Space Charge and Transverse Instabilities

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special thanks - to E.Metral

SC'2013, April 18 2013, CERN

Transverse Collective Dipole Instabilities at Strong Space Charge

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Möhl–Schönauer Equation

- In 1974, D. Möhl and H. Schönauer suggested to describe coasting beam oscillations by the following equation (MSE):

$$\frac{d^2 x_i}{dt^2} + \Omega_i^{\,2} Q_i^{\,2} x_i + 2\Omega_0^{\,2} Q_0 \left[Q_c \bar{x} + Q_{sc}\left(x_i - \bar{x} \right) \right] = 0 \,.$$

lattice　　wake　　space charge

$$Q_{sc} = Q_{sc}(\mathbf{J}_i); \qquad Q_{sc}(0) = -\frac{N r_0}{4\pi\beta^2\gamma^3\varepsilon_{\mathrm{rms}}} \,.$$

action

Coherent tune shift, in a frequency domain:

$$Q_c = -i \frac{N r_0 \beta_x}{\gamma C} \frac{Z_x}{Z_0} \,; \qquad Z_0 = \frac{4\pi}{c} = 377\,\mathrm{Ohm}$$

MSE: Dispersion Relation, Coasting Beam

$$\int \frac{\left[Q_c - Q_{sc}(\mathbf{J})\right] f_x J_x d\Gamma}{\Delta Q_l(\mathbf{J},\hat{p}) + Q_{sc}(\mathbf{J}) - \nu - i\,0} = 1;$$

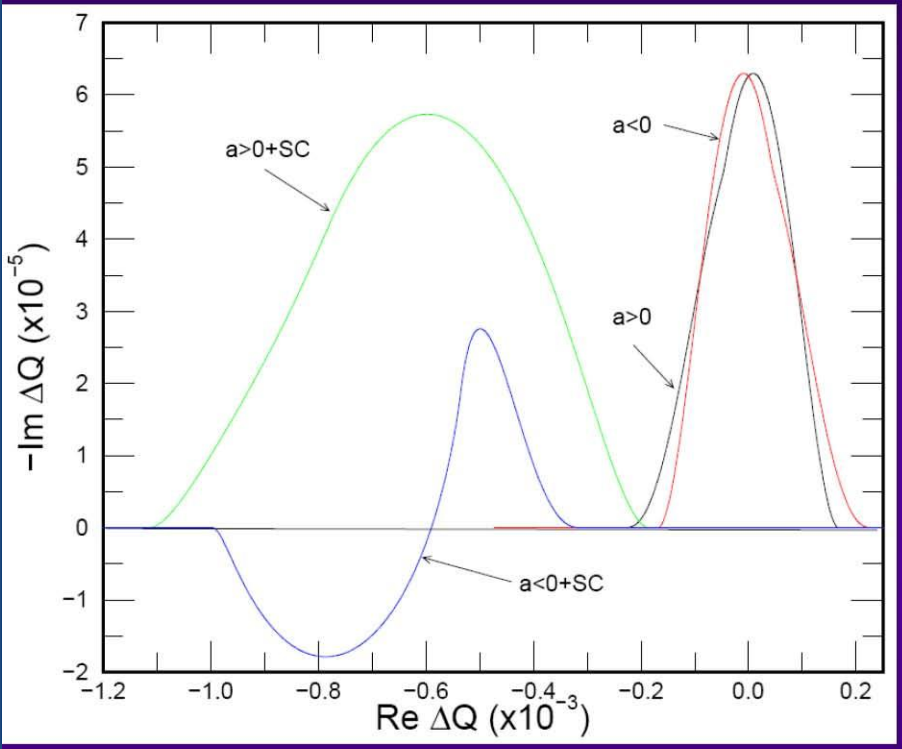
$$d\Gamma \equiv dJ_x dJ_y d\hat{p}; \quad \hat{p} = \frac{\delta p}{p};$$

