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Modelling with Space-Charge

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and

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Reference: *"Computer Simulation Using Particles"*, R.W. Hockney and J.W. Eastwood (UKAEA Culham and Reading University).

ICFA Space-Charge Workshop
April 2003

Historical

- 1980's - inertial confinement fusion, behaviour of beams under intense space charge
  - Lawson, Reiser, Keefe, Wangler..
- 1990's - spallation neutron sources: EHF, JHF, ESS, SNS
  - Machida, Holmes ...
- 2000's - proton drivers, neutrino factory, Fermilab upgrades
- 2010's - ...

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Message: Don't run codes blindly and assume the answers are always right. Question the results. It pays to know what is in the codes, what they calculate and how. In accelerator modelling, all-purpose "black boxes" don't always work

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Overview

- Principles of simulation with space-charge
- Calculation of space-charge forces
- Analytical models
- Formulation of results
- Benchmarking and validation
- Use of codes
- Illustrations
- List of codes

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Basic Equations (Paraxial)

$$x'' + k_x(s)x - \frac{q}{m_0\gamma^3\beta^2c^2} E_x(x,y,z,s) = 0 \quad (1)$$

$$y'' + k_y(s)y - \frac{q}{m_0\gamma^3\beta^2c^2} E_y(x,y,z,s) = 0 \quad (2)$$

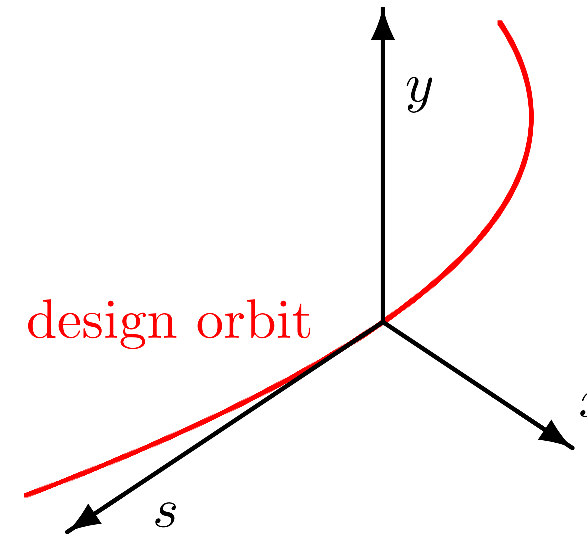
$$z'' + k_z(s)z - \frac{q}{m_0\gamma^3\beta^2c^2} E_z(x,y,z,s) = 0 \quad (3)$$

Particle coordinates (x, y, z) with respect to frame whose motion is given by s

Factor $1/\gamma^2$ from electrostatic-magnetostatic effects

Other factor γ from relativistic mass $m = m_0\gamma$

Space-charge field $\mathbf{E}$ from Maxwell's equation:



$$\nabla \cdot \mathbf{E} = \frac{q}{\epsilon_0}\, n(x,y,z,s) \quad (4)$$

where $n(x, y, z, s)$ is the number density of the beam distribution.

$n(x, y, z, s)$ given by the particle density $f(x, y, z, x', y', z', s)$ in six-dimensional phase space, which must satisfy the Vlasov equation

$$\frac{\partial f}{\partial s} + (\mathbf{x}' \cdot \nabla) f - (\mathbf{k} - \frac{q}{m_0 \gamma^3 \beta^2 c^2} \mathbf{E}) \cdot \nabla_{\mathbf{x}'} f = 0, \tag{5}$$

through

$$n = \iiint f(x, y, z, x', y', z', s)\, \mathrm{d}x'\, \mathrm{d}y'\, \mathrm{d}z'. \tag{6}$$

The total number of particles in the beam is

$$N = \iiint n(x, y, z, s)\, \mathrm{d}x\, \mathrm{d}y\, \mathrm{d}z. \tag{7}$$

This is a complete set of seven coupled equations in which the distribution determines the forces, which determine the motion, which determines the distribution, and so on.

Recall: For a 2D uniform beam with elliptical cross section $\frac{x^2}{a^2} + \frac{y^2}{b^2} \leqslant 1$, space-charge forces are linear and given by

$$\mathbf{E} = \frac{Nq}{\pi\epsilon_0(a+b)}\left(\frac{x}{a}, \frac{y}{b}\right),$$

where N is the number of particles per unit length.

Equations of particle motion and envelope equations are then:

$$\begin{aligned}
x'' + k_x(s)x - \frac{2K}{a+b}\frac{x}{a} &= 0 \\
y'' + k_y(s)y - \frac{2K}{a+b}\frac{y}{b} &= 0 \\
a'' + k_x(s)a - \frac{\epsilon_x^2}{a^3} - \frac{2K}{a+b} &= 0 \\
b'' + k_b(s)b - \frac{\epsilon_y^2}{b^3} - \frac{2K}{a+b} &= 0
\end{aligned}$$

$K = \frac{I}{I_0}\frac{2}{(\beta\gamma)^3}$ is the Perveance and $I_0 = \frac{4\pi\epsilon_0 m_0 c^3}{q}$

For non-linear beams, rms beam size is $\tilde{a} = \sqrt{\langle x^2 \rangle}$ and rms evolution equations are

$$\frac{\mathrm{d}^2 \tilde{a}}{\mathrm{d}s^2} + k_x(s)\tilde{a} - \frac{\tilde{\epsilon}^2}{\tilde{a}^3} - \frac{q}{m_0 \gamma^3 \beta^2 c^2} \frac{\langle x E_x \rangle}{\tilde{a}} = 0 \tag{1}$$

$$\frac{\mathrm{d}^2}{\mathrm{d}s^2}\tilde{\epsilon}^2 = \frac{2q}{m_0 \gamma^3 \beta^2 c^2} \left[\langle x^2 \rangle \langle x' E_x \rangle - \langle x x' \rangle \langle x E_x \rangle \right] \tag{2}$$

$$\text{where} \qquad \tilde{\epsilon} = \sqrt{\langle x^2 \rangle \langle x'^2 \rangle - \langle x x' \rangle^2} \quad \text{is the rms emittance.} \tag{3}$$

Sacherer showed that for ellipsoidal particle densities of the form

$$n(x, y, z, s) = n\left(\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} \right)$$

the averages $\langle x E_x \rangle, \langle y E_y \rangle$ etc depend only very weakly on the exact charge distribution.

So the general rms envelope equation is the same as for an equivalent KV beam.

Envelope Codes

2D (transverse) codes use envelope equations for KV beam:

$$a'' + k_x(s)a - \frac{\epsilon_x^2}{a^3} - \frac{2K}{a+b} = 0$$

$$b'' + k_y(s)b - \frac{\epsilon_y^2}{b^3} - \frac{2K}{a+b} = 0$$

$$K = \frac{I}{I_0}\frac{2}{(\beta\gamma)^3}$$ is the Perveance

Define $\mathbf{X} = \begin{bmatrix} a \\ a' \\ b \\ b' \end{bmatrix} \implies \frac{\mathrm{d}\mathbf{X}}{\mathrm{d}s} = \frac{\mathrm{d}}{\mathrm{d}s}\begin{bmatrix} a \\ a' \\ b \\ b' \end{bmatrix} = \begin{bmatrix} a' \\ -k_x(s)a + \frac{2K}{a+b} + \frac{\epsilon_x^2}{a^3} \\ b' \\ -k_y(s)b + \frac{2K}{a+b} + \frac{\epsilon_y^2}{b^3} \end{bmatrix}$

Integrate using standard numerical packages based on, for example, Runge-Kutta techniques.

Output either as beam sizes or Twiss parameters defined by $\beta_x = a^2/\epsilon_x$, $\alpha_x = -aa'/\epsilon_x$ etc

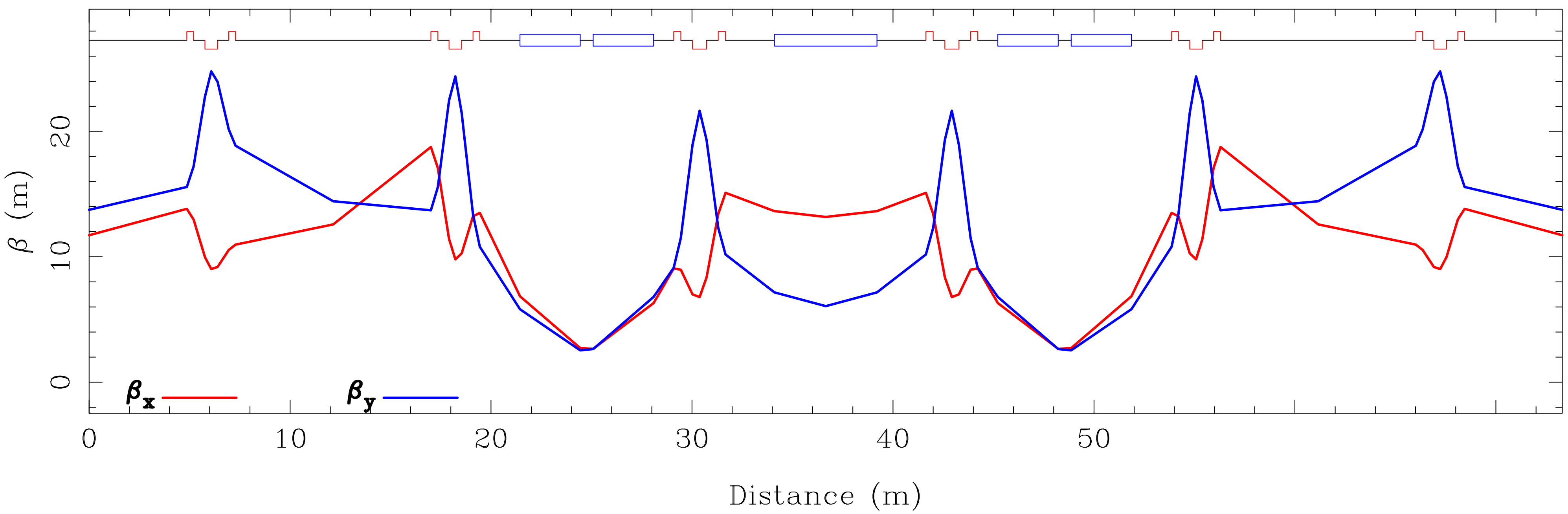
Examples of Envelope Codes

KVBL (Prior) Includes matching, parameter optimisation, graphical output; switches to standard matrix methods in absence of space-charge

WinAgile (Bryant) Similar to KVBL but Windows based with GUI

Trace2D Old code from Los Alamos, based on matrices with space charge kicks in the middle of elements

Trace3D Development of Trace2d to include momentum effects; works from a keyboard implemented graphics system.



β-functions with space-charge from the ESS

KVBL: ESS Funnel (1996)

- 57 mA linac current (H^-)
- Achromatic in all planes
- Uniform optics to minimise non-linear space-charge effects

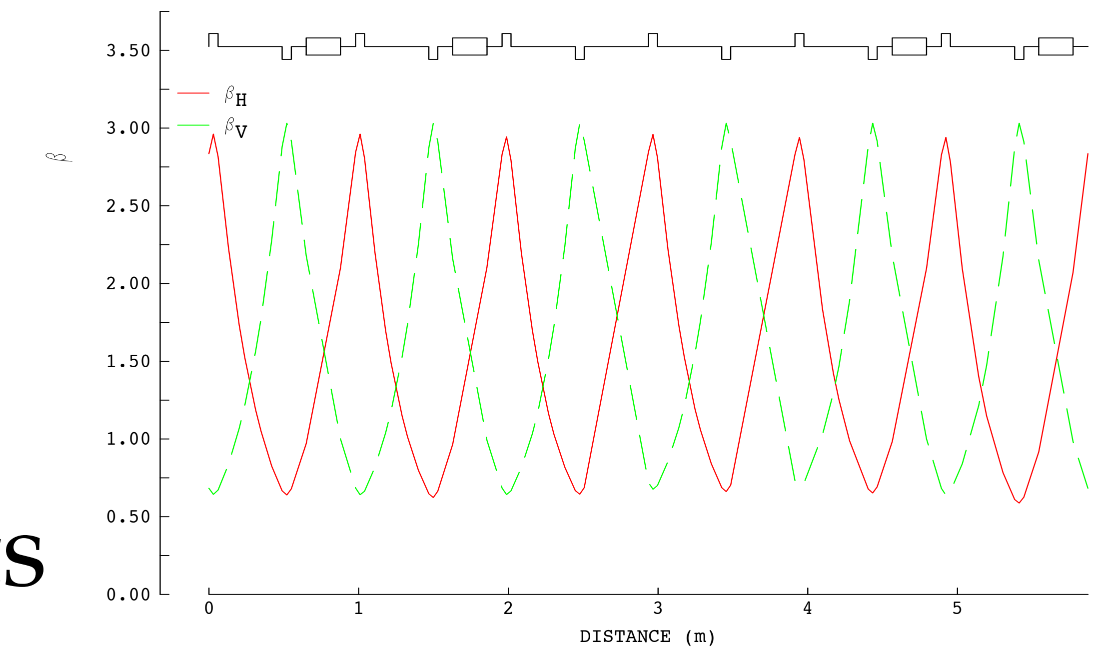
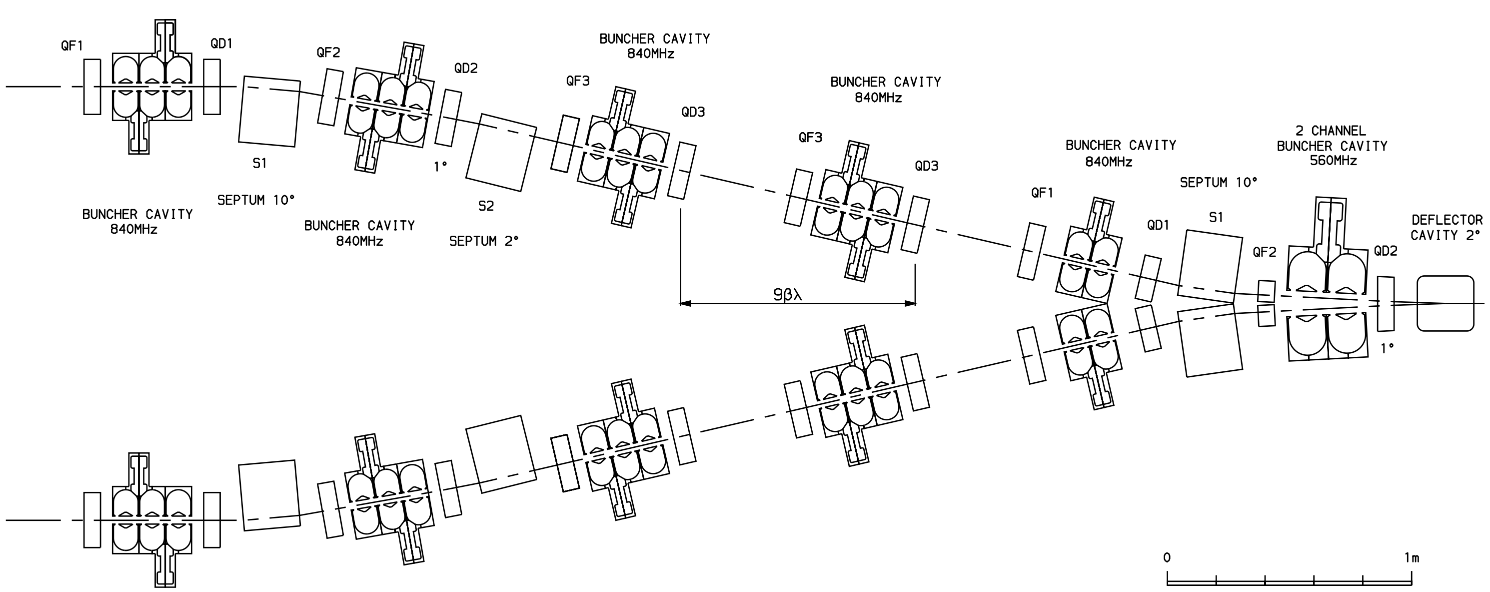
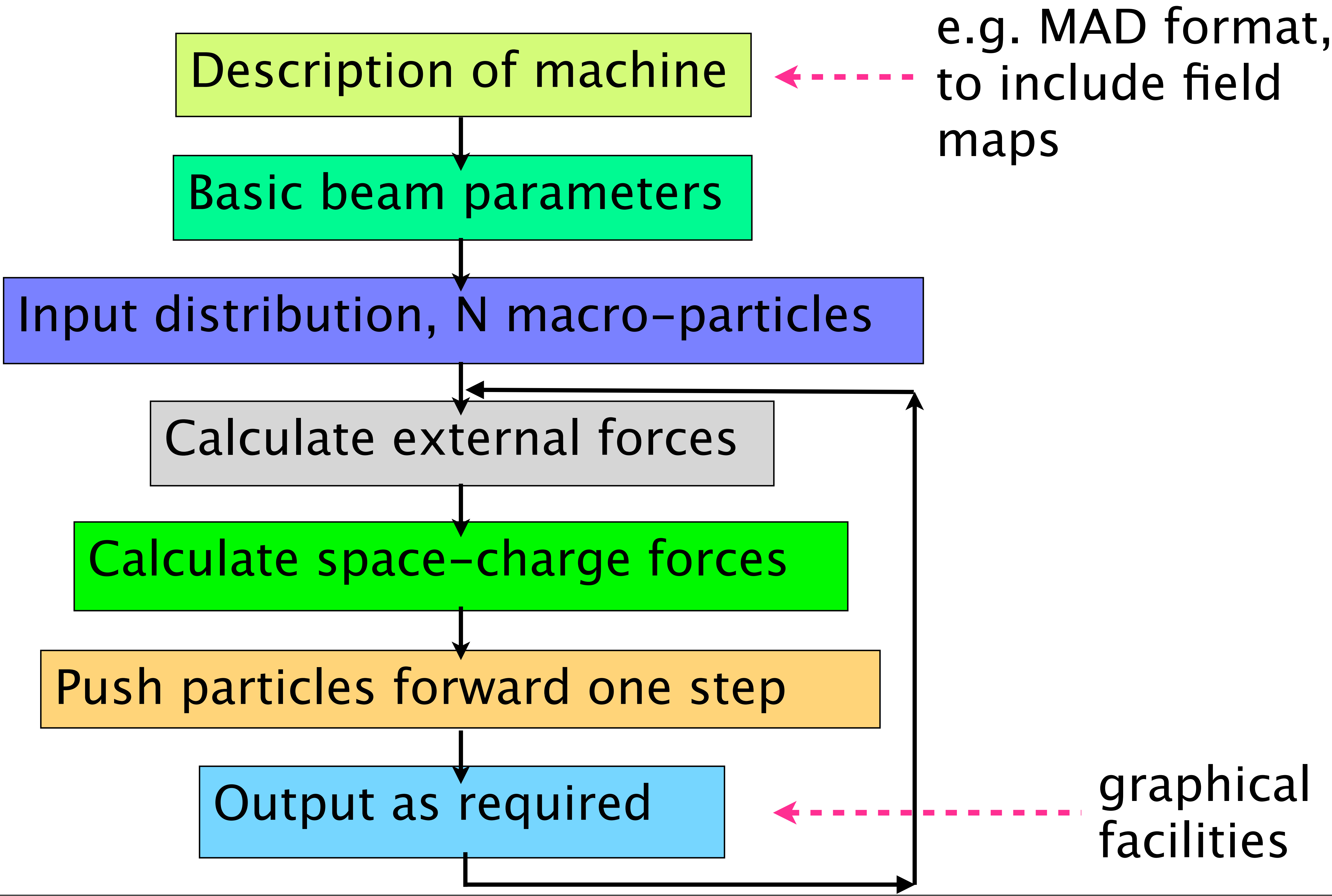


Figure 3: Focusing parameters for the ESS funnel



Enabled emittance growth to be reduced to <1% cf ~50% in early LANL experiments

Basic Tracking Procedures



Input Beam Distribution

- How many macro-particles are needed to model a beam of 10^{10}-10^{14} real particles?
  - sufficiently many for good statistics
  - predicted effects should not be a consequence of reduced number of particles, statistical errors, rounding and interpolation errors.
- Most space-charge codes now use ~10^5-10^6 simulation particles; some runs have been made with 10^7-10^8.

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Initial Particle Distribution

- Can be read from a given dataset, for example from a previous run, or can be based on physical data.
- Can be a model distribution - KV, Waterbag, Gaussian, semi-Gaussian etc
- Also include stationary distributions (self-consistent functions of the Hamiltonian H)
  - generate a normalised distribution; then scale and rotate as appropriate
  - may need to change coordinates, e.g Cartesians to 4D-polar system
  - can be fitted to given beam sizes or created as an rms equivalent beam

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Method based on $f(x)\,\mathrm{d}x = \mathrm{d}F(x)$ where $F'(x) = f(x)$; this may not always be possible.

Method of Ratio of Uniform Deviates:

The density function $f(x)$ can be generated through a uniform filling of the region

$$0 < u < \sqrt{f\left(\frac{v}{u}\right)}$$

of the two-dimensional (u, v) plane with a random number generator. Then $x = \frac{v}{u}$ has the desired density function.

Example: The Cauchy Distribution $f(x) = \frac{1}{\pi}\frac{1}{1+x^2}$.

The sampling region is

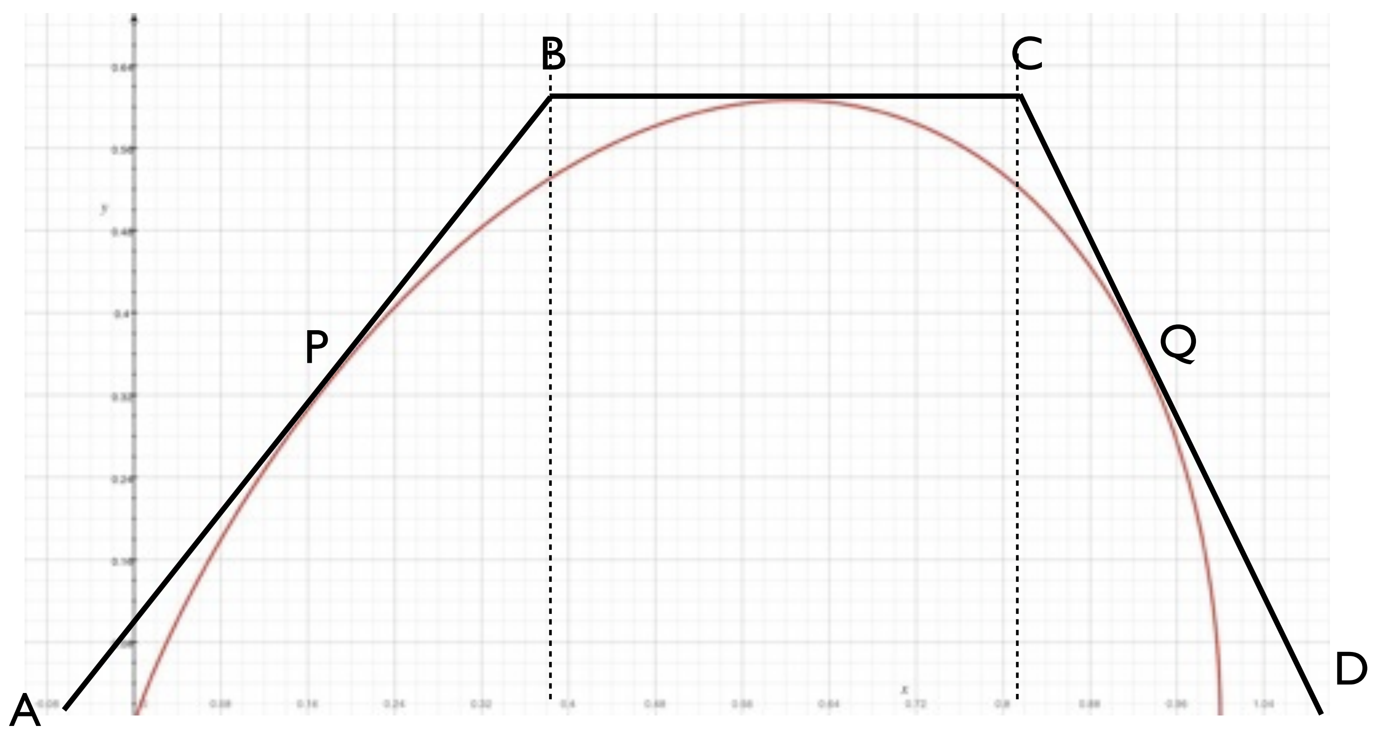
$$\left\{(u,v) : 0 \le u \le \left[1 + \left(\frac{v}{u}\right)^2\right]^{-\frac{1}{2}}\right\} = \{(u,v) : u^2 + v^2 \le 1, u \ge 0\}.$$

Half circle, centred on origin, radius 1.

Example: Gaussian Distribution $f(x) = \dfrac{1}{\sqrt{2\pi}} \mathrm{e}^{-\frac{1}{2}x^2}$.

The sampling region is

$$\left\{(u,v) : 0 \le u \le \mathrm{e}^{-\frac{v^2}{2u^2}}\right\} = \{(u,v) : v \le \sqrt{-2u^2 \ln u},\ 0 \le u \le 1\}.$$



Sampling method: adjust tangents at P, Q to minimise area ABCD.
$\Longrightarrow$ Probability that a point within ABCD is also within required region is 0.922. Method uses some simple initial pre-sampling to check that a point (u,v) lies in region; if so, then a more accurate check.

Approach turns out to be faster than most other methods and also gives a very good model of the required distribution.

Solution of Equations of Motion

Equations of motion of the form $\mathbf{x}'' = \mathbf{F}(s, \mathbf{x}, \mathbf{x}')$

Euler (forward) difference method: $\begin{bmatrix} \mathbf{x}_{n+1} \\ \mathbf{x}'_{n+1} \end{bmatrix} = \begin{bmatrix} \mathbf{x}_n + h\mathbf{x}'_n \\ \mathbf{x}'_n + h\mathbf{F}(s_n, \mathbf{x}_n, \mathbf{x}'_n) \end{bmatrix}$

h is step length

Accuracy is only $O(h^2)$

Could use *Runge-Kutta* method accurate to $O(h^4)$, but requires extra storage and 4 calculations of $\mathbf{F}$ for each particle instead of one. Can be reduced by Blum's method, but unlikely to give a viable method for modelling $\gtrsim 10^5$ particles in a realistic time.

Set $\mathbf{X} = \begin{bmatrix} \mathbf{x} \\ \mathbf{x}' \end{bmatrix}$, $\quad \frac{\mathrm{d}\mathbf{X}}{\mathrm{d}s} = \mathbf{f}(s, \mathbf{X}) = \begin{bmatrix} \mathbf{x}' \\ \mathbf{F}(s, \mathbf{x}, \mathbf{x}') \end{bmatrix}$

$$\left.\begin{aligned} \mathbf{k_1} &= h\mathbf{f}(s_n, \mathbf{X}_n) \\ \mathbf{k_2} &= h\mathbf{f}(s_n + \tfrac{1}{2}h, \mathbf{X}_n + \tfrac{1}{2}\mathbf{k_1}) \\ \mathbf{k_3} &= h\mathbf{f}(s_n + \tfrac{1}{2}h, \mathbf{X}_n + \tfrac{1}{2}\mathbf{k_2}) \\ \mathbf{k_4} &= h\mathbf{f}(s_n + h, \mathbf{X}_n + \mathbf{k_3}) \end{aligned}\right\} \Longrightarrow \mathbf{X}_{n+1} = \mathbf{X}_n + \tfrac{1}{6}(\mathbf{k_1} + 2\mathbf{k_2} + 2\mathbf{k_3} + \mathbf{k_4})$$

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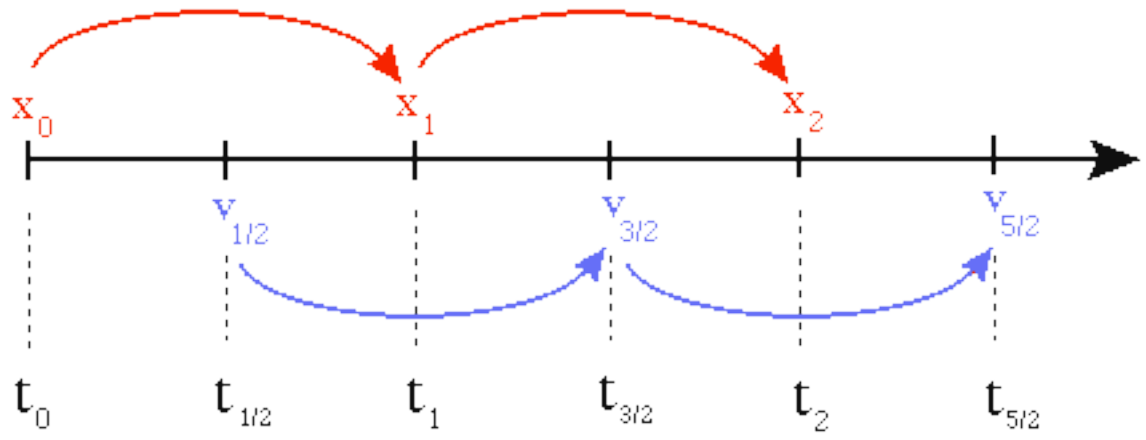
Symplectic Leap-frog Integration Scheme



Interleave position and velocity half a step out of phase, and leap-frog coordinates forward in distance or time.

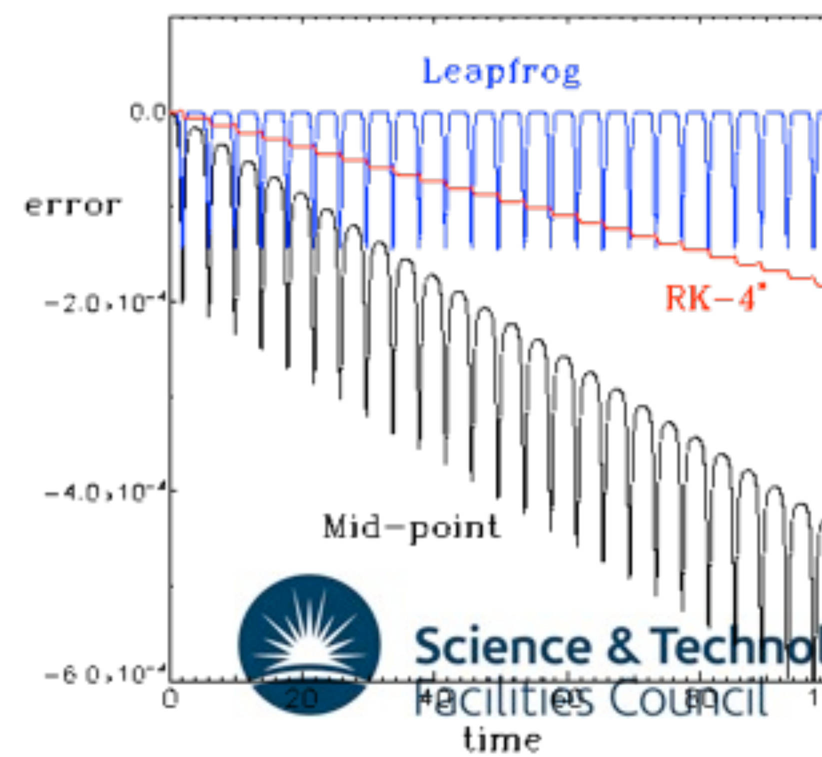
$$\mathbf{x}_{n+1} = \mathbf{x}_n + h\mathbf{x'}_{n+\frac{1}{2}}$$

Error is $O(h^3)$

$$\mathbf{x'}_{n+\frac{1}{2}} = \mathbf{x'}_{n-\frac{1}{2}} + \frac{1}{2}h\left[\mathbf{F}(s_n, \mathbf{x_n}, \mathbf{x'}_{n-\frac{1}{2}}) + \mathbf{F}(s_n, \mathbf{x_n}, \mathbf{x'}_{n+\frac{1}{2}})\right]$$

Step length must be chosen to allow plasma oscillations to be represented ($\omega_p h/\beta c \ll 2$)

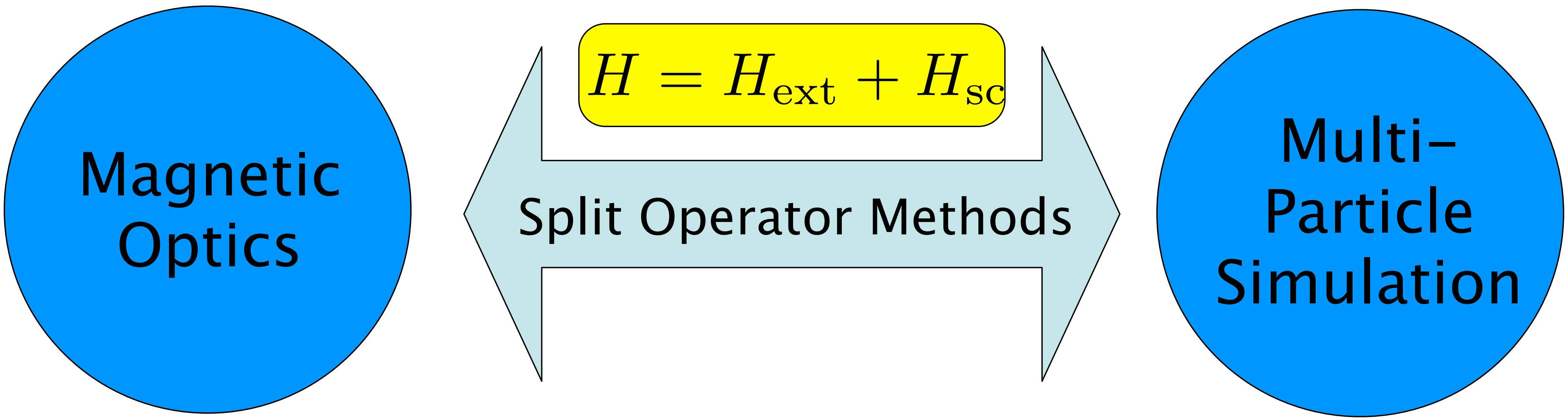
Method is extremely stable. It has a time-reversible property, which avoids long-term drift caused by systematic errors that could mask the true solution. Note: even 4th order Runge-Kutta suffers.



Split Operator Approach

$H = H_{\text{ext}}$ $\qquad$ $H = H_{\text{sc}}$

$$H = H_{\text{ext}} + H_{\text{sc}}$$



$\mathcal{M} = \mathcal{M}_{\text{ext}}$ $\qquad$ $\mathcal{M} = \mathcal{M}_{\text{sc}}$

$$\mathcal{M}(t) = \mathcal{M}_{\text{ext}}\left(t/2\right)\,\mathcal{M}_{\text{sc}}(t)\,\mathcal{M}_{\text{ext}}\left(t/2\right) + \mathrm{O}\left(t^3\right)$$

Philosophy:

- Do not take tiny steps to push ~10^7–10^8 particles
- Do take tiny steps to compute maps, then push particles with maps.

Acknowledgement: Rob Ryne/Ji Qiang – IMPACT

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Space-Charge Calculations

Many different approaches, most based on approximations.

- Use Coulomb forces between pairs of particles to calculate forces at each step.
- Assume variation of space-charge with distance is small; impose space-charge kicks once or twice per element, but track using non-space-charge methods otherwise.
- Use KV linear space-charge formula, scaled by longitudinal line density.
- Calculate space-charge potential from Poisson's equation in beam-frame, either in 3D or 2+1D.

Note: some methods ignore boundary effects

Space-Charge Options

- Coulomb approach is $O(N^2)$ and very time consuming. Problems when particles move too close together $\implies$ cut-off distance needed (Debye length). Also open to rounding errors, especially on the axes where transverse forces should sum to zero.
  e.g 2D circular uniform beam, $N = 50,000$ simulation particles: calculations take several minutes and only 60% within 10% error band.

- 2D+1D approach gives good results (beam split into 2D slices for transverse forces, then longitudinal force from line density).

- Finite difference method represents sophisticated approach but inclusion of boundaries (image effects) not easy.

- Finite elements provide most flexible approach and can call on huge range of engineering expertise.

