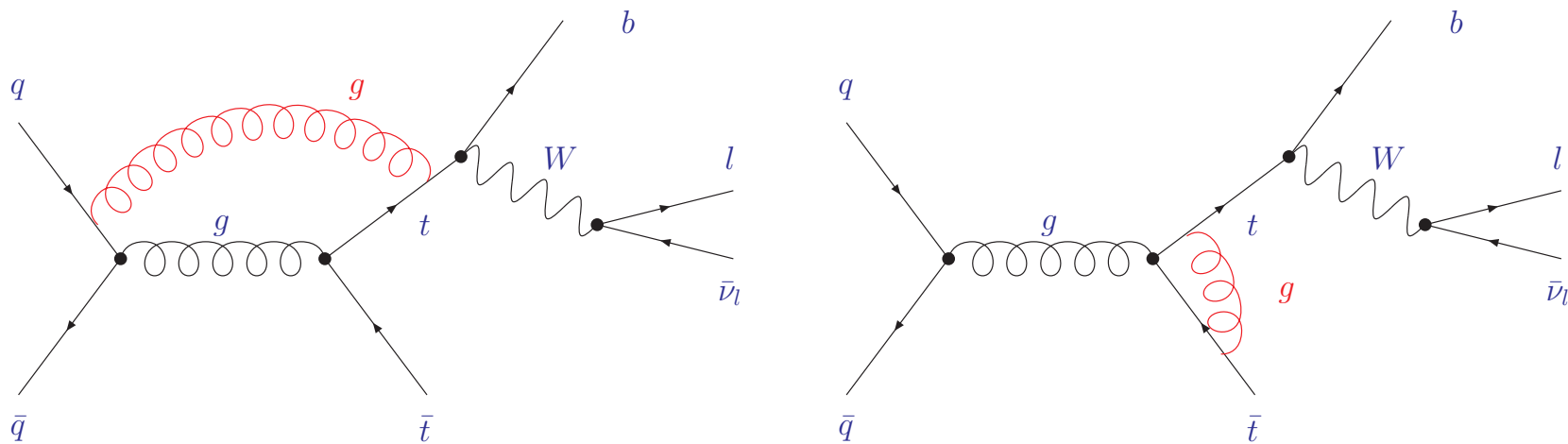


- Parton shower MC
 - emission probability modeled by Sudakov exponential with cut-off Q_0
 - leading logarithmic accuracy

$$\Delta(Q^2, Q_0^2) = \exp\left(-C_F \frac{\alpha_s}{2\pi} \ln\left(\frac{Q^2}{Q_0^2}\right)\right)$$