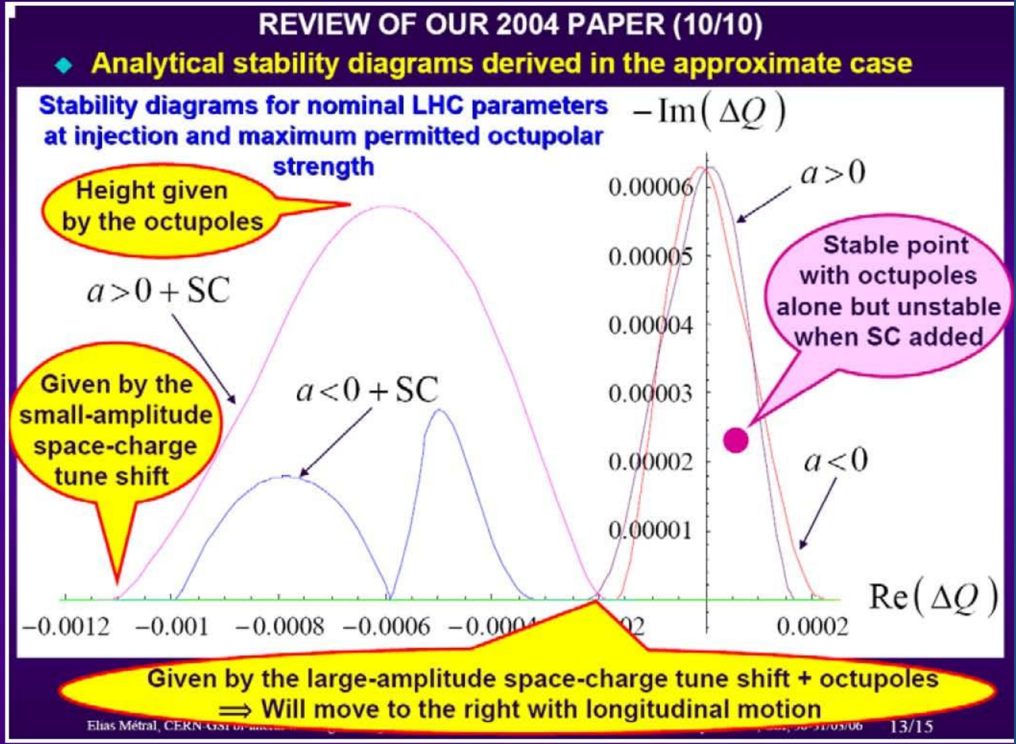
$$f_x \equiv \frac{\partial f}{\partial J_x}; \quad \int f d\Gamma = 1; \quad \nu \equiv \frac{\omega}{\Omega_0} - (n + Q_0)\,.$$

Stability diagram method is applied for the stability analysis

Result from King Ng (12/08/06)

Our previous result (EM & FR)





⇒ **Everything is the same except for the first bump when a<0+SC (which has the same shape but is negative)**

⇒ **To be checked…**

Elias Metral - 2006

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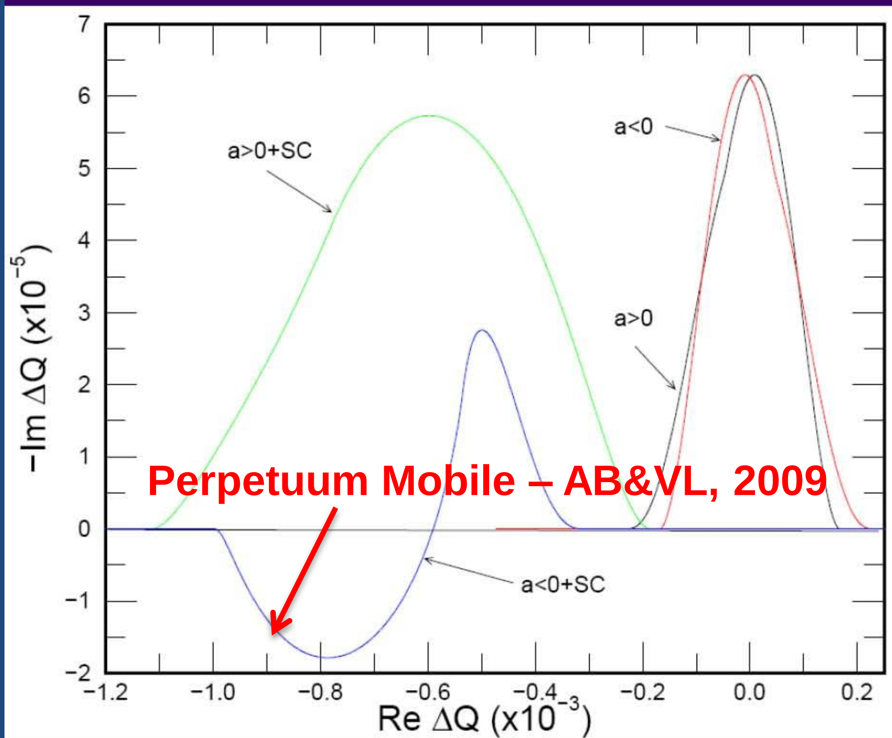