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Example: SIMPSONS

- Solves Poisson's equation in cylindrical coordinates for perfectly conducting circular pipe:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \Phi}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2\Phi}{\partial\phi^2}=-\frac{\rho}{\epsilon_0\gamma^2}$$

- Charge distribution is Fourier transformed in azimuthal direction

$$\left.\begin{aligned}\Phi &= \sum_m \phi_m \mathrm{e}^{im\phi} \\ \frac{\rho}{\epsilon_0\gamma^2} &= \sum_m n_m \mathrm{e}^{im\phi}\end{aligned}\right\} \Longleftrightarrow \left\{\begin{aligned}\phi_m &= \frac{1}{2\pi}\int_0^{2\pi}\Phi(r,z,\phi)\mathrm{e}^{-im\phi}\,\mathrm{d}\phi \\ n_m &= \frac{1}{2\pi}\int_0^{2\pi}\frac{\rho(r,z,\phi)}{\epsilon_0\gamma^2}\mathrm{e}^{-im\phi}\,\mathrm{d}\phi\end{aligned}\right.$$

- Then $\quad n_m(r,z) = -\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\phi_m(r,z)\right)+\frac{m^2}{r^2}\phi_m(r,z)$

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SIMPSONS (cont)

- Solution with $\phi_m = 0$ on $r = b$ is

$$\phi_m = W(r) - \left(\frac{r}{b}\right)^m W(b), \qquad \phi_{-m} = \phi_m^*, \; m \geq 0$$

where

$$W(r,z) = \begin{cases} \displaystyle\int_0^r \ln\left(\frac{r}{r'}\right) n_m(r',z) r' \, \mathrm{d}r' & \text{for } m = 0 \\ \displaystyle\frac{r^m}{2m}\int_0^r n_m(r',z) r'^{1-m} \, \mathrm{d}r' - \frac{r^{-m}}{2m}\int_0^r n_m(r',z) r'^{1+m} \, \mathrm{d}r' & \text{for } m \neq 0 \end{cases}$$

- In practice, integral is replaced by summation over grid points.
- Then $\vec{E} = \nabla\Phi = \sum_m \left(\frac{\partial \phi_m}{\partial r}, \frac{im}{r}\phi_m, \frac{\partial \phi_m}{\partial z}\right) \mathrm{e}^{im\phi}$

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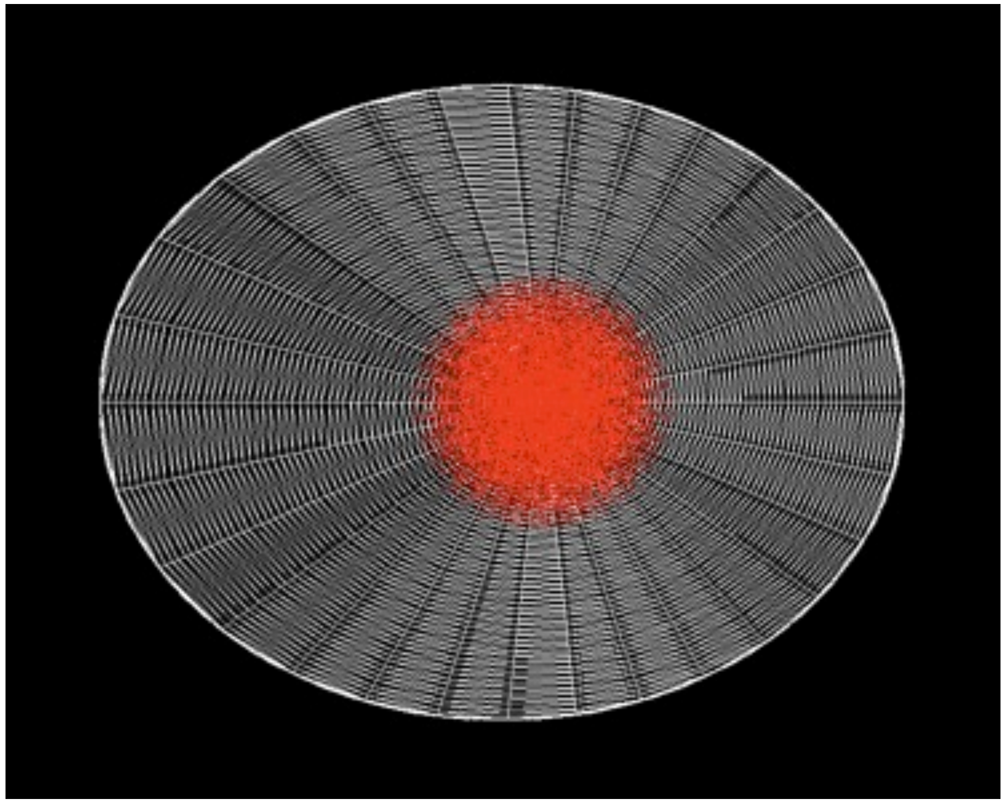
Finite Element Approach

Based on the variational problem:

$$\delta\pi(\phi) = 0, \text{ where } \pi(\phi) = \frac{1}{2}\int_V |\nabla\phi|^2 \, \mathrm{d}V + \frac{1}{\epsilon_0}\int_V \rho\phi \, \mathrm{d}V - \int_{\partial V} \bar{\phi}_n \phi \, \mathrm{d}S$$

Equivalent to $\nabla^2\phi = -\frac{\rho}{\epsilon_0}$ in V, $\phi = \bar{\phi}$, $\frac{\partial\phi}{\partial n} = \bar{\phi}_n$ on ∂V

Cover region with mesh to fit boundaries:



Fit a polynomial to each mesh:

$$\begin{aligned}
\phi &= a_0 && \text{order 0} \\
& \quad +a_1 x + a_2 y + a_3 z && \text{order 1} \\
& \quad +a_4 x^2 + a_5 y^2 + a_6 z^2 && \\
& \quad\quad +a_7 xy + a_8 xz + a_9 yz && \text{order 2} \\
& \quad + \ldots && \\
&= \textstyle\sum_i f_i(\xi_j)\phi_i &&
\end{aligned}$$

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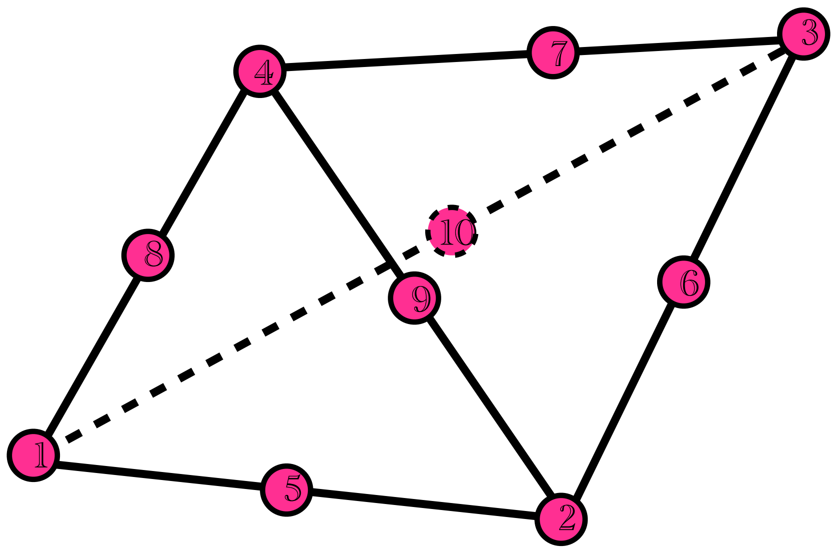
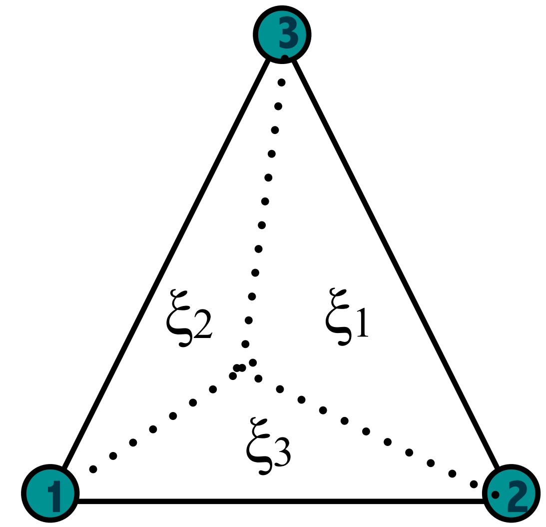
f_i are *shape functions*, ξ_i are *areal coordinates*. Look for a complete set to chosen order. For example to order 2, 10 unknowns $(a_0, \ldots, a_9)$, so need 10 nodes:





Variational problem turns into sparse matrix equation for potential at nodes given by

$$K_{ij}\phi_j = Q_i, \qquad (1)$$

where Q_i come from charge distribution

$$\rho(\mathbf{x}) = \sum_{\text{particles } i} q_i \delta(\mathbf{x} - \mathbf{x}_i) \qquad \Longrightarrow \int \rho\phi \, \mathrm{d}V = \sum_{\text{particles } i} q_i \phi(\mathbf{x}_i).$$

(K_{ij}) depends only on mesh. (1) is solved by standard methods (Gaussian elimination, triple factoring, conjugate gradient etc).

Space-charge forces calculated from

$$\mathbf{F} = -\nabla\phi = -\sum_{i,j} \frac{\partial f_i}{\partial \xi_j} \phi_i \nabla \xi_j .$$

- Since *stiffness matrix* (K_{ij}) depends only on mesh, can be set up and pre-inverted, giving a fast, simple "black-box" for space-charge calculations

$$\phi_i = \sum_j K_{ij}^{-1} \rho_j$$

Standard Test: 2D uniform circular beam, $N = 50,000$ macro-particles

- 3rd Order, $\sim$ 400 mesh elements, find $>$ 95% within 10% error band
- 1st order, $\sim$ 3000 mesh elements, find $>$ 90% within 10% error band

- Simple conversion to different coordinate systems.
  - For example, 2D transverse (x, y) Poisson solver to axisymmetric 3D (r, z) code through $x \to r$, $y \to z$, $\mathrm{d}x\,\mathrm{d}y \to r\,\mathrm{d}r\,\mathrm{d}z$
- 2D code easily converted to 3D, triangular elements to tetrahedra
- Fits all boundaries likely in accelerators. For analytical purposes, "infinite" boundaries modelled using super-elements or matching to $\ln r$ (2D) or $1/r$ (3D) potentials at sufficiently large distances.
- Longitudinal boundary conditions generally periodic (bunch to bunch in linacs or rings)
- May require large amount of storage (not a problem nowadays) and CPU
  - can be parallelised using method of *static condensation* to split into substructures

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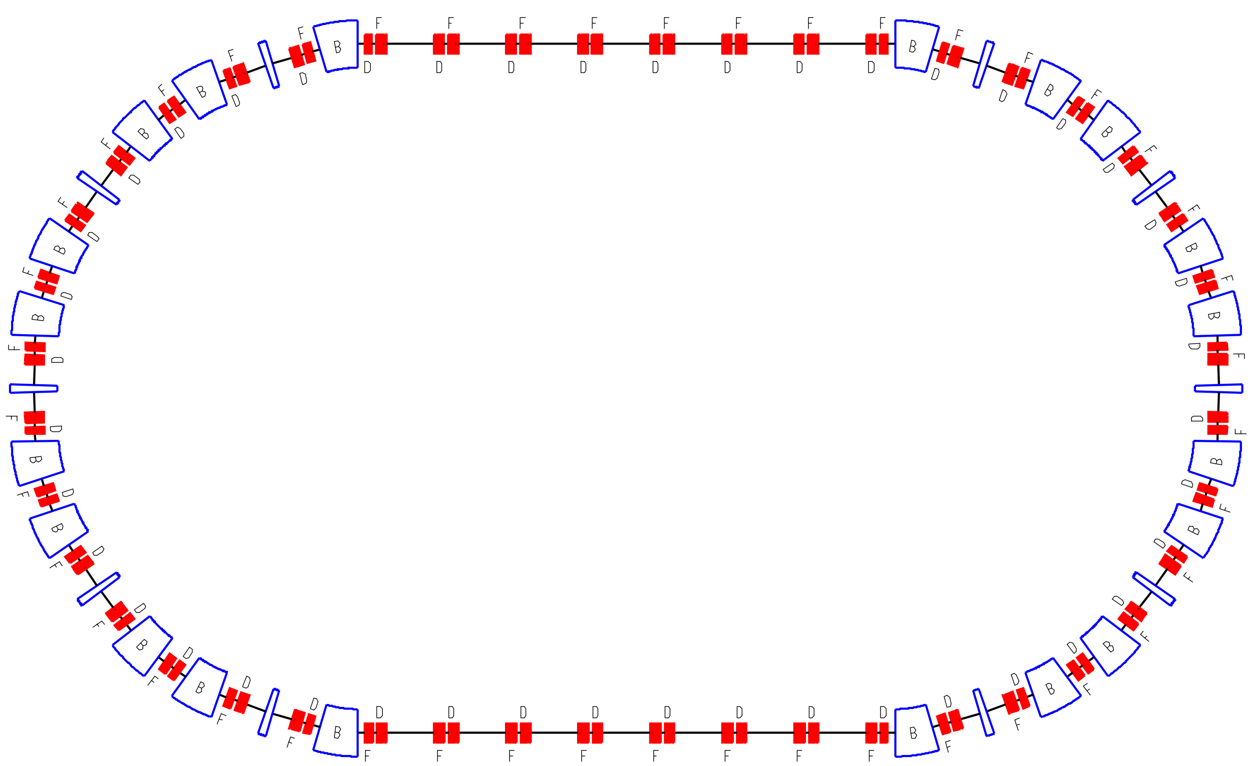
Example: Multiturn Injection at FNAL

Injection into a 0.4-8 GeV RCS designed as replacement for Fermilab booster

- 400 MeV H^- injection
- 45 injection turns, 474 m ring
- Phase space painting
  - Horizontal orbit bumps
  - Vertical variation of beam angle

Basic injection parameters and painting optimisation using codes MISxxx (C.Prior)



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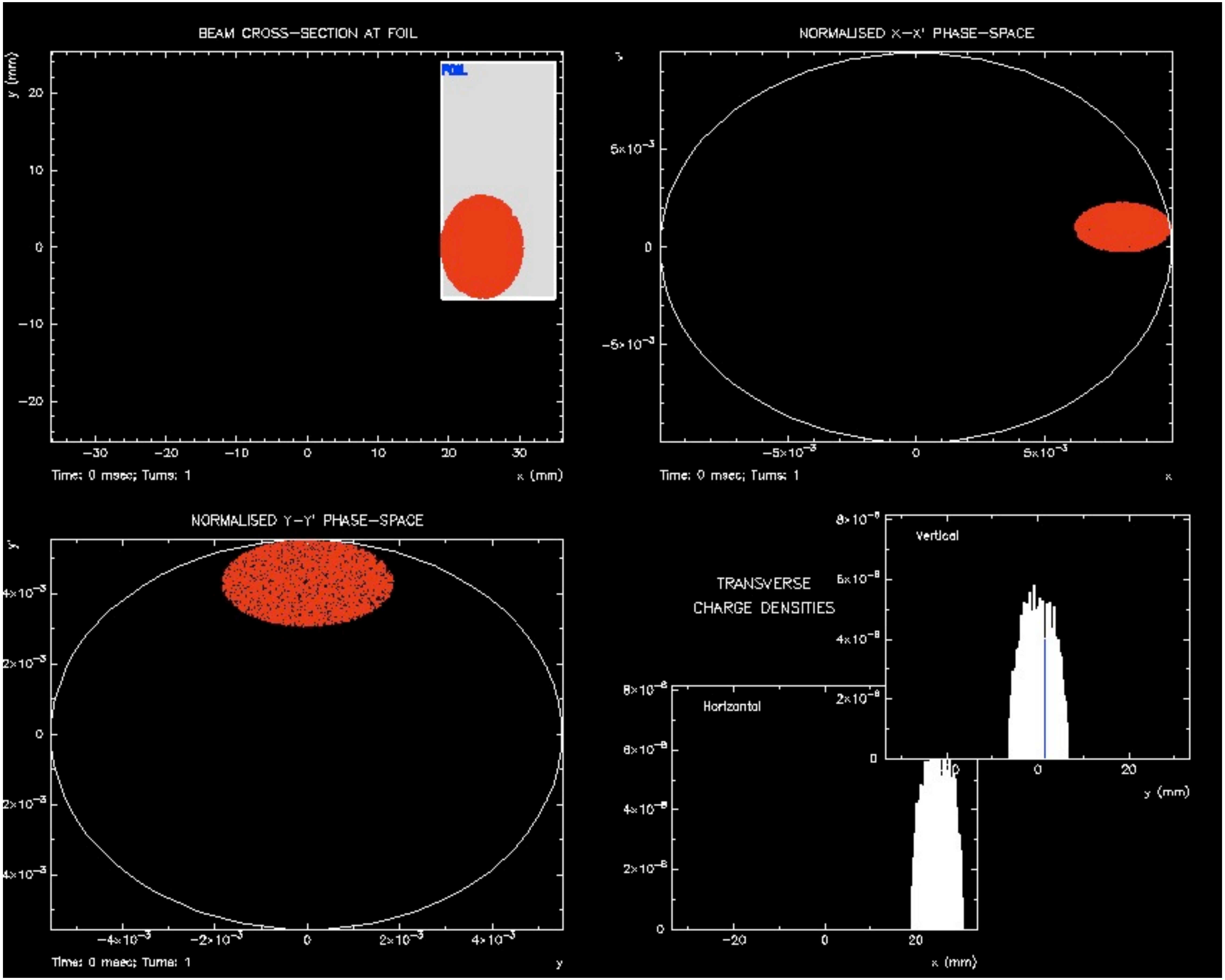
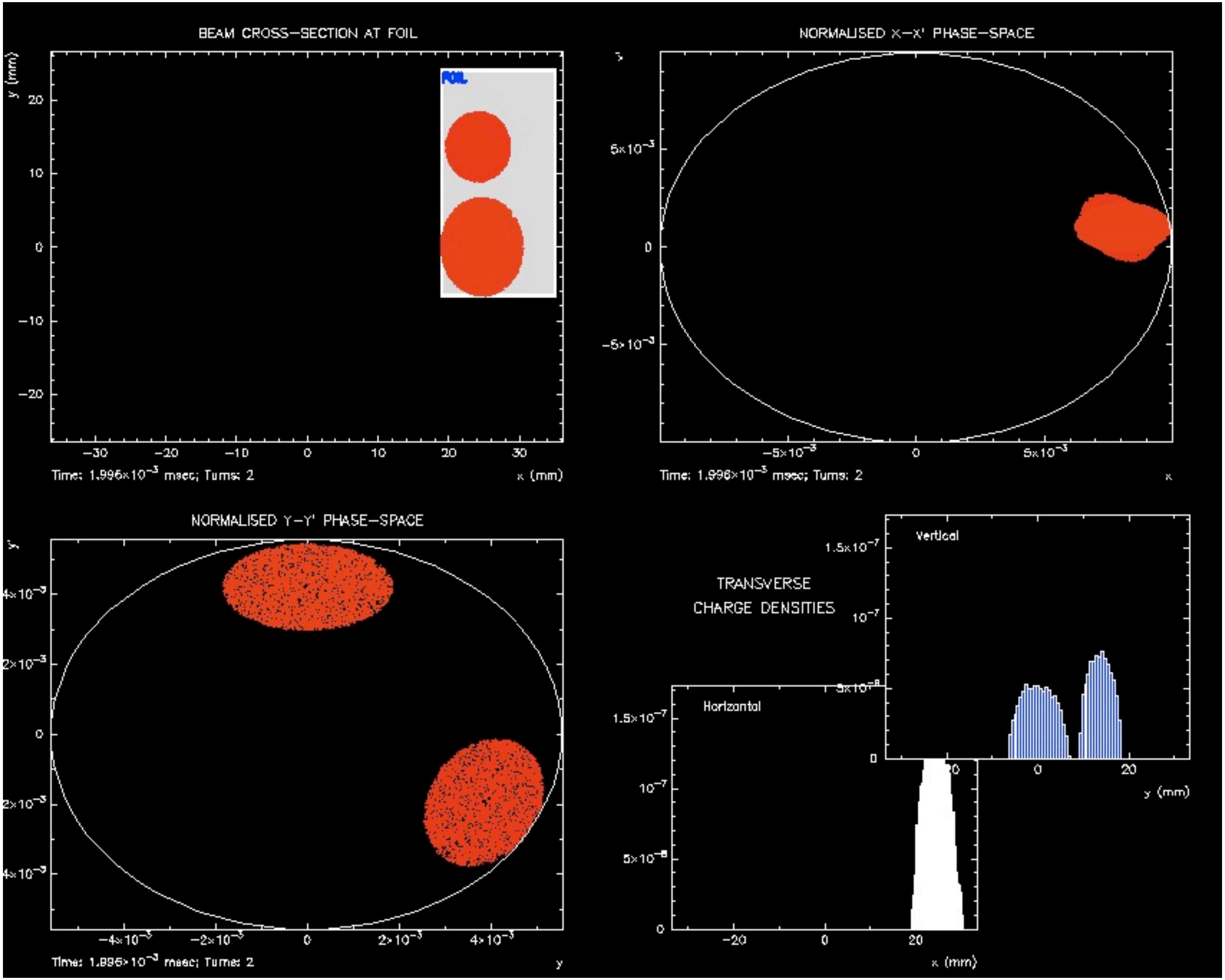
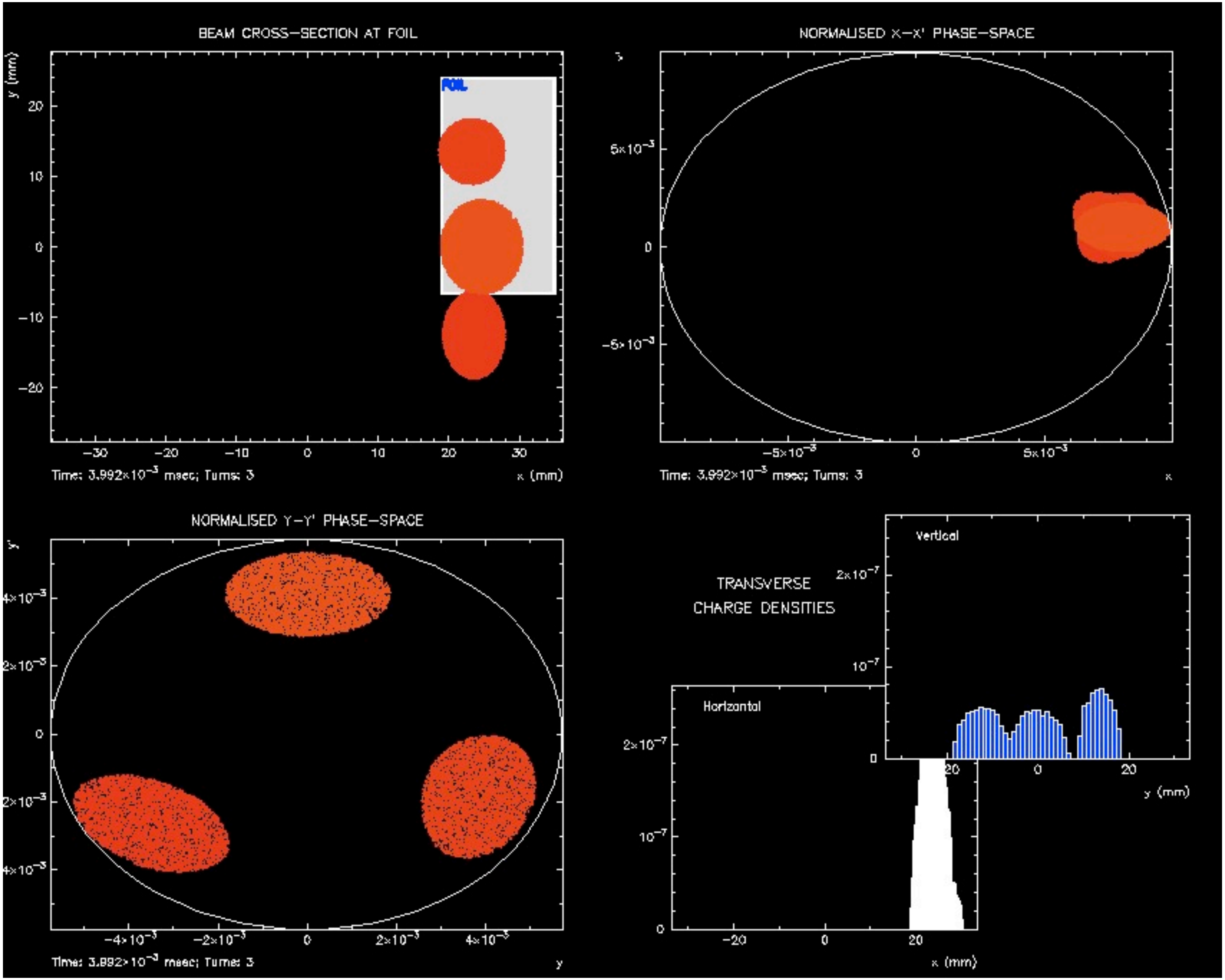
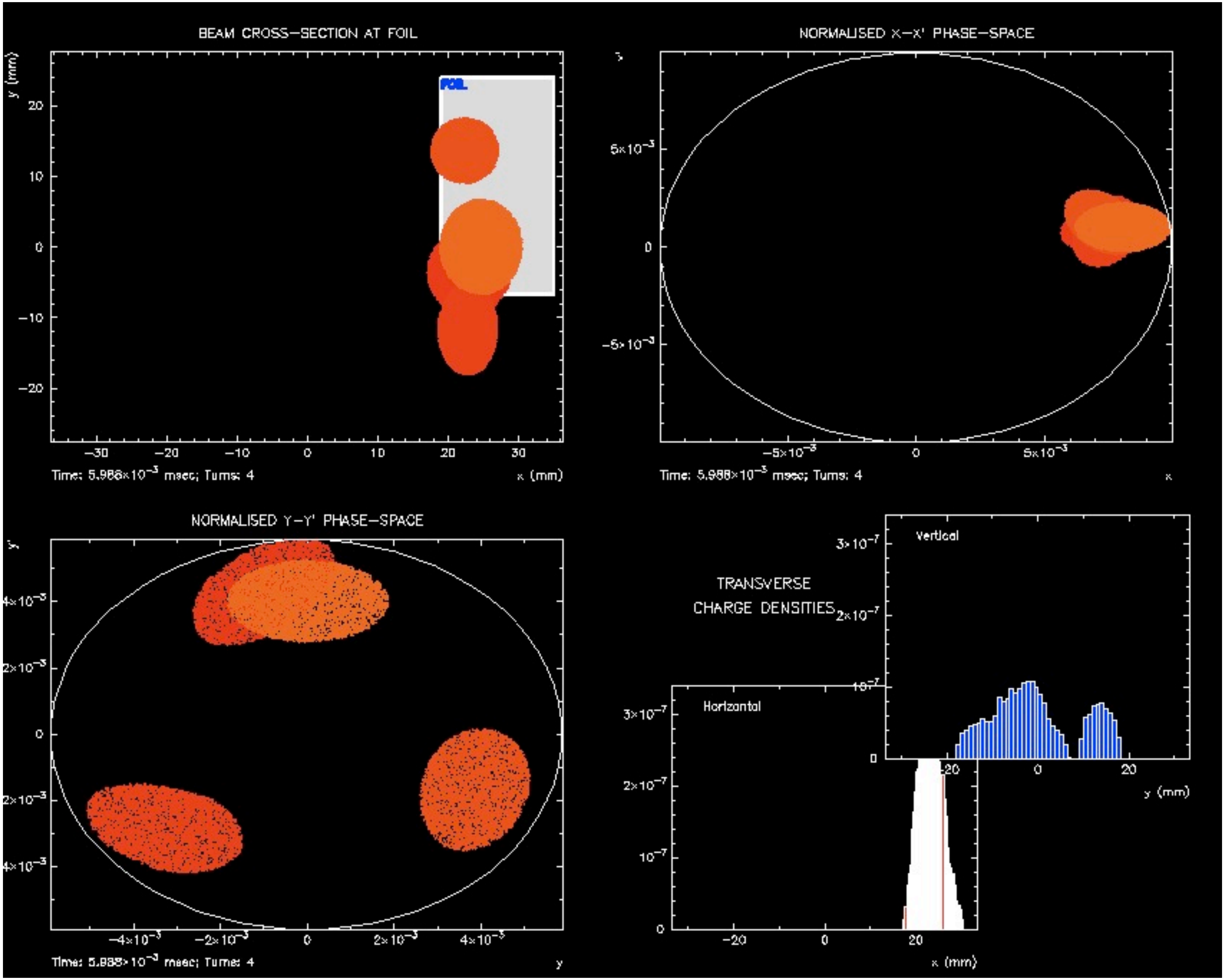
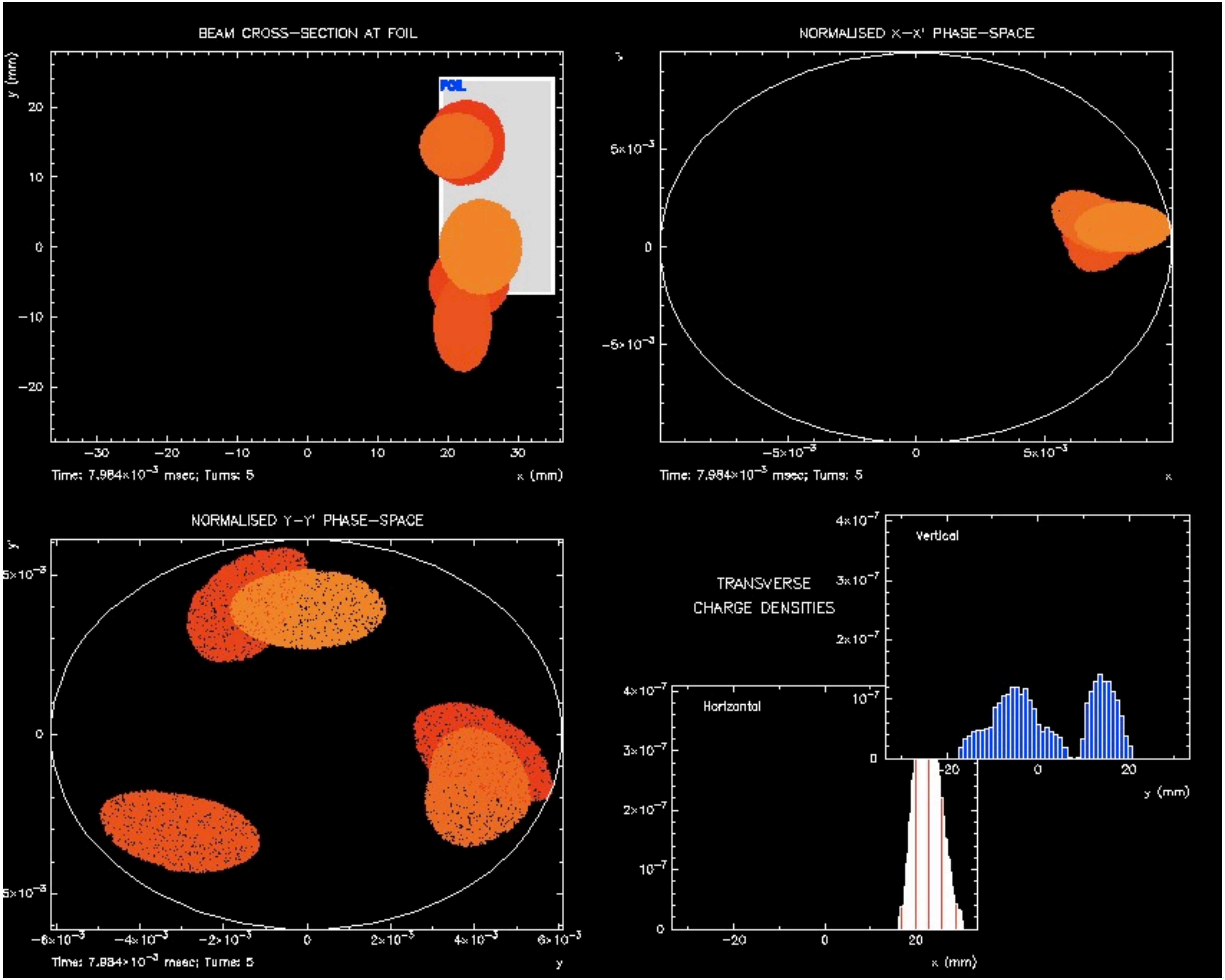


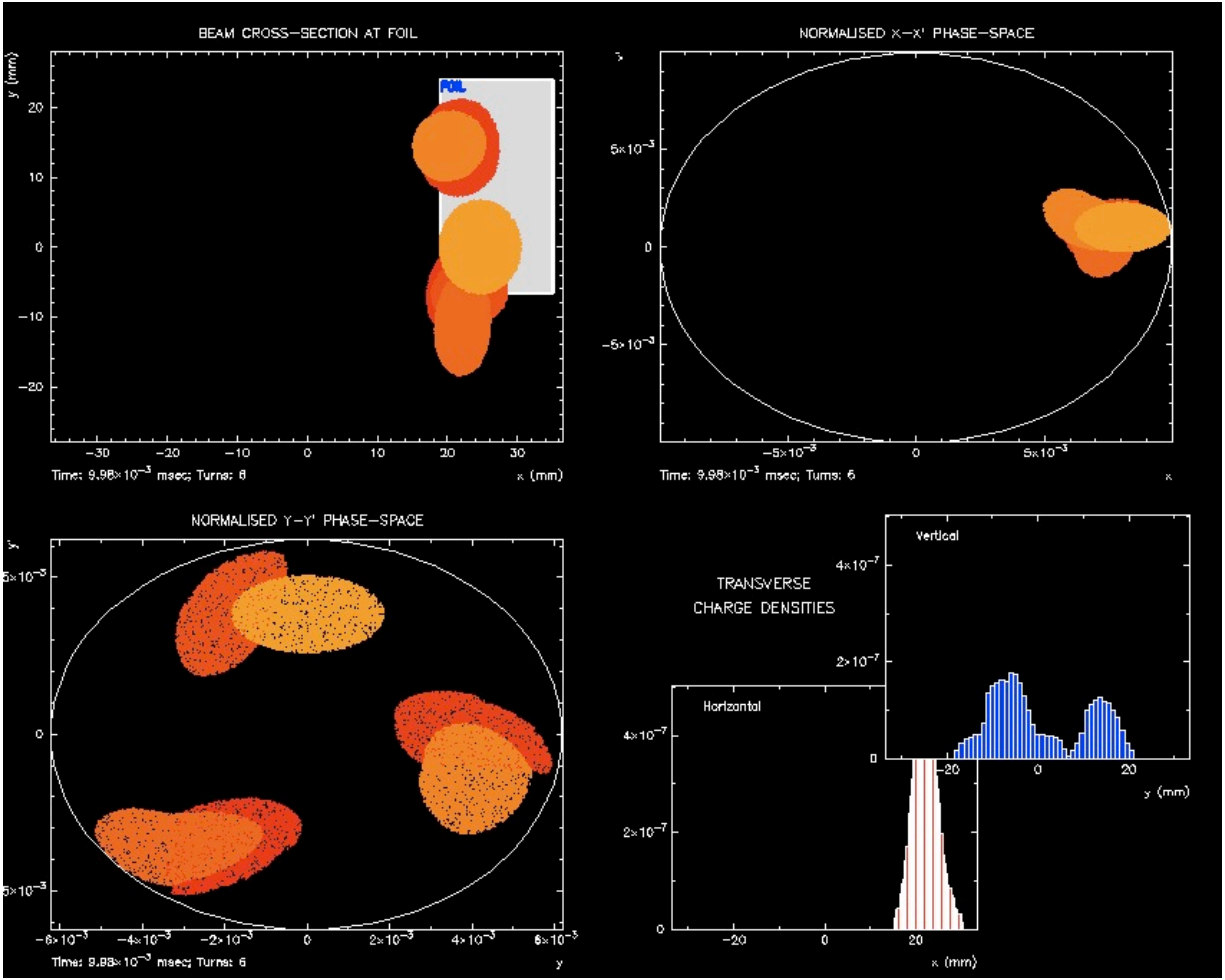
BEAM CROSS-SECTION AT FOIL
y (mm)
FOIL
x (mm)
Time: 0 msec; Turns: 1
NORMALISED X-X' PHASE-SPACE
Time: 0 msec; Turns: 1
NORMALISED Y-Y' PHASE-SPACE
Time: 0 msec; Turns: 1
TRANSVERSE
CHARGE DENSITIES
Vertical
y (mm)
Horizontal
x (mm)

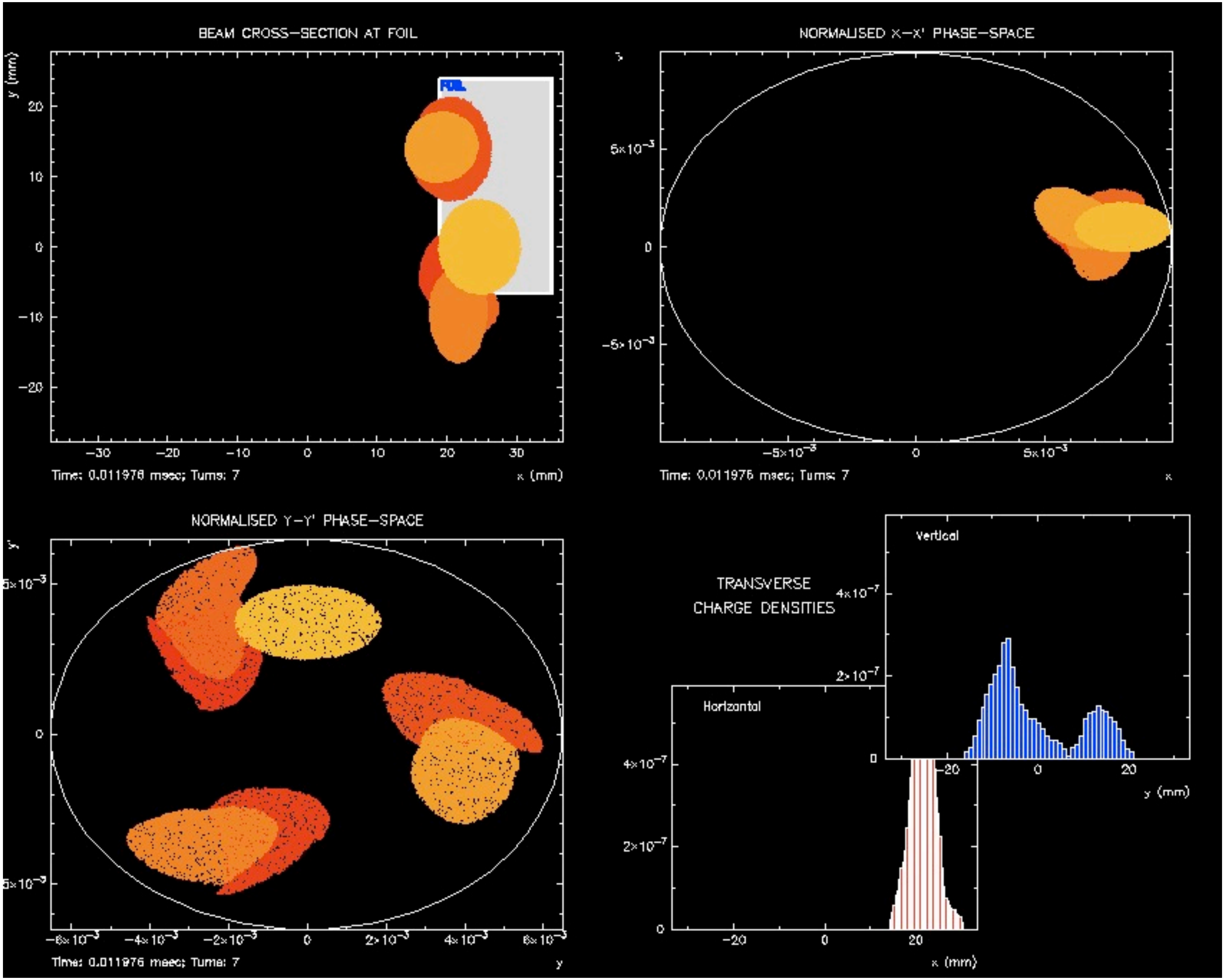

BEAM CROSS-SECTION AT FOIL
FOIL
y (mm)
x (mm)
Time: 1.996×10⁻³ msec; Turns: 2
NORMALISED X-X' PHASE-SPACE
Time: 1.996×10⁻³ msec; Turns: 2
NORMALISED Y-Y' PHASE-SPACE
Time: 1.996×10⁻³ msec; Turns: 2
TRANSVERSE
CHARGE DENSITIES
Vertical
Horizontal
y (mm)
x (mm)

