Simulation of High Intensity Beams

Ji Qiang

Lawrence Berkeley National Laboratory

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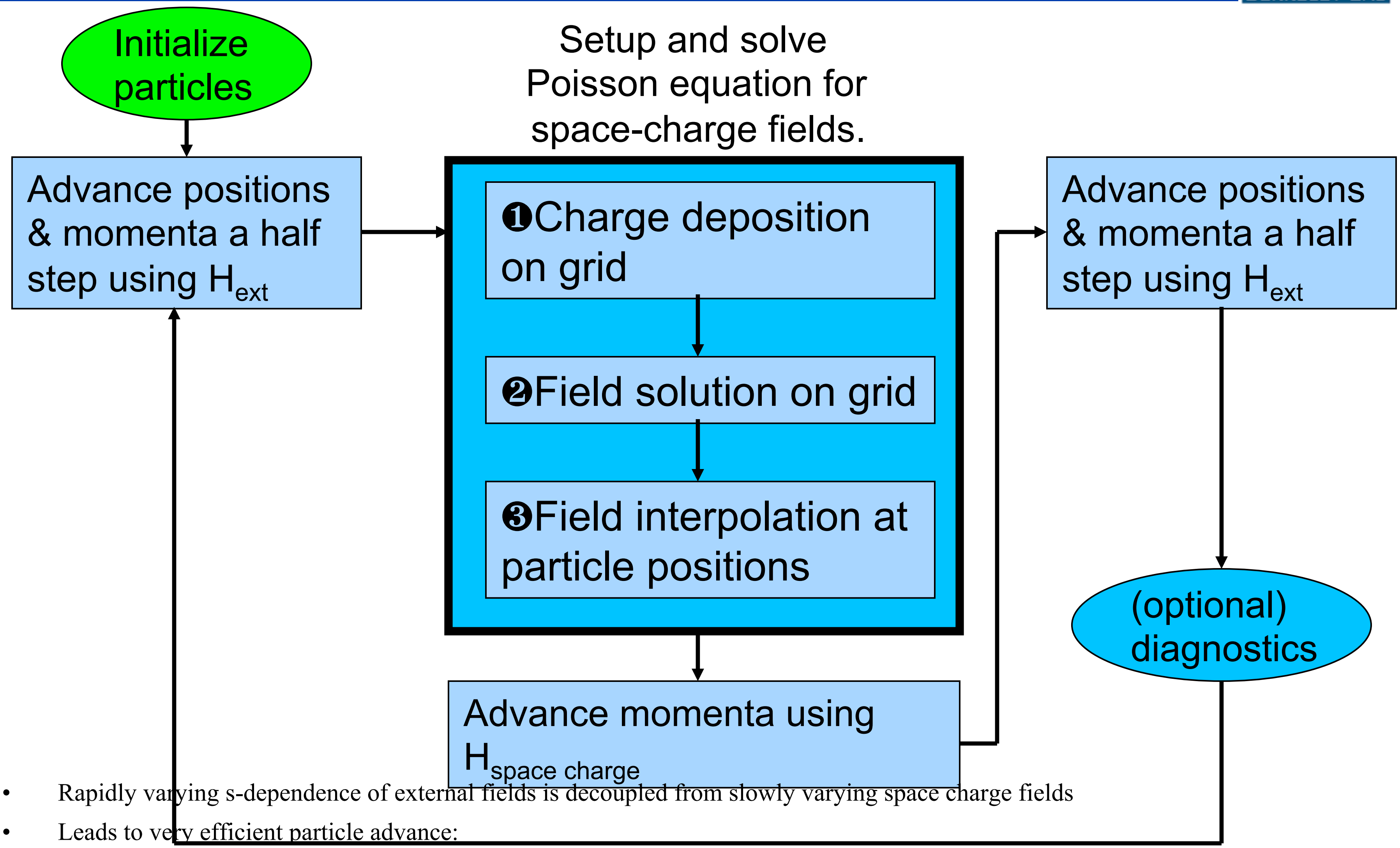
Outline

- Computational model
- Parallel implementation
- Simulation of space-charge driven coupling resonance at PS
- Simulation of space-charge effects at PS2
- Simulation of space-charge driven microbunching instability at an electron linac

Particle-In-Cell Simulation with Split-Operator Method



- Rapidly varying s-dependence of external fields is decoupled from slowly varying space charge fields
- Leads to very efficient particle advance:
  - Do not take tiny steps to push millions - billions of particles
  - Do take tiny steps to compute maps; then push particles w/ maps

Different Boundary/Beam Conditions Need Different Efficient Numerical Algorithms O(Nlog(N)) or O(N)

FFT based Green function method:

- Standard Green function: low aspect ratio beam
- Shifted Green function: separated particle and field domain
- Integrated Green function: large aspect ratio beam
- Non-uniform grid Green function: 2D radial non-uniform beam

Fully open boundary conditions

Spectral-finite difference method:

2D open boundary
Transverse regular pipe with longitudinal open

Multigrid spectral-finite difference method:

Transverse irregular pipe

J. Qiang, S. Paret, "Poisson solvers for self-consistent multiparticle simulations,"

Green Function Solution of Poisson's Equation (open boundary conditions)

$$\phi(r) = \int G(r,r')\rho(r')dr' \; ; \; r = (x, y, z)$$

$$\phi(r_i) = h\sum_{i'=1}^{N} G(r_i - r_{i'})\rho(r_{i'})$$

$$G(x,y,z) = 1/\sqrt{(x^2 + y^2 + z^2)}$$

Direct summation of the convolution scales as N^2 !!!!
N – ($N_xN_yN_z$) grid number in each dimension

Hockney's Algorithm /zero padding:- scales as (2N)log(2N)

- Ref: Hockney and Easwood, ***Computer Simulation using Particles***, McGraw-Hill Book Company, New York, 1985.