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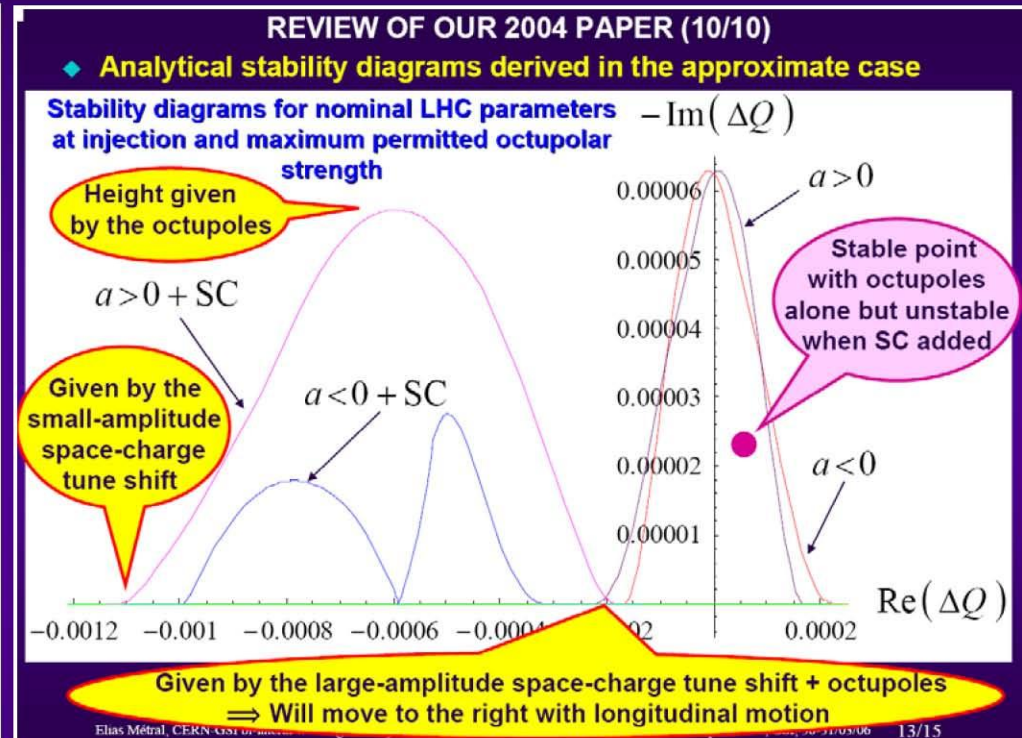

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MSE Applicability

$$\frac{d^2 x_i}{dt^2} + \Omega_i^{\,2} Q_i^{\,2} x_i + 2\Omega_0^{\,2} Q_0 \left[Q_c \bar{x} + Q_{sc} \left(x_i - \bar{x} \right) \right] = 0\,.$$

lattice wake space charge

This equation assumes the beam cross-section does not change at oscillations:

$$Q_{sc} = Q_{sc}(\mathbf{J}_i, \cancel{t}) = Q_{sc}(\mathbf{J}_i).$$

When all the lattice tunes are same, $\Omega_i Q_i = \Omega_0 Q_0 \;\Rightarrow x_i(t) = \bar{x}(t)$, this assumption is correct.