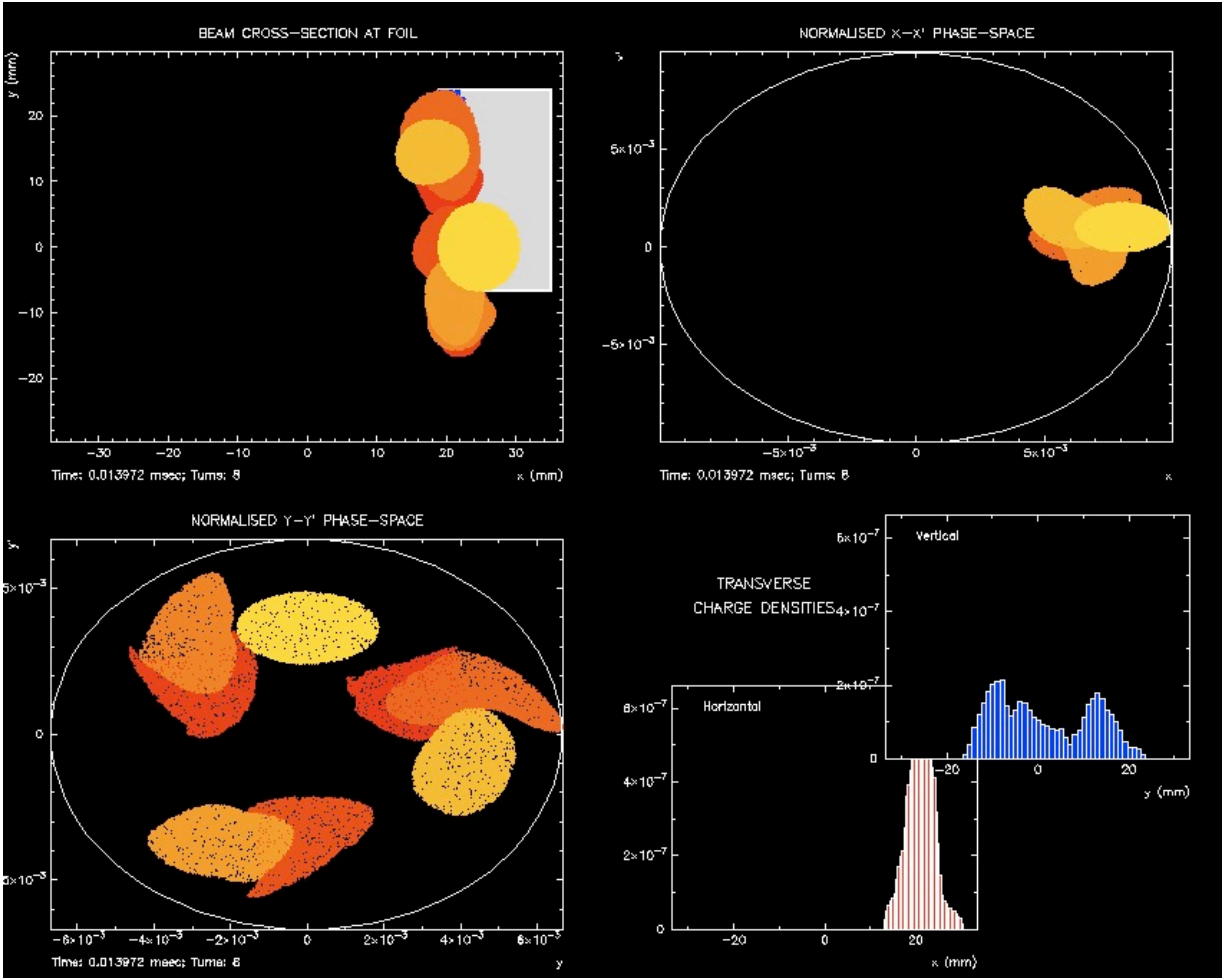

BEAM CROSS-SECTION AT FOIL
FOIL
y (mm)
x (mm)
Time: 3.992×10⁻³ msec; Turns: 3
NORMALISED X-X' PHASE-SPACE
NORMALISED Y-Y' PHASE-SPACE
TRANSVERSE
CHARGE DENSITIES
Vertical
Horizontal
y (mm)
x (mm)

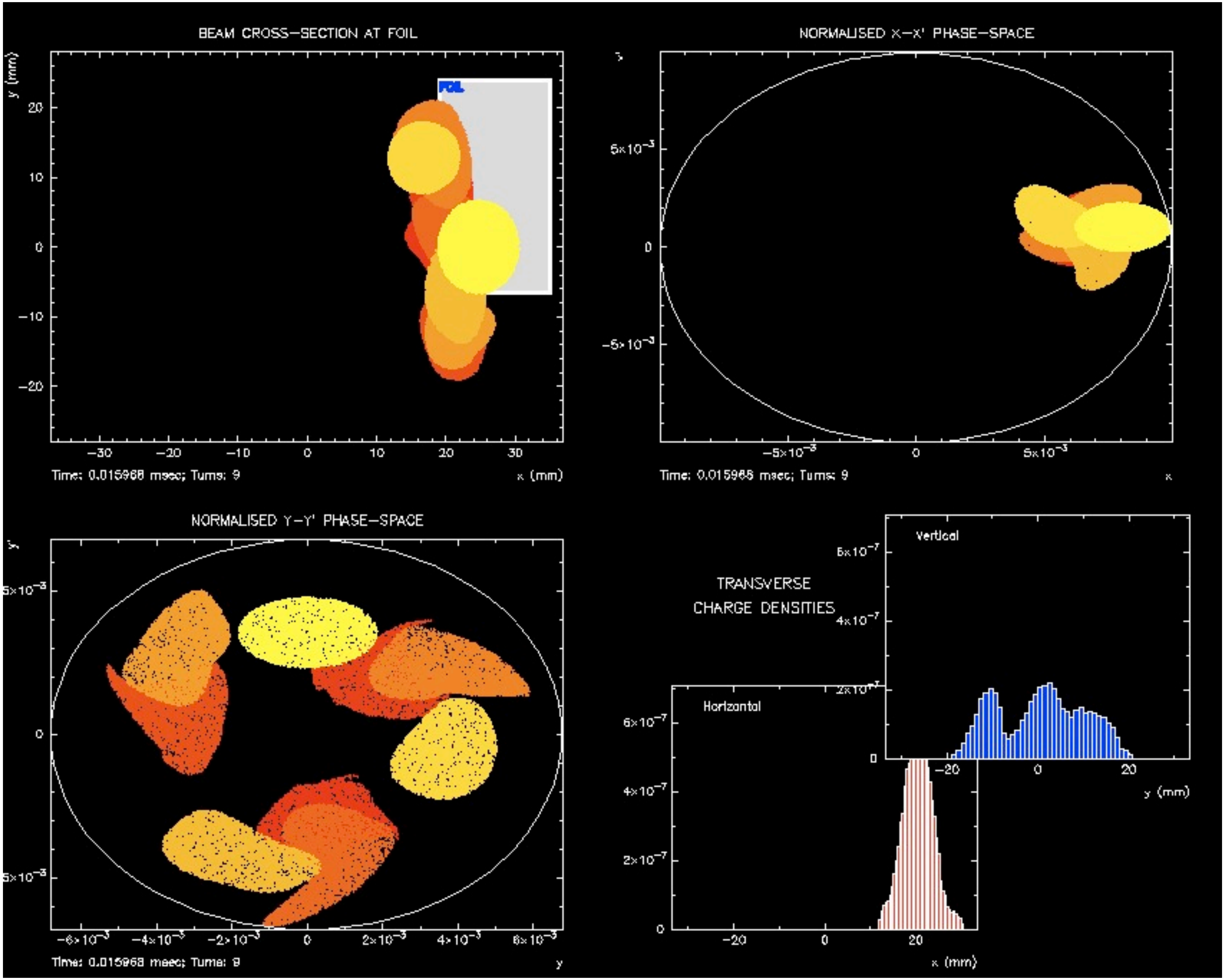

BEAM CROSS-SECTION AT FOIL
FOIL
y (mm)
x (mm)
Time: 5.988×10⁻³ msec; Turns: 4
NORMALISED X-X' PHASE-SPACE
Time: 5.988×10⁻³ msec; Turns: 4
NORMALISED Y-Y' PHASE-SPACE
Time: 5.988×10⁻³ msec; Turns: 4
TRANSVERSE
CHARGE DENSITIES
Vertical
y (mm)
Horizontal
x (mm)


BEAM CROSS-SECTION AT FOIL
FOIL
y (mm)
x (mm)
Time: 7.984×10⁻³ msec; Turns: 5
NORMALISED X-X' PHASE-SPACE
Time: 7.984×10⁻³ msec; Turns: 5
NORMALISED Y-Y' PHASE-SPACE
Time: 7.984×10⁻³ msec; Turns: 5
TRANSVERSE
CHARGE DENSITIES
Vertical
Horizontal
y (mm)
x (mm)

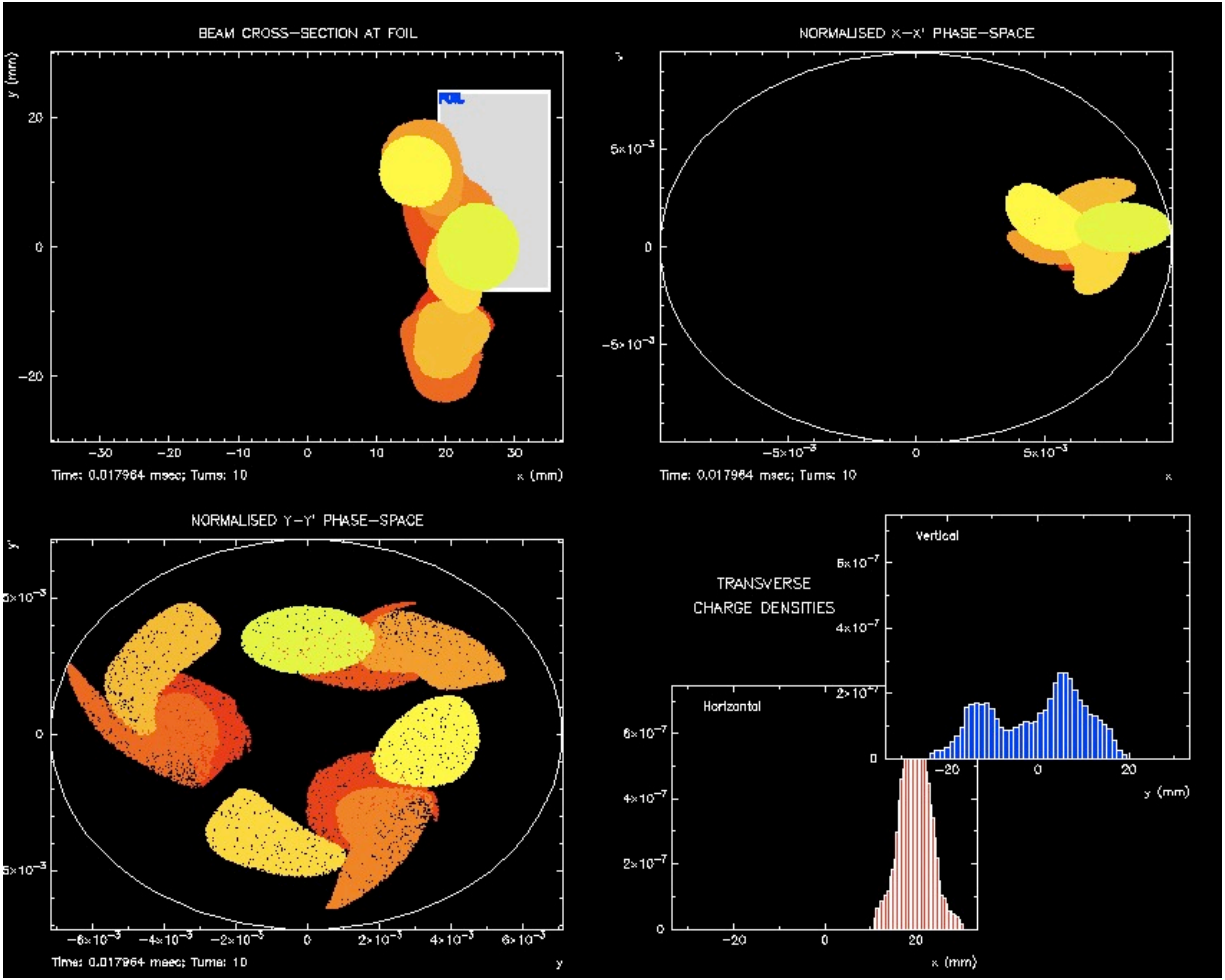

BEAM CROSS-SECTION AT FOIL
FOIL
y (mm)
x (mm)
Time: 9.98×10^-3 msec; Turns: 6
NORMALISED X-X' PHASE-SPACE
Time: 9.98×10^-3 msec; Turns: 6
NORMALISED Y-Y' PHASE-SPACE
Time: 9.98×10^-3 msec; Turns: 6
TRANSVERSE
CHARGE DENSITIES
Vertical
y (mm)
Horizontal
x (mm)

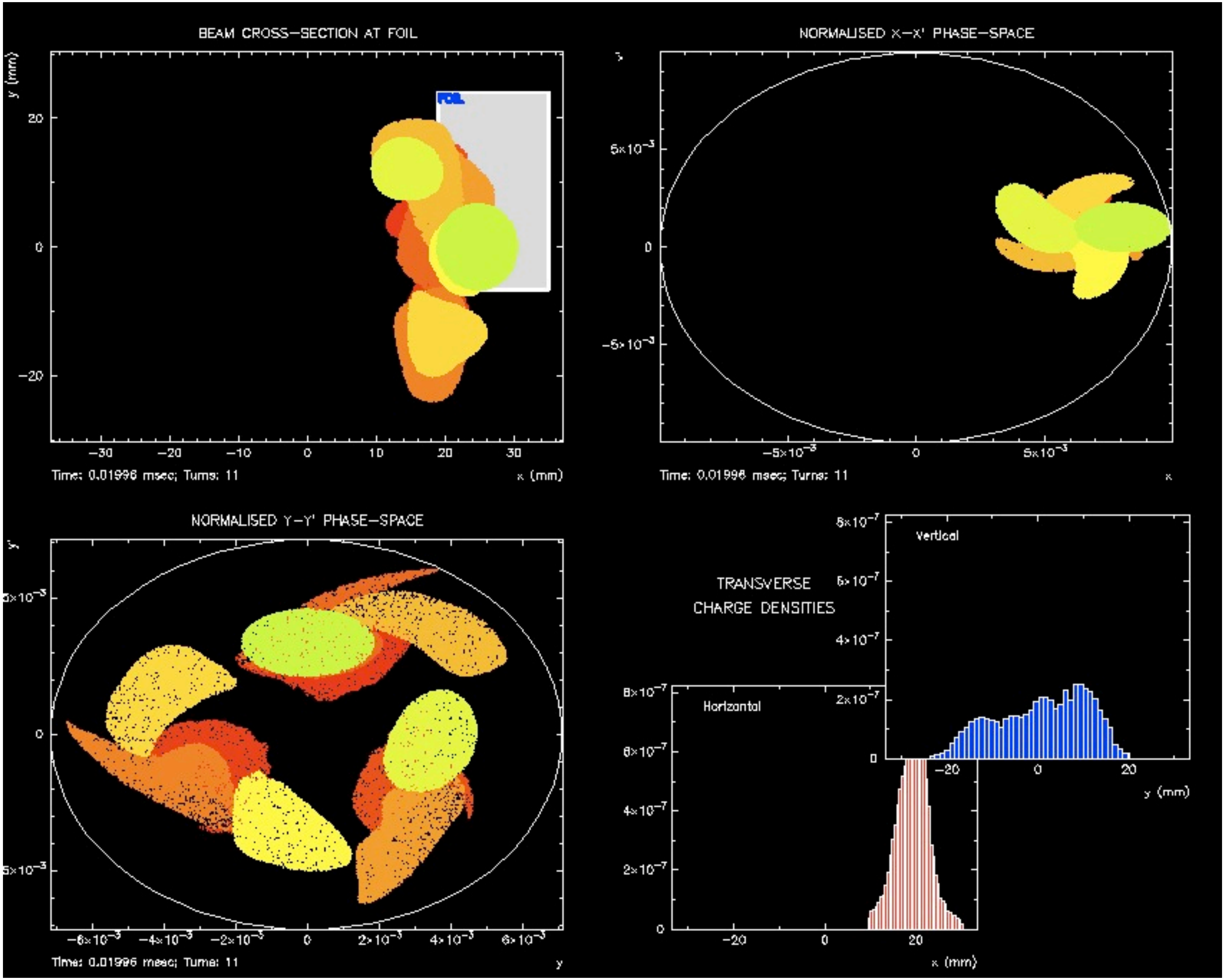

BEAM CROSS-SECTION AT FOIL
FOIL
y (mm)
x (mm)
Time: 0.011976 msec; Turns: 7
NORMALISED X-X' PHASE-SPACE
x
Time: 0.011976 msec; Turns: 7
NORMALISED Y-Y' PHASE-SPACE
y
Time: 0.011976 msec; Turns: 7
TRANSVERSE
CHARGE DENSITIES
Vertical
y (mm)
Horizontal
x (mm)


BEAM CROSS-SECTION AT FOIL
y (mm)
x (mm)
Time: 0.013972 msec; Turns: 8
NORMALISED X-X' PHASE-SPACE
Time: 0.013972 msec; Turns: 8
NORMALISED Y-Y' PHASE-SPACE
Time: 0.013972 msec; Turns: 8
TRANSVERSE
CHARGE DENSITIES
Vertical
Horizontal
y (mm)
x (mm)

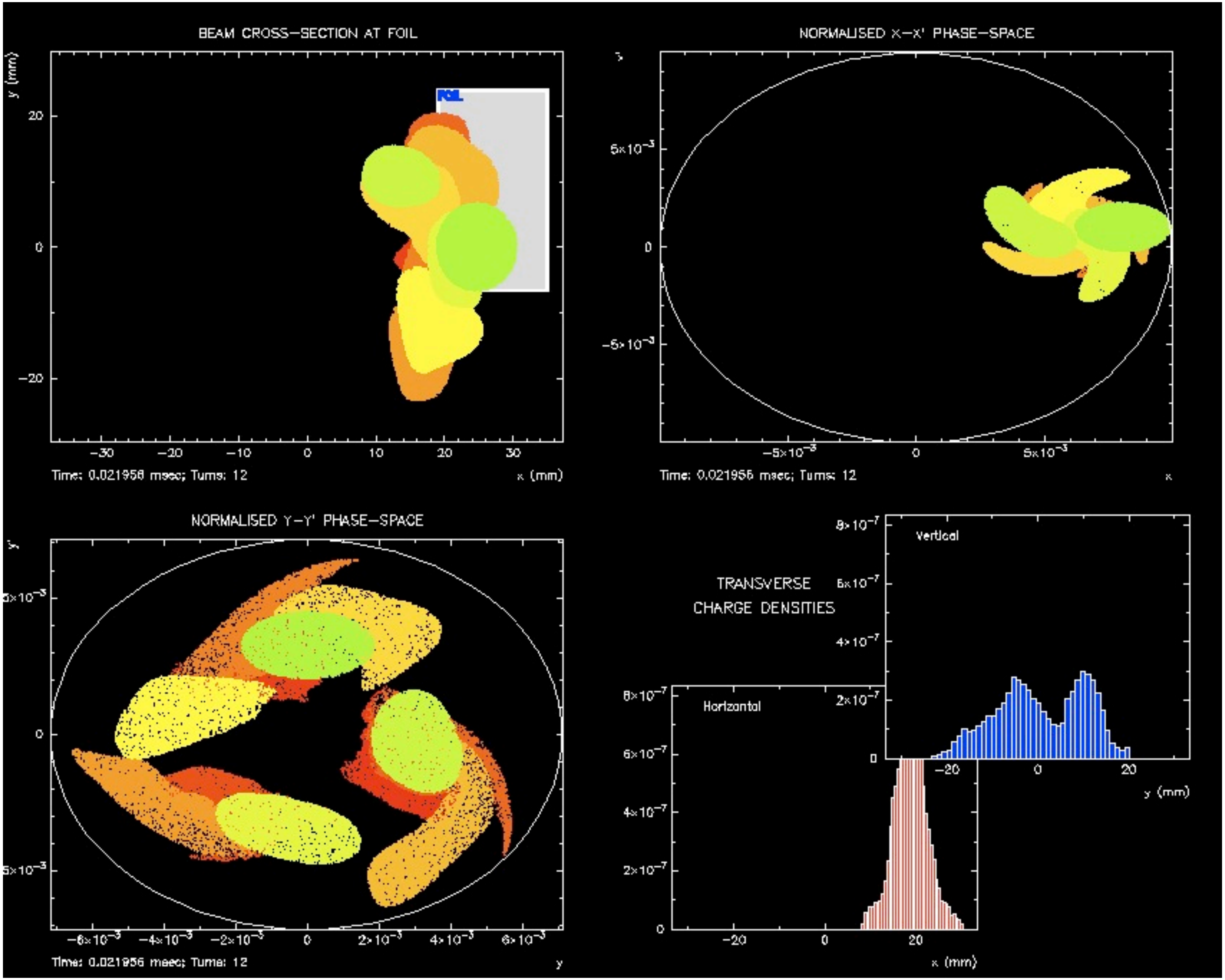


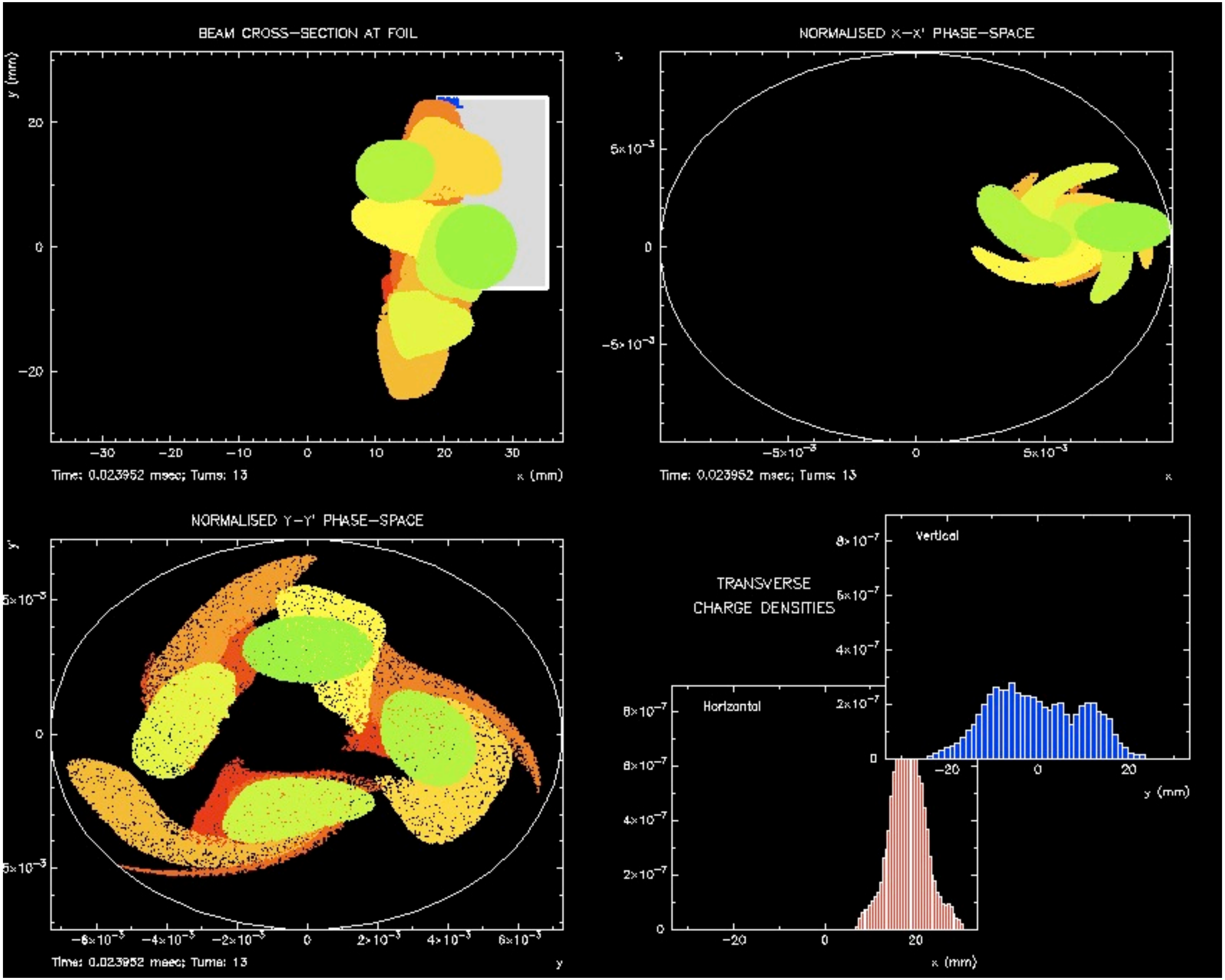
BEAM CROSS-SECTION AT FOIL
y (mm)
FOIL
x (mm)
Time: 0.015968 msec; Turns: 9
NORMALISED X-X' PHASE-SPACE
x
Time: 0.015968 msec; Turns: 9
NORMALISED Y-Y' PHASE-SPACE
y
Time: 0.015968 msec; Turns: 9
TRANSVERSE
CHARGE DENSITIES
Vertical
y (mm)
Horizontal
x (mm)

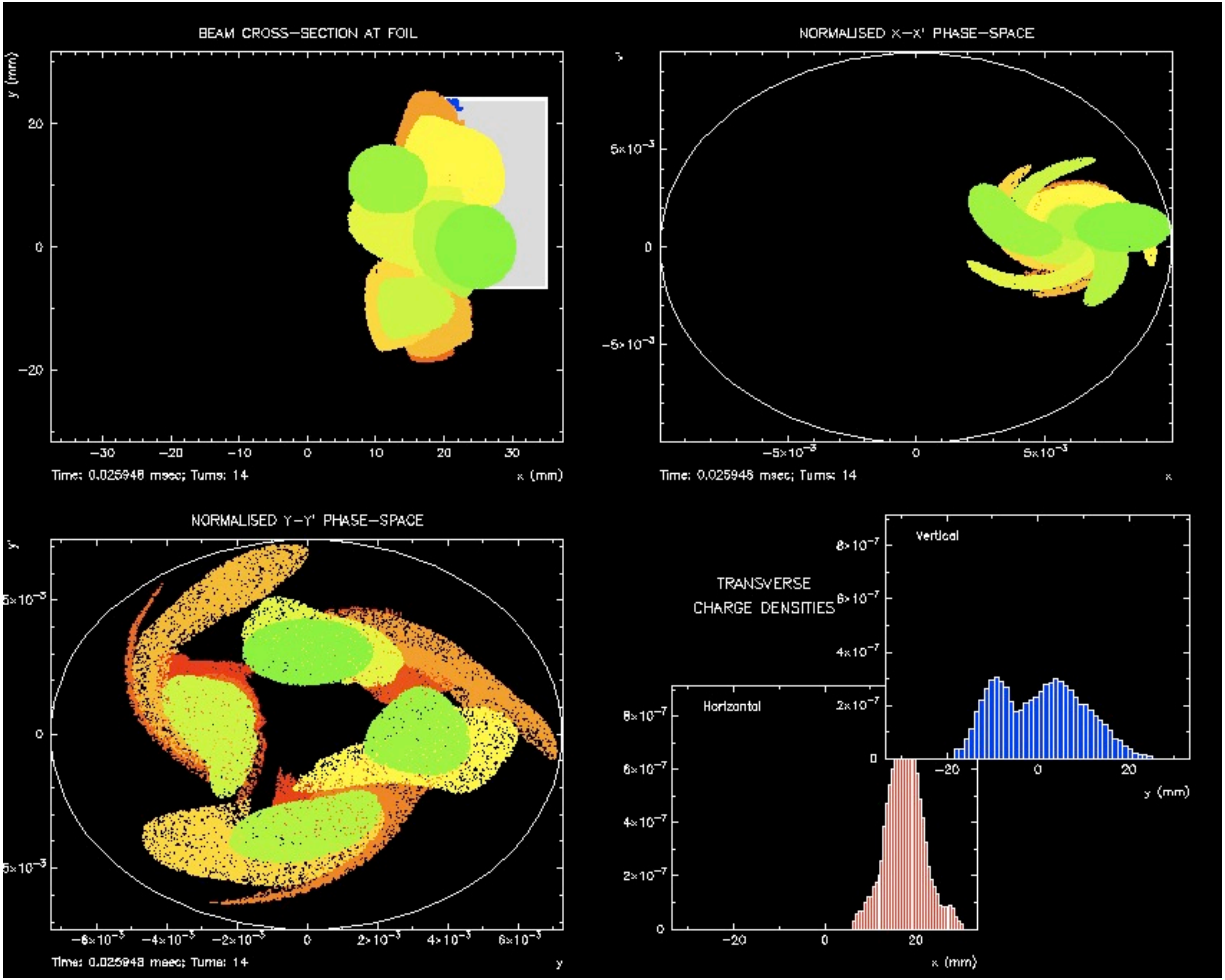

BEAM CROSS-SECTION AT FOIL
FOIL
y (mm)
x (mm)
Time: 0.017964 msec; Turns: 10
NORMALISED X-X' PHASE-SPACE
Time: 0.017964 msec; Turns: 10
NORMALISED Y-Y' PHASE-SPACE
Time: 0.017964 msec; Turns: 10
TRANSVERSE
CHARGE DENSITIES
Vertical
y (mm)
Horizontal
x (mm)

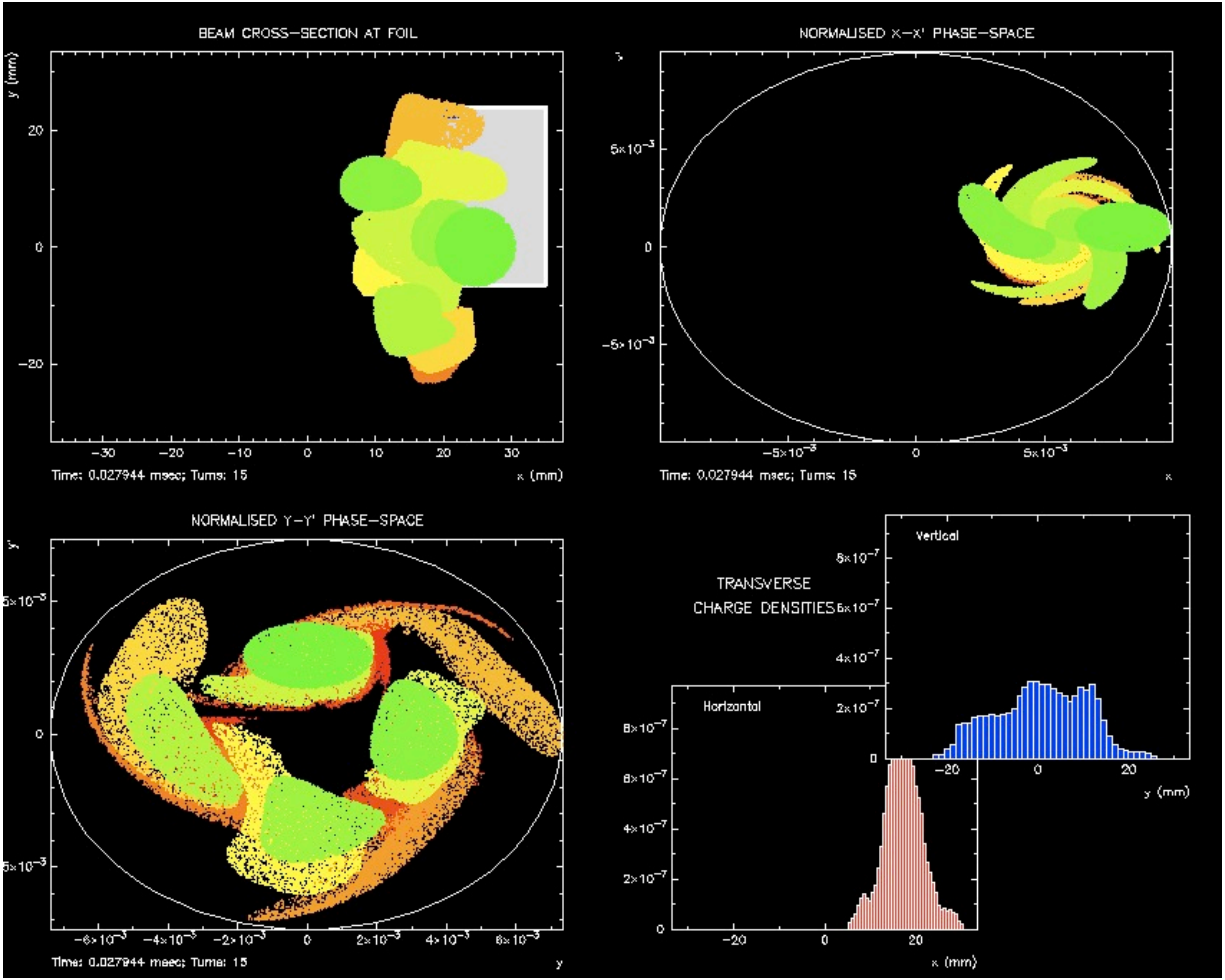


BEAM CROSS-SECTION AT FOIL
FOIL
y (mm)
x (mm)
Time: 0.01996 msec; Turns: 11
NORMALISED X-X' PHASE-SPACE
Time: 0.01996 msec; Turns: 11
NORMALISED Y-Y' PHASE-SPACE
Time: 0.01996 msec; Turns: 11
TRANSVERSE
CHARGE DENSITIES
Vertical
y (mm)
Horizontal
x (mm)

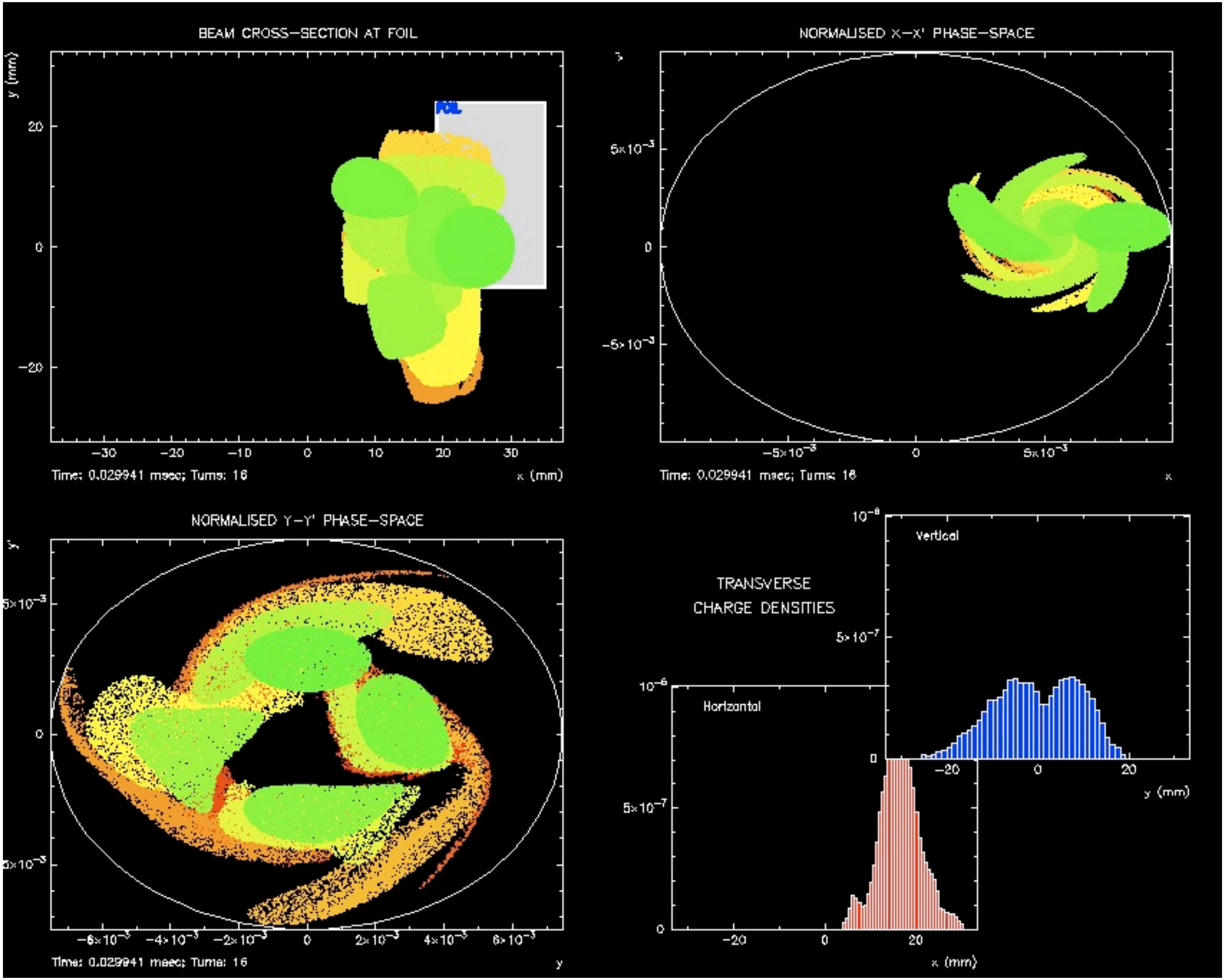

BEAM CROSS-SECTION AT FOIL
y (mm)
FOIL
x (mm)
Time: 0.021956 msec; Turns: 12
NORMALISED X-X' PHASE-SPACE
Time: 0.021956 msec; Turns: 12
NORMALISED Y-Y' PHASE-SPACE
Time: 0.021956 msec; Turns: 12
TRANSVERSE
CHARGE DENSITIES
Vertical
y (mm)
Horizontal
x (mm)