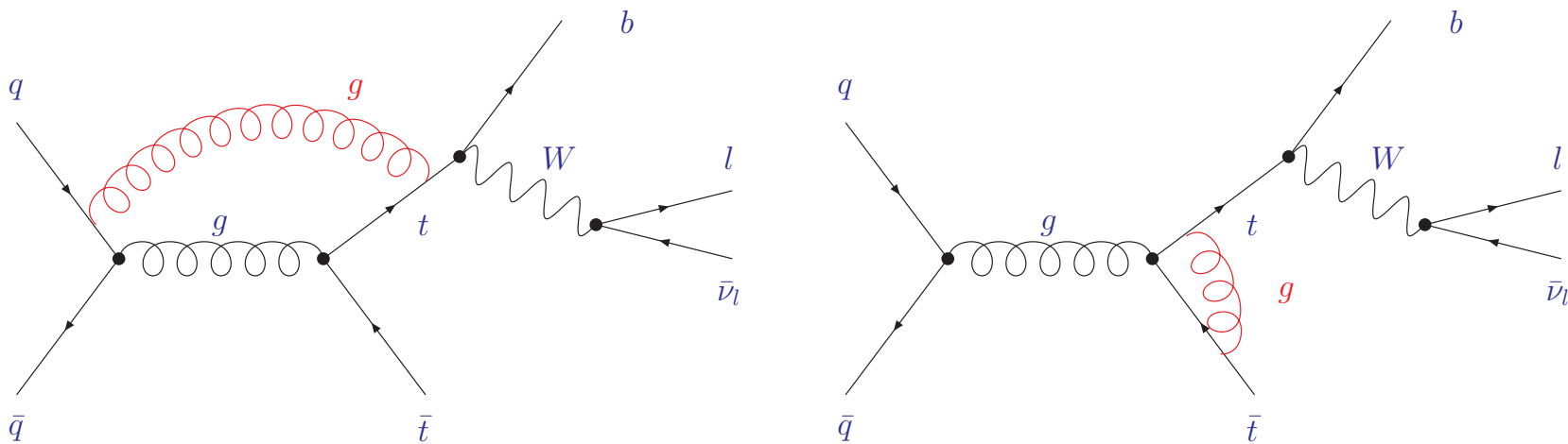
Radiative corrections

- Virtual corrections (examples): gluon exchange
 - box diagram (left) and vertex corrections (right)
 - infrared divergences cancel against real emission contributions

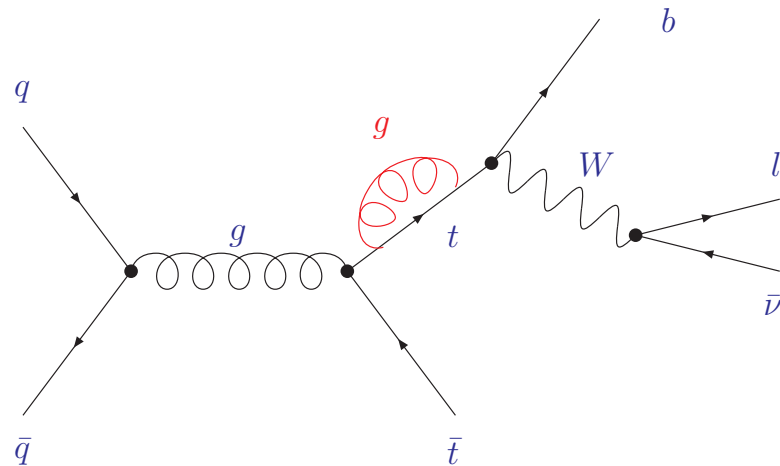


Radiative corrections

- Virtual corrections (examples): gluon exchange
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- Mass renormalization from self-energy corrections to top quark

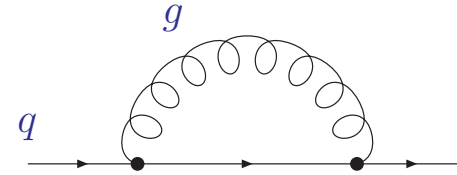


Mass measurement

Pole mass

- Based on (unphysical) concept of top-quark being a free parton

$$\not{p} - m_q - \Sigma(p, m_q) \Big|_{p^2 = m_q^2}$$



- heavy-quark self-energy $\Sigma(p, m_q)$ receives contributions from regions of all loop momenta – also from momenta of $\mathcal{O}(\Lambda_{QCD})$
- Definition of pole mass ambiguous up to corrections $\mathcal{O}(\Lambda_{QCD})$
 - bound from lattice QCD: $\Delta m_q \geq 0.7 \cdot \Lambda_{QCD} \simeq 200 \text{ MeV}$
Bauer, Bali, Pineda '11