Thus, MSE requires lattice tune spread to be small enough:

$$\delta(\Omega_i Q_i) \ll \Omega_0 Q_{sc}$$

Burov, Lebedev, Phys. Rev. ST-AB 12, 034201 (2009)

Instability Thresholds, Coasting Beam

$$\int \frac{\left[Q_c - Q_{sc}(\mathbf{J})\right] f_x J_x d\Gamma}{\Delta Q_l(\mathbf{J},\hat{p}) + Q_{sc}(\mathbf{J}) - \nu - i\,0} = 1;$$

$$d\Gamma \equiv dJ_x dJ_y d\hat{p}; \quad \hat{p} = \frac{\delta p}{p};$$

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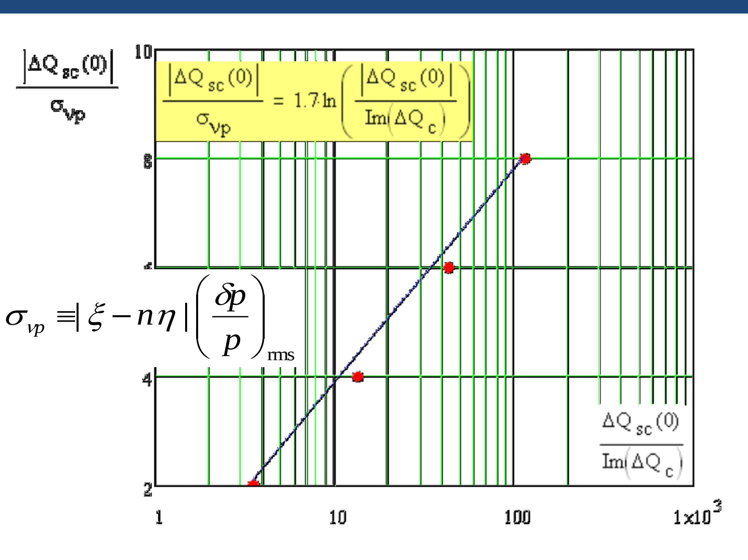
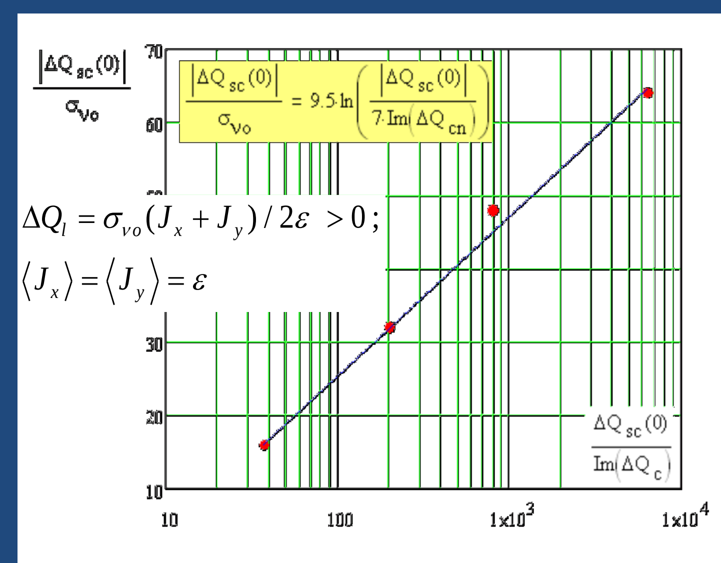
Chromatic threshold



$$\sigma_{\nu p} \equiv |\,\xi - n\eta\,| \left(\frac{\delta p}{p}\right)_{\rm rms}$$

Octupolar threshold



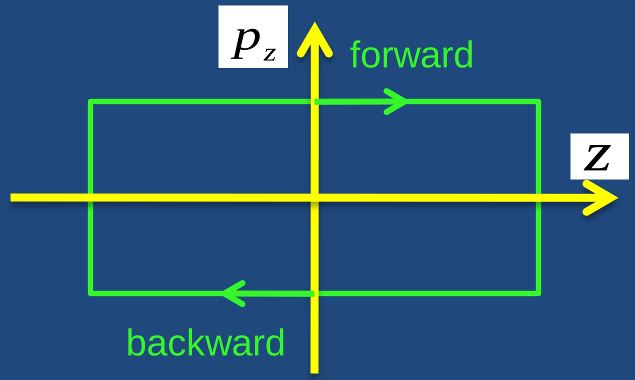
$$\Delta Q_l = \sigma_{\nu o}(J_x + J_y)/2\varepsilon > 0;$$

$$\langle J_x \rangle = \langle J_y \rangle = \varepsilon$$

Square Well Model (SWM)

- For a square potential well and KV transverse distribution, the head-tail modes with space charge were described by Mike Blaskiewicz (1998).