BEAM CROSS-SECTION AT FOIL
y (mm)
x (mm)
Time: 0.023952 msec; Turns: 13
NORMALISED X-X' PHASE-SPACE
Time: 0.023952 msec; Turns: 13
NORMALISED Y-Y' PHASE-SPACE
Time: 0.023952 msec; Turns: 13
TRANSVERSE
CHARGE DENSITIES
Vertical
y (mm)
Horizontal
x (mm)

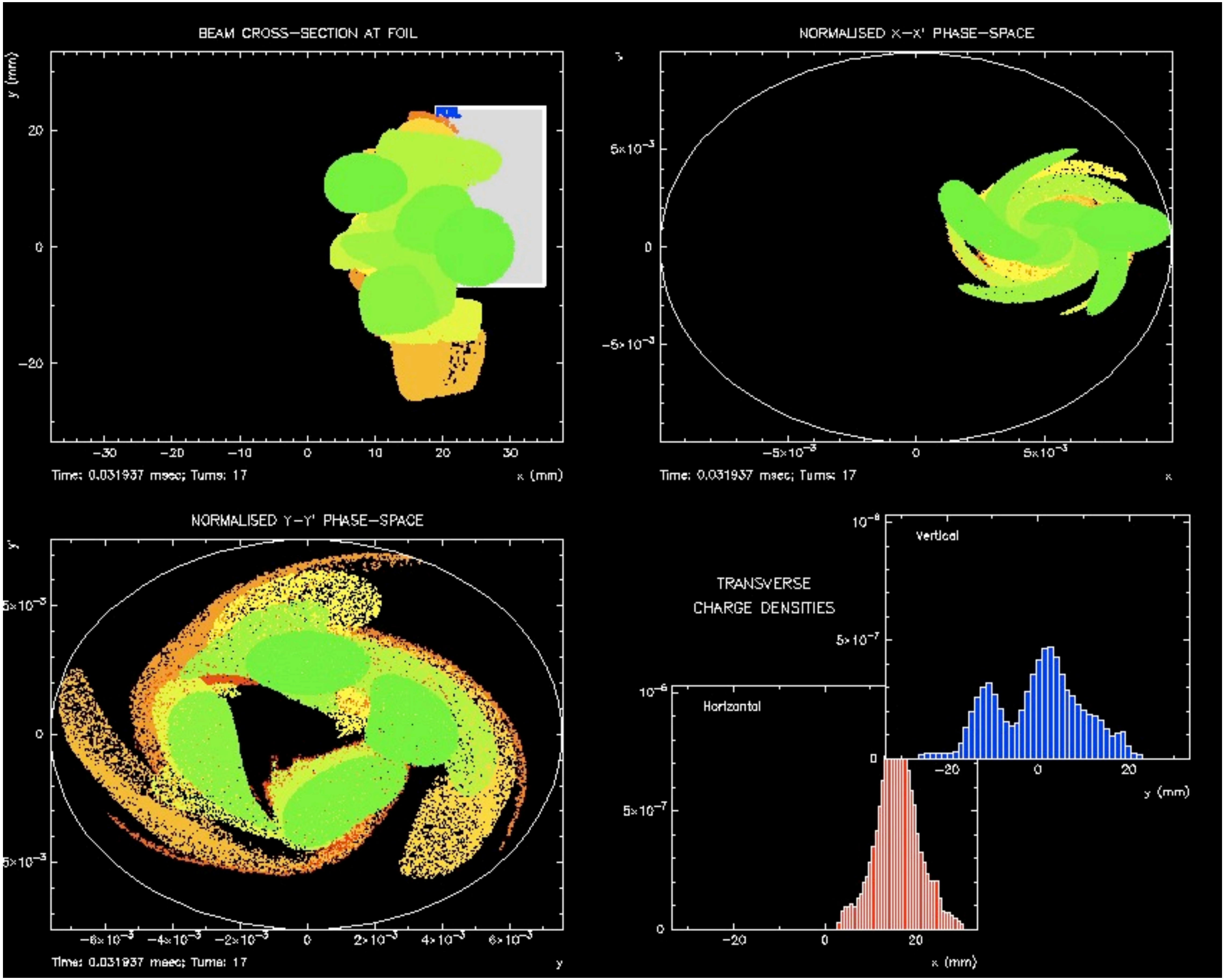

BEAM CROSS-SECTION AT FOIL
Time: 0.025948 msec; Turns: 14
NORMALISED X-X' PHASE-SPACE
Time: 0.025948 msec; Turns: 14
NORMALISED Y-Y' PHASE-SPACE
Time: 0.025948 msec; Turns: 14
TRANSVERSE
CHARGE DENSITIES
Vertical
Horizontal
x (mm)
y (mm)



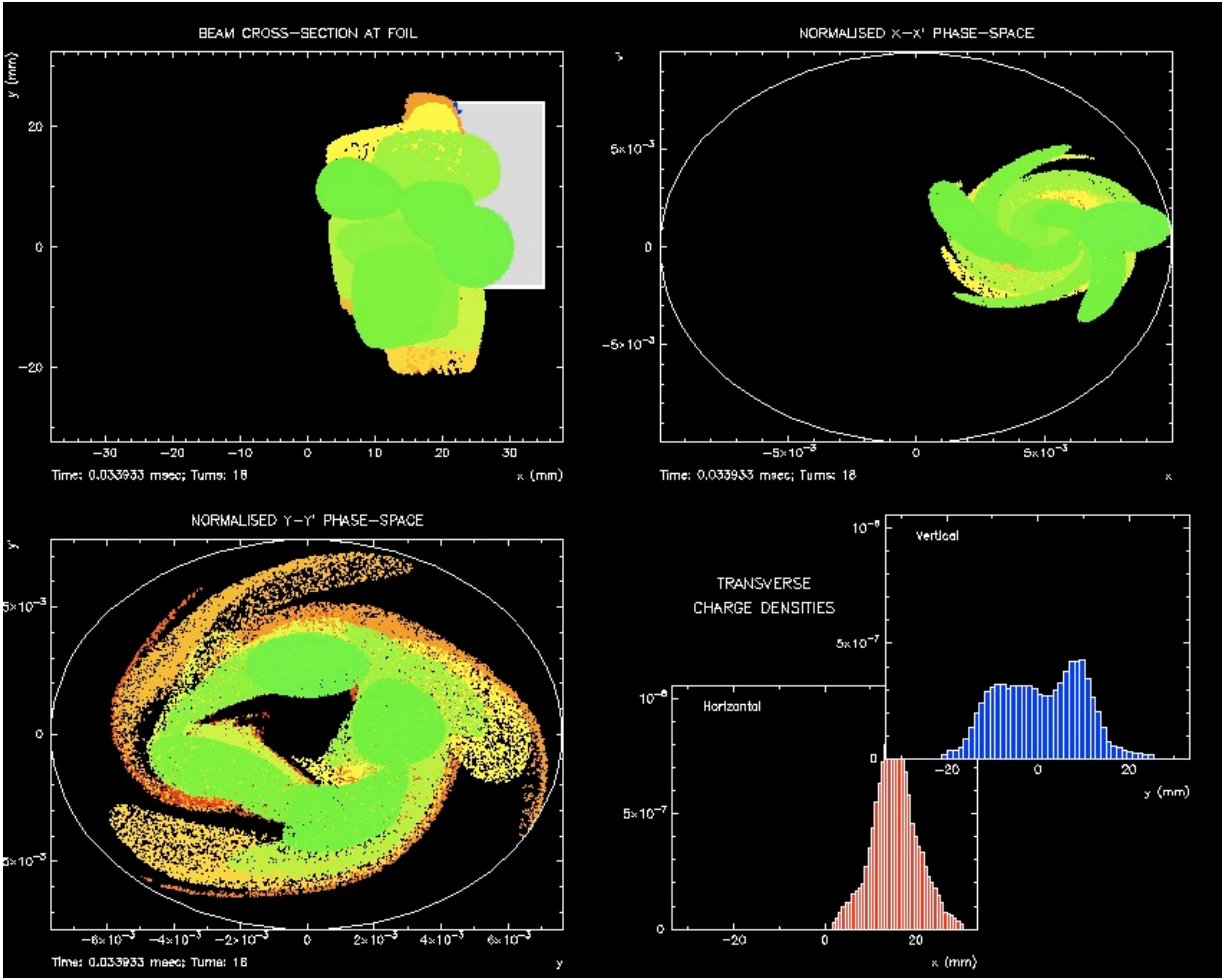
BEAM CROSS-SECTION AT FOIL
y (mm)
x (mm)
Time: 0.027944 msec; Turns: 15
NORMALISED X-X' PHASE-SPACE
x
Time: 0.027944 msec; Turns: 15
NORMALISED Y-Y' PHASE-SPACE
y
Time: 0.027944 msec; Turns: 15
TRANSVERSE
CHARGE DENSITIES
Vertical
y (mm)
Horizontal
x (mm)

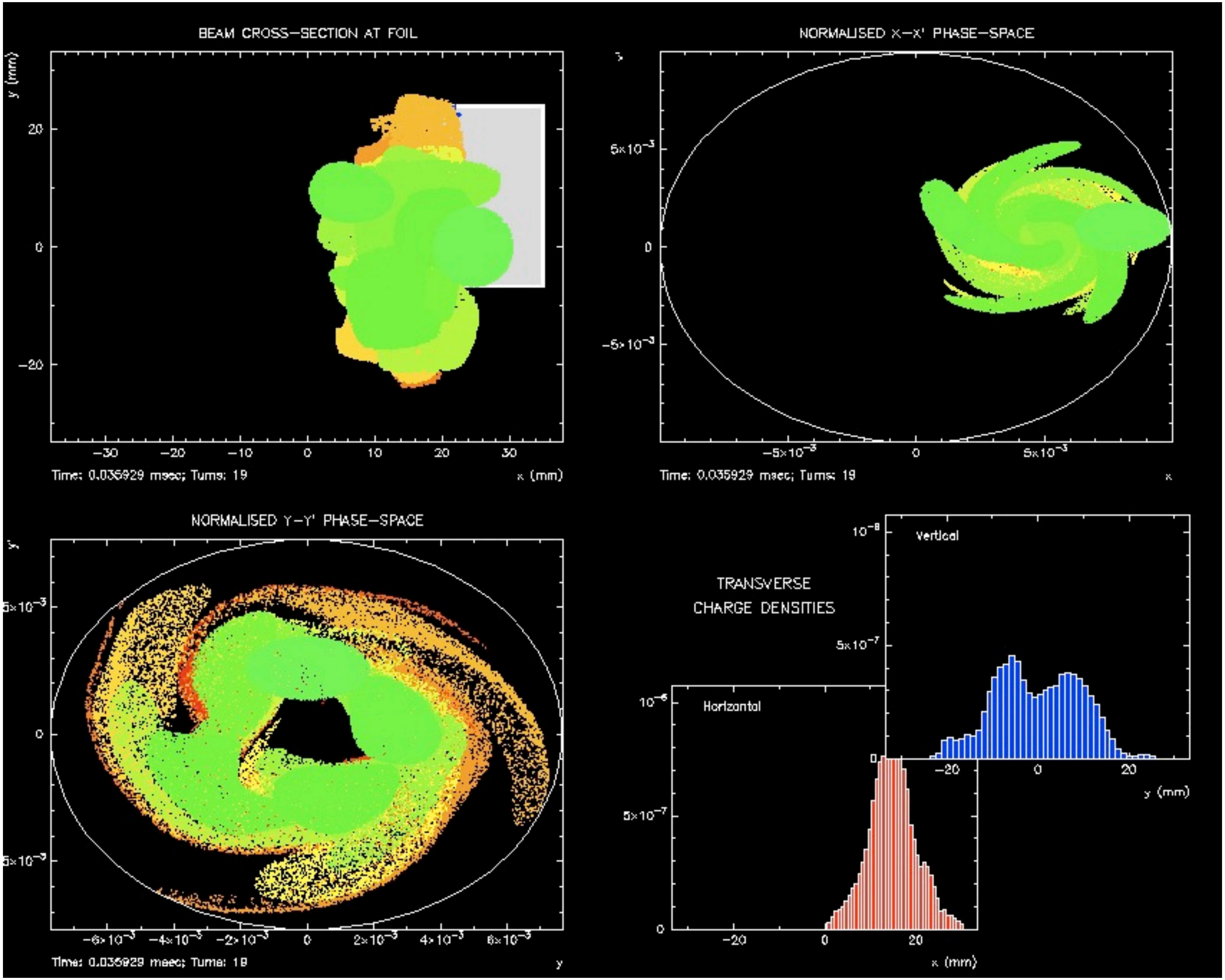


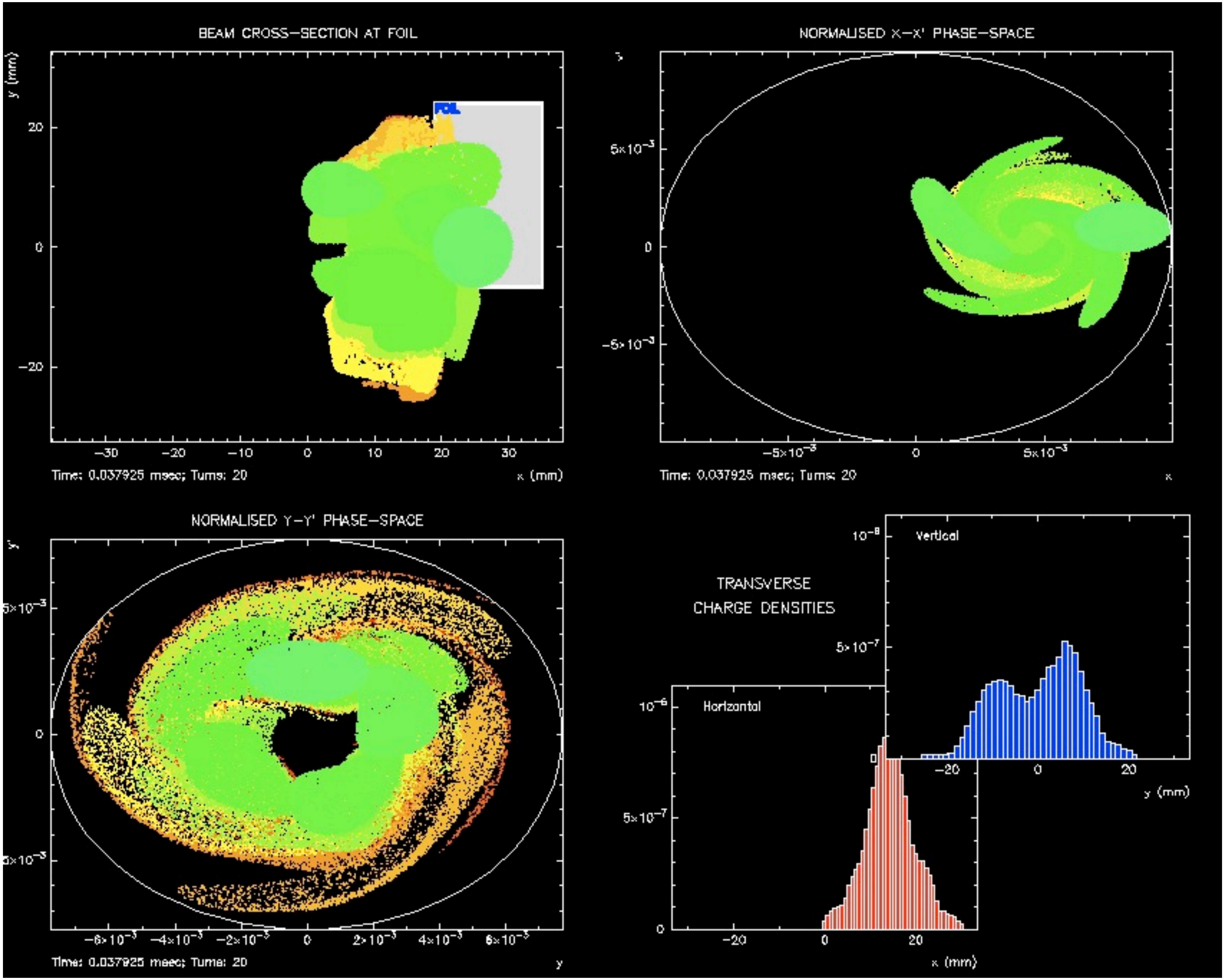
BEAM CROSS-SECTION AT FOIL
FOIL
y (mm)
x (mm)
Time: 0.029941 msec; Turns: 16
NORMALISED X-X' PHASE-SPACE
Time: 0.029941 msec; Turns: 16
NORMALISED Y-Y' PHASE-SPACE
Time: 0.029941 msec; Turns: 16
TRANSVERSE
CHARGE DENSITIES
Vertical
y (mm)
Horizontal
x (mm)

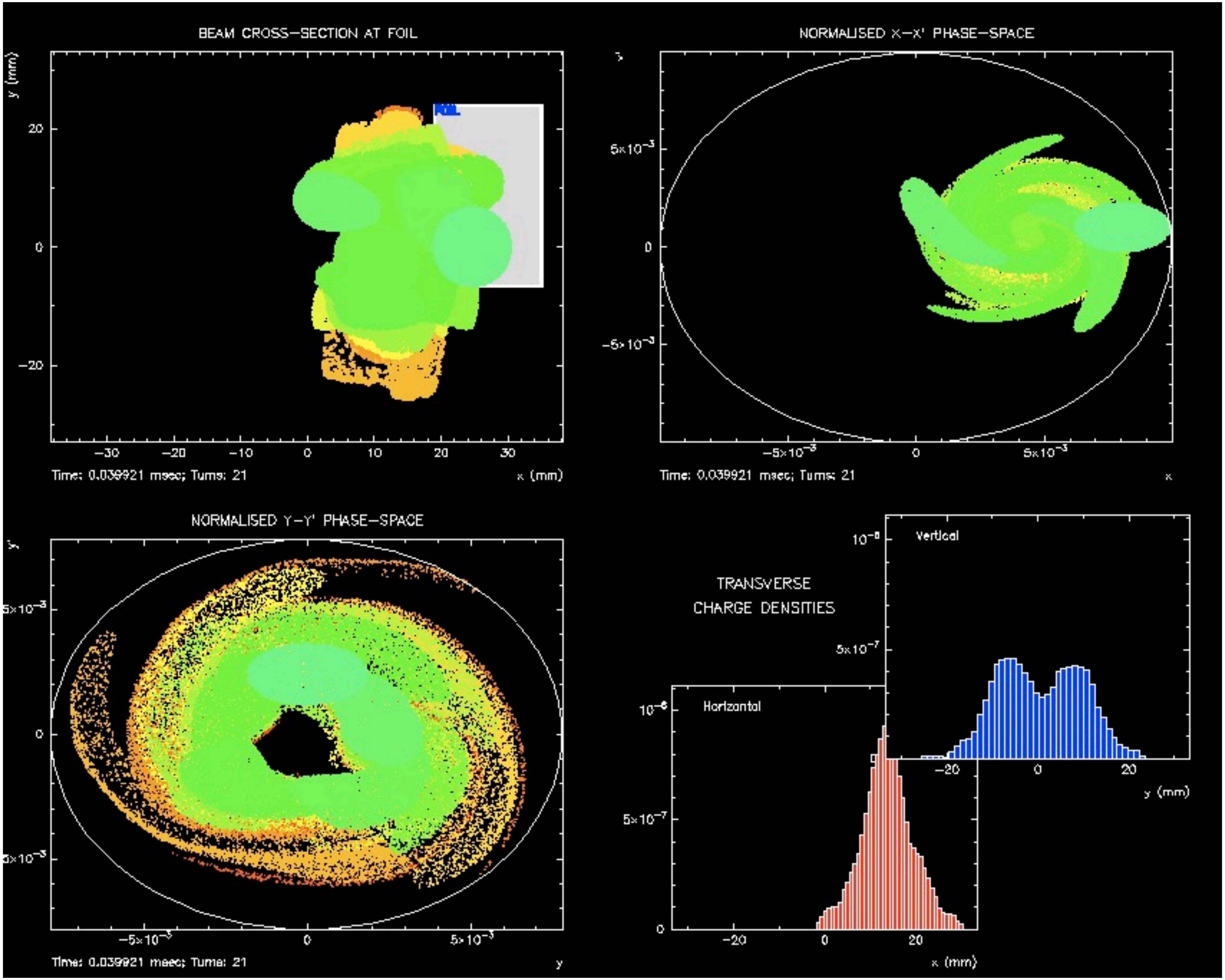


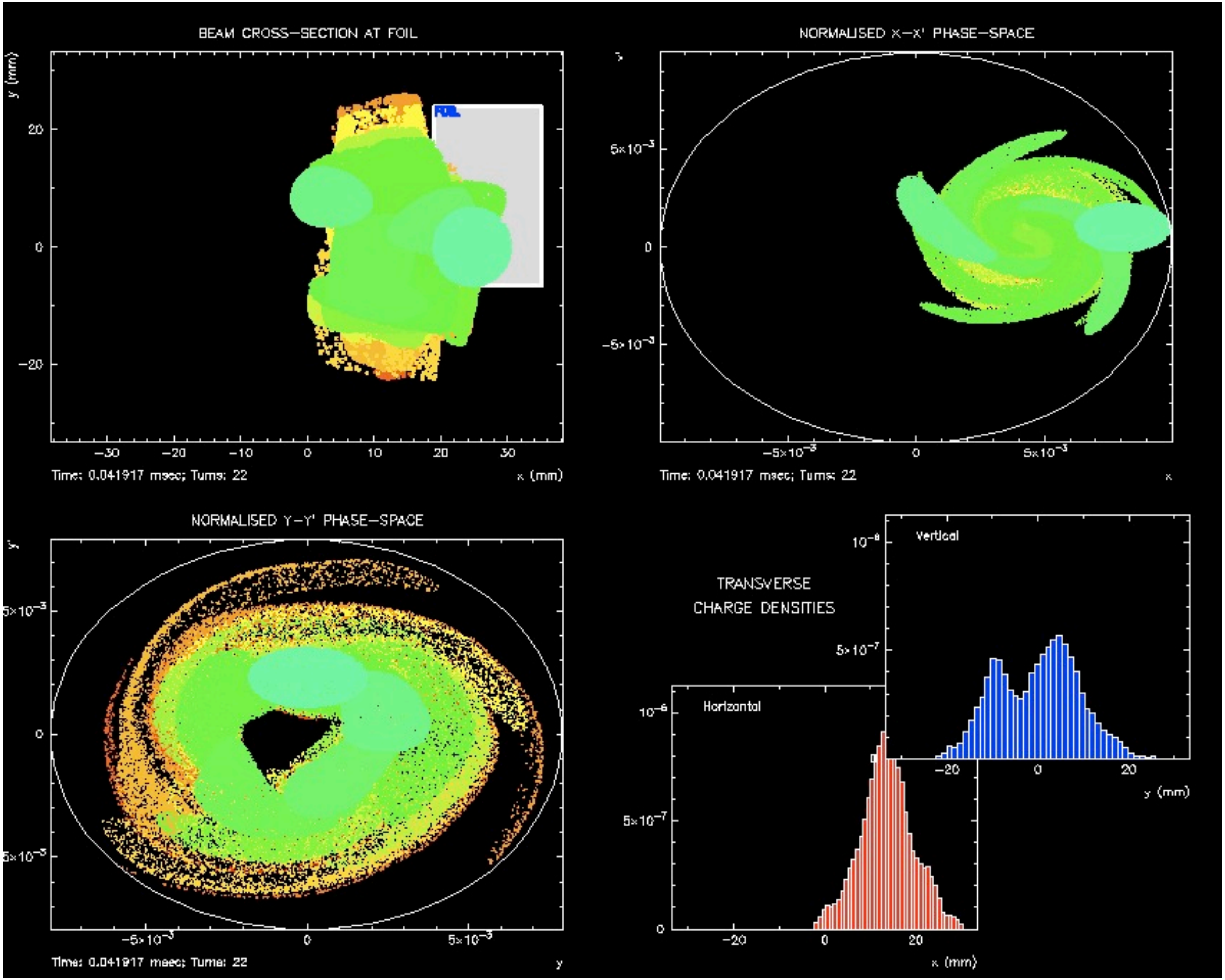
BEAM CROSS-SECTION AT FOIL
y (mm)
x (mm)
Time: 0.031937 msec; Turns: 17
NORMALISED X-X' PHASE-SPACE
x
Time: 0.031937 msec; Turns: 17
NORMALISED Y-Y' PHASE-SPACE
y
Time: 0.031937 msec; Turns: 17
TRANSVERSE
CHARGE DENSITIES
Vertical
y (mm)
Horizontal
x (mm)


BEAM CROSS-SECTION AT FOIL
y (mm)
x (mm)
Time: 0.033933 msec; Turns: 18
NORMALISED X-X' PHASE-SPACE
Time: 0.033933 msec; Turns: 18
NORMALISED Y-Y' PHASE-SPACE
Time: 0.033933 msec; Turns: 18
TRANSVERSE
CHARGE DENSITIES
Vertical
y (mm)
Horizontal
x (mm)

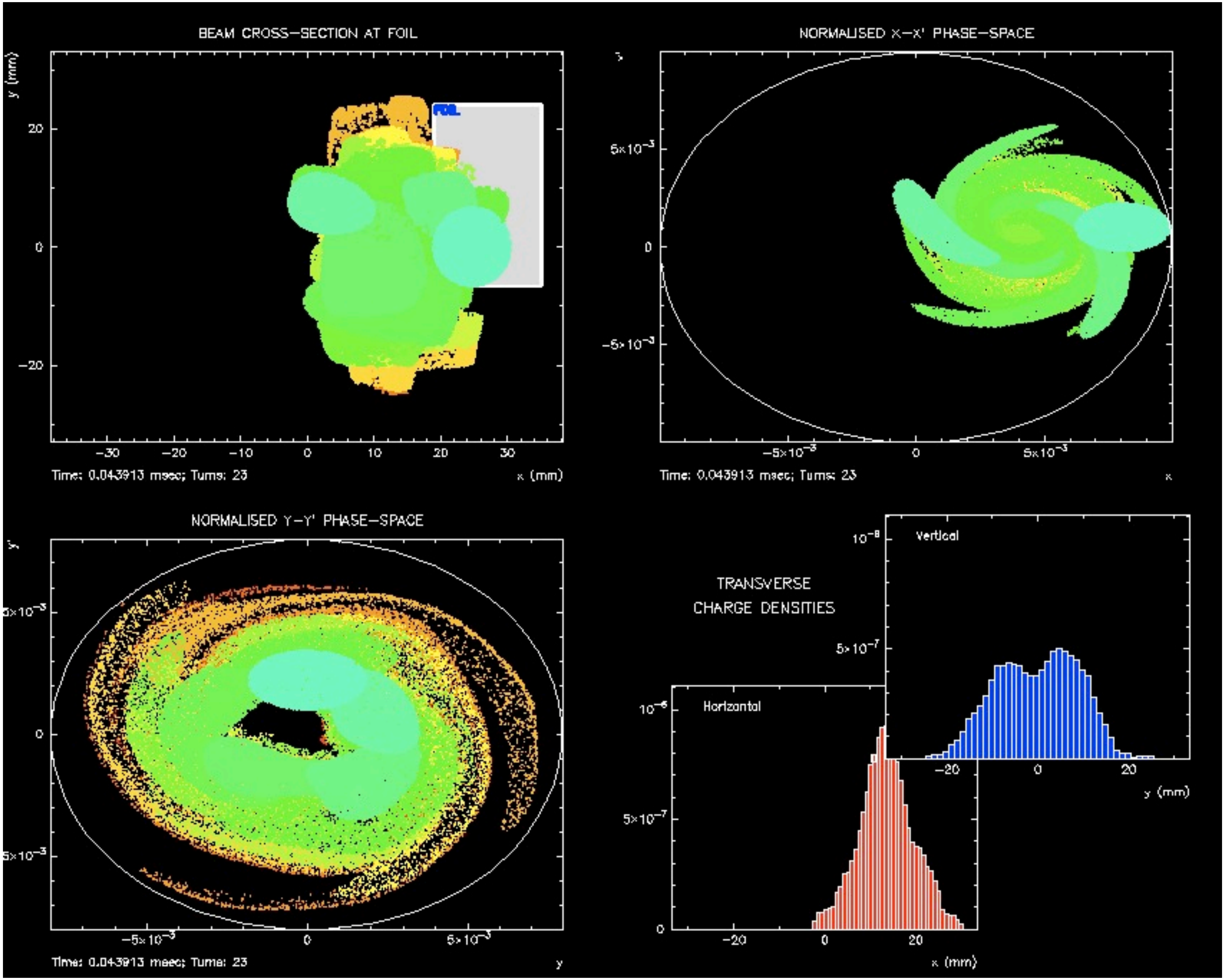

BEAM CROSS-SECTION AT FOIL
y (mm)
x (mm)
Time: 0.035929 msec; Turns: 19
NORMALISED X-X' PHASE-SPACE
Time: 0.035929 msec; Turns: 19
NORMALISED Y-Y' PHASE-SPACE
Time: 0.035929 msec; Turns: 19
TRANSVERSE
CHARGE DENSITIES
Vertical
y (mm)
Horizontal
x (mm)

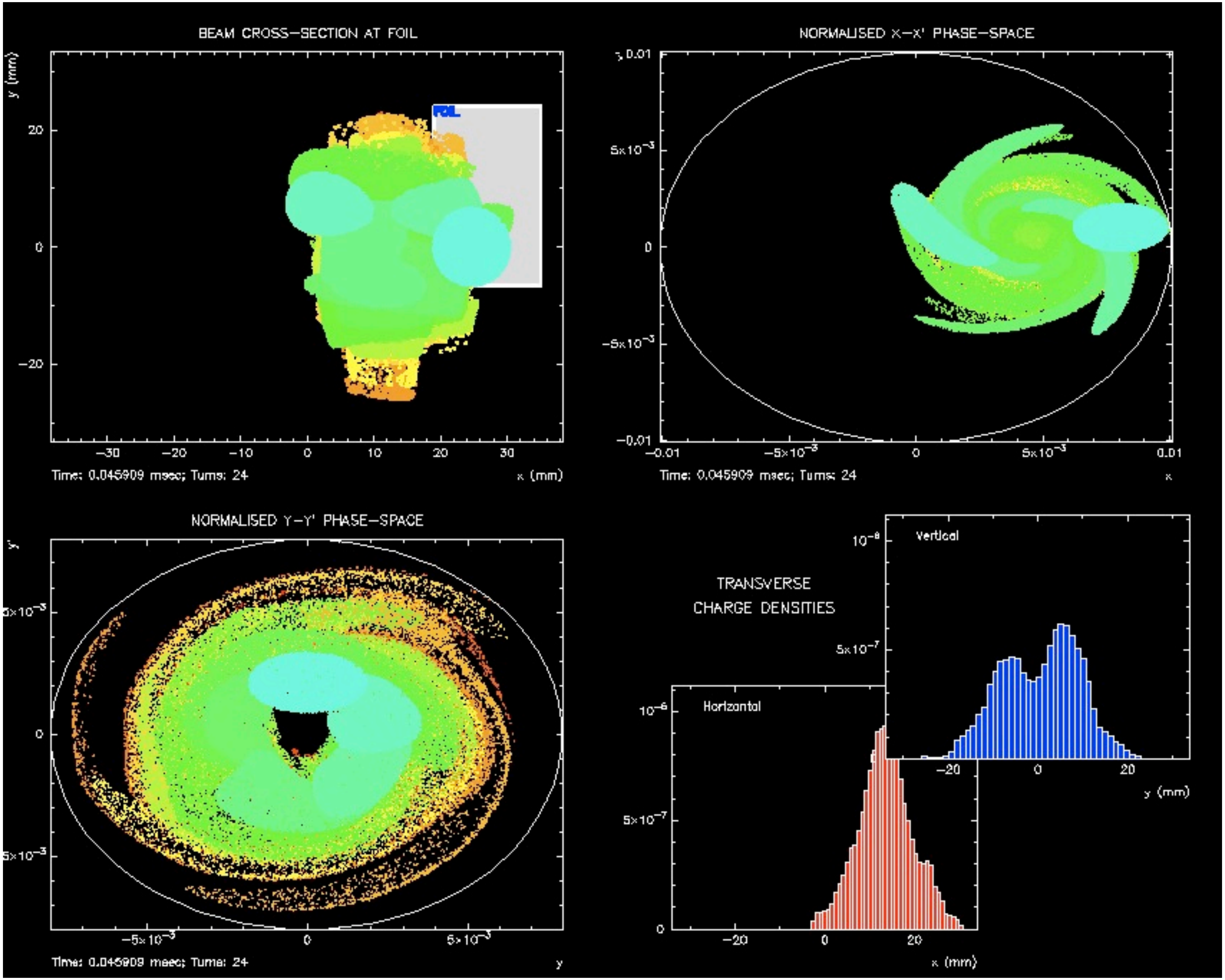

BEAM CROSS-SECTION AT FOIL
FOIL
y (mm)
x (mm)
Time: 0.037925 msec; Turns: 20
NORMALISED X-X' PHASE-SPACE
Time: 0.037925 msec; Turns: 20
NORMALISED Y-Y' PHASE-SPACE
Time: 0.037925 msec; Turns: 20
TRANSVERSE
CHARGE DENSITIES
Vertical
Horizontal
y (mm)
x (mm)

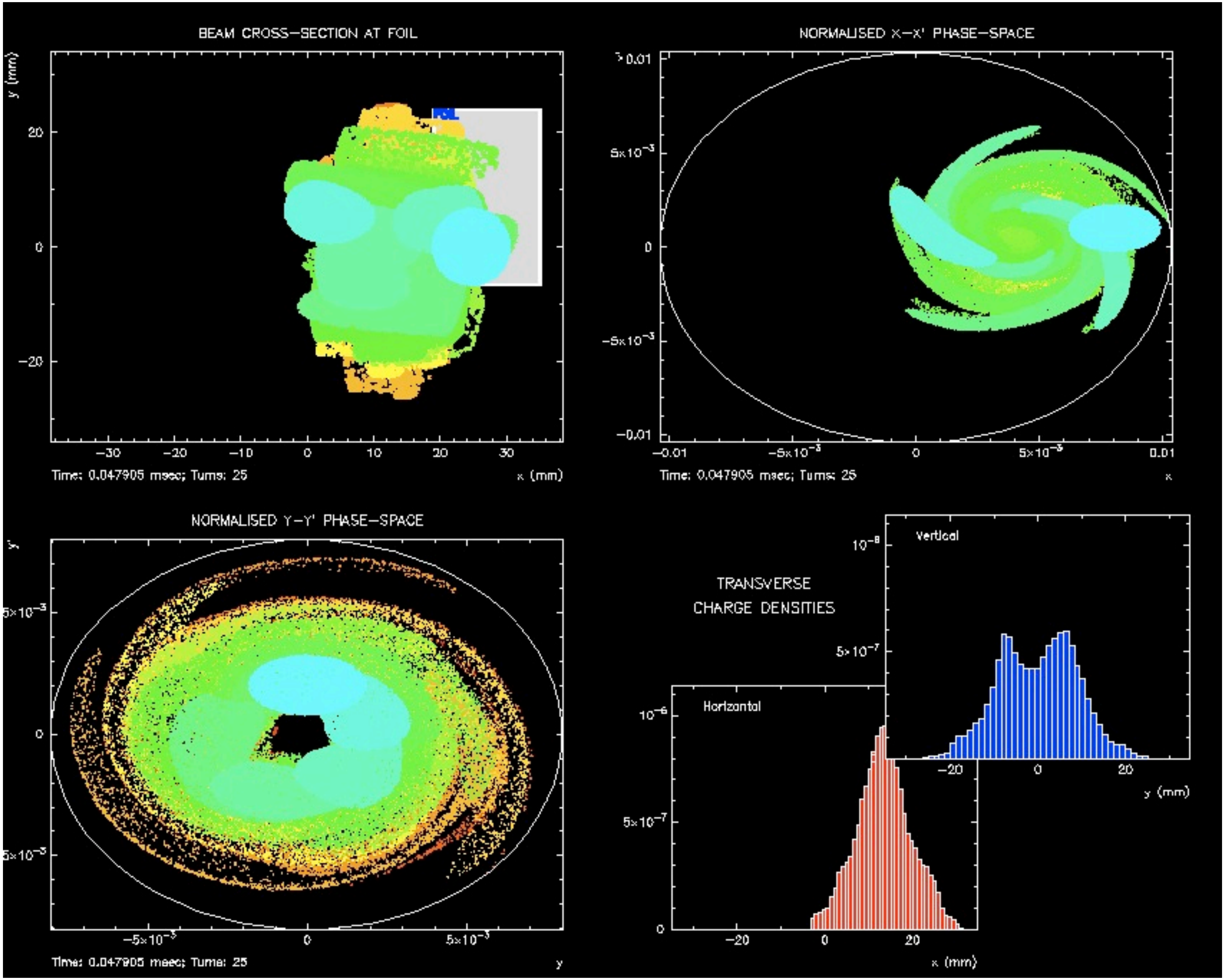

BEAM CROSS-SECTION AT FOIL
y (mm)
x (mm)
Time: 0.039921 msec; Turns: 21
NORMALISED X-X' PHASE-SPACE
Time: 0.039921 msec; Turns: 21
NORMALISED Y-Y' PHASE-SPACE
Time: 0.039921 msec; Turns: 21
TRANSVERSE
CHARGE DENSITIES
Vertical
y (mm)
Horizontal
x (mm)