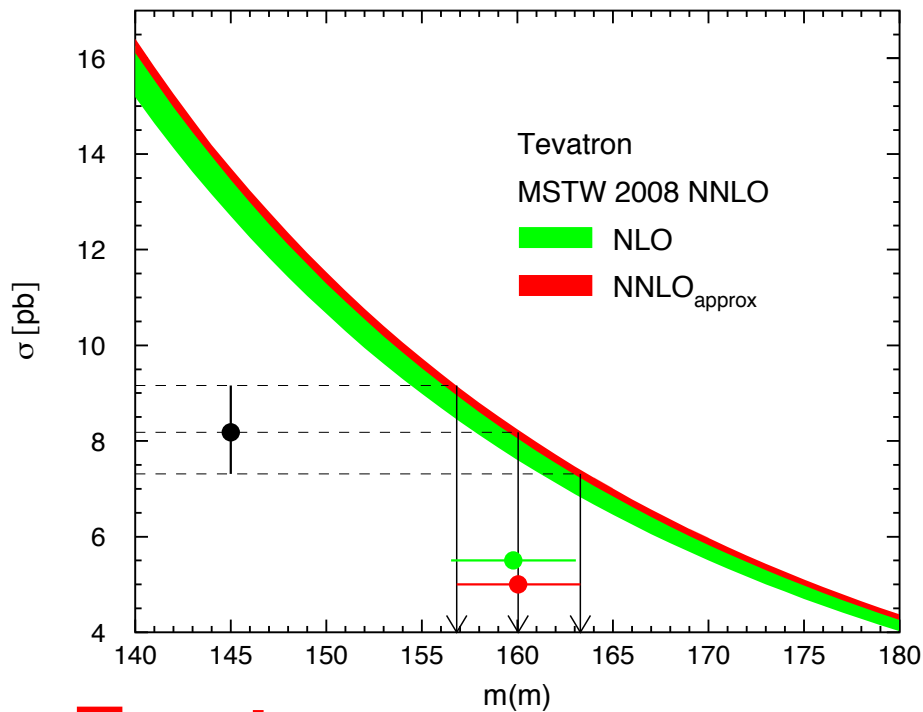
Running quark masses

- \overline{MS} mass definition $m(\mu_R)$ realizes running mass (scale dependence)
 - short distance mass probes at scale of hard scattering
 $m_{\text{pole}} = m_{\text{short distance}} + \delta m$
 - conversion between m_{pole} and \overline{MS} mass $m(\mu_R)$ perturbation theory

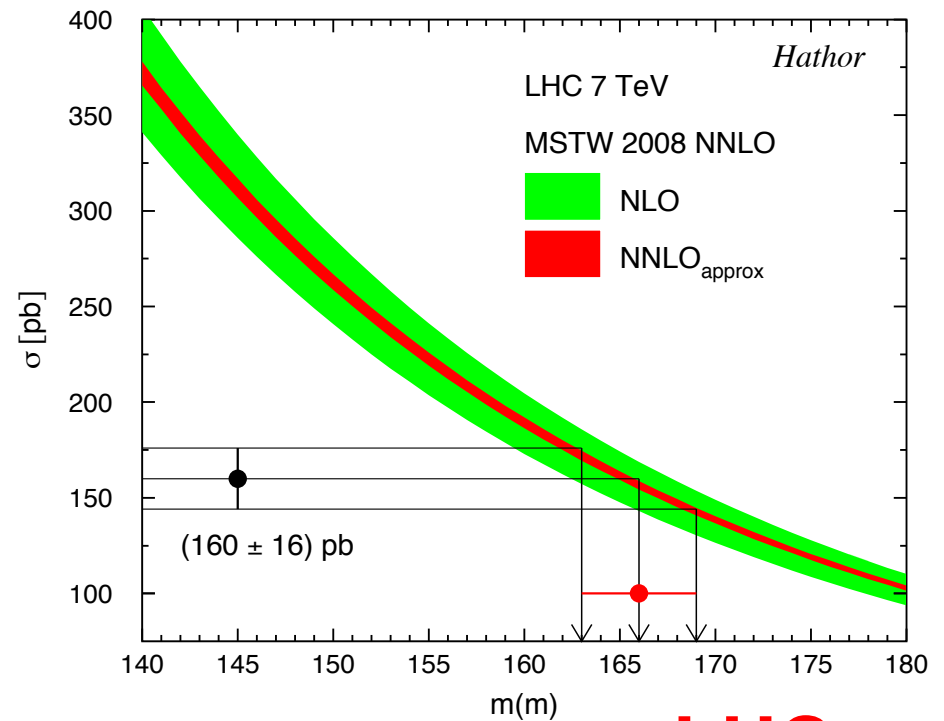
Top mass from total cross section

- Total top quark cross section as function of \overline{MS} mass
 - good apparent convergence of perturbative expansion
 - small theoretical uncertainty from scale variation