- For the air-bag distribution, there are two particle fluxes in the synchrotron phase space:



- Since $Q_{sc} = \text{const}$, MSE is easier to solve in this case.

SWM: coherent tunes, no wakes

general result:

$$\nu_{k\pm} = \nu_b - \frac{Q_{sc}}{2} \pm \sqrt{\frac{Q_{sc}^2}{4} + k^2 Q_s^2} \quad ; \quad k = 0, 1, 2\ldots$$

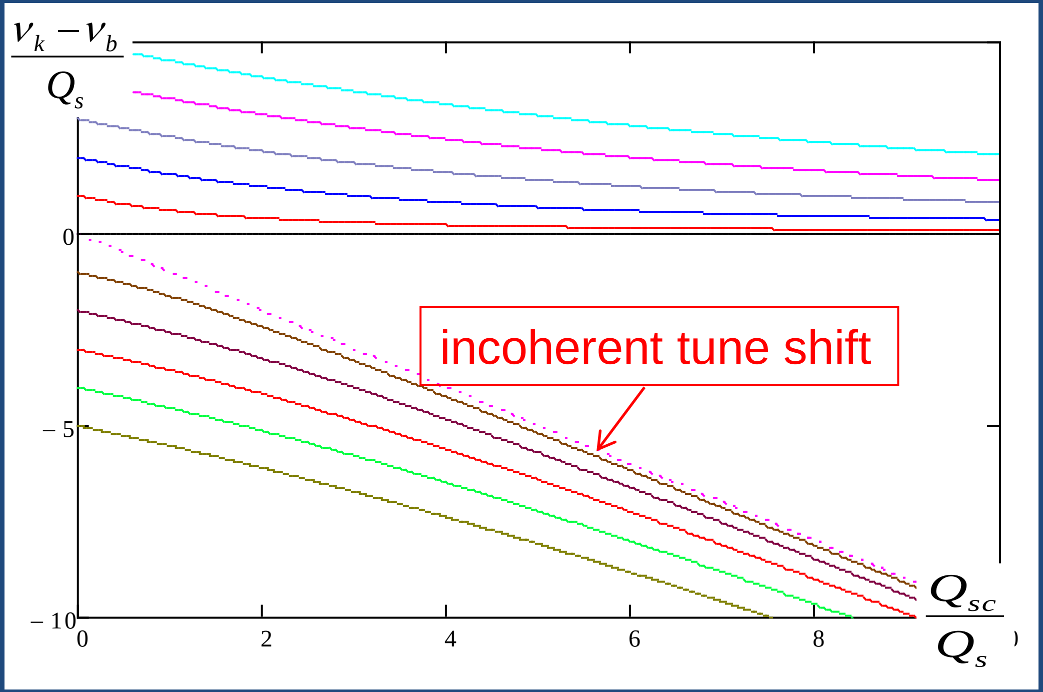
high space charge:

$$\nu_{k+} = \nu_b + \frac{k^2 Q_s^2}{Q_{sc}} \quad ; \quad k = 0,1,2\ldots$$

$$\nu_{k-} = \nu_b - Q_{sc} - \frac{k^2 Q_s^2}{Q_{sc}}$$

small space charge:

$$\nu_k = \nu_b + kQ_s - \frac{Q_{sc}}{2} \quad ; \quad k = \pm 1, \pm 2\ldots$$



At high space charge, the + and - fluxes oscillate in phase for the positive modes and out of phase for the negative modes.

Only positive modes matter for high space charge.

SWM: Landau damping

- For the square-well model, the dispersion equation is similar to the coasting beam case, except the chromaticity is dropped (Blaskiewicz, 2003; Burov, 2009):

$$-\int \frac{Q_{sc}(\mathbf{J})\left[Q_{sc}(\mathbf{J})+\nu\right] f_x J_x d\Gamma}{\left[Q_{sc}(\mathbf{J})-\Delta Q_l(\mathbf{J})+\nu+i\,0\right]^2-(kQ_s)^2}=1;$$

$$d\Gamma \equiv dJ_x dJ_y dQ_s; \quad f_x \equiv \frac{\partial f}{\partial J_x}; \quad \int f d\Gamma = 1;$$

- Thus, for the resonant particles

$$k\Delta Q_s + \Delta Q_l = Q_{sc}\ .$$

- For the strong space charge, density of these particle is low, so Landau damping is suppressed.