$$\phi_c(r_i) = h\sum_{i'=1}^{2N} G_c(r_i - r_{i'})\rho_c(r_{i'})$$

$$\phi(r_i) = \phi_c(r_i) \quad \text{for i} = 1, N$$

Integrated Green Function Solution of Poisson's Equation (large aspect ratio beam with open boundary conditions)

$$\phi_c(r_i) = \sum_{i'=1}^{2N} G_i(r_i - r_{i'})\rho_c(r_{i'})$$

$$G_i(r,r') = \oint G_s(r,r')\,dr'$$

integrated Green function

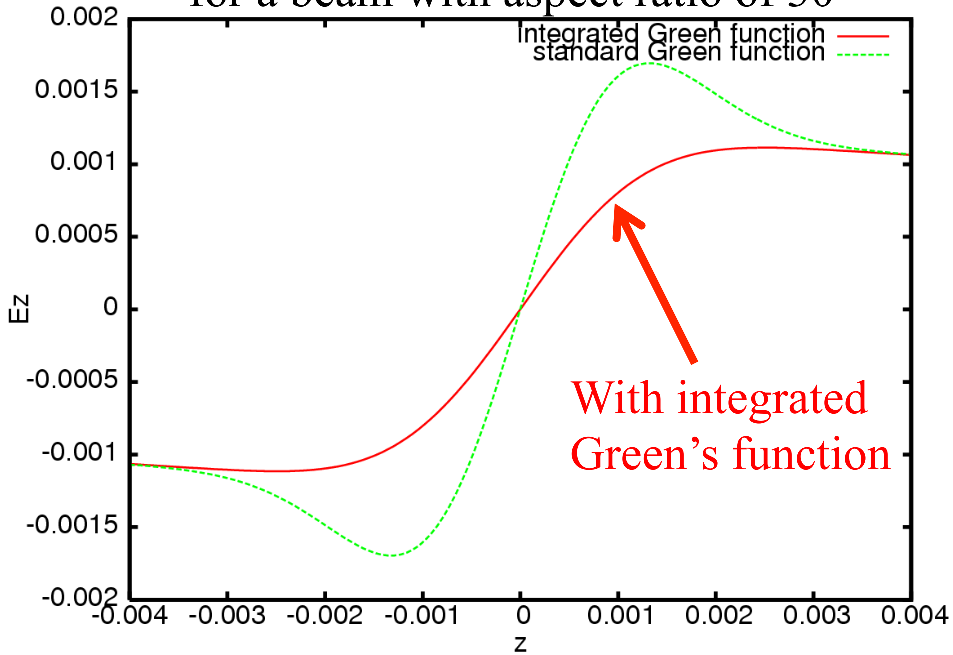
$$G_s(x,y,z) = 1/\sqrt{(x^2+y^2+z^2)}$$

standard Green function

Integrated Green's function is needed for modeling large aspect ratio beams!

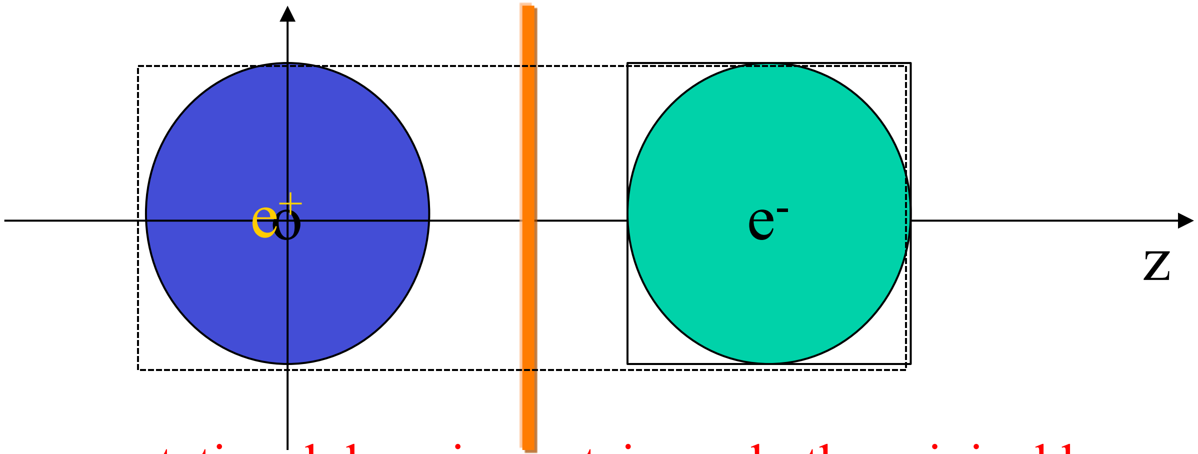
(O(N log N))

Comparison between the IG and SG for a beam with aspect ratio of 30



K. Ohmi, Phys. Rev. E 62 (2000) 7287.
R. D. Ryne, ICFA Beam DynamicsMini Workshop on Space Charge Simulation, Trinity College, Oxford, 2003
J. Qiang, et al., PRST-AB 9, 044204 (2006).
V. Ivanov, Int. J. Mod. Phys. A, 24, p. 869 (2009).

Efficient Shifted Green Function Method to Calculate Image Space-Charge Effects



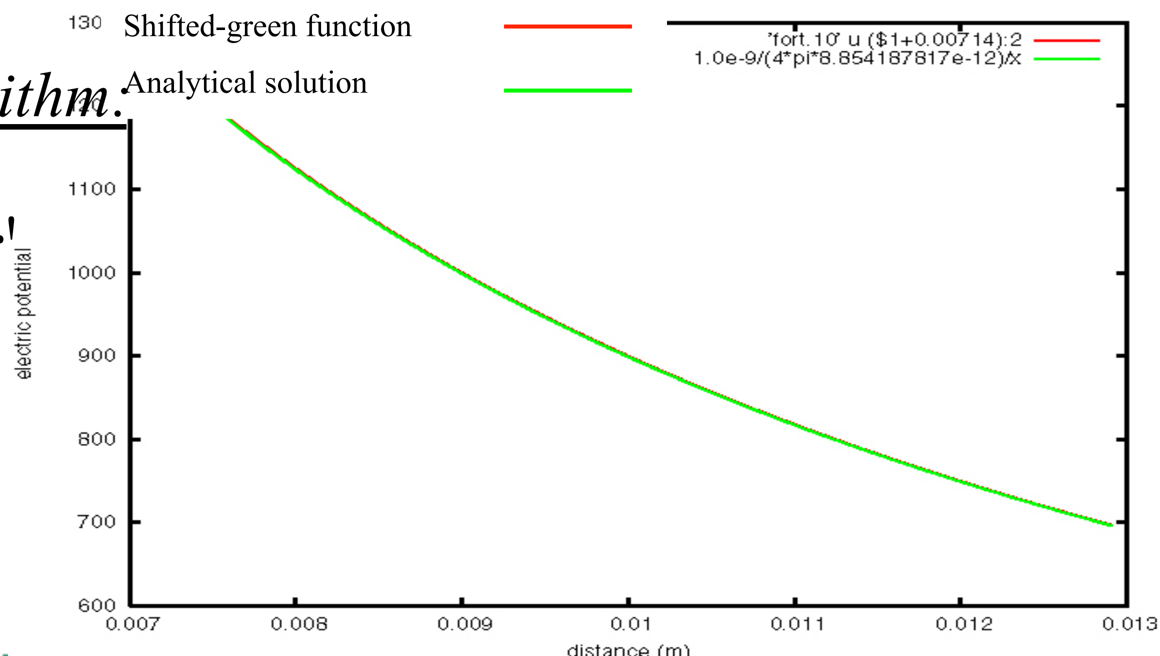
computational domain contains only the original beam