Langenfeld, S.M., Uwer '09



Tevatron



LHC

The fine print

- Dependence on mass renormalization scheme
 - recent study with potential subtracted mass m_t^{PS} , unstable top-quarks and Γ_t/m_t effects: $\Delta m_t \geq 500 \dots 900 \text{ MeV}$ (affects also kinematical mass extraction) Falgari, Papanastasiou, Signer '13

- Intrinsic limitation of sensitivity in total cross section

$$\left| \frac{\Delta \sigma_{t\bar{t}}}{\sigma_{t\bar{t}}} \right| \simeq 5 \times \left| \frac{\Delta m_t}{m_t} \right|$$

- Cross section at LHC has correlation of m_t , $\alpha_S(M_Z)$, gluon PDF

$$\sigma_{t\bar{t}} \sim \alpha_s^2 m_t^2 g(x) \otimes g(x)$$

- effective parton $\langle x \rangle \sim 2m_t/\sqrt{s} \sim 2.5 \dots 5 \cdot 10^{-2}$
- fit with fixed values of m_t and $\alpha_S(M_Z)$ carries significant bias
Czakon, Mangano, Mitov, Rojo '13
- Fit with correlations
 - $g(x)$ and $\alpha_S(M_Z)$ already well constrained by global fit (no changes)
 - $m_t(m_t) = 159.7 \text{ GeV}$ fitted with $\chi^2/NDP = 3/5$
Alekhin, Blümlein, S.M. [in progress]

Tevatron

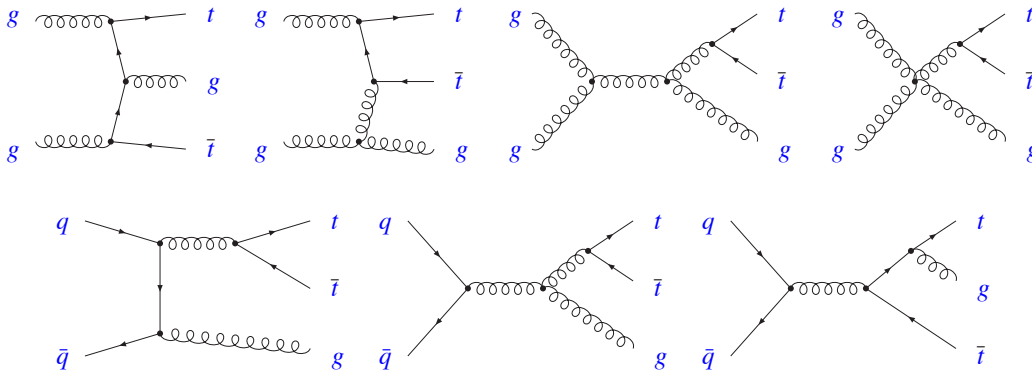
- Determine top quark mass from Tevatron cross section data
 - $\sigma_{t\bar{t}} = 7.56^{+0.63}_{-0.56}$ pb D0 coll. arXiv:1105.5384
 - $\sigma_{t\bar{t}} = 7.50^{+0.48}_{-0.48}$ pb CDF coll. CDF-note-9913
- Fit of m_t for individual PDFs
 - parton luminosity at Tevatron driven by $q\bar{q}$
 - \overline{MS} -scheme for $m_t^{\overline{MS}}(m_t)$, then scheme transformation to pole mass m_t^{pole} at NNLO