MSE for General Bunched Beam

- After a substitution $x_i(\theta) = y_i(\theta)\exp(-i\zeta\tau_i(\theta))$ with a new variable y,

$$\zeta = -\xi/\eta\,, \qquad \eta = \gamma_t^{-2} - \gamma^{-2}\,, \qquad \xi = dQ_x / d(\Delta p/p)\,,$$

  the chromatic term disappears from MSE, going instead into the wake term (no octupoles!):

$$\dot{y}_i(\theta) = iQ_{sc}(\tau_i(\theta))\left[y_i(\theta) - \bar{y}(\theta,\tau_i(\theta))\right] - i\kappa\hat{\mathbf{W}}\bar{y}$$

$$\kappa = \frac{r_0 R}{4\pi\beta^2\gamma Q_b}\,; \qquad \hat{\mathbf{W}}\bar{y} = \int_\tau^\infty W(\tau - s)\exp(i\zeta(\tau - s))\rho(s)\bar{y}(s)ds\,.$$

- Thus, for no-wake case, the bunched beam modes do not depend on the chromaticity, except the head-tail modulation $\propto \exp(-i\zeta\tau)$.

- Applicable when $Q_{sc} \gg kQ_s$, all other tune shifts

MSE, Arbitrary Bunch

2nd order ordinary IDE follows (A. Burov, 2009; Balbekov, 2009):

$$\frac{d}{d\tau}\left(u^2 \frac{d\bar{y}}{d\tau}\right) + \nu_k Q_{sc}\bar{y} = \kappa Q_{sc}\hat{W}\bar{y}\,; \qquad u^2 \equiv \frac{\int_{-\infty}^{\infty} \mathrm{v}^2 f(\mathrm{v},\tau)d\mathrm{v}}{\int_{-\infty}^{\infty} f(\mathrm{v},\tau)d\mathrm{v}}\,; \qquad \bar{y}'(\pm\infty) = 0.$$

$$\hat{\mathbf{W}}\bar{y} = \int_{\tau}^{\infty} W(\tau - s)\exp(i\zeta(\tau - s))\rho(s)\bar{y}(s)ds\,.$$

$Q_{sc} = Q_{sc}(\tau)$ is cross-section averaged.

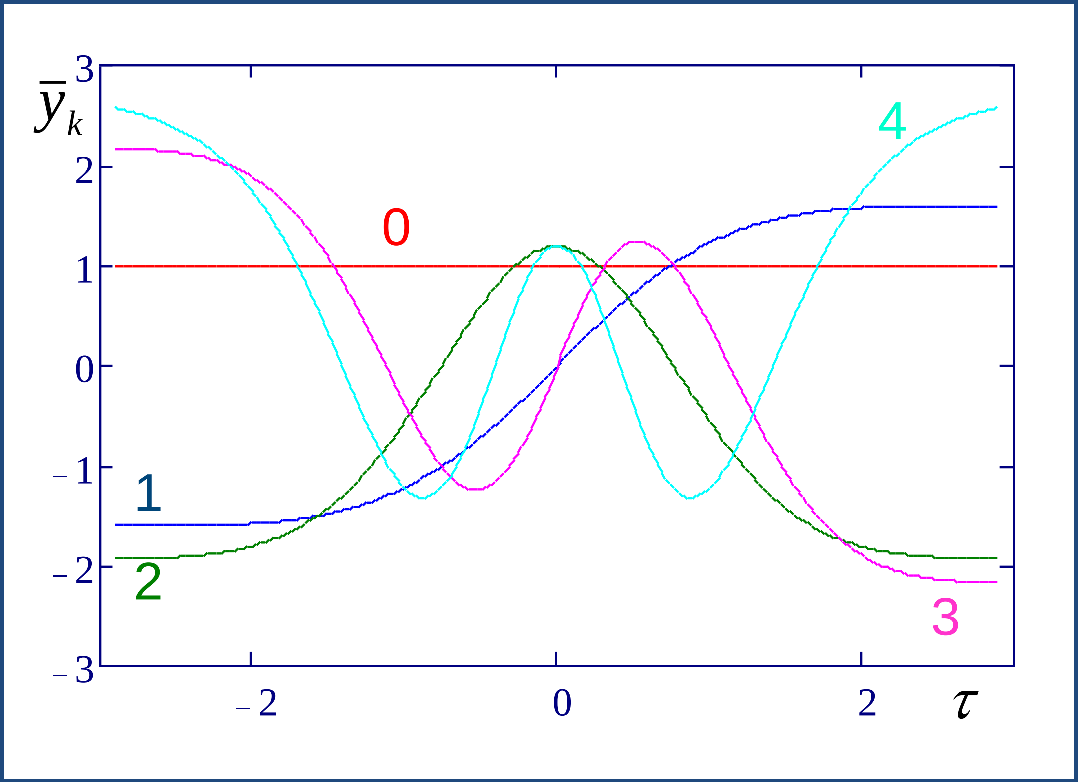
Damper can be treated as an imaginary/complex wake.