Shifted Green function Algorithm:

$$\phi_F(r) = \int G_s(r,r')\rho(r')dr'$$

$$G_s(r,r') = G(r + r_s, r')$$

(O(N logN))



J. Qiang, et al., PRST-AB 9, 044204 (2006).

3D Poisson Solver with Transverse Rectangular Pipe

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = -\frac{\rho}{\epsilon_0}$$

with boundary conditions

$$\begin{aligned}
\phi(x=0, y, z) &= 0, \\
\phi(x=a, y, z) &= 0, \\
\phi(x, y=0, z) &= 0, \\
\phi(x, y=b, z) &= 0, \\
\phi(x, y, z=\pm\infty) &= 0,
\end{aligned}$$

$$\rho(x,y,z) = \sum_{l=1}^{N_l} \sum_{m=1}^{N_m} \rho^{lm}(z) \sin(\alpha_l x) \sin(\beta_m y),$$

$$\phi(x,y,z) = \sum_{l=1}^{N_l} \sum_{m=1}^{N_m} \phi^{lm}(z) \sin(\alpha_l x) \sin(\beta_m y),$$

where

$$\rho^{lm}(z) = \frac{4}{ab} \int_0^a \int_0^b \rho(x,y,z) \sin(\alpha_l x) \sin(\beta_m y),$$

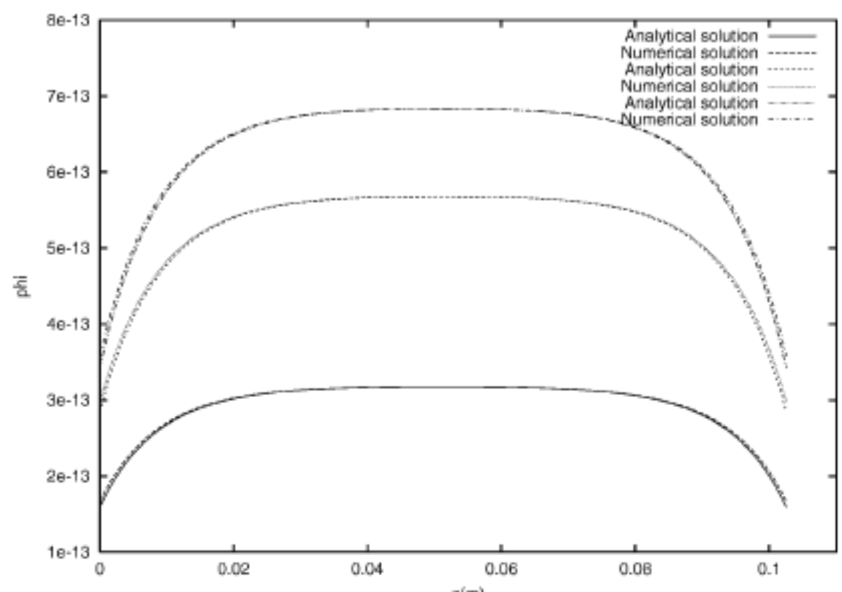
$$\phi^{lm}(z) = \frac{4}{ab} \int_0^a \int_0^b \phi(x,y,z) \sin(\alpha_l x) \sin(\beta_m y),$$

$$\frac{\partial^2 \phi^{lm}(z)}{\partial z^2} - \gamma_{lm}^2 \phi^{lm}(z) = -\frac{\rho^{lm}(z)}{\epsilon_0},$$

$$\frac{\phi_{n+1}^{lm} - 2\phi_n^{lm} + \phi_{n-1}^{lm}}{h_z^2} - \gamma_{lm}^2 \phi_n^{lm} = -\frac{\rho_n^{lm}}{\epsilon_0},$$

$$\begin{aligned}
\phi_{-1}^{lm} &= \exp(-\gamma_{lm} h_z)\phi_0^{lm}, \qquad n = 0, \\
\phi_{N+1}^{lm} &= \exp(-\gamma_{lm} h_z)\phi_N^{lm}, \qquad n = N.
\end{aligned}$$

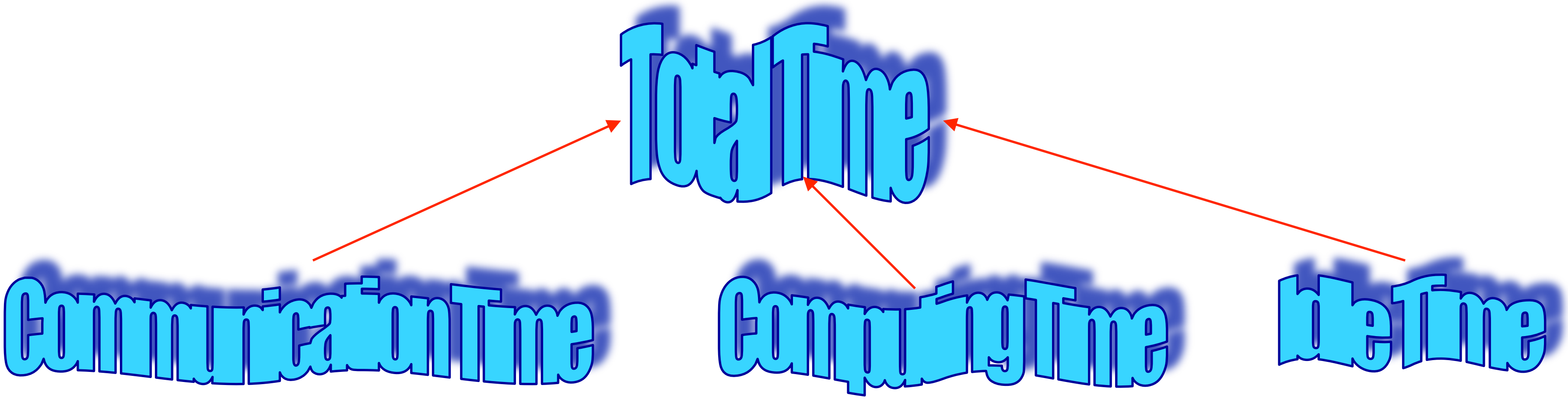
Numerical Solutions vs. Analytical Solutions



z (m)