	ABM11	JR09	MSTW08	NN21
$m_t^{\overline{MS}}(m_t)$	$162.0^{+2.3+0.7}_{-2.3-0.6}$	$163.5^{+2.2+0.6}_{-2.2-0.2}$	$163.2^{+2.2+0.7}_{-2.2-0.8}$	$164.4^{+2.2+0.8}_{-2.2-0.2}$
m_t^{pole}	$171.7^{+2.4+0.7}_{-2.4-0.6}$	$173.3^{+2.3+0.7}_{-2.3-0.2}$	$173.4^{+2.3+0.8}_{-2.3-0.8}$	$174.9^{+2.3+0.8}_{-2.3-0.3}$
(m_t^{pole})	$(169.9^{+2.4+1.2}_{-2.4-1.6})$	$(171.4^{+2.3+1.2}_{-2.3-1.1})$	$(171.3^{+2.3+1.4}_{-2.3-1.8})$	$(172.7^{+2.3+1.4}_{-2.3-1.2})$

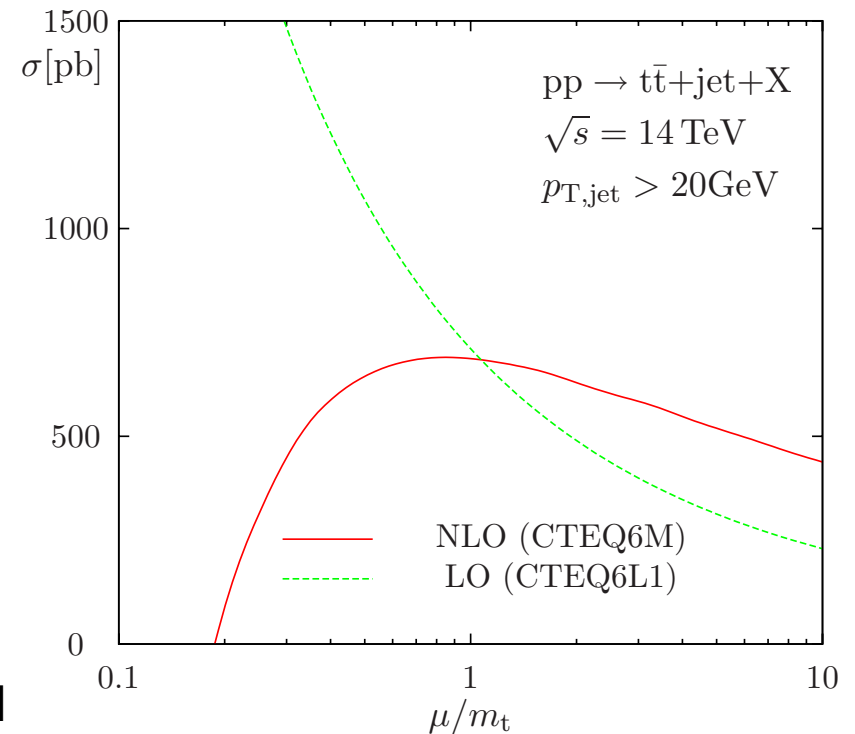
- Good consistency within errors for $m_t^{\text{pole}} = 171.7 \dots 174.9$ at NNLO

Top-quark pairs with one jet

- LHC: large rates for production of $t\bar{t}$ -pairs with additional jets
- NLO QCD corrections for $t\bar{t} + 1\text{jet}$ Dittmaier, Uwer, Weinzierl '07-'08
 - scale dependence greatly reduced at NLO
 - corrections for total rate at scale $\mu_r = \mu_f = m_t$ are almost zero



- Additional jet raises kinematical threshold
 - invariant mass $\sqrt{s_{t\bar{t}+1\text{jet}}}$