Without wake, the orthogonality condition is satisfied:

$$\int d\tau \rho(\tau)\bar{y}_i(\tau)\bar{y}_j(\tau) = \delta_{ij}$$

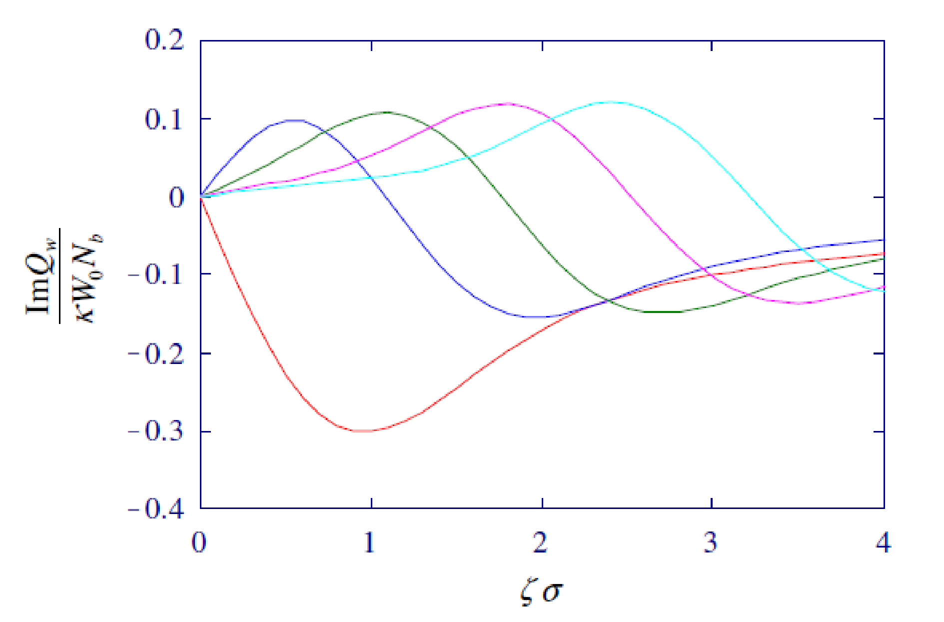
With wake, these functions serve as a complete orthonormal basis

No-wake (space charge only) modes, Gaussian bunch



These modes do not look too different from no-space charge case.
The only significant difference is that for strong space charge, modes are counted by a single integer, while conventional zero-space-charge modes require 2 integers: for azimuthal and radial numbers (due to their possible variations along synchrotron phase and action).

Weak Head-Tail for Strong SC



Coherent growth rates for the Gaussian bunch

with the constant wake W_0 as functions of the head-tail phase $\chi = \zeta\sigma$, for the lowest mode 0 (red), mode 1 (blue), mode 2 (green), mode 3 (magenta), and mode 4 (cyan).

The rates are in units of $N_b W_0$.