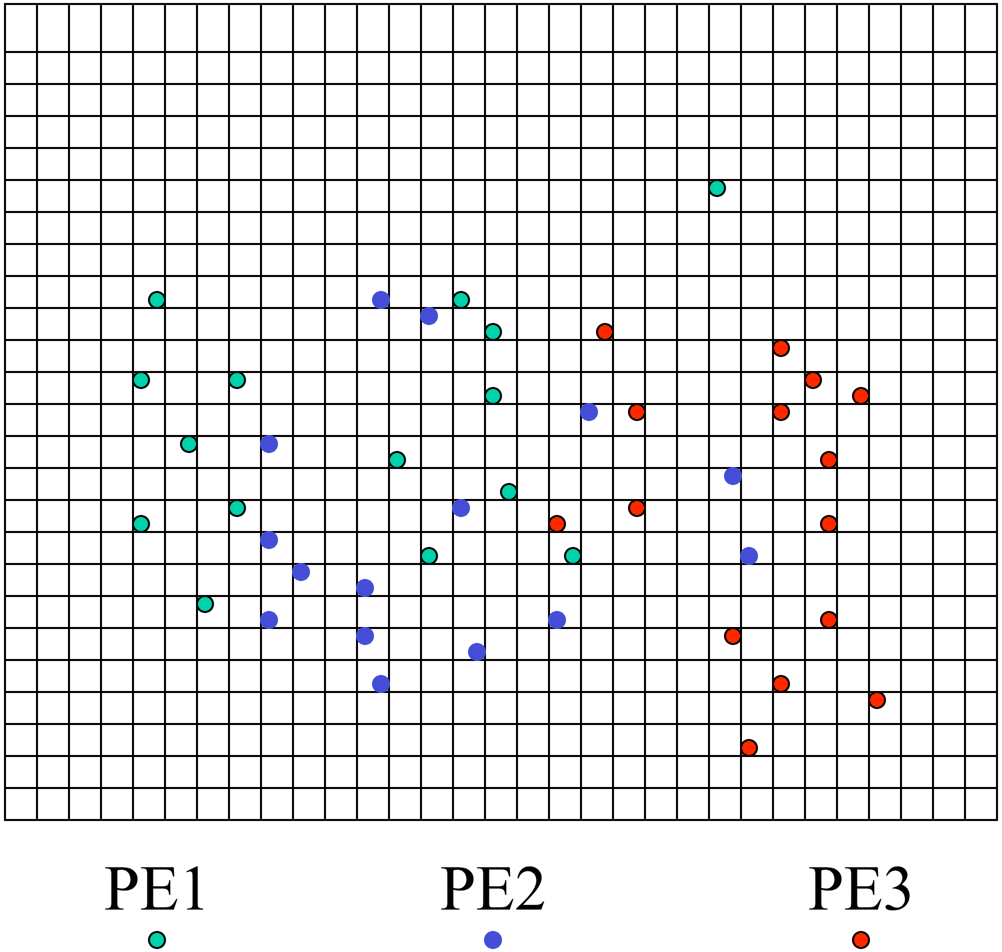
J. Qiang, and R. Ryne, Comput. Phys. Comm. 138, p. 18 (2001).