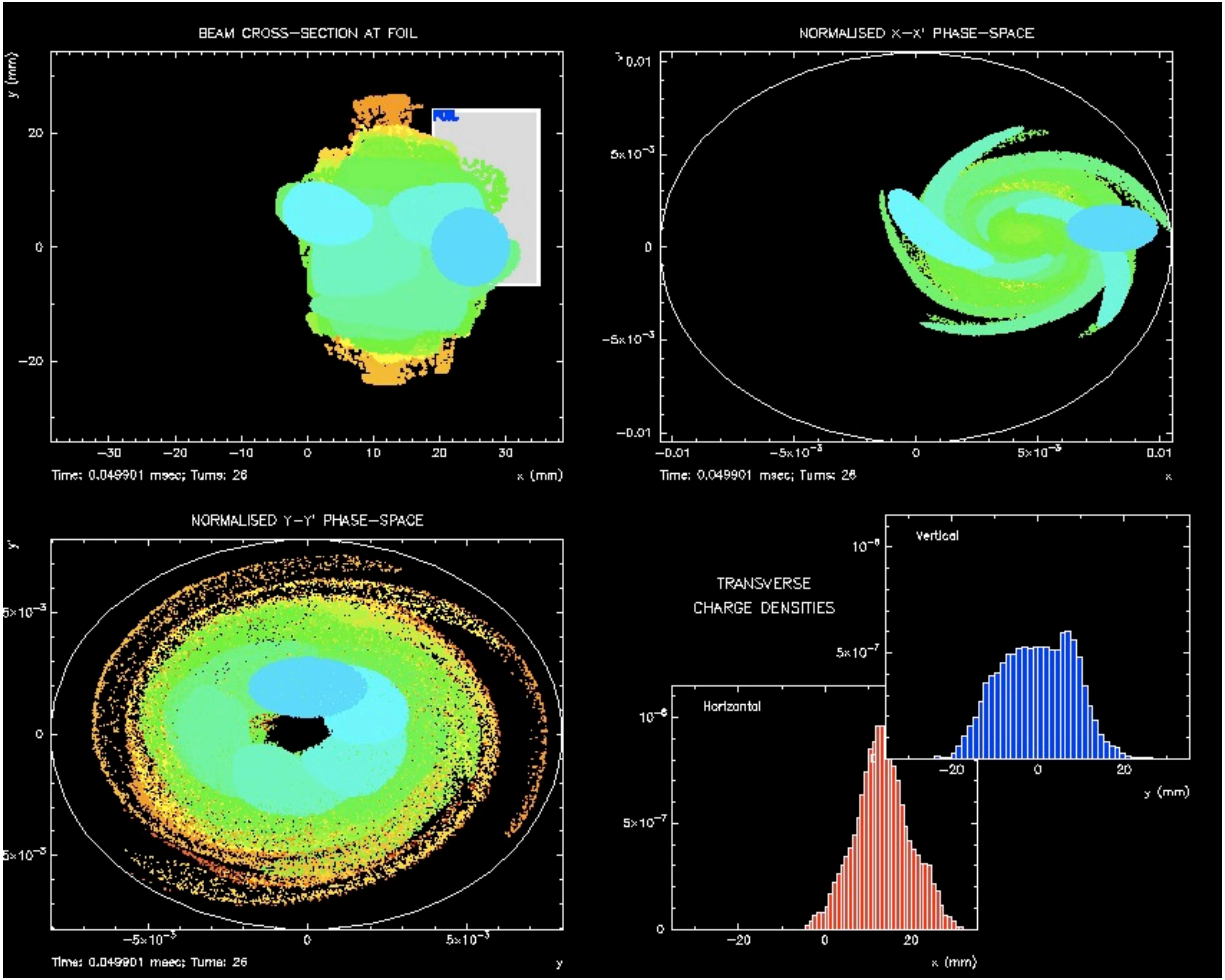


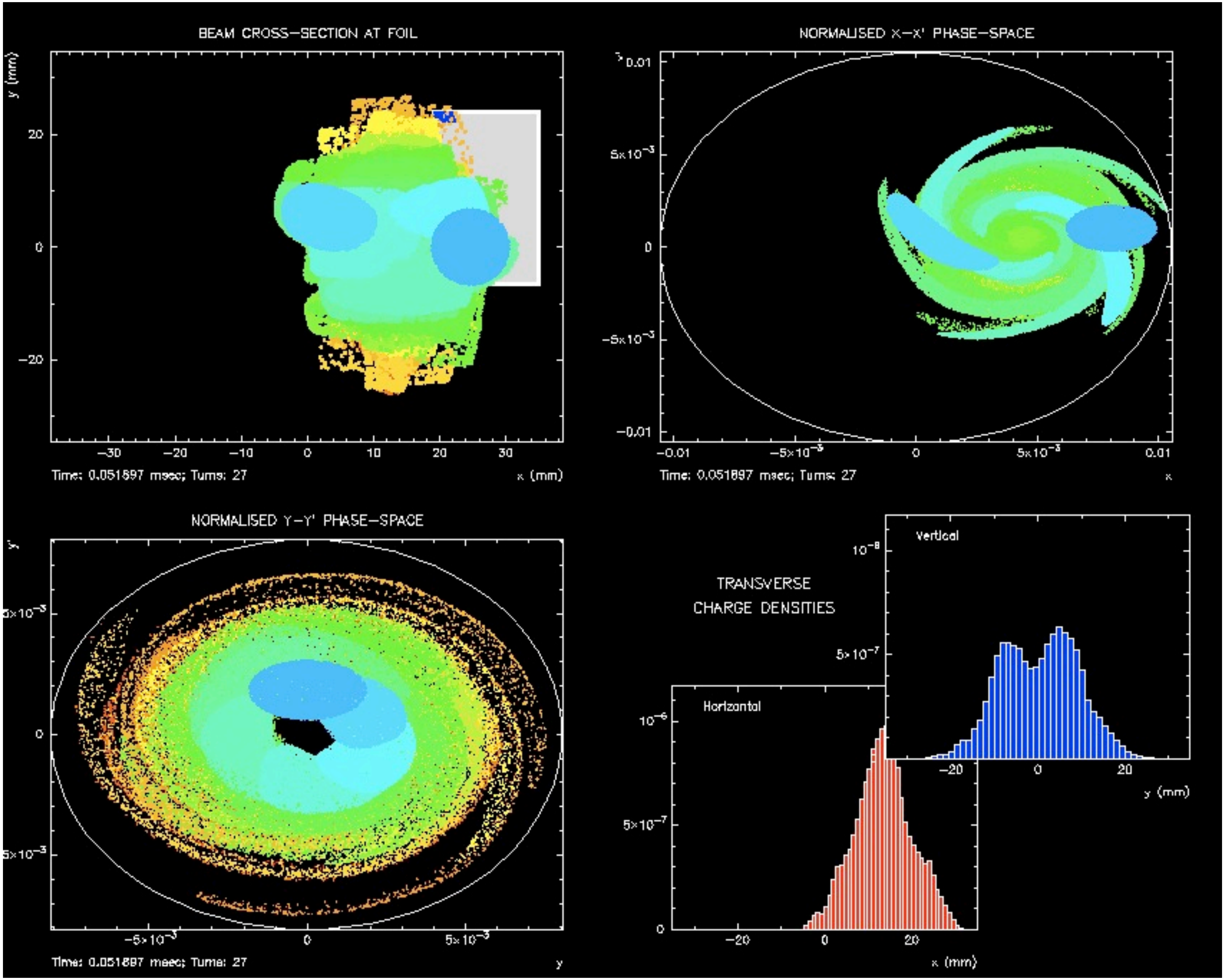
BEAM CROSS-SECTION AT FOIL
y (mm)
FOIL
20
0
−20
−30
−20
−10
0
10
20
30
x (mm)
Time: 0.041917 msec; Turns: 22
NORMALISED X-X' PHASE-SPACE
5×10⁻³
0
−5×10⁻³
x
NORMALISED Y-Y' PHASE-SPACE
y
TRANSVERSE
CHARGE DENSITIES
Vertical
Horizontal
y (mm)
x (mm)

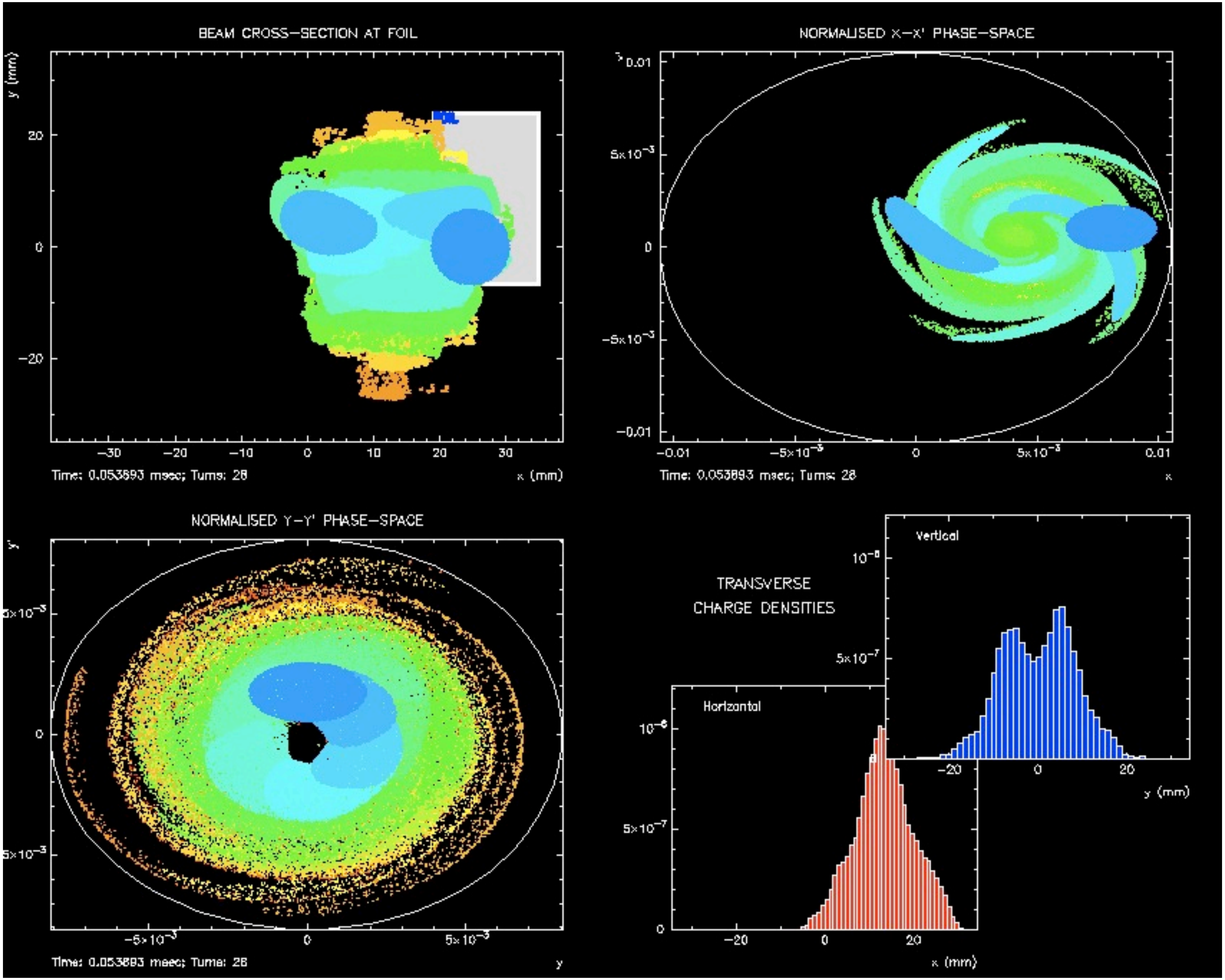

BEAM CROSS-SECTION AT FOIL
FOIL
y (mm)
x (mm)
Time: 0.043913 msec; Turns: 23
NORMALISED X-X' PHASE-SPACE
Time: 0.043913 msec; Turns: 23
NORMALISED Y-Y' PHASE-SPACE
Time: 0.043913 msec; Turns: 23
TRANSVERSE
CHARGE DENSITIES
Vertical
Horizontal
y (mm)
x (mm)

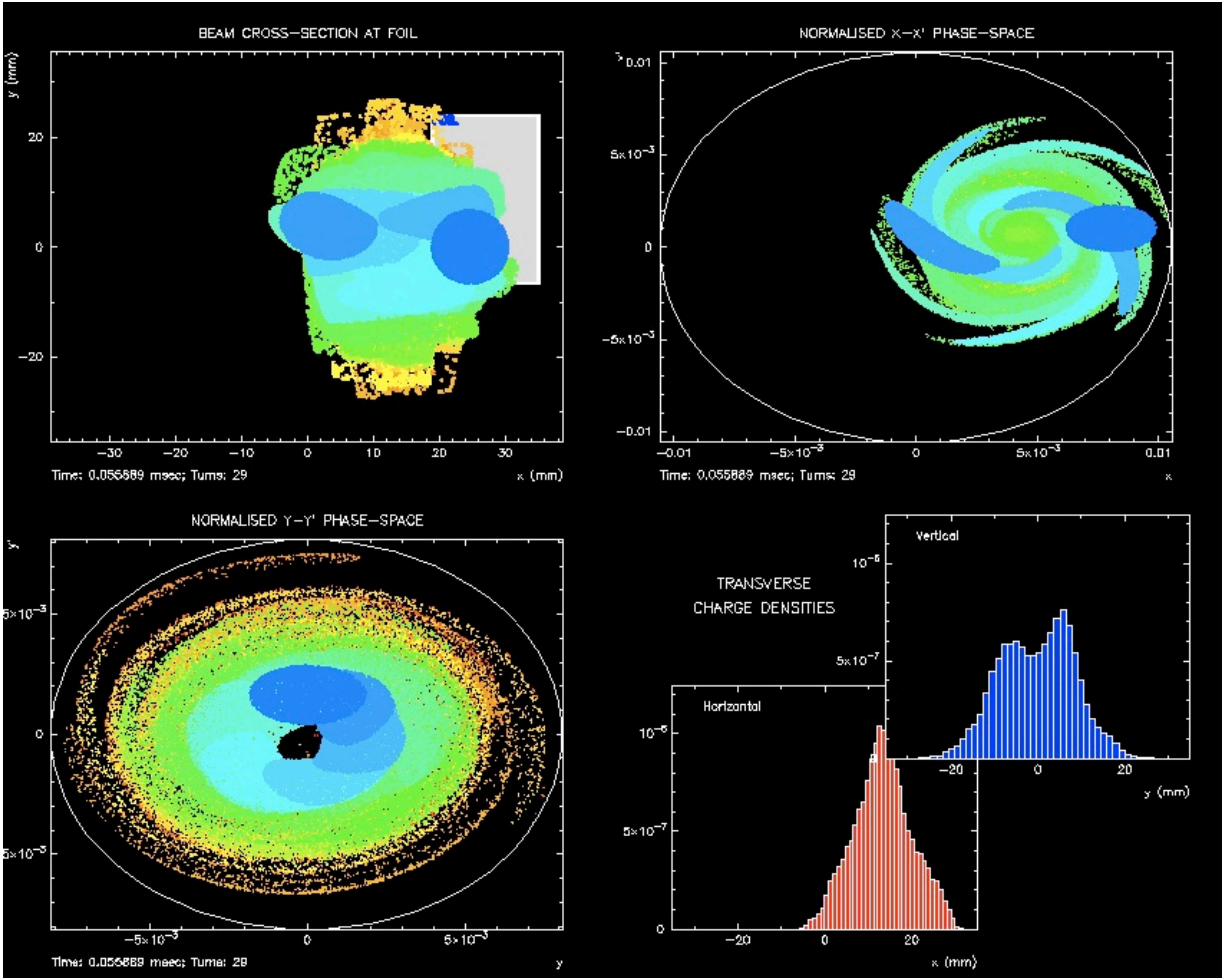

BEAM CROSS-SECTION AT FOIL
FOIL
y (mm)
x (mm)
Time: 0.045909 msec; Turns: 24
NORMALISED X-X' PHASE-SPACE
x
Time: 0.045909 msec; Turns: 24
NORMALISED Y-Y' PHASE-SPACE
y
Time: 0.045909 msec; Turns: 24
TRANSVERSE
CHARGE DENSITIES
Vertical
y (mm)
Horizontal
x (mm)

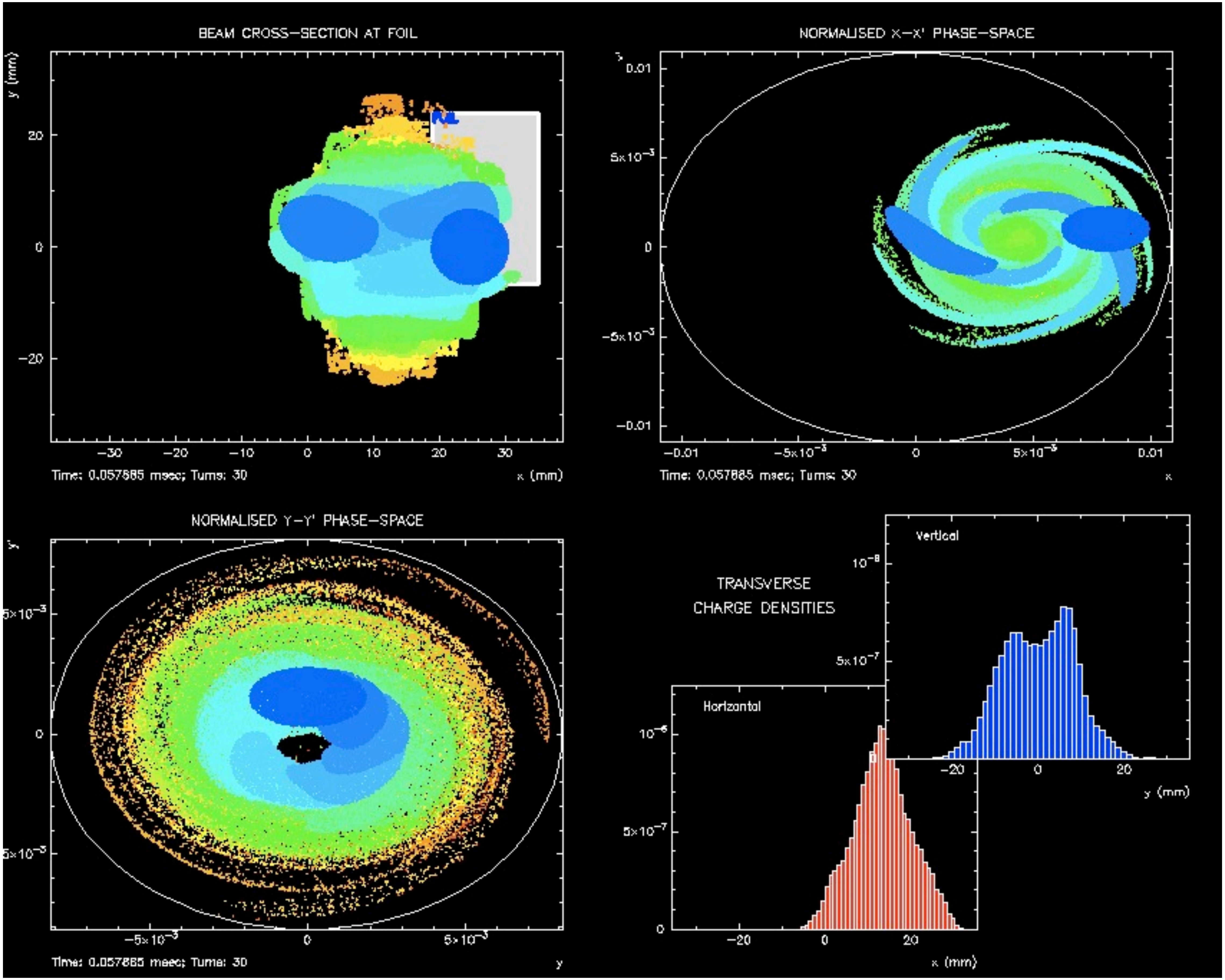

BEAM CROSS-SECTION AT FOIL
FOIL
y (mm)
x (mm)
Time: 0.047905 msec; Turns: 25
NORMALISED X-X' PHASE-SPACE
Time: 0.047905 msec; Turns: 25
NORMALISED Y-Y' PHASE-SPACE
Time: 0.047905 msec; Turns: 25
TRANSVERSE
CHARGE DENSITIES
Vertical
Horizontal
y (mm)
x (mm)

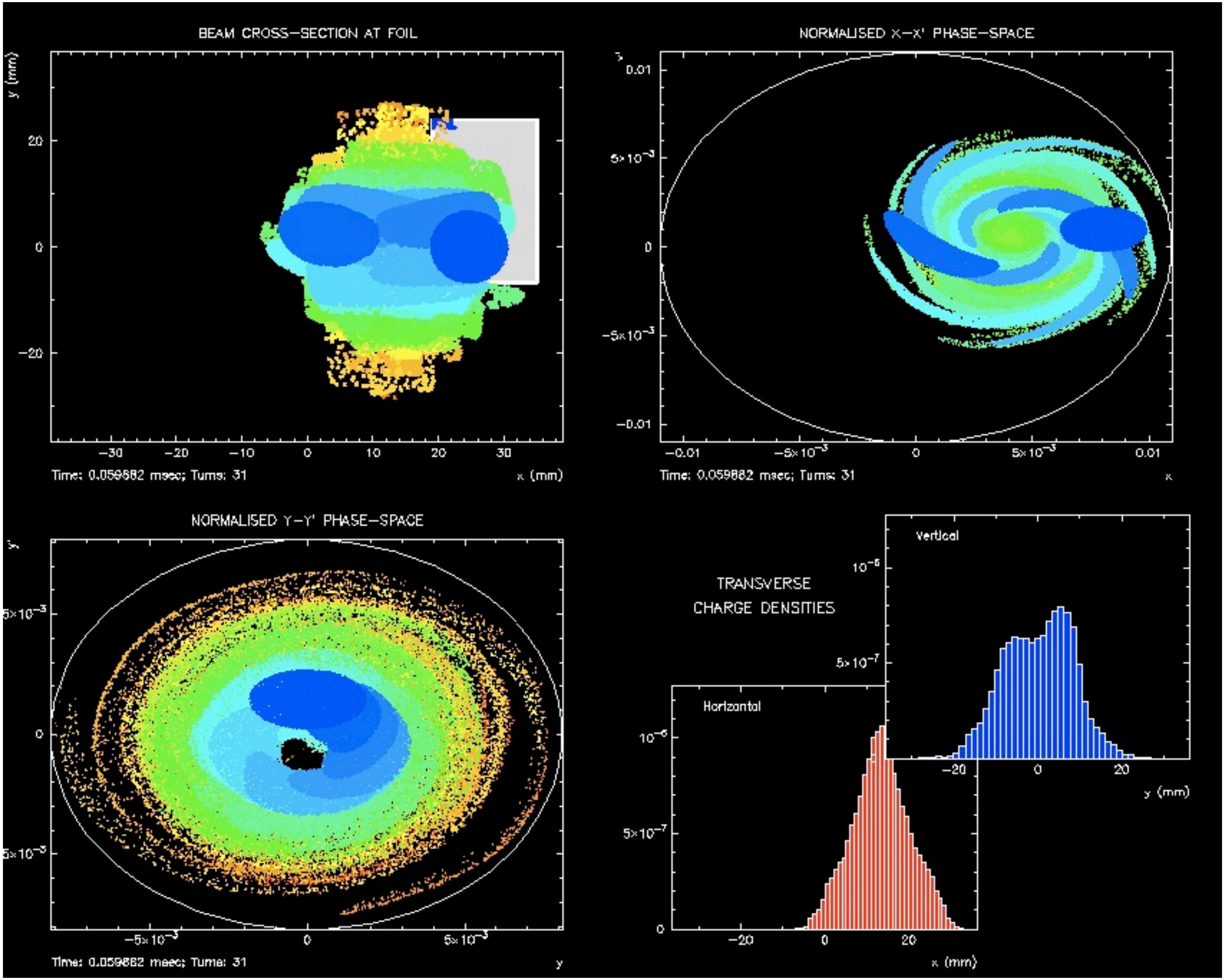

BEAM CROSS-SECTION AT FOIL
y (mm)
FOIL
x (mm)
Time: 0.049901 msec; Turns: 26
NORMALISED X-X' PHASE-SPACE
x
Time: 0.049901 msec; Turns: 26
NORMALISED Y-Y' PHASE-SPACE
y
Time: 0.049901 msec; Turns: 26
TRANSVERSE
CHARGE DENSITIES
Vertical
y (mm)
Horizontal
x (mm)

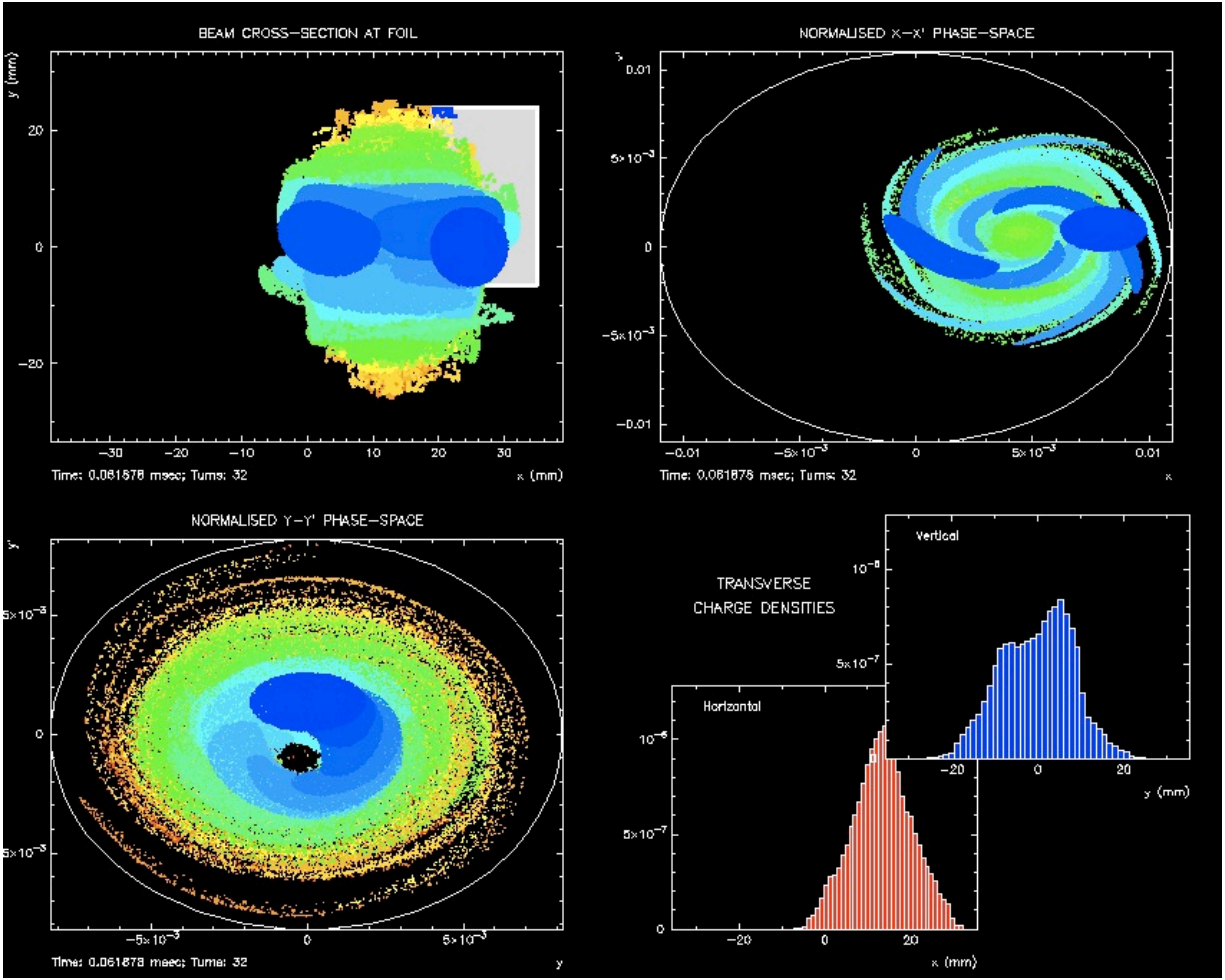

BEAM CROSS-SECTION AT FOIL
y (mm)
x (mm)
Time: 0.051897 msec; Turns: 27
NORMALISED X-X' PHASE-SPACE
x
Time: 0.051897 msec; Turns: 27
NORMALISED Y-Y' PHASE-SPACE
y
Time: 0.051897 msec; Turns: 27
TRANSVERSE
CHARGE DENSITIES
Vertical
y (mm)
Horizontal
x (mm)

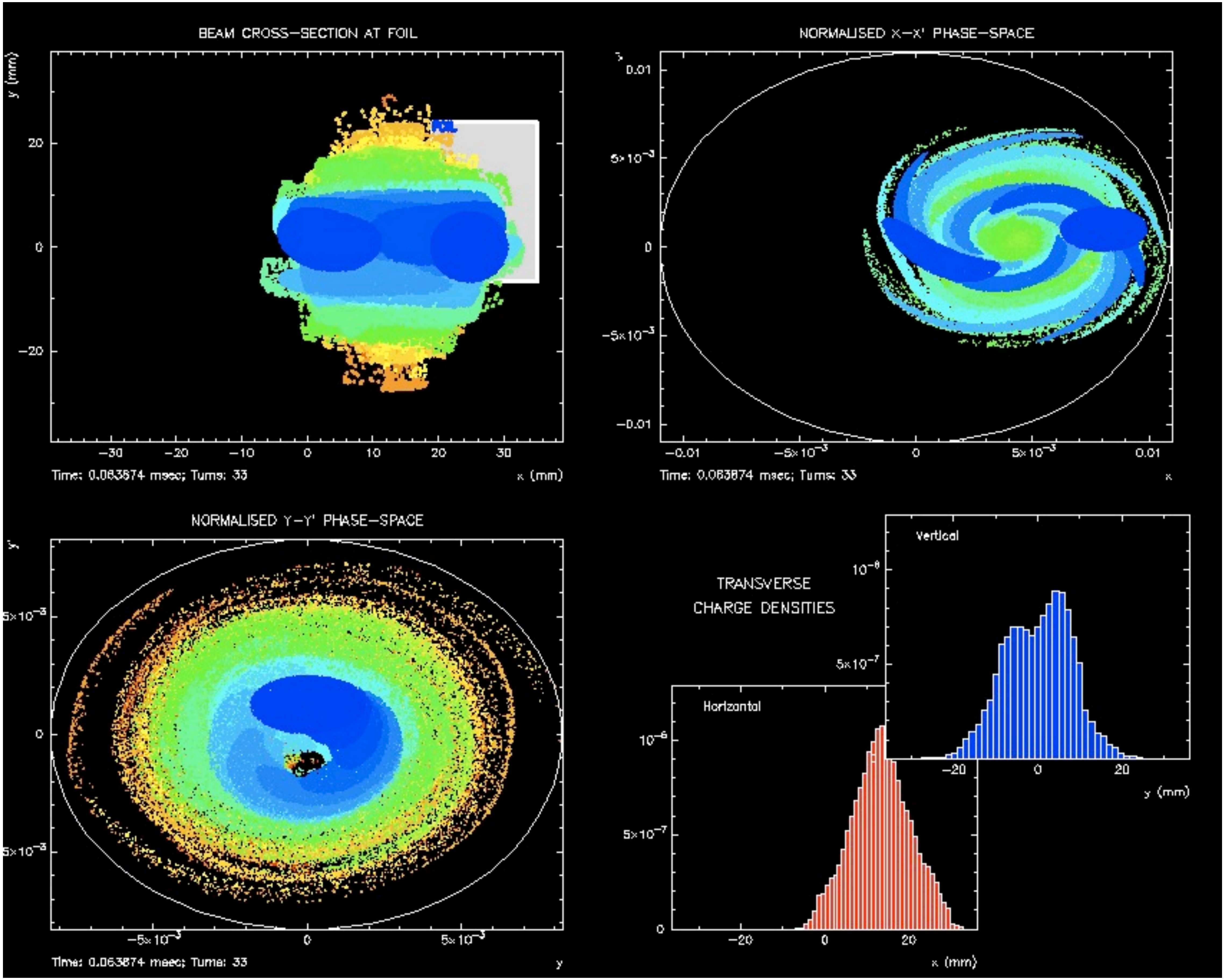

BEAM CROSS-SECTION AT FOIL
y (mm)
x (mm)
Time: 0.053893 msec; Turns: 28
NORMALISED X-X' PHASE-SPACE
Time: 0.053893 msec; Turns: 28
NORMALISED Y-Y' PHASE-SPACE
Time: 0.053893 msec; Turns: 28
TRANSVERSE
CHARGE DENSITIES
Vertical
y (mm)
Horizontal
x (mm)


BEAM CROSS-SECTION AT FOIL
y (mm)
x (mm)
Time: 0.055889 msec; Turns: 29
NORMALISED X-X' PHASE-SPACE
Time: 0.055889 msec; Turns: 29
NORMALISED Y-Y' PHASE-SPACE
Time: 0.055889 msec; Turns: 29
TRANSVERSE
CHARGE DENSITIES
Vertical
Horizontal
y (mm)
x (mm)

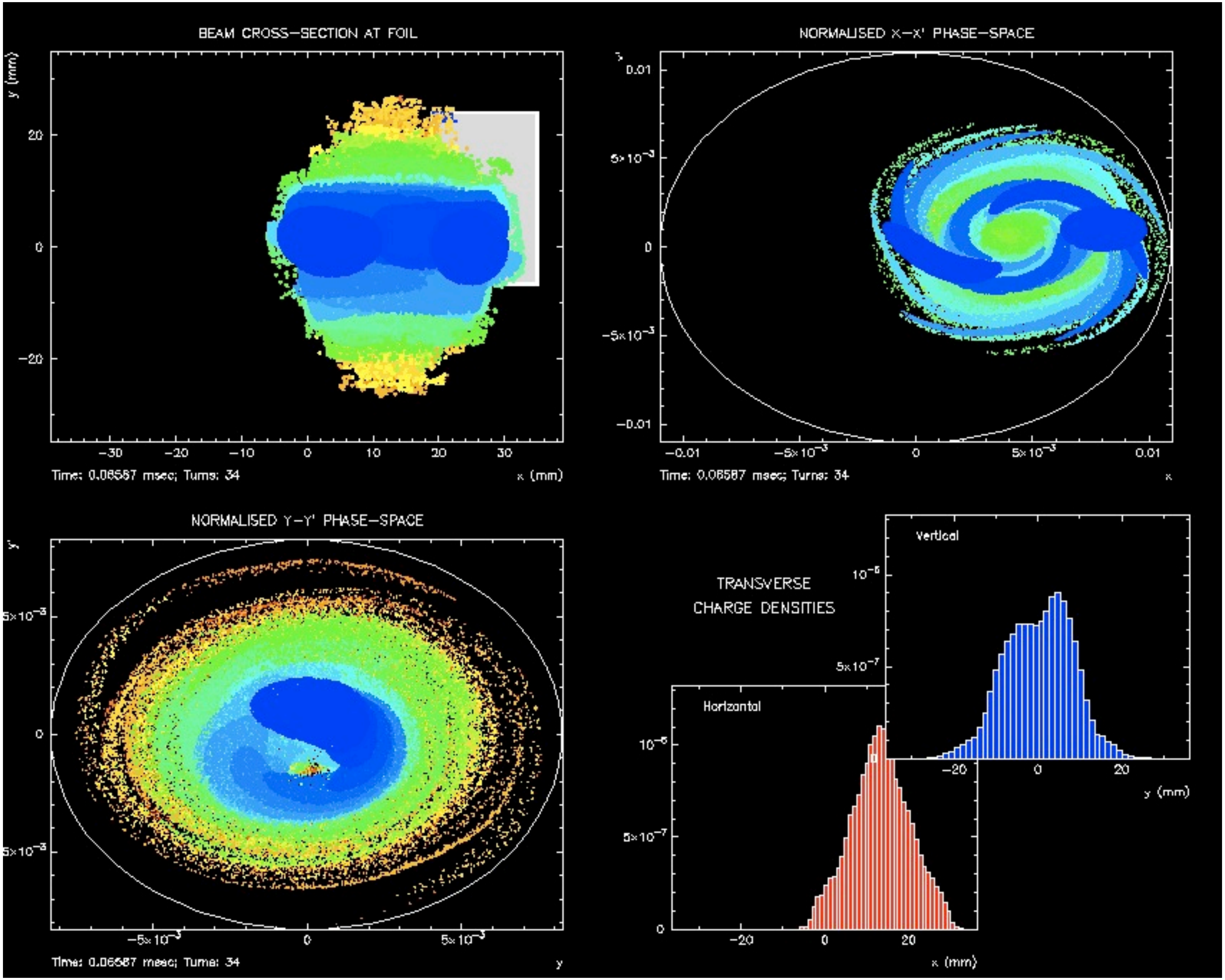

BEAM CROSS-SECTION AT FOIL
y (mm)
x (mm)
Time: 0.057885 msec; Turns: 30
NORMALISED X-X' PHASE-SPACE
x
Time: 0.057885 msec; Turns: 30
NORMALISED Y-Y' PHASE-SPACE
y
Time: 0.057885 msec; Turns: 30
TRANSVERSE
CHARGE DENSITIES
Vertical
y (mm)
Horizontal
x (mm)