Mass measurement with $t\bar{t}$ + jet-samples

- Mass measurement with new observable

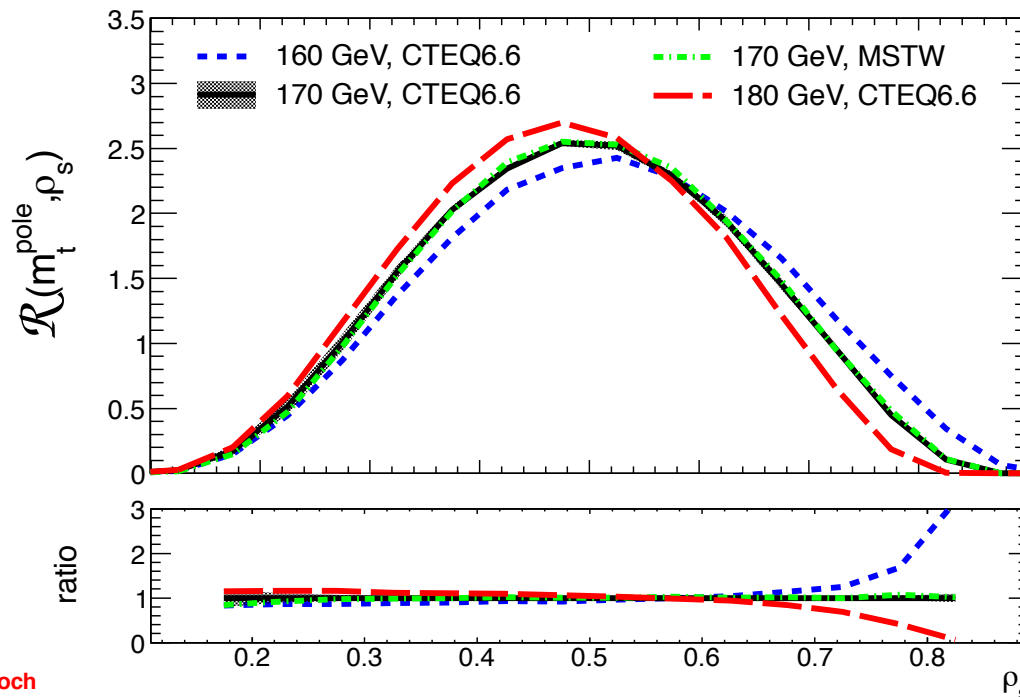
Alioli, Fernandez, Fuster, Irlles, S.M., Uwer, Vos '13

- variable $\rho_s = \frac{2 \cdot m_0}{\sqrt{s_{t\bar{t}+1\text{jet}}}}$ with invariant mass of $t\bar{t}$ + 1jet system and fixed scale $m_0 = 170$ GeV

- Normalized-differential $t\bar{t}$ + jet cross section

$$\mathcal{R}(m_t, \rho_s) = \frac{1}{\sigma_{t\bar{t}+1\text{jet}}} \frac{d\sigma_{t\bar{t}+1\text{jet}}}{d\rho_s}(m_t, \rho_s)$$

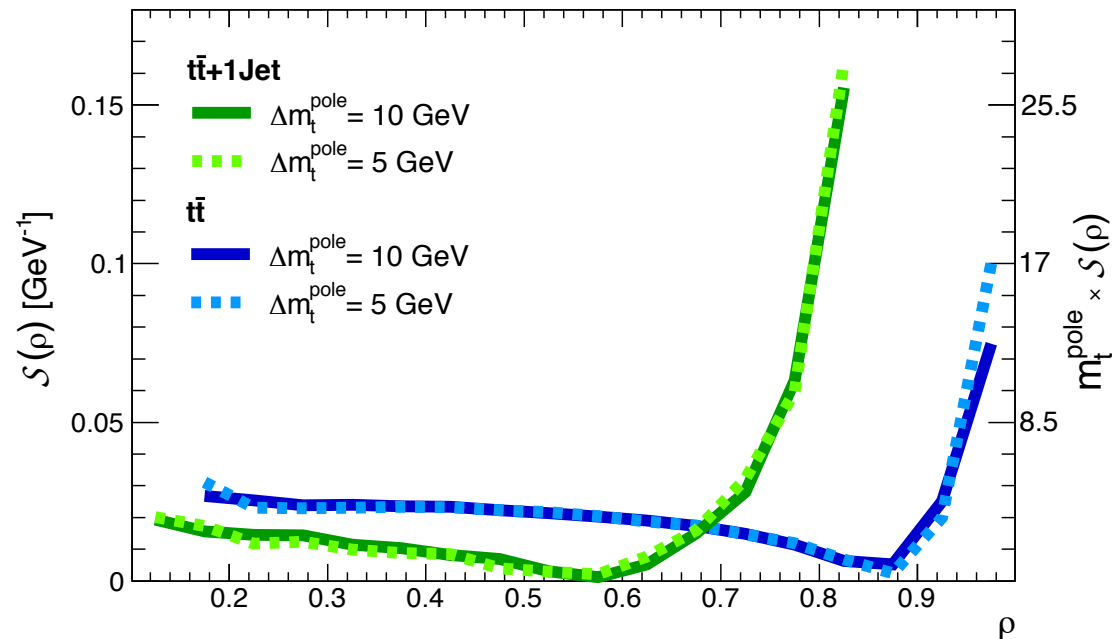
- significant mass dependence for $0.4 \leq \rho_s \leq 0.5$ and $0.7 \leq \rho_s$