Parallel Implementation

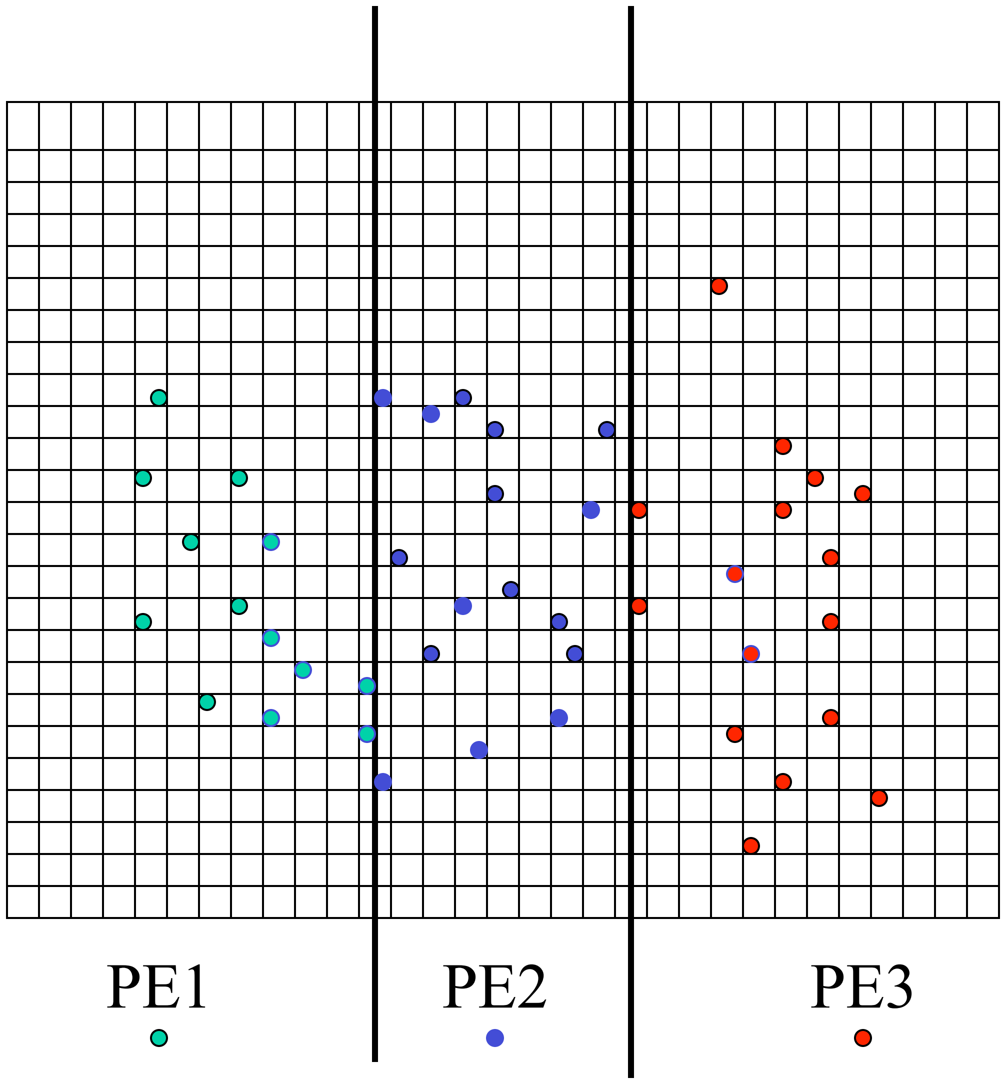


- Parallel Implementation of PIC Simulation
  - Particle decomposition
  - Domain decomposition
  - Particle-field decomposition