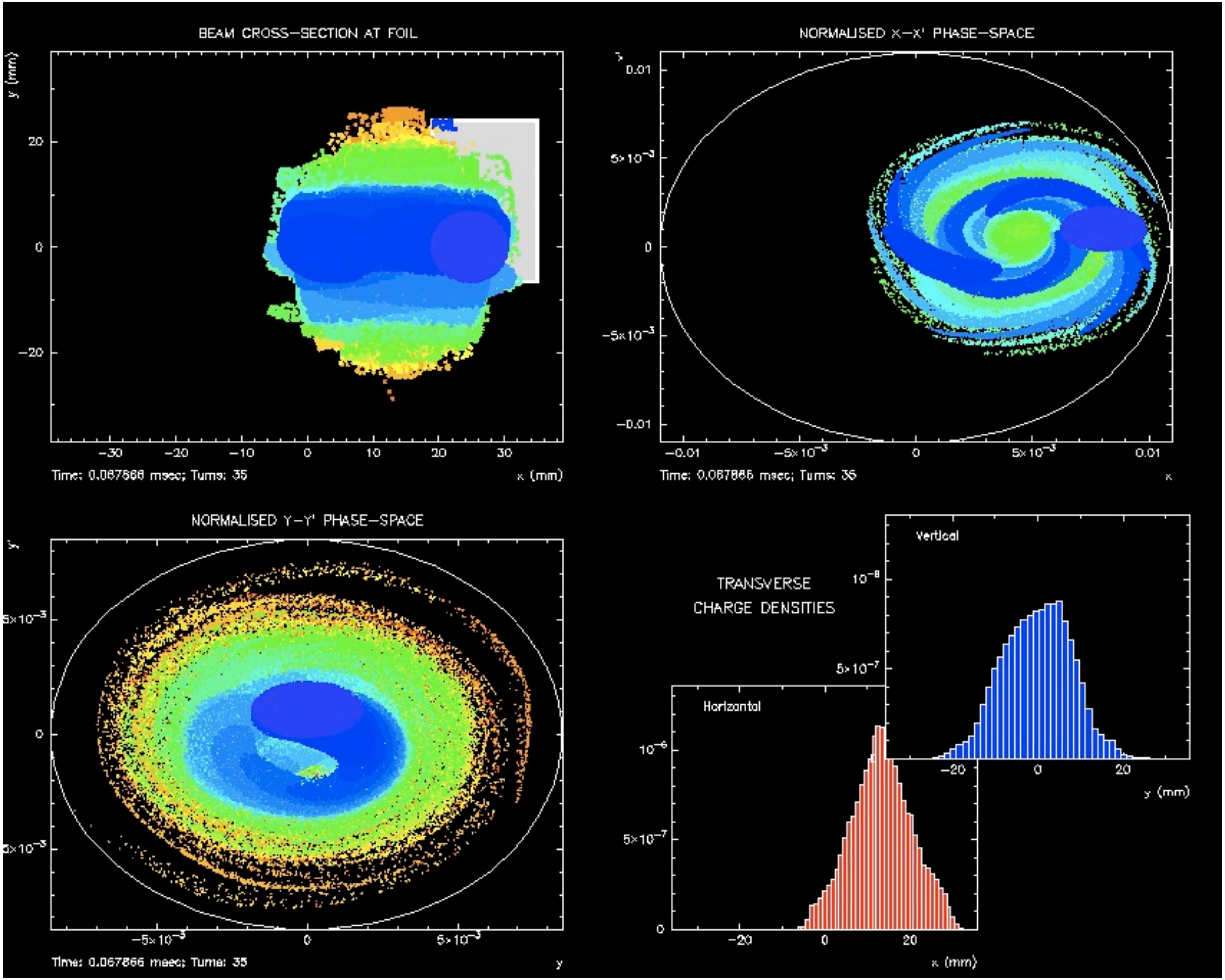

BEAM CROSS-SECTION AT FOIL
y (mm)
x (mm)
Time: 0.059882 msec; Turns: 31
NORMALISED X-X' PHASE-SPACE
x
Time: 0.059882 msec; Turns: 31
NORMALISED Y-Y' PHASE-SPACE
y
Time: 0.059882 msec; Turns: 31
TRANSVERSE
CHARGE DENSITIES
Vertical
y (mm)
Horizontal
x (mm)

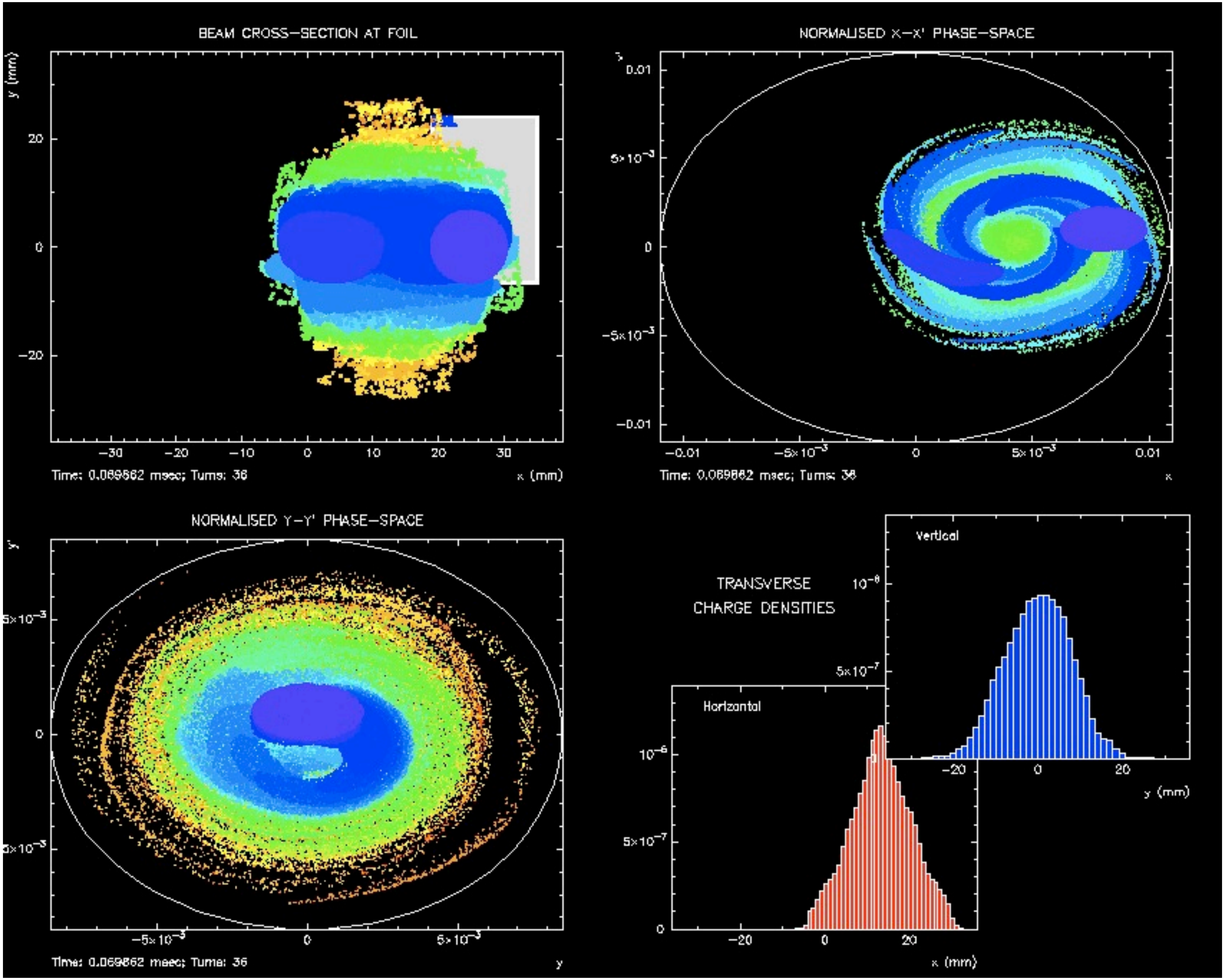

BEAM CROSS-SECTION AT FOIL
NORMALISED X-X' PHASE-SPACE
NORMALISED Y-Y' PHASE-SPACE
TRANSVERSE
CHARGE DENSITIES
FOIL
Vertical
Horizontal
y (mm)
x (mm)
Time: 0.061878 msec; Turns: 32



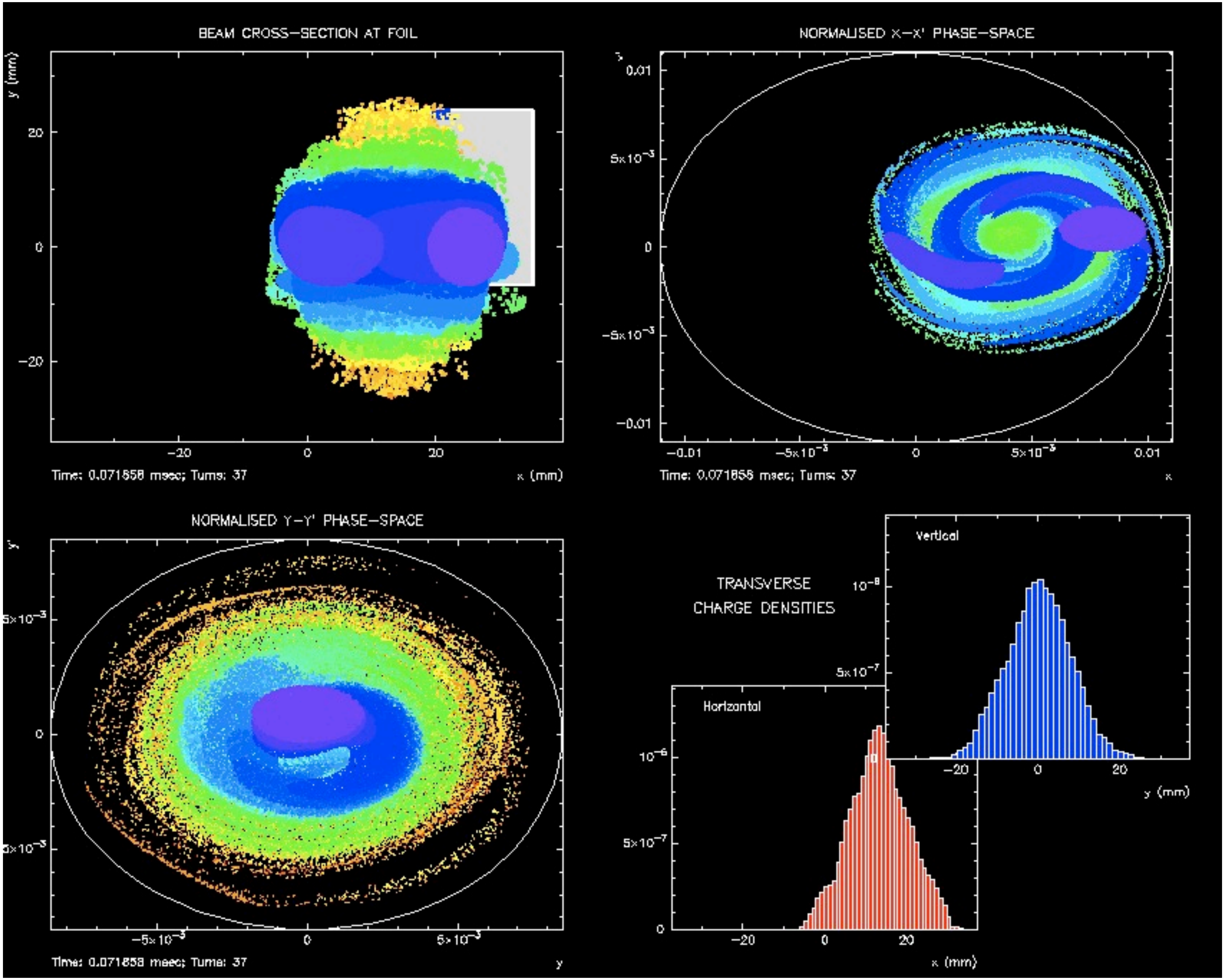
BEAM CROSS-SECTION AT FOIL
y (mm)
x (mm)
FOIL
Time: 0.063874 msec; Turns: 33
NORMALISED X-X' PHASE-SPACE
x
Time: 0.063874 msec; Turns: 33
NORMALISED Y-Y' PHASE-SPACE
y
Time: 0.063874 msec; Turns: 33
TRANSVERSE
CHARGE DENSITIES
Vertical
y (mm)
Horizontal
x (mm)

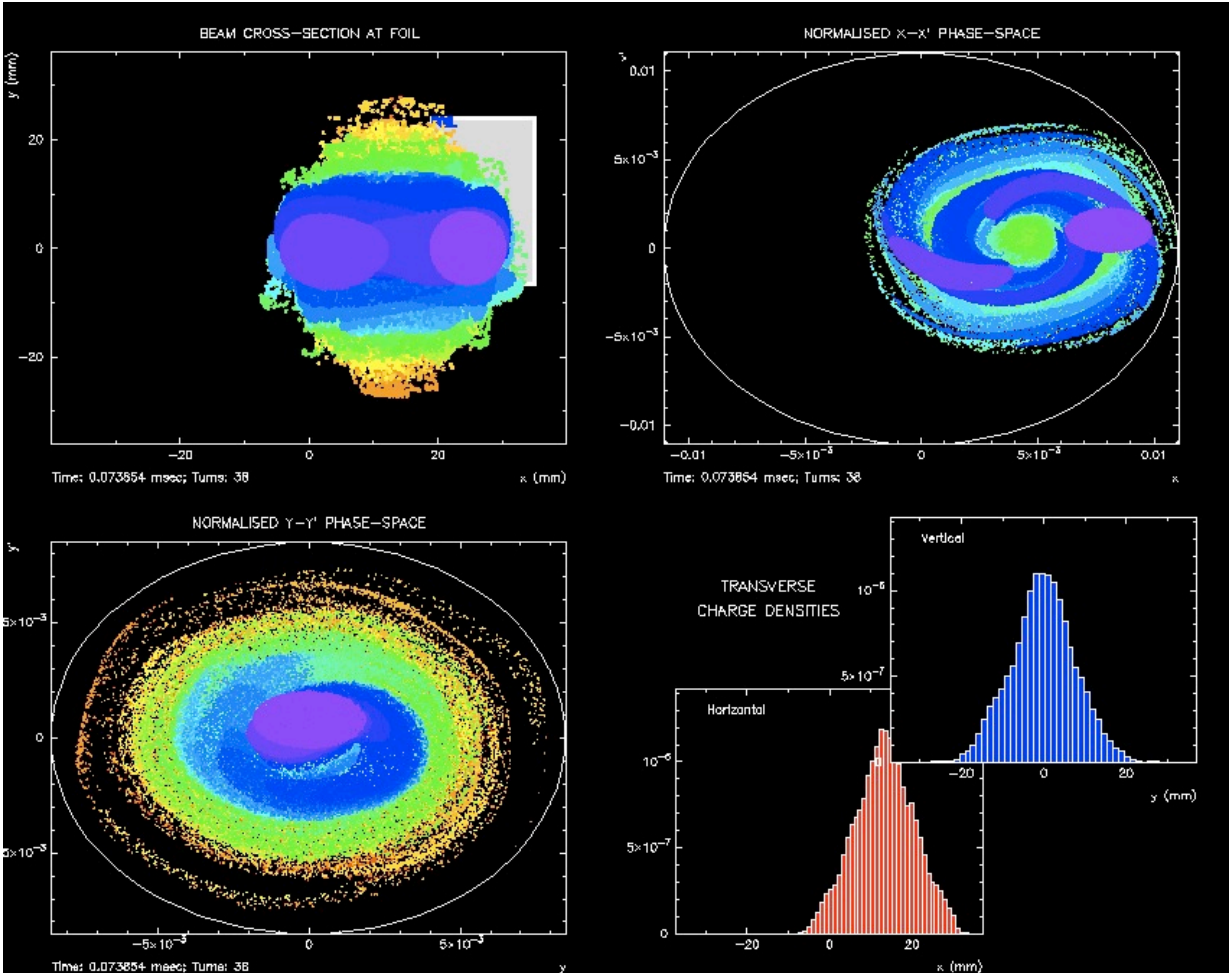

BEAM CROSS-SECTION AT FOIL
y (mm)
x (mm)
Time: 0.06567 msec; Turns: 34
NORMALISED X-X' PHASE-SPACE
x
Time: 0.06567 msec; Turns: 34
NORMALISED Y-Y' PHASE-SPACE
y
Time: 0.06567 msec; Turns: 34
TRANSVERSE
CHARGE DENSITIES
Vertical
y (mm)
Horizontal
x (mm)

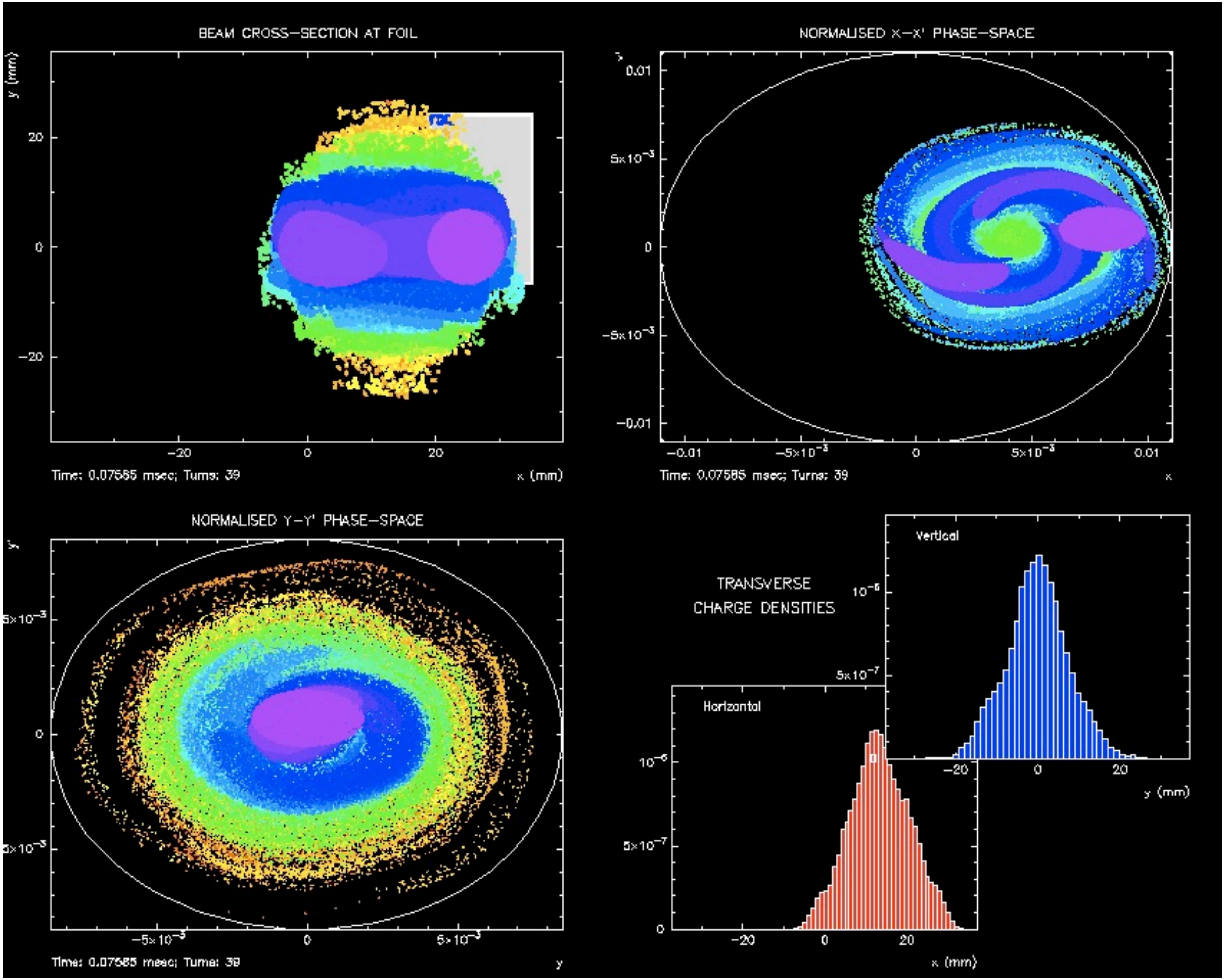

BEAM CROSS-SECTION AT FOIL
y (mm)
x (mm)
Time: 0.067866 msec; Turns: 35
NORMALISED X-X' PHASE-SPACE
Time: 0.067866 msec; Turns: 35
NORMALISED Y-Y' PHASE-SPACE
Time: 0.067866 msec; Turns: 35
TRANSVERSE
CHARGE DENSITIES
Vertical
Horizontal
y (mm)
x (mm)

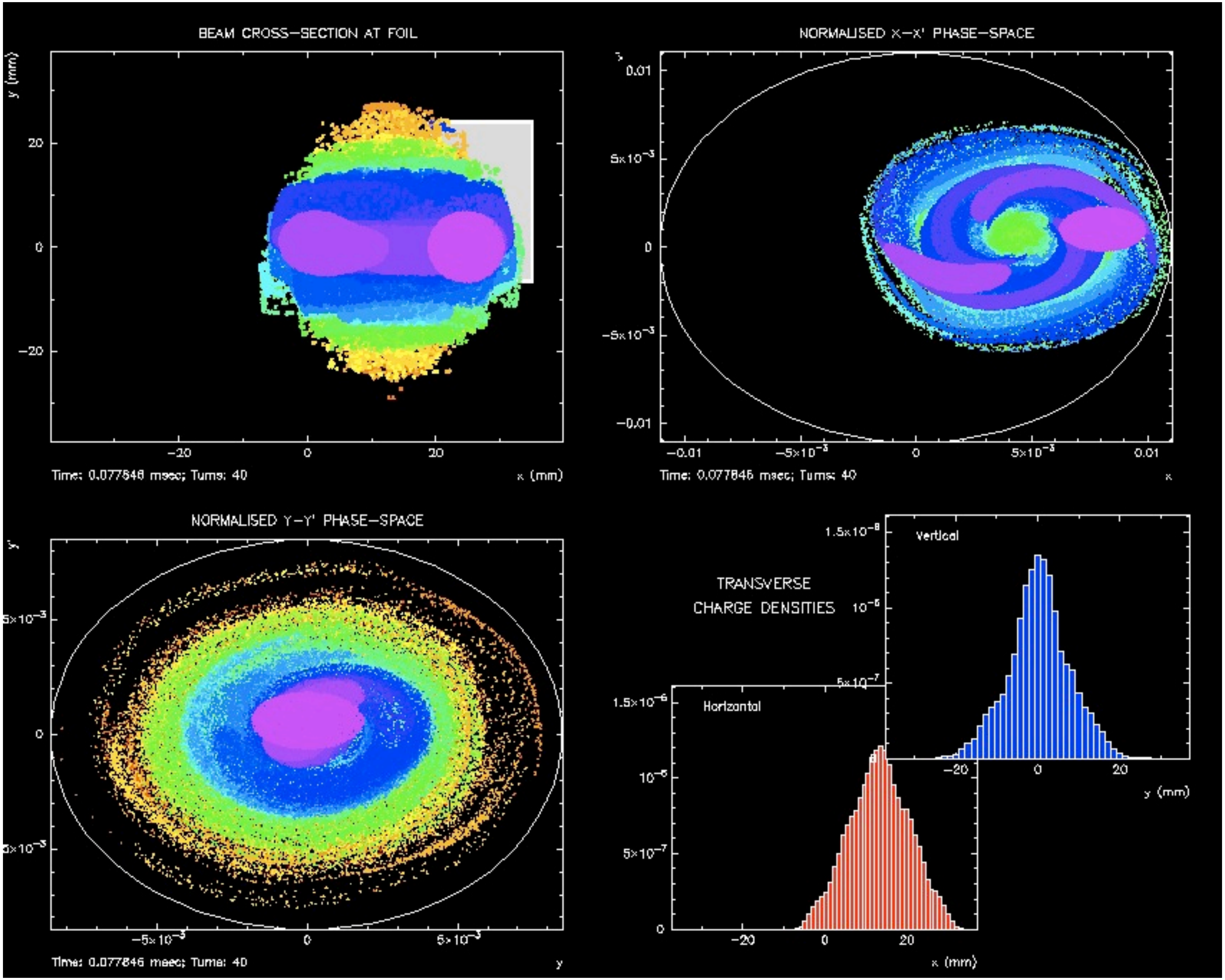


BEAM CROSS-SECTION AT FOIL
y (mm)
x (mm)
Time: 0.069862 msec; Turns: 36
NORMALISED X-X' PHASE-SPACE
x
Time: 0.069862 msec; Turns: 36
NORMALISED Y-Y' PHASE-SPACE
y'
y
Time: 0.069862 msec; Turns: 36
TRANSVERSE
CHARGE DENSITIES
Vertical
y (mm)
Horizontal
x (mm)

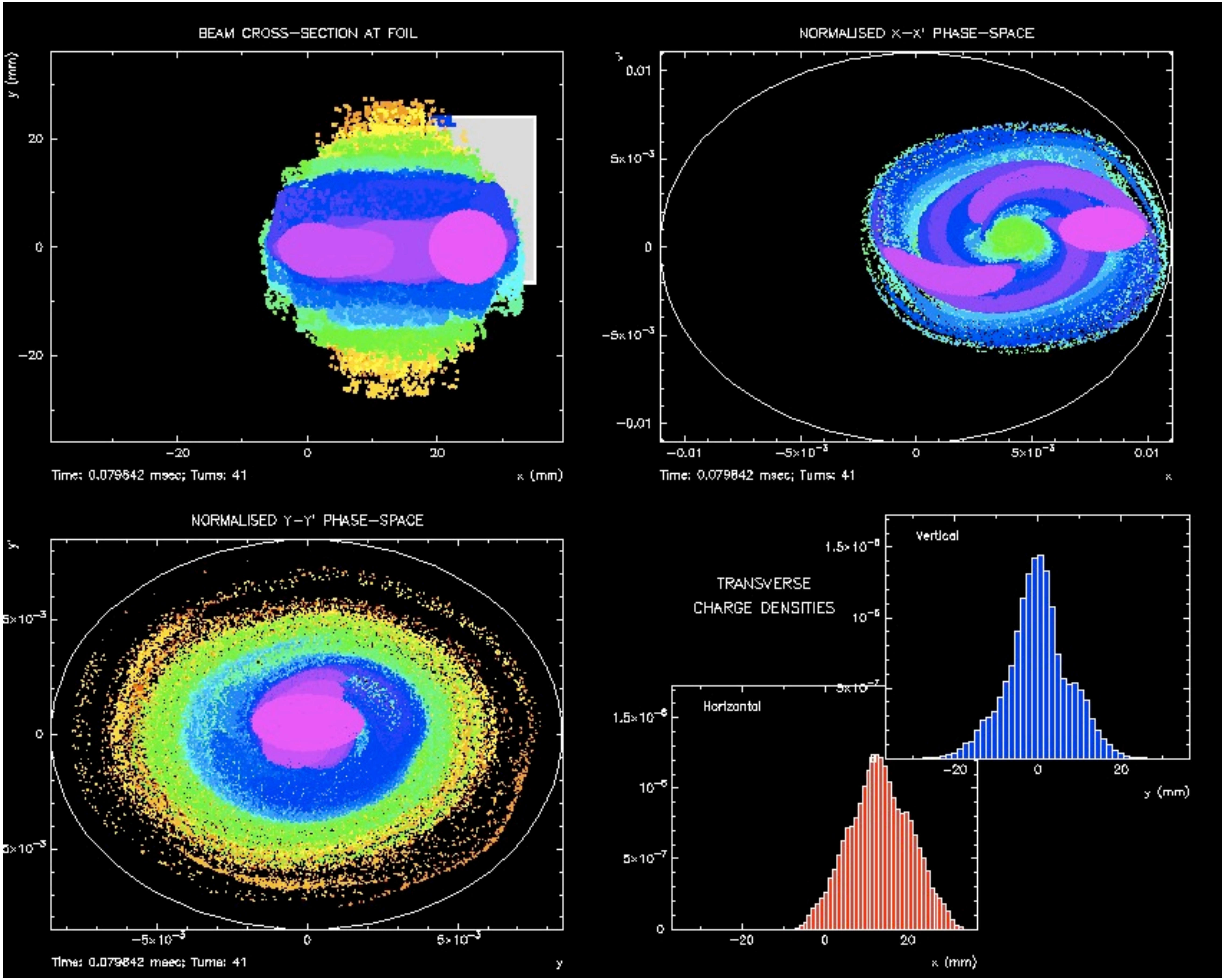

BEAM CROSS-SECTION AT FOIL
y (mm)
x (mm)
Time: 0.071858 msec; Turns: 37
NORMALISED X-X' PHASE-SPACE
Time: 0.071858 msec; Turns: 37
NORMALISED Y-Y' PHASE-SPACE
Time: 0.071858 msec; Turns: 37
TRANSVERSE
CHARGE DENSITIES
Vertical
y (mm)
Horizontal
x (mm)

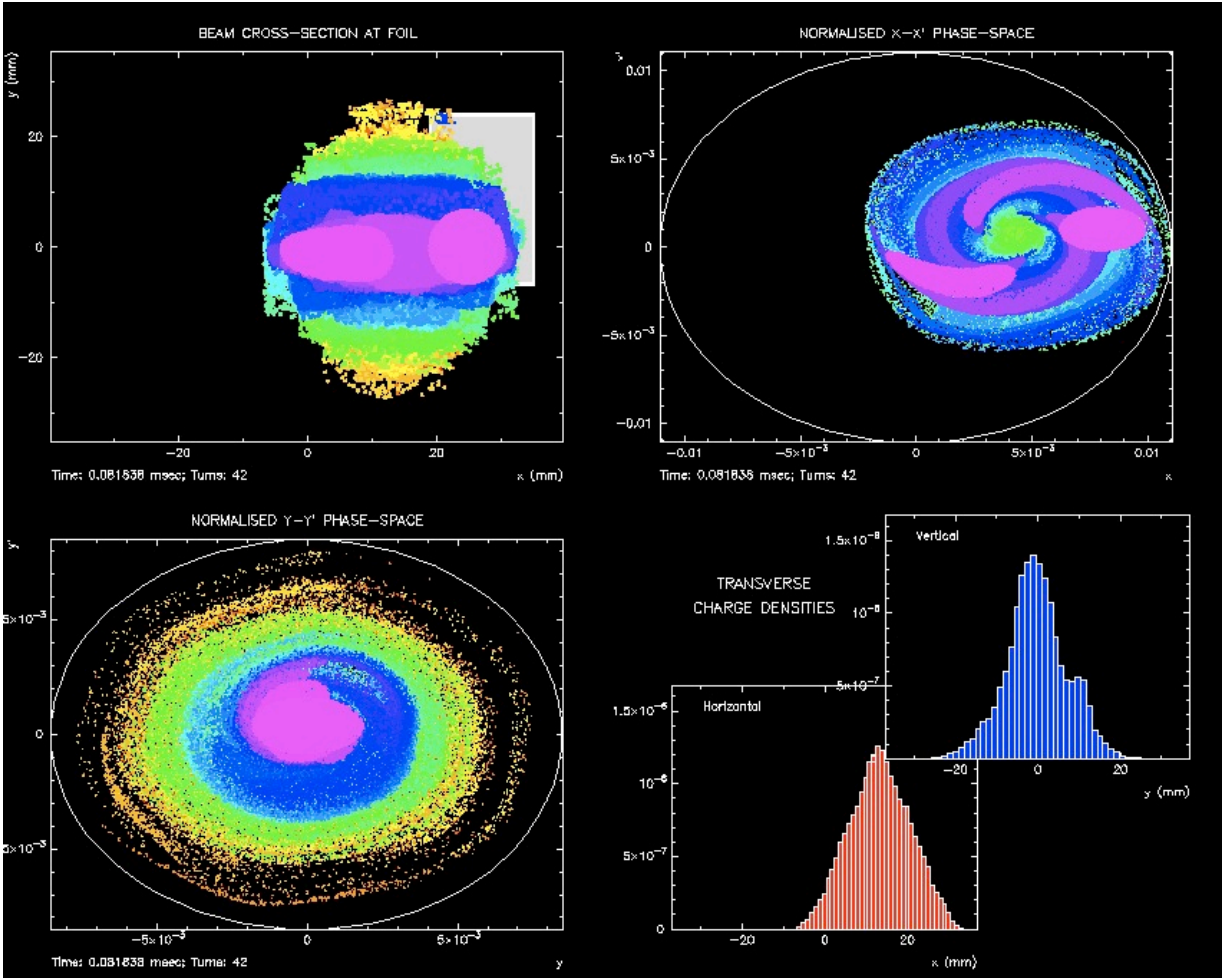

BEAM CROSS-SECTION AT FOIL
y (mm)
x (mm)
Time: 0.073854 msec; Turns: 38
NORMALISED X-X' PHASE-SPACE
x
Time: 0.073854 msec; Turns: 38
NORMALISED Y-Y' PHASE-SPACE
y'
y
Time: 0.073854 msec; Turns: 38
TRANSVERSE
CHARGE DENSITIES
Vertical
y (mm)
Horizontal
x (mm)

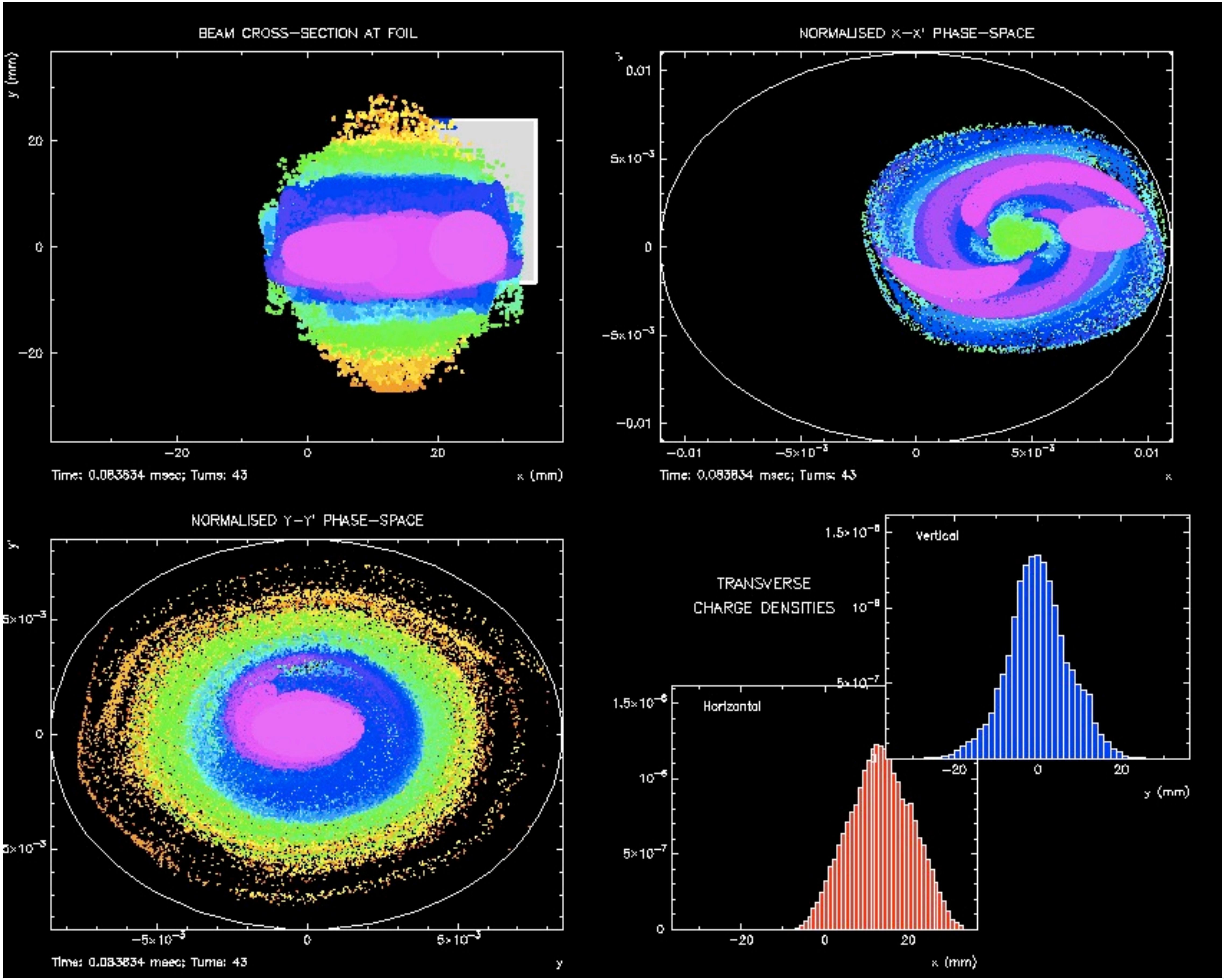

BEAM CROSS-SECTION AT FOIL
y (mm)
x (mm)
Time: 0.07585 msec; Turns: 39
NORMALISED X-X' PHASE-SPACE
Time: 0.07585 msec; Turns: 39
NORMALISED Y-Y' PHASE-SPACE
Time: 0.07585 msec; Turns: 39
TRANSVERSE
CHARGE DENSITIES
Vertical
Horizontal
y (mm)
x (mm)