- Differential cross section $\mathcal{R}(m_t, \rho_s)$
 - good perturbative stability, small theory uncertainties, small dependence on experimental uncertainties, ...
- Sensitivity to top-quark mass very good

$$\left| \frac{\Delta \mathcal{R}}{\mathcal{R}} \right| \simeq (m_t \mathcal{S}) \times \left| \frac{\Delta m_t}{m_t} \right|$$

- increased sensitivity for system $t\bar{t} + \text{jet}$ compared to $t\bar{t}$



Upshot

- Precision determination of well-defined top-quark mass m_t possible
 - alternative to inclusive cross sections

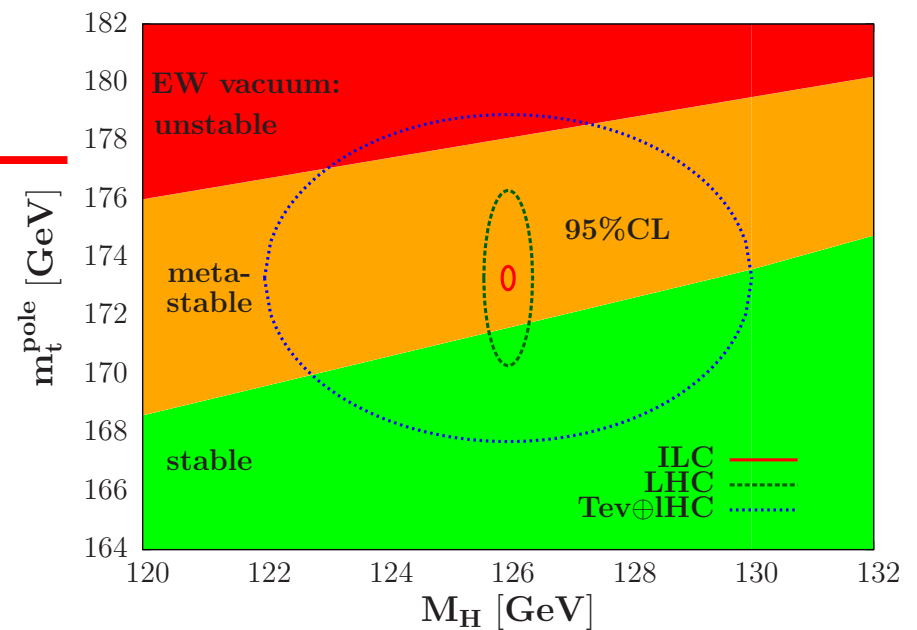
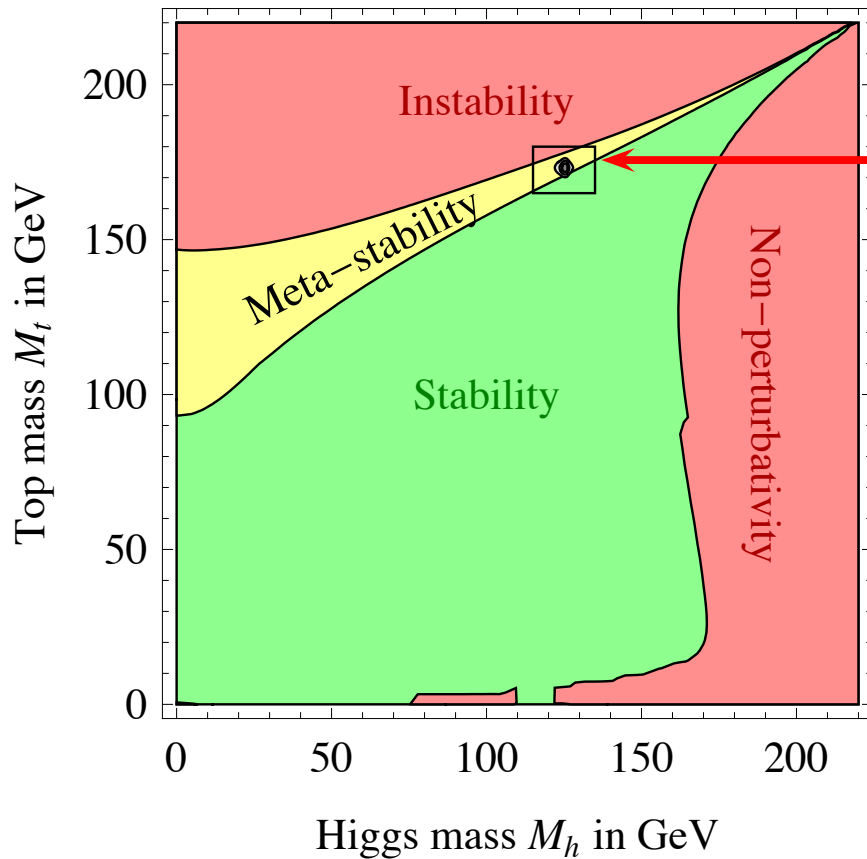
Implications on electroweak vacuum