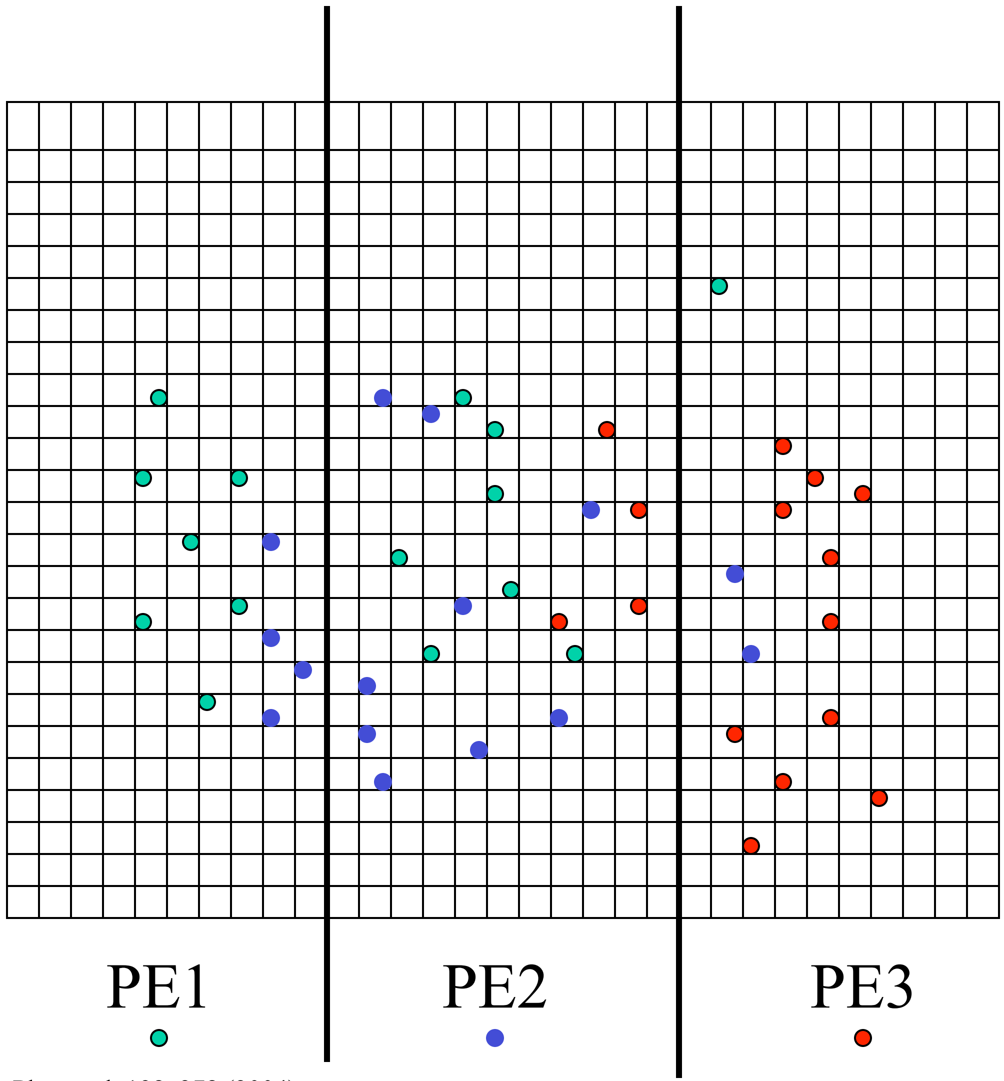
Particle Decomposition



Domain Decomposition



Particle-Field Decomposition



J. Qiang, M. Furman, and R. Ryne, J. Comp. Phys. vol. 198, 278 (2004).

Scalability with large number of macroparticles (500) per cell

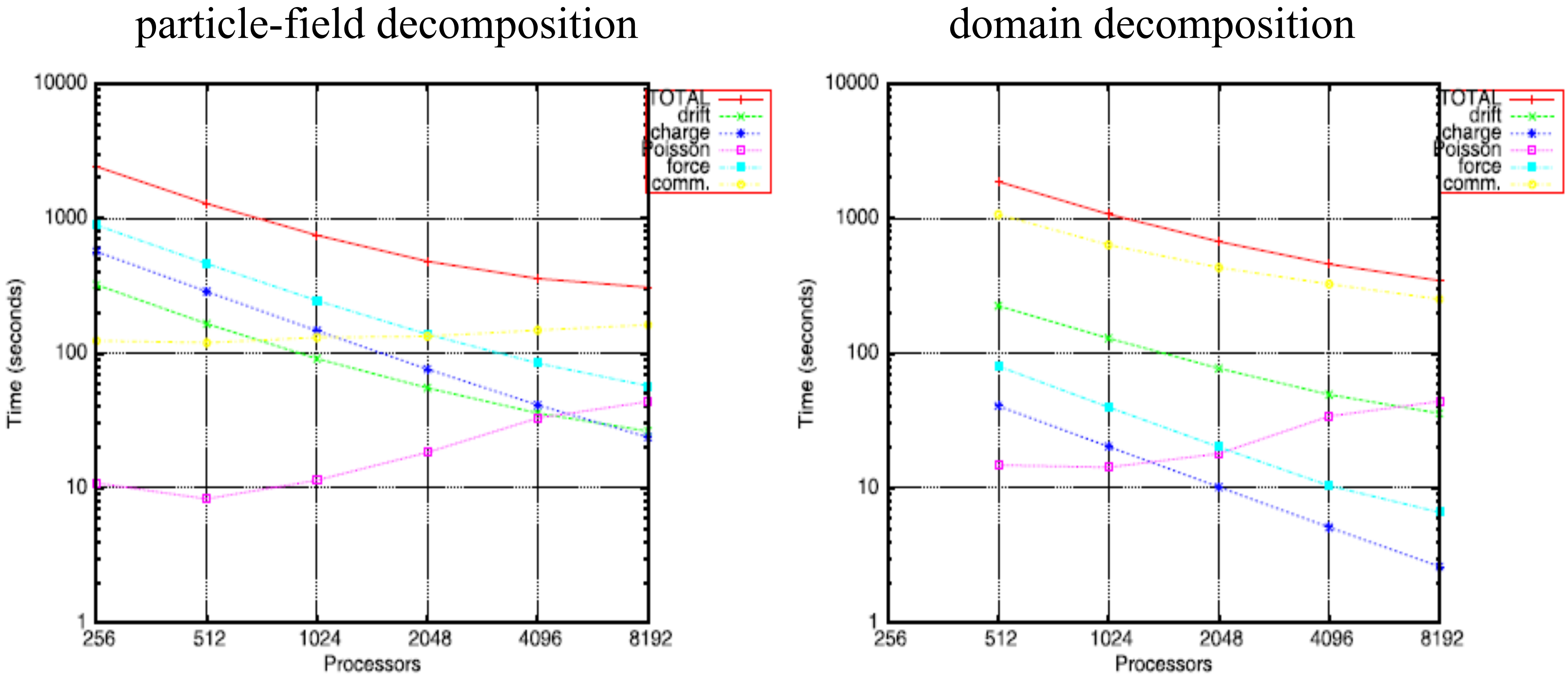


Fig. 5. Computing time as a function of number of processors on Cray XT4 using the particle-field decomposition scheme (left) and the domain decomposition scheme (right) in the strong scaling test A with 32 × 32 × 2048 grid points and 1 billion particles.

J. Qiang and X. Li, Comput. Phys. Comm., 181, 2024, (2010).

Scalability with small number of macroparticles (5) per cell

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particle-field decomposition domain decomposition

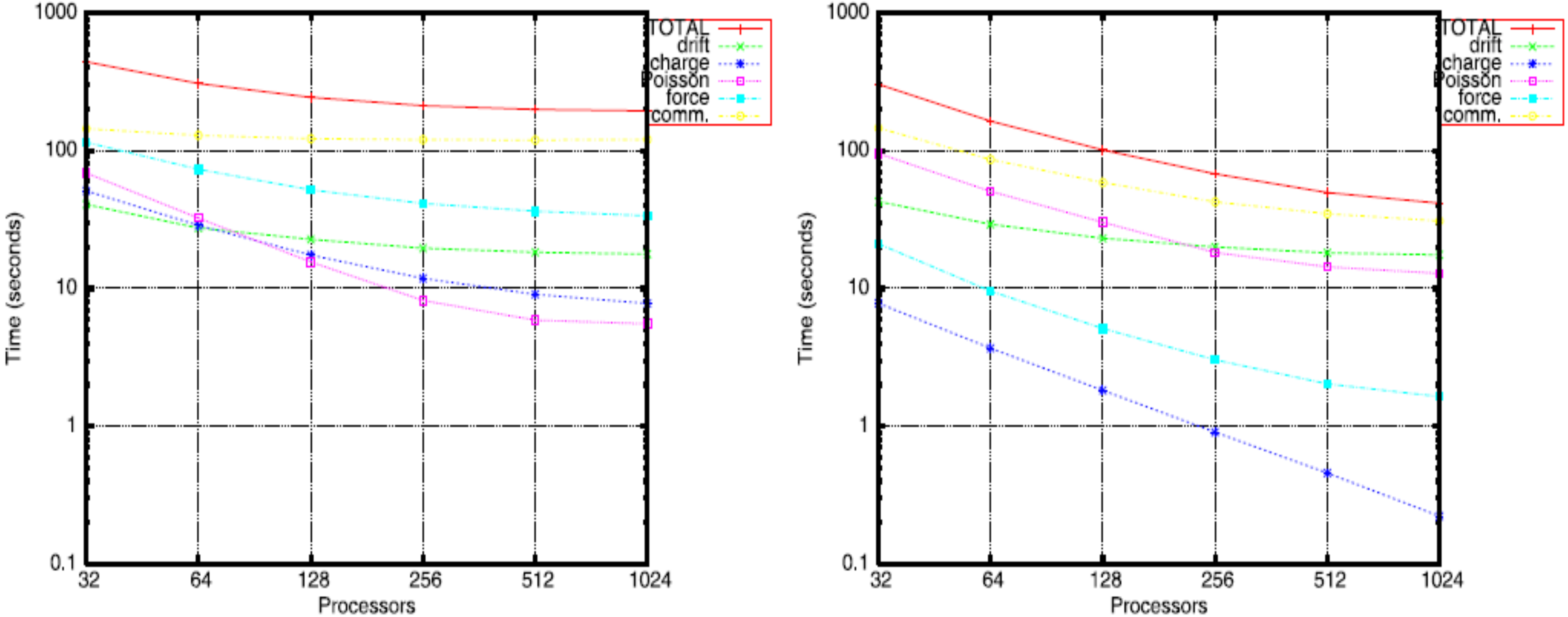


Fig. 6. Computing time as a function of number of processors on Cray XT4 using the particle-field decomposition scheme (left) and the domain decomposition scheme (right) in the strong scaling test B with 128 × 128 × 128 grid points and 10 million particles.