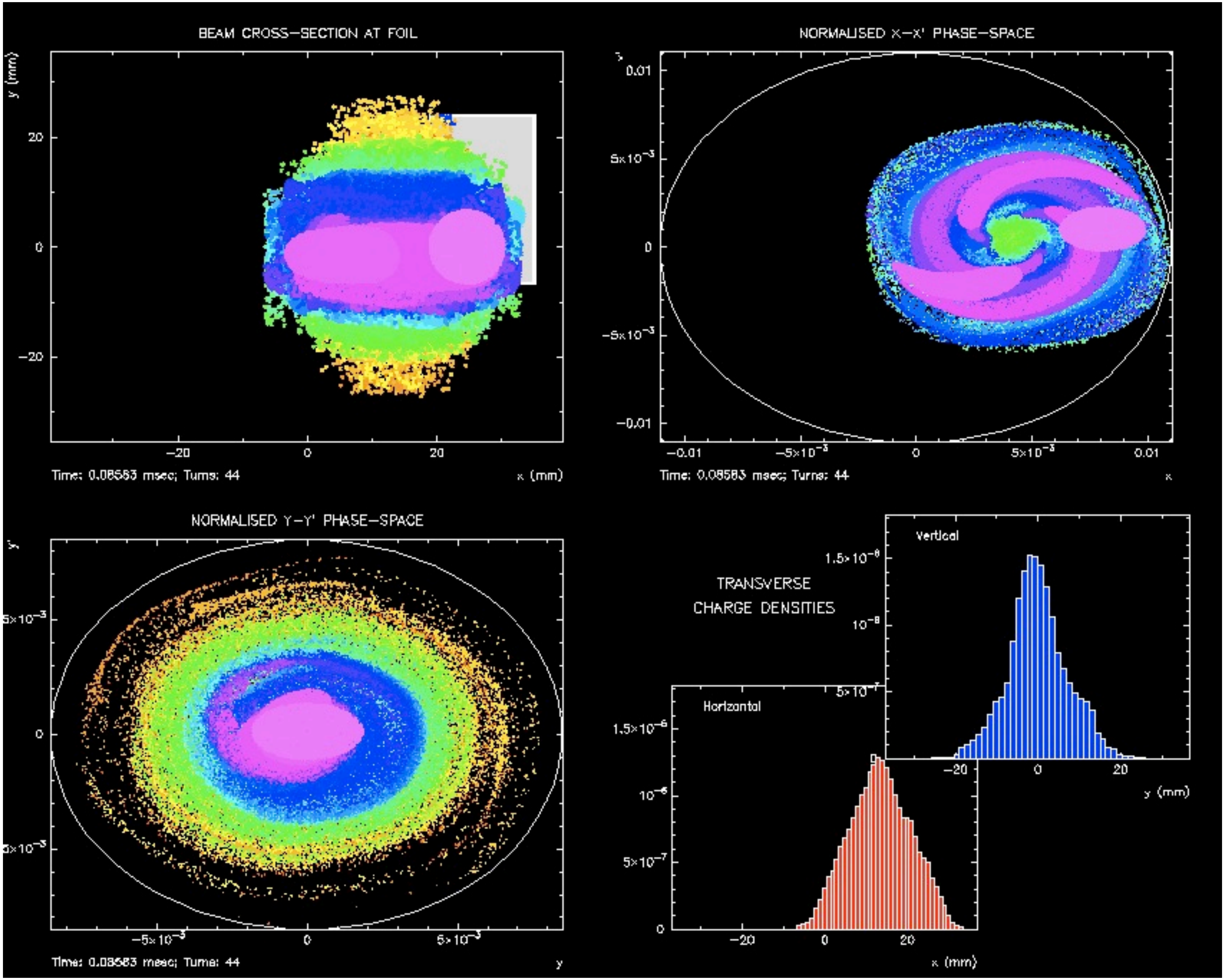


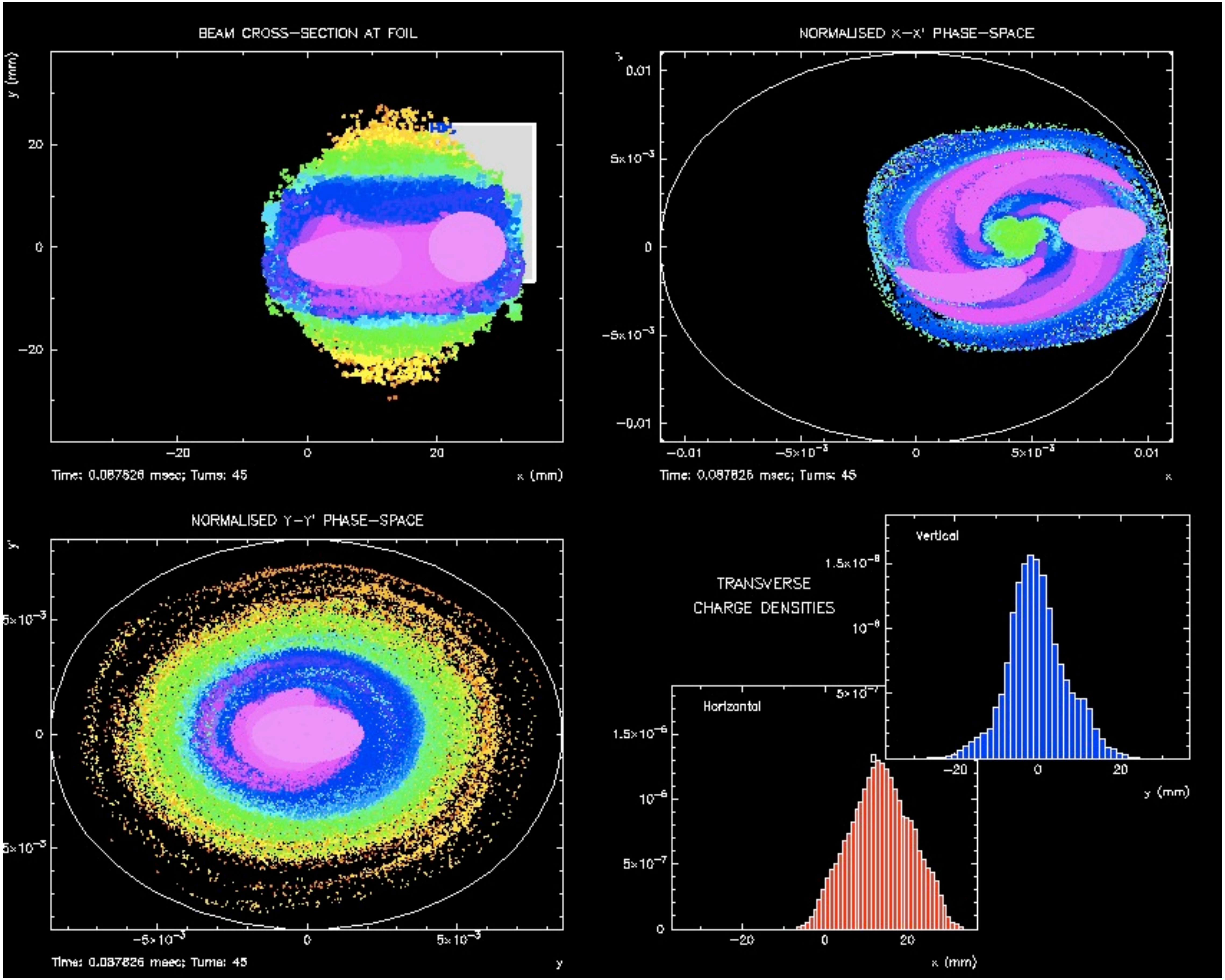
BEAM CROSS-SECTION AT FOIL
y (mm)
x (mm)
Time: 0.077846 msec; Turns: 40
NORMALISED X-X' PHASE-SPACE
x
Time: 0.077846 msec; Turns: 40
NORMALISED Y-Y' PHASE-SPACE
y
Time: 0.077846 msec; Turns: 40
TRANSVERSE
CHARGE DENSITIES
Vertical
y (mm)
Horizontal
x (mm)


BEAM CROSS-SECTION AT FOIL
y (mm)
FOIL
x (mm)
Time: 0.079842 msec; Turns: 41
NORMALISED X-X' PHASE-SPACE
x'
x
Time: 0.079842 msec; Turns: 41
NORMALISED Y-Y' PHASE-SPACE
y'
y
Time: 0.079842 msec; Turns: 41
TRANSVERSE
CHARGE DENSITIES
Vertical
y (mm)
Horizontal
x (mm)

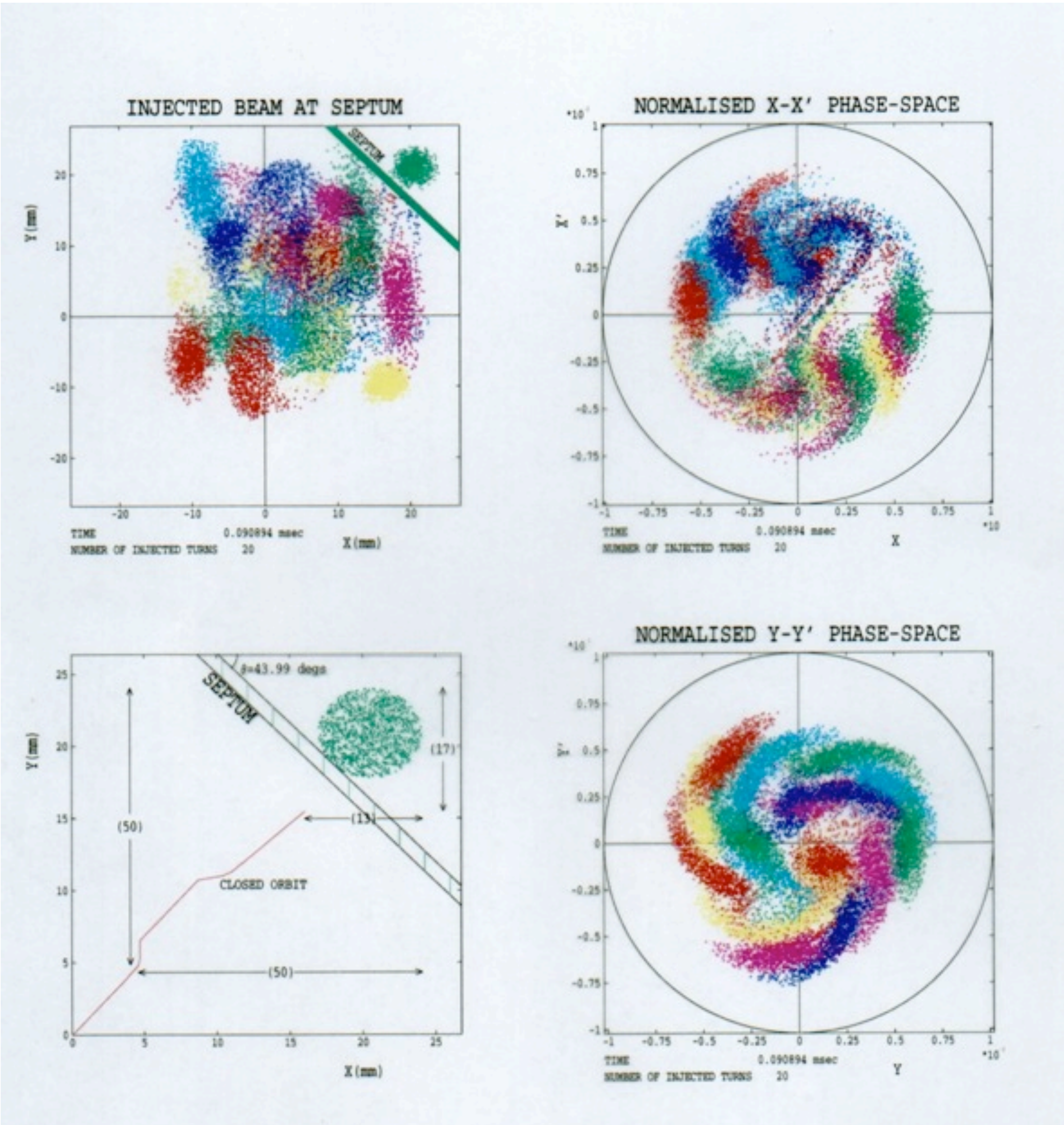

BEAM CROSS-SECTION AT FOIL
y (mm)
x (mm)
Time: 0.081838 msec; Turns: 42
NORMALISED X-X' PHASE-SPACE
x'
x
Time: 0.081838 msec; Turns: 42
NORMALISED Y-Y' PHASE-SPACE
y'
y
Time: 0.081838 msec; Turns: 42
TRANSVERSE
CHARGE DENSITIES
Vertical
y (mm)
Horizontal
x (mm)


BEAM CROSS-SECTION AT FOIL
y (mm)
x (mm)
Time: 0.083834 msec; Turns: 43
NORMALISED X-X' PHASE-SPACE
Time: 0.083834 msec; Turns: 43
NORMALISED Y-Y' PHASE-SPACE
Time: 0.083834 msec; Turns: 43
TRANSVERSE
CHARGE DENSITIES
Vertical
Horizontal
y (mm)
x (mm)


BEAM CROSS-SECTION AT FOIL
y (mm)
x (mm)
Time: 0.08583 msec; Turns: 44
NORMALISED X-X' PHASE-SPACE
Time: 0.08583 msec; Turns: 44
NORMALISED Y-Y' PHASE-SPACE
Time: 0.08583 msec; Turns: 44
TRANSVERSE
CHARGE DENSITIES
Vertical
y (mm)
Horizontal
x (mm)


BEAM CROSS-SECTION AT FOIL
NORMALISED X-X' PHASE-SPACE
Time: 0.087826 msec; Turns: 45
x (mm)
y (mm)
NORMALISED Y-Y' PHASE-SPACE
TRANSVERSE
CHARGE DENSITIES
Vertical
Horizontal

Heavy Ion Fusion (HIDIF)



Multiturn Injection modelled with TRACK2D

- 20 turns of 400 mA Bi^{+1} beam.
- Space-charge tune depression ~0.04.
- Two-plane injection using tilted electrostatic septum
- Note distortion of individual turns in phase space

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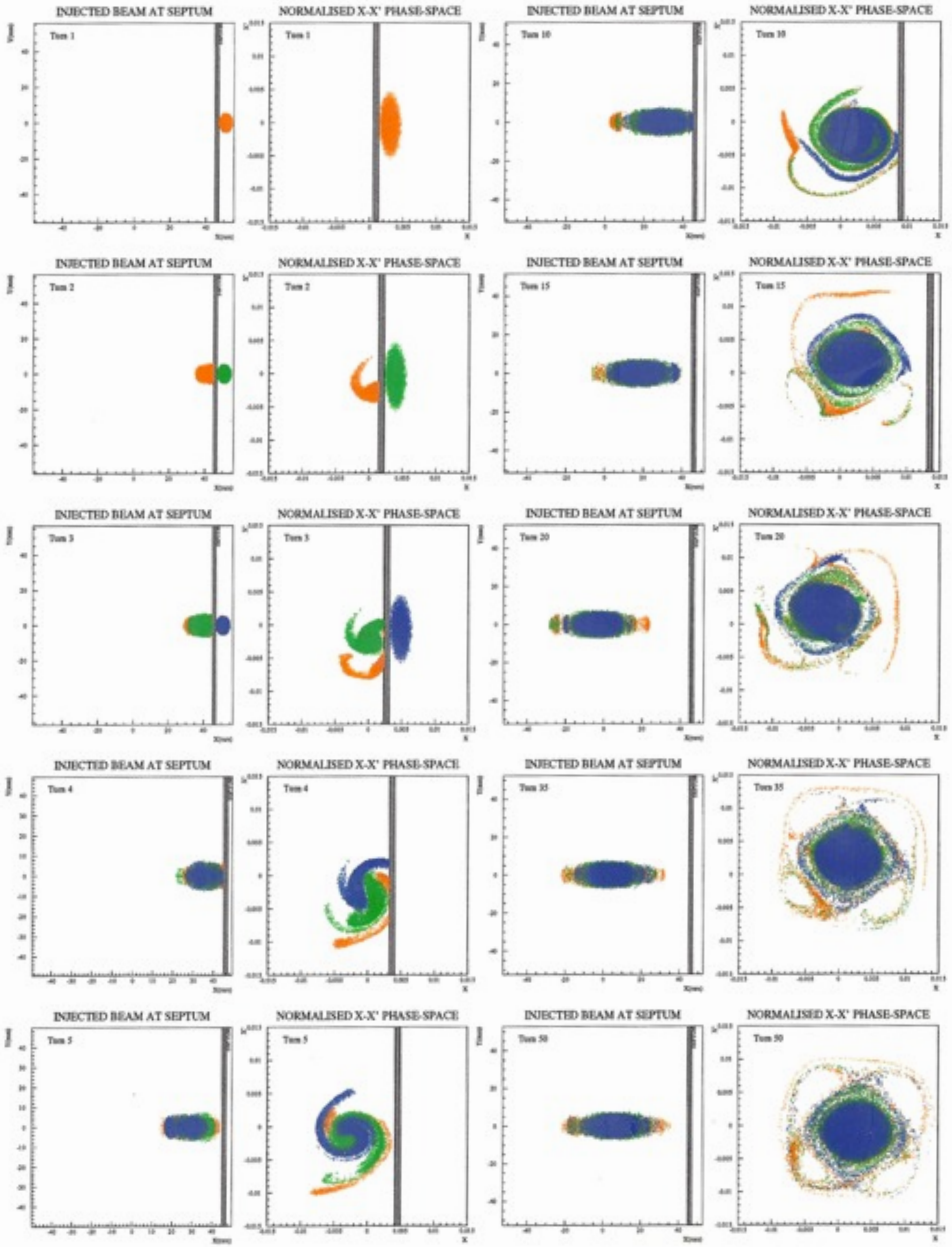

Figure 4.11: *Simulation of the three-turn betatron stacking process for the production of the LHC beam in the PS Booster. Clearly visible is a fourth order transverse resonance which builds up in the horizontal phase plane.*

Study from ~1998 of 3 turn stacking process in PS-Booster using TRACK2D

Shows beam loss at septum and development of a 4th order space-charge resonance

C. Prior & P. Knaus

Special Case: 1D Longitudinal Codes

$$\frac{\mathrm{d}\Delta\phi}{\mathrm{d}t} = \frac{h\omega_0^2\eta}{\beta^2\mathcal{E}}\left(\frac{\Delta\mathcal{E}}{\omega_0}\right), \qquad \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\Delta\mathcal{E}}{\omega_0}\right) = \frac{q}{2\pi}\left[V(\phi) - V(\phi_s) + U_s(\phi)\right]$$

$$U_s = -q\beta cR\left[\frac{g_0}{2\beta}\frac{Z_0}{\gamma^2} - \omega_0 L\right]\frac{\partial\lambda}{\partial s}$$

Tracking uses symplectic mapping with space charge calculated from derivative of the line density.

CODES: ESME (FNAL), LONG1D (TRIUMF), TRACK1D (RAL)

A major issue in a high intensity proton accelerator is building up the beam intensity through several turns of injection. For the SNS, $N_{turns} = 1600$ turns are required; for the ESS, $N_{turns} \sim 1000$. For reliable results, need ~ 5000 particles per turn, so $\gtrsim 5 \times 10^6$ overall.

One solution is to use "painting" technique with variable charge build-up. Restricts total to $\sim 10^5$ particles, yet has ~ 5000 per turn.

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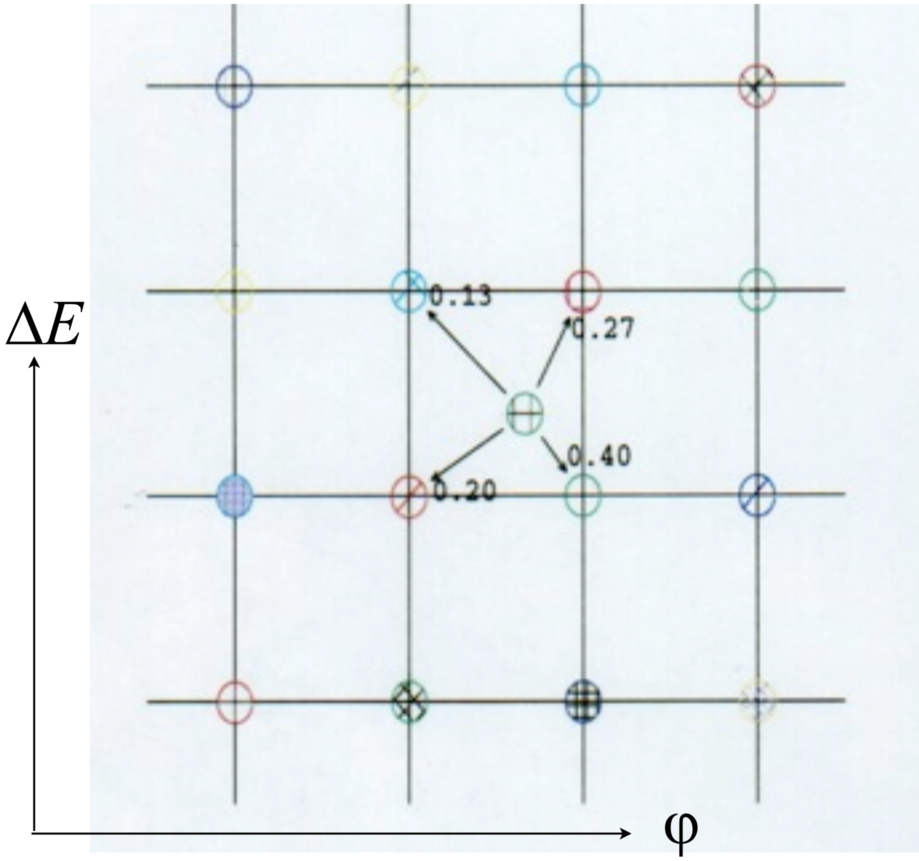
Charge Assignment to Grid



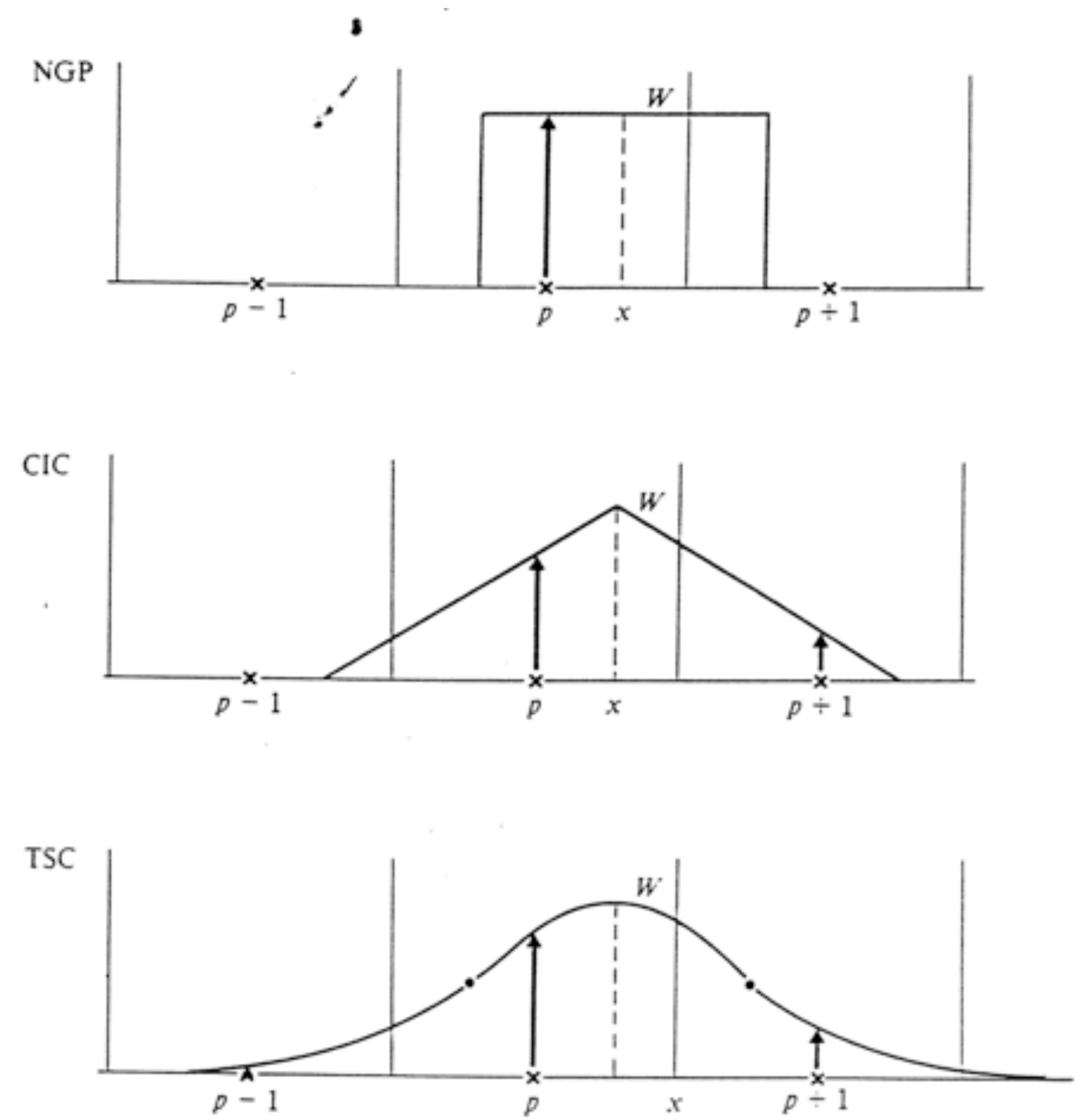


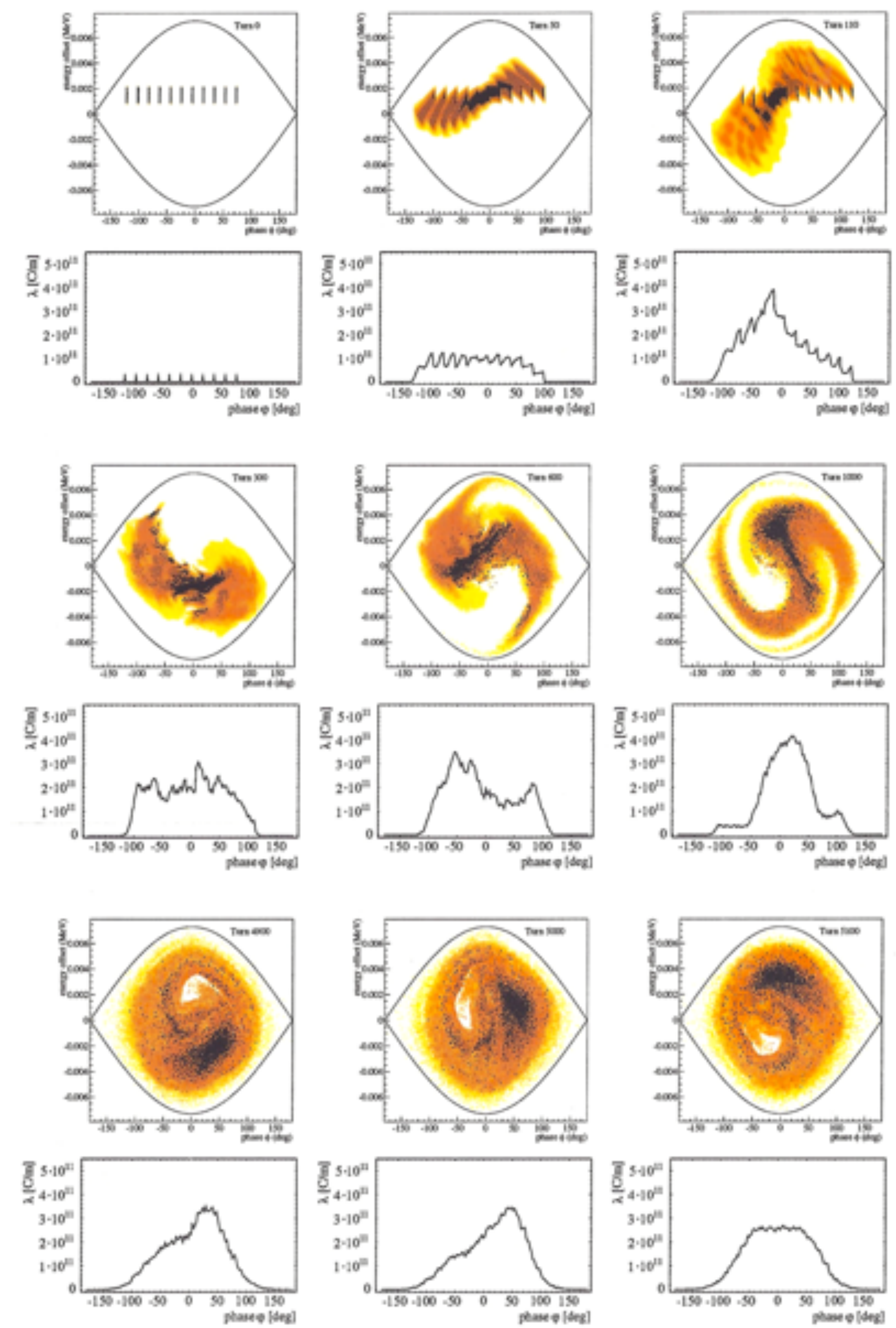
Figure 5-5 The assignment function shape interpretation of charge assignment. The fraction of charge assigned from a particle at position x to a given mesh point is equal to the value of the assignment function W at that point.

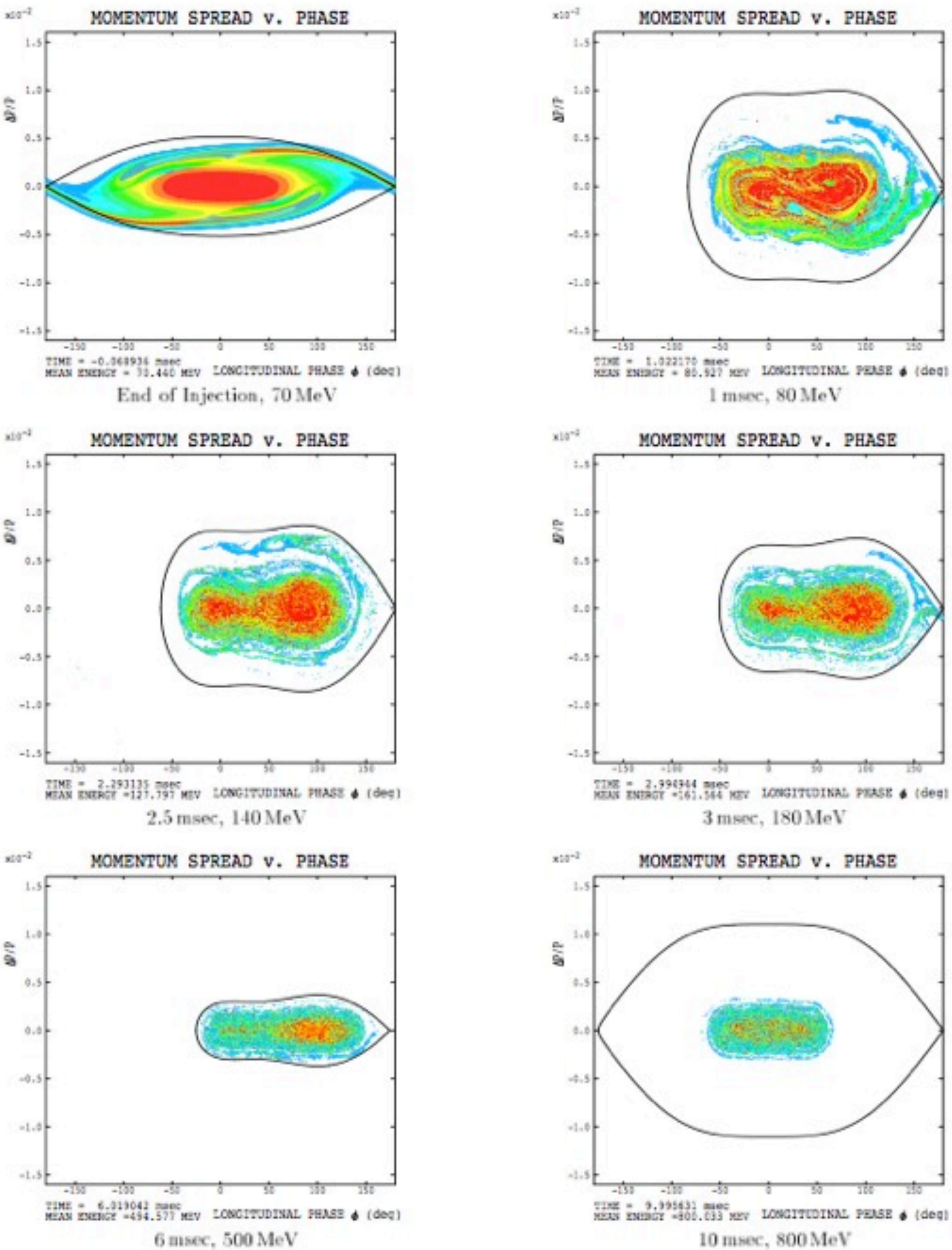
TRACK1D assignment uses Triangular Shaped Cloud (TSC). Line density smoothed with cubic splines to remove statistical effects. Includes corrections to counteract artificial spreading of the beam.

Longitudinal Study of CERN-PS

C. Prior & P. Knaus, ~1998

- Off-energy injection
- 11 micro-bunches per bucket
- Tracked for ~10 synchrotron periods
- No beam loss, good distribution





Longitudinal phase space plots for acceleration of 3×10^{13} protons in the ISIS synchrotron from 70 MeV to 800 MeV with dual harmonic RF system.

Dual harmonic (h=2/4) injection, trapping and acceleration in ISIS. 150 turns of 70-800 MeV beam

C. Prior, ICANS 1996

Available Codes I

IMPACT	Rob Ryne, Ji Qiang (LBL); mainly a linac code with new MaryLie developments for modelling rings
TRACEWIN	Nicolas Pichoff (CEA); Windows linac code
ORBIT	Jeff Holme, Sarah Cousineau (SNS); ring code developed for SNS; exists in different versions at ORNL, FNAL, BNL
SIMPSONS	Shinji Machida (RAL); used for J-PARC modelling
ACCSIM	Fred Jones (TRIUMF); developed from matrix applications with space-charge kicks
TRACKxD	Chris Prior (RAL); $x = 1, 2, 3$; used for ISIS, ESS, SNS, HIDIF, Neutrino Factory and other modelling
GPT	Bas van der Geer (Pulsar Physics); Commercial code developed initially for high intensity, very short electron bunches.

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Codes II

OPAL	Andreas Adelmann (PSI); major advances; fully 3D; developed mainly for cyclotron studies
BEST	Hong Qin (PPPL); based on δf-method. Used for investigating two-stream instabilities.
Micromap	Ingo Hofmann, Giuliano Franchetti (GSI); used for modelling FAIR and for comparison with earlier theories.
WARP	David Grote, Alex Friedmann (LLNL); fully 3D, mainly for early stages of acceleration, from ion gun.
PATH	Alessandra Lombardi (CERN); originally written to model the muon beam for the CERN neutrino factory, later developed for protons.
VADOR	Eric Sonnendrucker (Strasbourg); Vlasov solver
PARMILA	Jim Billen (LANL); long established linac code; works in association with Trace3D; also PARMELA.