- Relation between Higgs mass m_H and top quark mass m_t
 - condition of absolute stability of electroweak vacuum $\lambda(\mu) \geq 0$
 - extrapolation of Standard Model up to Planck scale M_P
 - $\lambda(M_P) \geq 0$ implies lower bound on Higgs mass m_H

$$m_H \geq 129.2 + 1.8 \times \left(\frac{m_t^{\text{pole}} - 173.2 \text{ GeV}}{0.9 \text{ GeV}} \right) - 0.5 \times \left(\frac{\alpha_s(M_Z) - 0.1184}{0.0007} \right) \pm 1.0 \text{ GeV}$$

- recent NNLO analyses Bezrukov, Kalmykov, Kniehl, Shaposhnikov '12; Degrassi, Di Vita, Elias-Miro, Espinosa, Giudice et al. '12
 - uncertainty in results due to α_s and m_t (pole mass scheme)
- Top quark mass from Tevatron in well-defined scheme
 - $m_t^{\overline{\text{MS}}}(m_t) = 163.3 \pm 2.7 \text{ GeV}$ implies in pole mass scheme $m_t^{\text{pole}} = 173.3 \pm 2.8 \text{ GeV}$
 - good consistency of mass value between different PDF sets

Fate of the universe



Degrassi, Di Vita, Elias-Miro, Espinosa, Giudice et al. '12; Alekhin, Djouadi, S.M. '12; Masina '12

- Uncertainty in Higgs bound due to m_t from in \overline{MS} scheme
 - bound relaxes $m_H \geq 129.4 \pm 5.6$ GeV
 - “fate of universe” still undecided

Summary

Top quark mass

- Top quark mass is parameter of Standard Model Lagrangian
- Measurements of m_t require careful definition of observable
- Radiative corrections at higher orders mandatory for scheme definition

Current measurements

- Kinematic reconstruction
 - very precise value, but only leading order/leading logarithm
 - lacking renormalization scheme definition
- Mass dependence from cross section measurements
 - (N)NLO QCD determination available
 - correlation of m_t with $\alpha_S(M_Z)$, parton distributions
- \overline{MS} mass
 - improved convergence and better scale stability for total cross section

Future challenge

- Study of new observables which meet all theory requirements
- Joint effort theory and experiment