IMPACT Code Suite

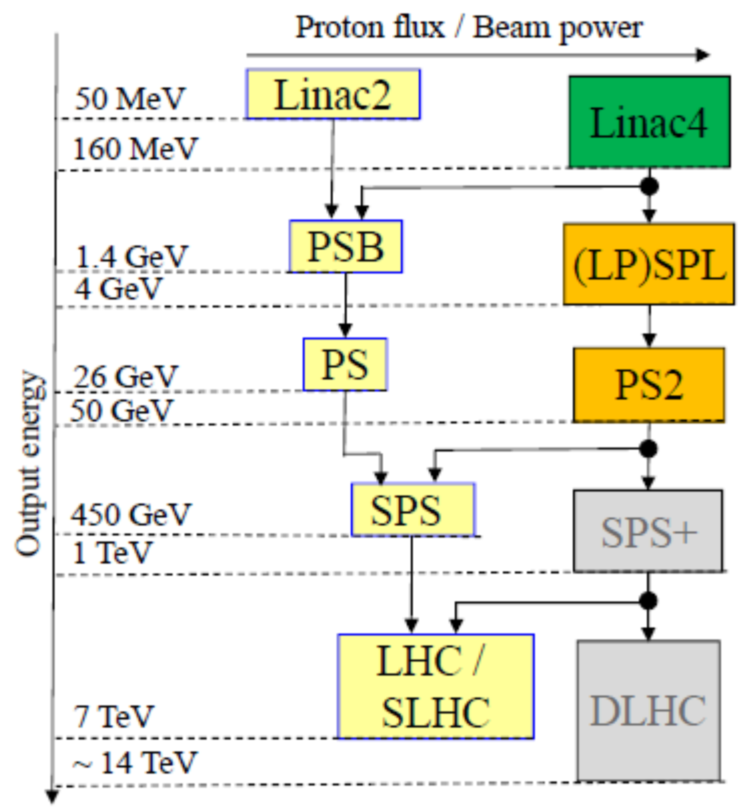
- IMPACT-Z: parallel PIC code (z-code)
- IMPACT-T: parallel PIC code (t-code)
- Envelope code, pre- and post-processors,…
- Key Features
  - Detailed RF accelerating and focusing model, beam line elements of linac and ring
  - Multiple 3D Poisson solvers
    - Variety of boundary conditions: transverse round/ rectangular, longitudinal open/periodic, …
    - 3D shifted-integrated Green Function
  - Multi-charge states, multi-bunches
  - Machine error studies and steering
  - Wakefields
  - CSR/ISR
  - Gas ionization
  - Run on both serial and multiple processor computers

Space-Charge Driven Coupling Resonance at PS

- Proton-Synchrotron (PS) is amonc injectors the oldest, and will continu the LHC at least for the next 25 yea
- Space-charge effects is a dominar limiting the bunch intensity.
- Montague Resonance:

  $2\ Qx - 2\ Qy = 0$
- can cause particle due to

  unequal aperture size in horizontal and vertical dimensions.
- benchmark space-charge codes

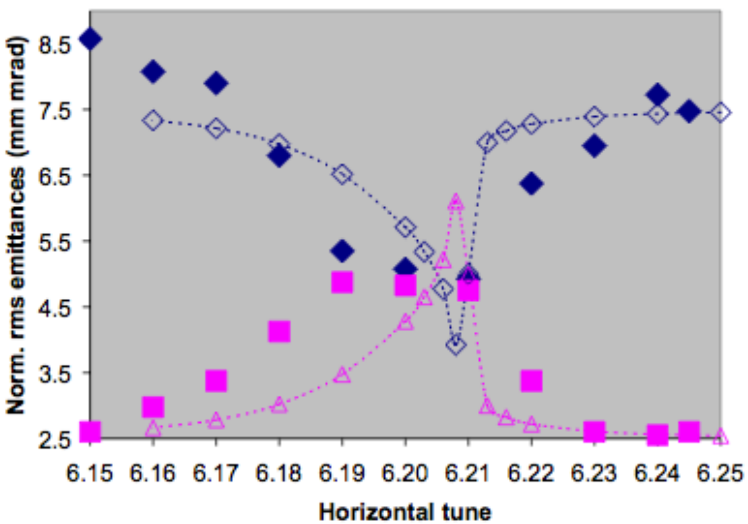


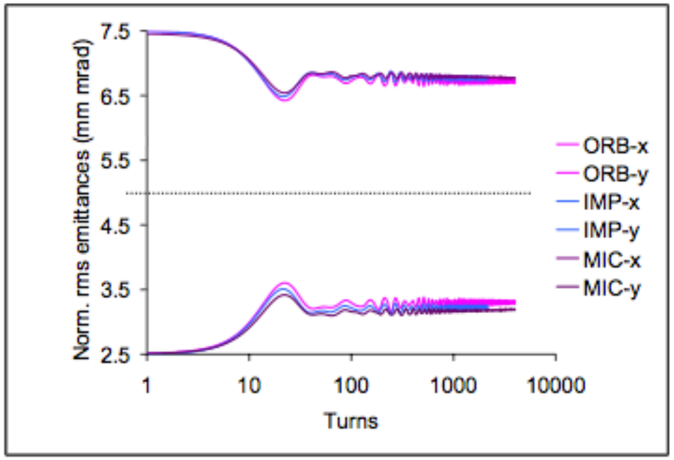
Refs: B. W. Montague, CERN-Report No. 68-38, CERN, 1968.
E. Metral et al., Proc. of EPAC 2004, p. 1894.
I. Hofmann et al., Proc. of EPAC 2004, p. 1960.

Space-Charge Driven Coupling Resonance at PS

“For this purpose increasing levels of complexity have been planned with simulations, first in 2D approximation and up to 2000 turns:
- step (1) in constant focusing approximation; ✓
- step (2) using a linearized version of the AG lattice; ✓
- step (3) using the fully nonlinear lattice of the PS [7]); ✓
- step (4) the 21/2 D or 3D bunched beam simulation including all lattice effects;
- step (5) extension up to the full 13,000 turns of the measurements provided that necessary CPU times – pre- sumably of the order of months – are not prohibitive.

At a later point, after suitable code optimization, the even more ambitious dynamical crossing may be addressed, preferably after new measurements are carried out over less than the demanding 44.000 turns of the 2003 experiment.” - I. Hofmann et al., Proceedings of 2005 PAC.