Burov, 2009

Vanishing TMCI

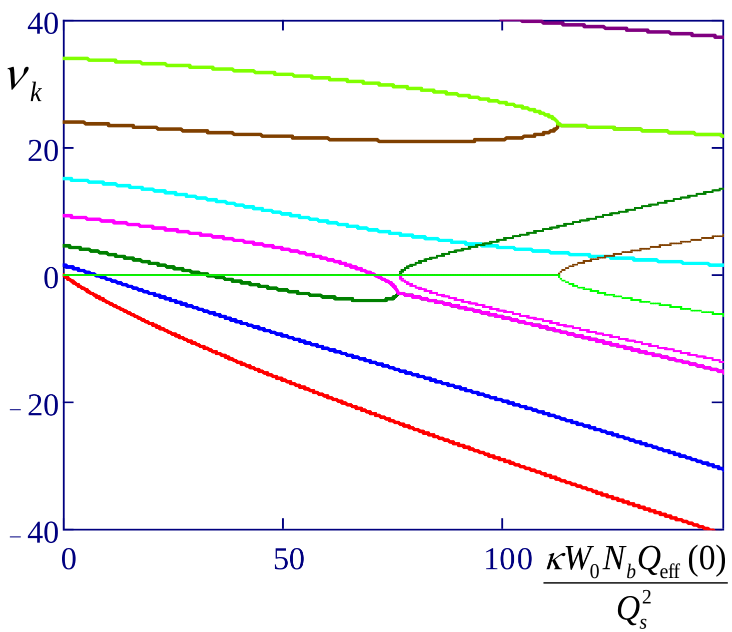
- While the conventional head-tail modes are numbered by integers,

$$\nu_k = kQ_s\ , k = 0, \pm 1, \pm 2, ...$$

the space charge modes are numbered by natural numbers:

$$\nu_k \propto k^2\ .$$

- This is a structural difference, leading to significant increase of the transverse mode coupling instability: the most affected lowest mode has no neighbor from below.



Coherent tunes of the Gaussian bunch for zero chromaticity and constant wake versus the wake amplitude.

Note high value of the TMCI threshold.

Landau Damping (LD)

- For strong space charge $Q_{sc} >> Q_s$, particles and modes are tune-separated by Q_{sc}, so their resonance seems to be impossible.

- However, this is not generally correct, since the SC tune shift goes to 0 at the bunch tails, where this resonance may happen.

- The higher is SC, the further to the tails it happens. Thus, space charge strongly suppresses LD.

- Positive octupolar tune shift provides more LD from high transverse amplitudes.

Landau damping – results

- According to (Burov, 2009), for Gaussian bunch intrinsic LD rate is estimated as:

$$\Lambda_k \cong 0.1 k^4 Q_s \left(\frac{Q_s}{Q_{sc}} \right)^3$$

(the numerical factor ~ 0.1 – a best fit of Vladimir Kornilov with his tracking simulations, 2010)

- Note: mode $k=0$ is not damped at all. It is stable though for the proper sign of the chromaticity. For coupled bunches – damper/octupoles are needed.

- With octupoles, an asymptotic estimation yields (never benchmarked so far!):

$$\Lambda_k \simeq 10 \frac{\Delta Q_{rms}^2}{Q_{sc}} .$$

Space Charge and E-Cloud

- Electron cloud generates both a destabilizing (broad-band) wake and stabilizing nonlinearity. Apparently, the latter should be higher, so e-cloud by itself should not drive a coherent instability.

- However, e-cloud nonlinearity may be partly cancelled by other nonlinearities, as it apparently happens at LHC (3-beam instability or beam-beam-beam effect? – Burov, 2013). If so, e-cloud wake drives the instability.

- If the space charge tune shift is higher than e-cloud's one, the latter is not important, so e-cloud drives instability.

- Apparently, the beam should be stabilized with higher e-density.
- However, e-cloud accumulation may be stopped at lower intensity by the instability itself.

Space Charge Trick

- Space charge tune shift limits beam emittances.

- However, there is a trick to reduce one of the two emittances down to “zero”, keeping another just sqrt(2) higher, and having the same space charge tune shift as for the conventional planar optics.

- This is achieved by means of circular optics.

- When needed, circular modes can be transferred to planar and back by means of skew triplets (Derbenev, Burov & Danilov).

- This trick looks extremely attractive for hadron colliders:

  A. Burov, “Circular modes for flat beams in LHC”, CERN AP Forum, Oct 1, 2012; Proc. HB’2012 workshop.

Summary

- SC effects on beam transverse oscillations can be described by MSE, applicable at strong SC both for coasting and bunched beams.

- With SC, bunch modes are counted by a single integer parameter (without SC it takes 2 of them).

- The eigensystem can be found from 1D ordinary IDE.

- TMCI is suppressed by SC, but LD is suppressed as well.

- SC should be expected to provoke e-cloud instability.

- SC limit to smaller emittance can be avoided: circular modes.

Many Thanks!