Goals

From: ACCSIM, Fred Jones (TRIUMF)

Plus:

- Full treatment of field maps
- Off-axis beams
- Full suite of routines for beam diagnostics
- Higher order effects
- Secondary particle effects
  - electron cloud
  - decays (muons)

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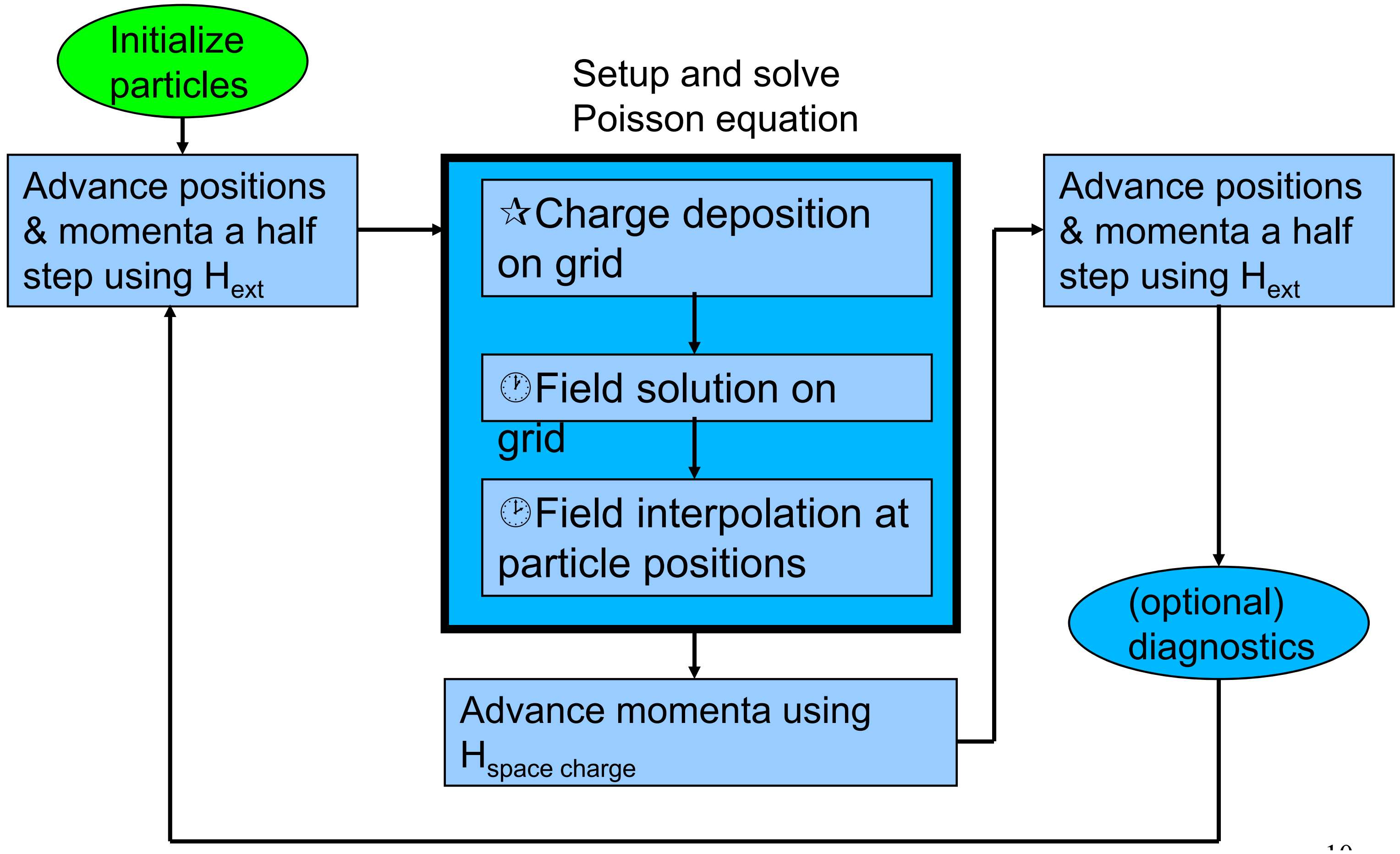
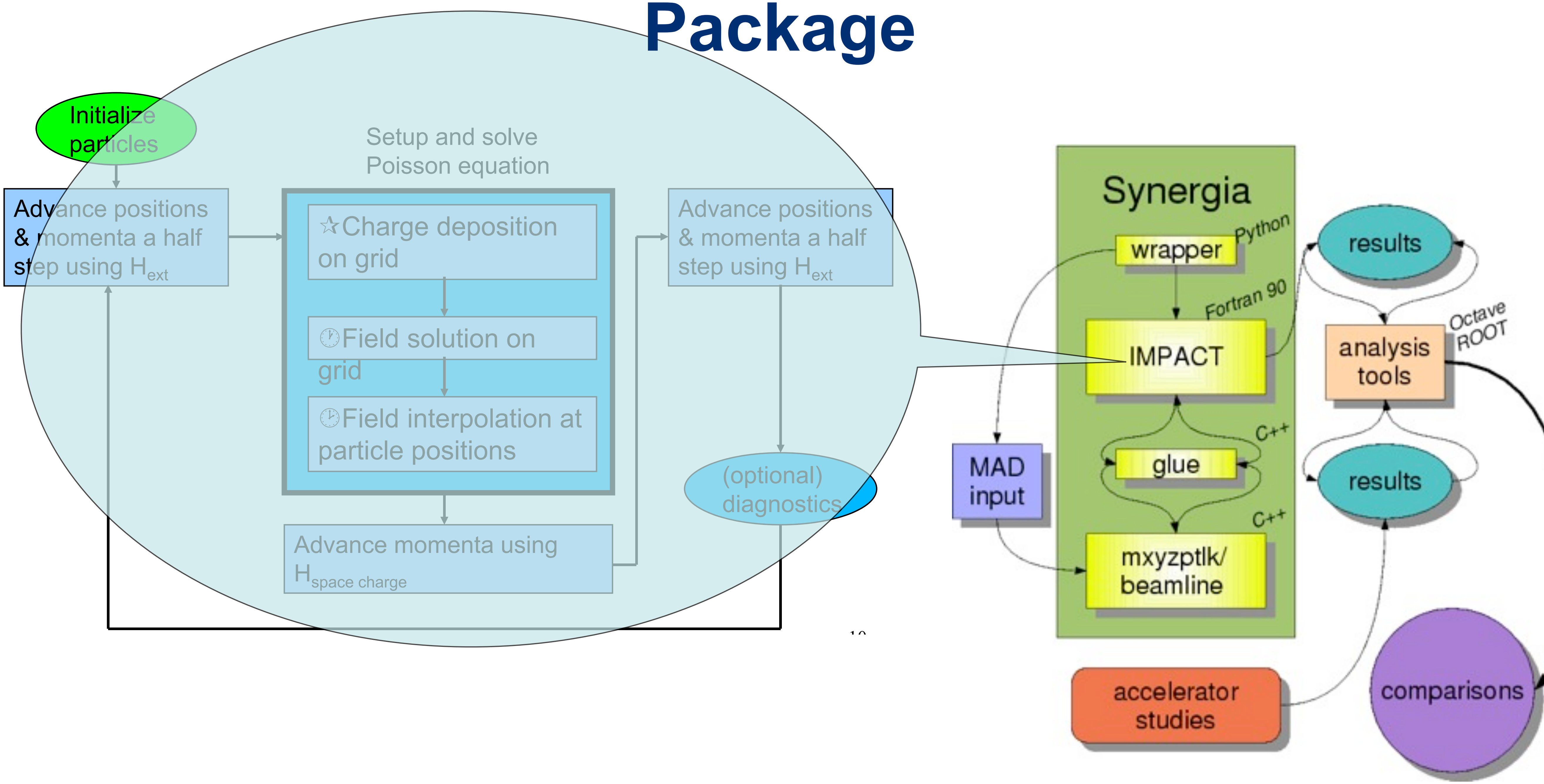
Components of a typical Beam Modelling Package



Components of a typical Beam Modelling Package



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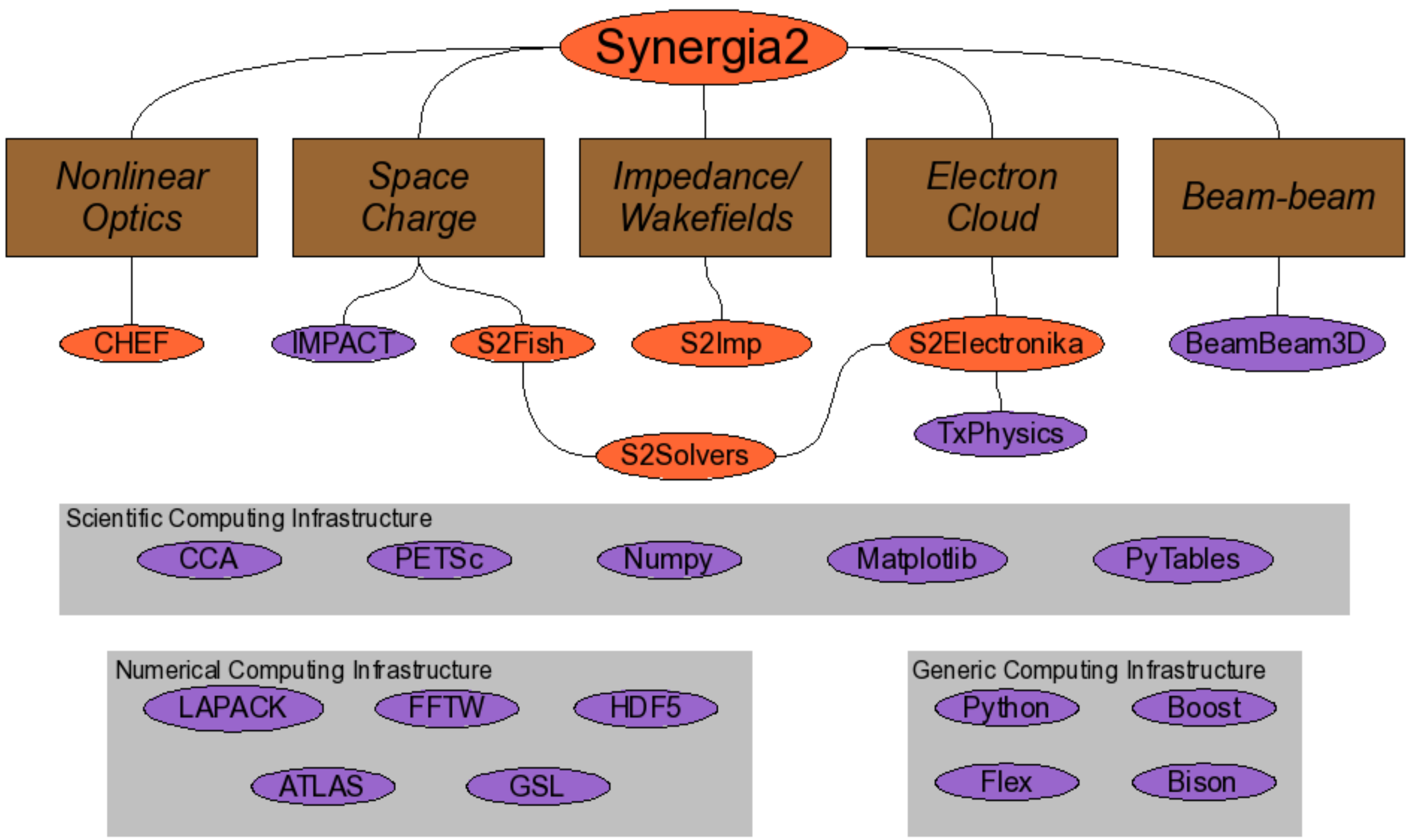
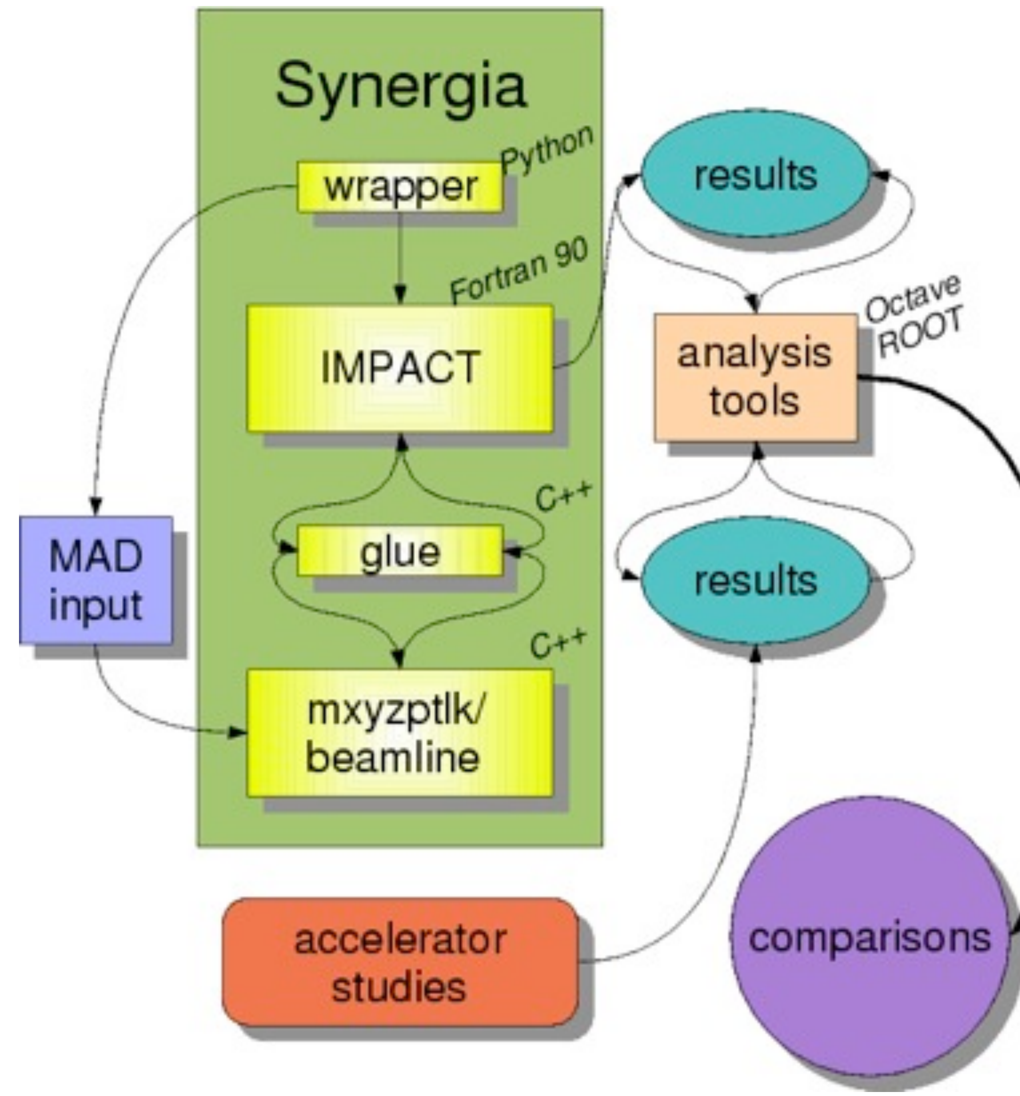
Components of a typical Beam Modelling Package

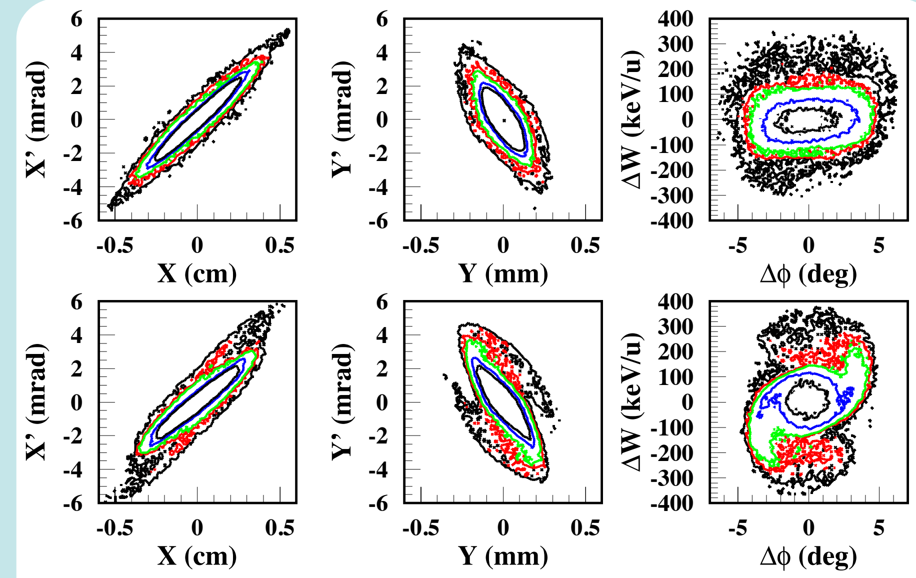




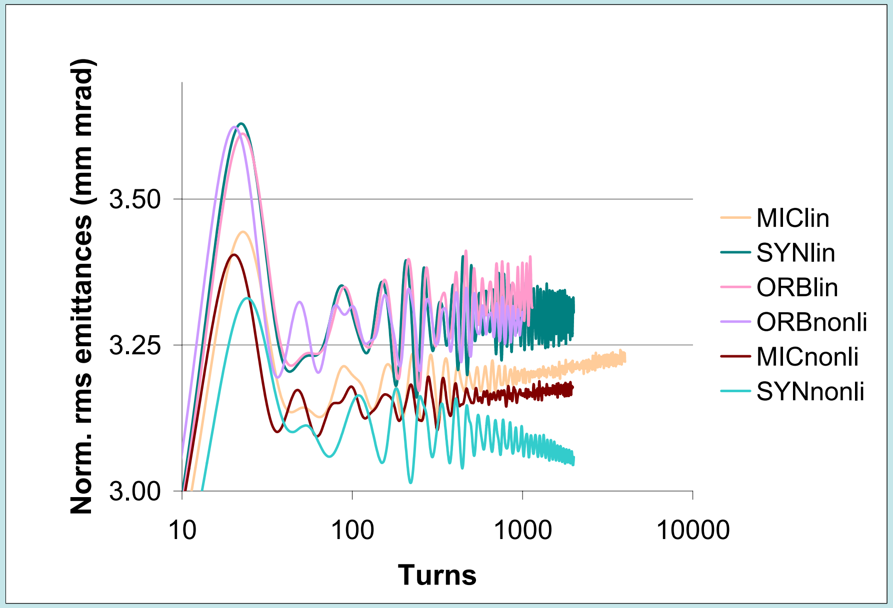
Synergia: Fermilab code, P. Spentzouris

How accurate are the codes?

- All codes should have a set of basic tests, preferably with known analytical solutions
- Benchmarking should cover
  - code v. code
  - code v. experiment
- Recent examples
  - Montague resonance tests with CERN PS, $2Q_h$-$2Q_v$=0 (ACCSIM, SYNERGIA, MICROMAP, SIMPSONS, IMPACT, ORBIT, SIMBAD)
  - HIPPI linac injector comparison
  - Electron cloud studies (PEHT, PEHTS, QUICKPIC, HEADTAIL)
  - Study of Hofmann resonances at KEK (IMPACT, TraceWin)

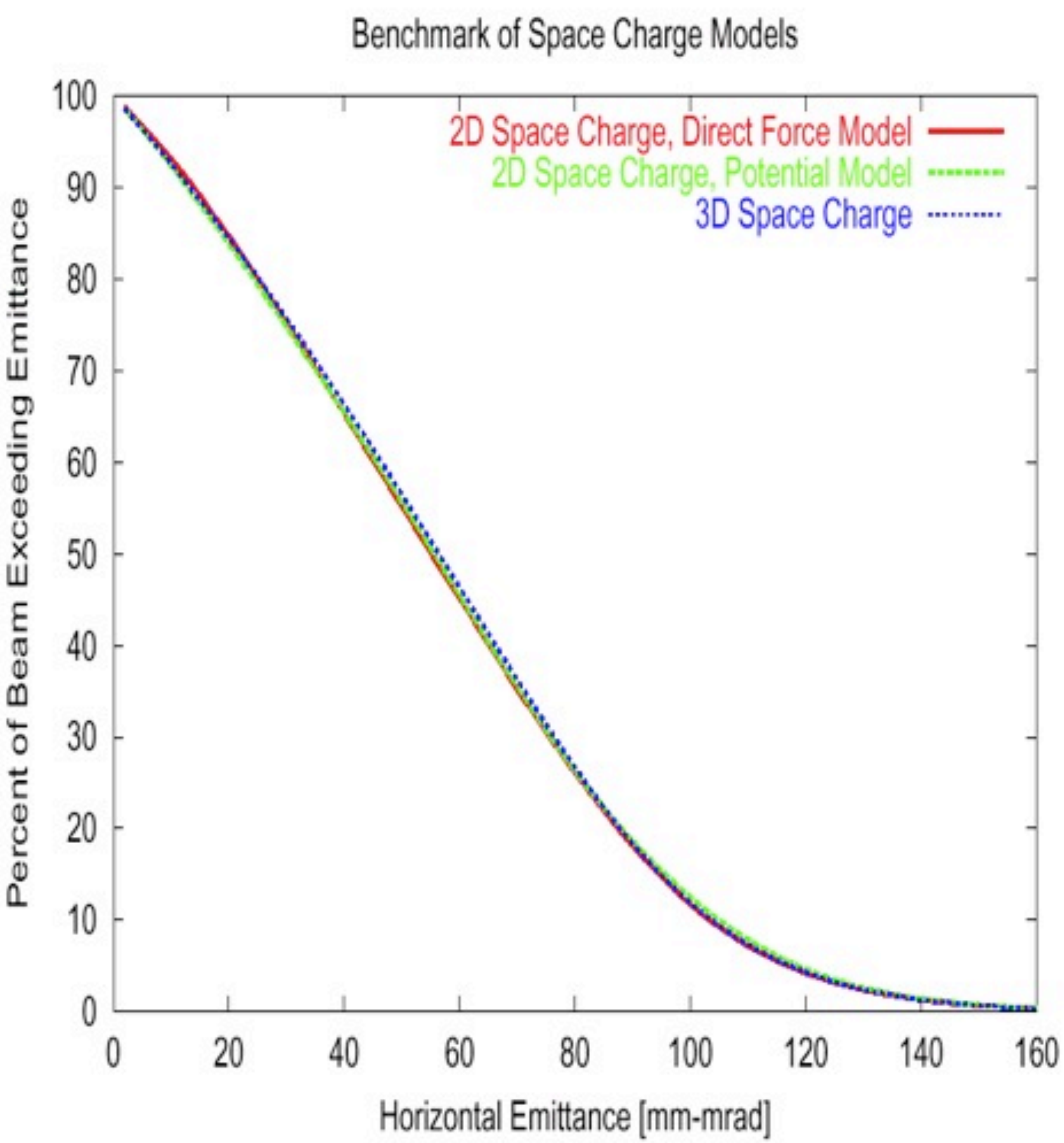


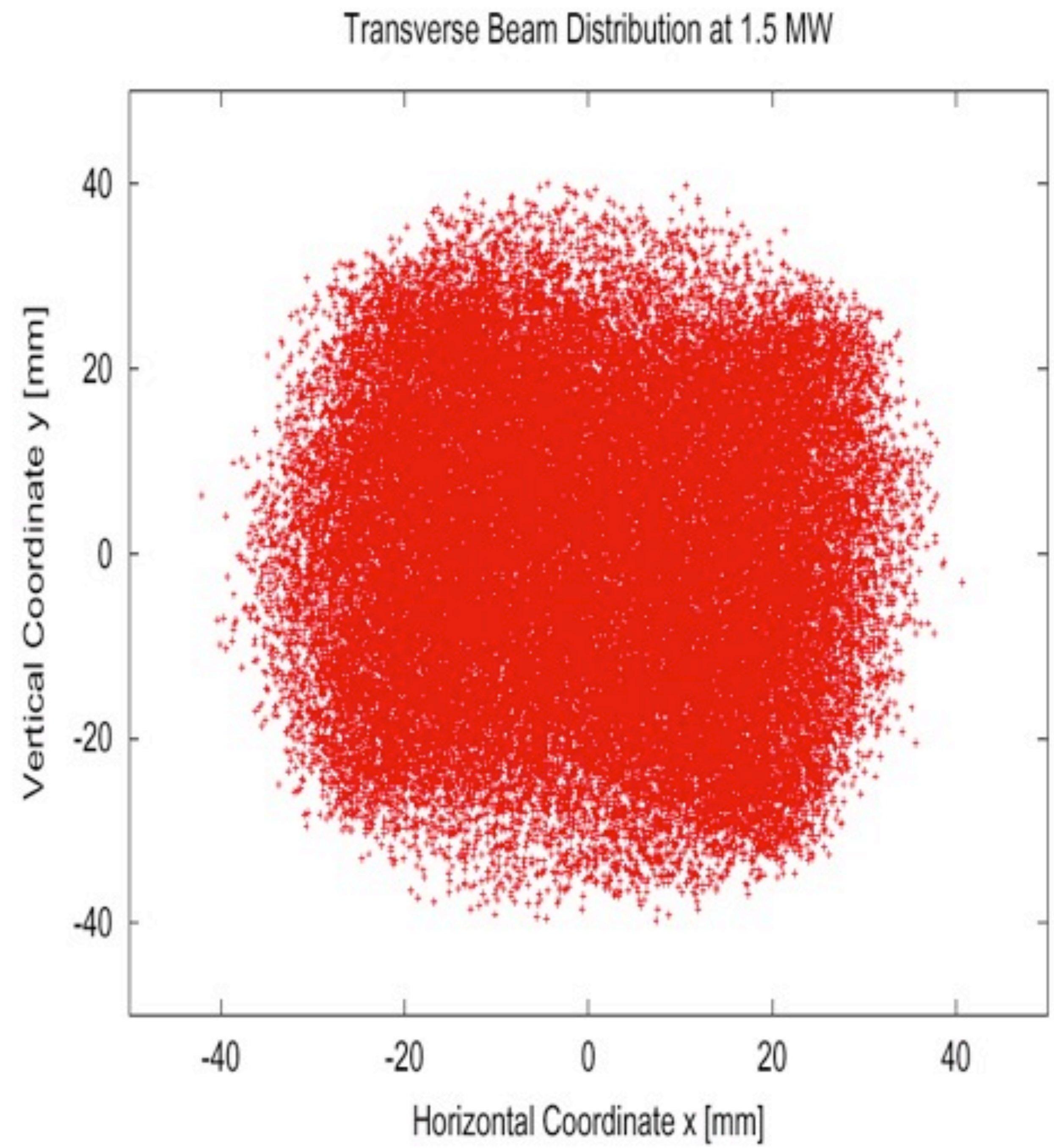
Longitudinal phase space: comparison between simulations using PARMILA and TRACK of SNS-DTL, 38 mA H- beam, 10^5 particles. Discrepancies caused by different fringe field models and space-charge routines.



See ICFA Beam Dynamics Newsletter 41, December 2006

ORBIT Application: SNS 1.44 MW Injection Space Charge Benchmark and Final Distribution





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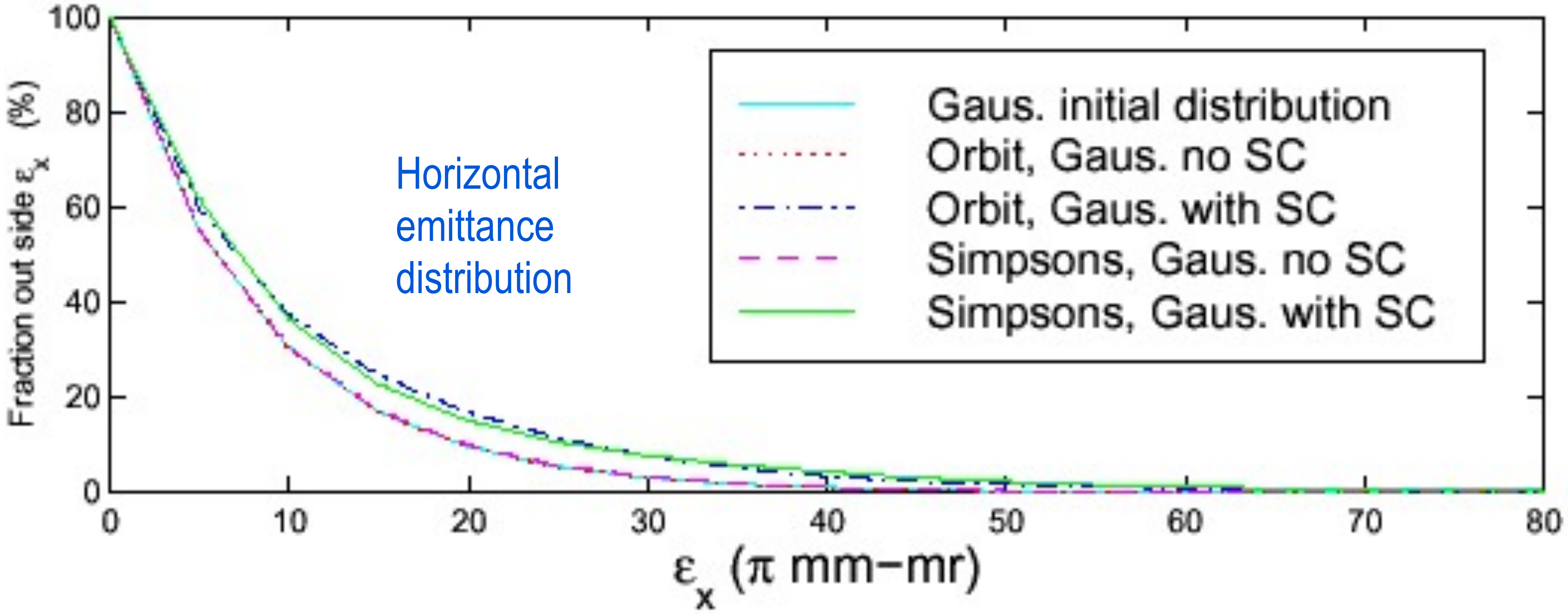
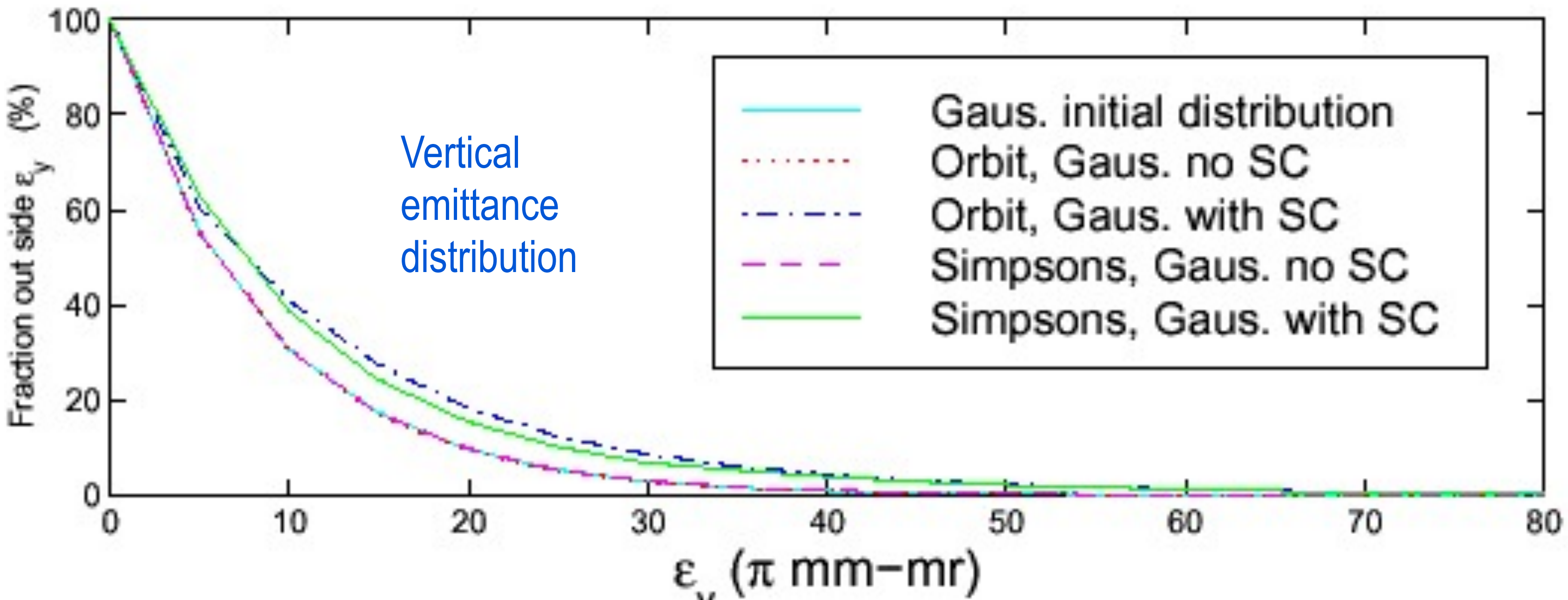
Code v. Code

- 10^5 macro particles
- peak current100A
- corresponding to 2 MW proton accumulation

SIMBAD/ORBIT (FFT) VS. SIMPSONS

Gaussian distribution, original SNS FODO lattice





Beam Kinetic Energy	1 GeV
Beam Average Power	1.0-2.0 MW
Proton Revolution Period	0.8413 μsec
Ring Circumference	220.688 m
Number of Turns Injected	1225
Beam Emittance $\varepsilon_{x,y}$	120 πmm-mr
Tunes ν_x / ν_y	1 1/290
Max. β_x / max. β_y	19.2 / 19.2 m
Dispersion X_p (max/min)	4.1 / 0.0 m

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Scope of Existing Codes with Space Charge

- Results/predictions
  - Beam profile measurements: CERN-PSB, KEK-PS, PSR
  - Injection losses (e.g. mismatch, emittance transfer)
  - Coherent resonances -- intensity limitation
  - Benchmarks with Accsim, Orbit, Simpsons, show long-term (50k turns) RMS matching to high degree of precision
- Study
  - Beam redistribution while preserving RMS matching
  - Intrinsic resonance due to space charge (sensitive to working point, observed independently in most codes)
  - Synchro-betatron effects -- space charge, chromaticity
  - Halo parameters and other amplitude measures
  - Stationary distributions (not widely used?)
  - Still a need for more accessible benchmarks and test cases

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Vlasov Solvers

- High intensity beams are usually modeled by the Vlasov equation.
- The distribution function $f(x, v, t)$ is given by

$$\frac{\partial f}{\partial t} + v \cdot \nabla_x f + \frac{q}{m}(E + v \times B) \cdot \nabla_v f = 0,$$

generally coupled with the Poisson or Maxwell equations.

- Numerical simulations are mostly performed using PIC method.

- Important noise in PIC methods especially in poorly populated regions of phase space makes it hard to see phenomena like e.g.
  - particle trapping (strong Landau damping) in plasmas
  - halo formation in beams
- Computers now powerful enough to do realistic physics using a grid in phase space.
- Provides alternative to PIC for code benchmarking.

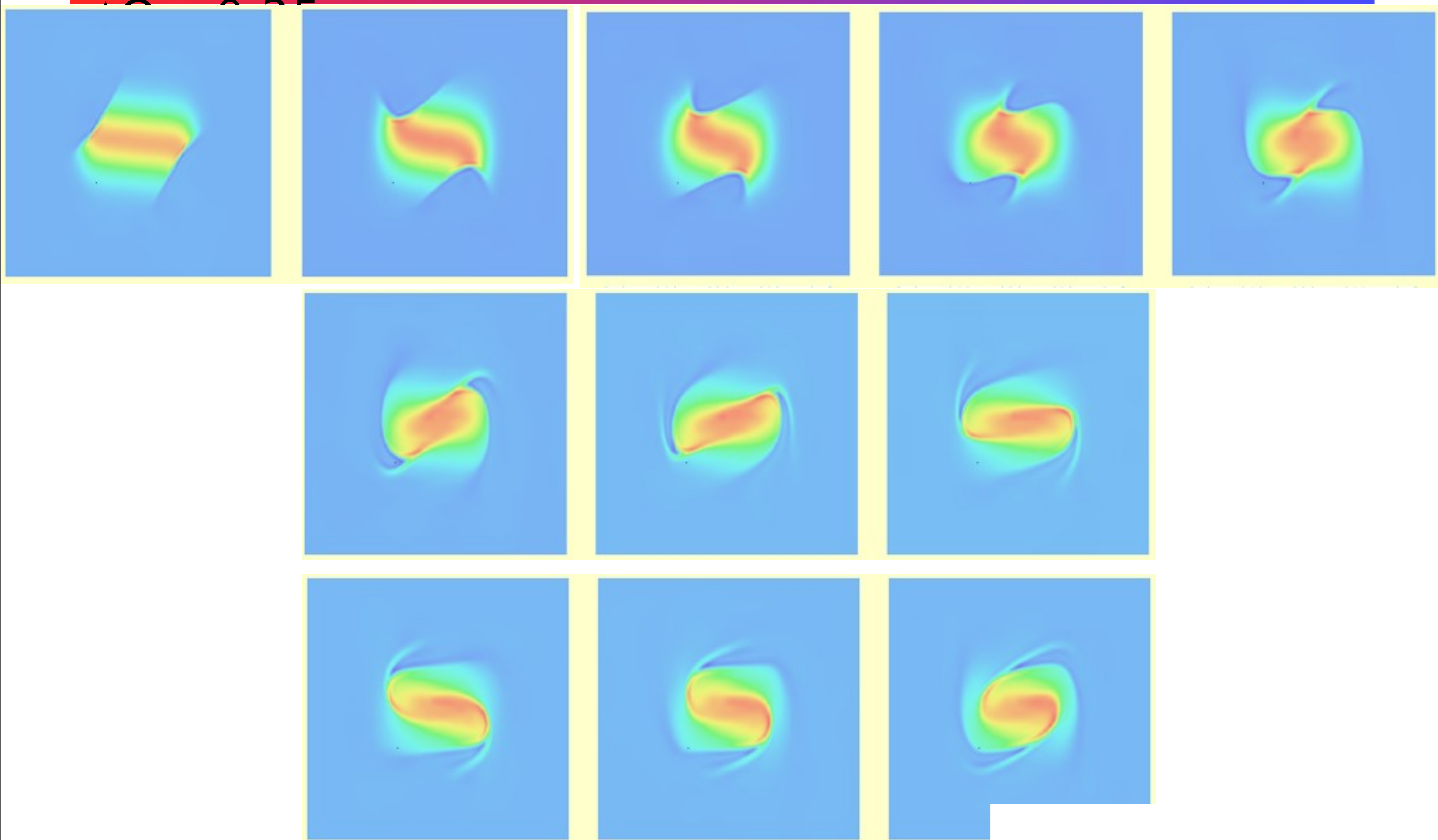
Difficult in >2D phase space because of very large number of grid points. Hence very slow.

Use symmetry or conserved quantities on characteristics

Optimise number of grid points for specific simulations

Proposal to develop 4D-->6D Vlasov solver with Jonathan Smith (Tech-X)and Hartree Centre, Daresbury

Example: Evolution of a Semi-Gaussian beam of 80keV Potassium ions in a uniform focussing channel,



Results from VADOR code, E. Sonnendrucker, Strasbourg

Spreadsheet of Space-Charge Codes I

Code	Language	Platform	GUI	Parallel	1D/2D/3D	Particles	linacs/rings
IMPACT	F90	Unix/Linux	no	MPI	3D	$> 10^6$	linacs
ML-IMPACT	F90	Unix/Linux/Mac	no	MPI	3D	$> 10^6$	linacs/rings
PARMILA	F90	Windows	no	no	2D/3D	10^4-10^5	linacs/ transfer lines
GPT	C, C++	Windows	yes	MPI scans	3D	10^6	linacs/FEL/ transfer lines
BEST	F90	Unix/Linux	python/IDL	MPI/ OpenMP	3D	$> 10^6$	linacs/rings
VADOR	C++	Unix/Linux	no	MPI	2D	n/a	linacs
SPUNCH	F77	Linux	no		1D	10^4	LEBT
PATH	F90	Windows	yes	no	3D	10^5	linacs/rings
TRACEWIN	C++	Windows	yes	no	2D/3D	10^5	linacs
DYNAC	F77	Linux/Unix/ Windows	no	no	2D/3D	10^5	linacs
Synergia	F90/C++/ Python	Unix	no	MPI	3D	$> 10^6$	linacs/rings
WARP	Python/ F77/F90/C	Linux/Unix/ Windows/Mac	Under dev	MPI	3D/rz/xy	up to 10^8	linacs/rings

Spreadsheet of Space-Charge Codes II

Code	Space Charge Solver	Boundaries/Images	Impedances	Field Maps	Integration order
IMPACT	spectral	open/periodic/ rectangular/circular	no	yes	2nd order in z
ML-IMPACT	spectral	elliptical/ polygon/lossy	yes	no	2nd in z 5th Runge-Kutta
PARMILA					
GPT	3D multigrid	open conductive rect. pipe, cathode	no	2D,3D	5th Runge-Kutta
BEST	spectral, FD	circular conducting wall	automatic/ external	no	user specified
VADOR	FFT	conductive wall any shape	no	no	2nd
SPUNCH	exact for disc-shaped particles	circular conducting wall	n/	n/a	1st
PATH	Schell, pt-to-pt	open	no	yes	?
TRACEWIN	Scheff/PICNIC/Gaussup	open	no	no	?
DYNAC	Scheff/Scherm/Hersc	open	no	yes	3rd analytical
Synergia	spectral (IMPACT)	open/periodic/ rectangular/circular	no	yes	2nd order in z
WARP	FFT, Cap matrix, multigrid, adaptive mesh, refined MG	square/round pipe, internal conductors, bent pipe, general	*ad hoc*	no	2nd order

Spreadsheet of Space-Charge Codes III

Code	t or s tracking	Graphics	Portability	Source code available?	Manual	Standard test cases
IMPACT	s	post proc. with MPI	all unix platforms	to collaborators	partial	yes
ML-IMPACT	s	post proc.		to collaborators	partial	yes
GPT	t	built in	portable except user interface	all beamline components	yes	yes
BEST	t	netcdf, IDL	any Linux	yes	no	yes
VADOR	s	GNUplot, openDX	any Linux	yes	almost	yes
SPUNCH	s	built in	fully portable	yes	no	yes
PATH	s	built in	any Windows	yes	yes	?
TRACEWIN	s	built in	any Windows	no	yes	?
DYNAC	t, s	GNUplot	fully portable	yes	yes	yes
Synergia	s	post proc. Root+Openinventor	all Linux	yes	in progress	not yet
WARP	t, s	PyGist 2D OpenDX 3D	portable	yes	online	yes

Final Message

SAVE EVERYTHING:

Even if you think the job is finished,
don't for one moment think you will
never need the data or the code again.