Physical Parameters of PS

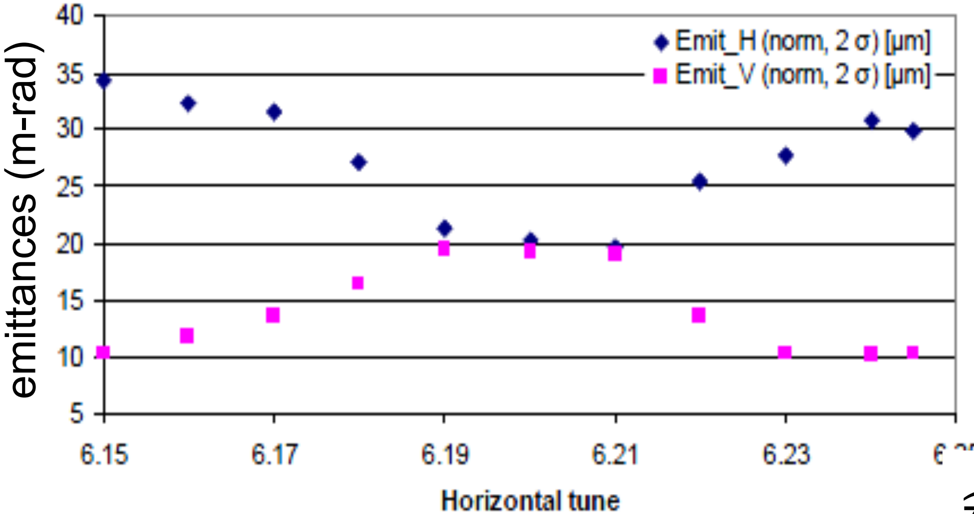
Physical parameters:

RF frequency = 3.5 MHz
RF voltage = 27 kV
Ek = 1.4 GeV
Emit_x = 7.5 mm-mrad
Emit_y = 2.5 mm-mrad
Rms bunch length = 45 ns
Rms dp/p = 1.7×10^{-3}

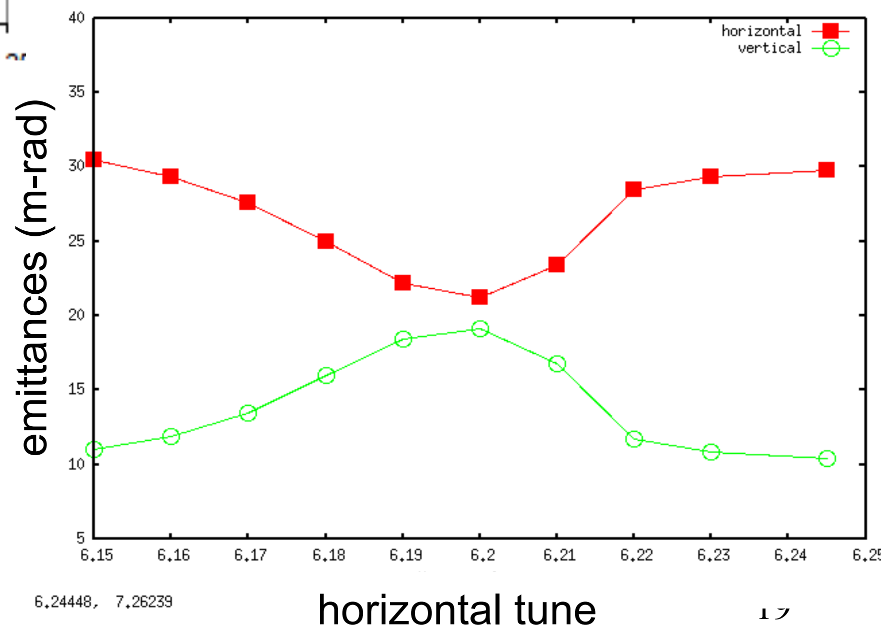
Horizontal tune: 6.15 – 6.245
Vertical tune: 6.21
Synchrotron period: 1.5 ms
Half Aperture = 7cm x 3.5cm
I = 1.0×10^{12}

Static Montague Resonance Crossing at PS

measurements

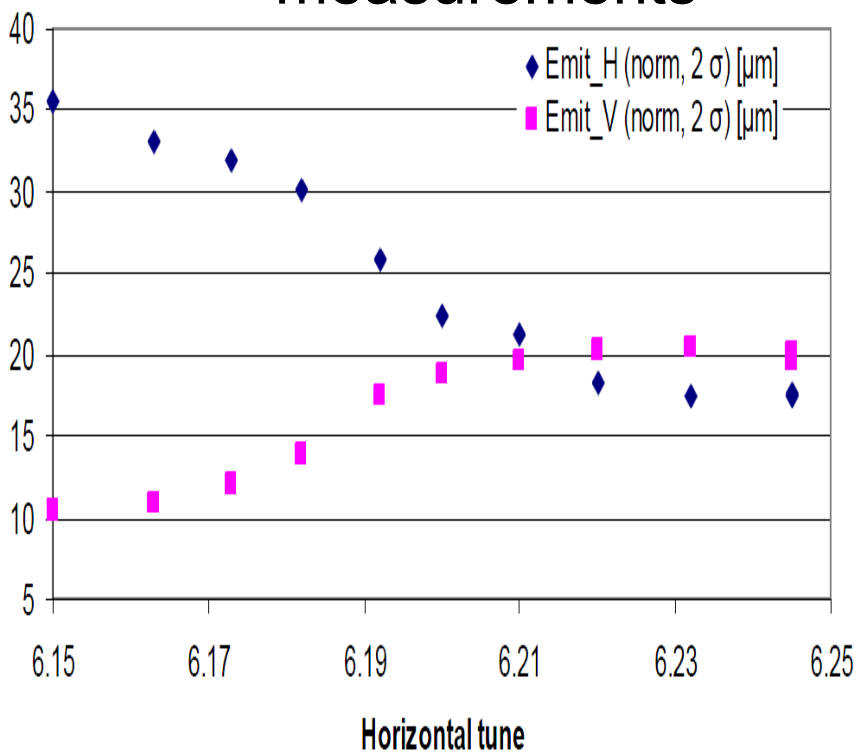


IMPACT simulation

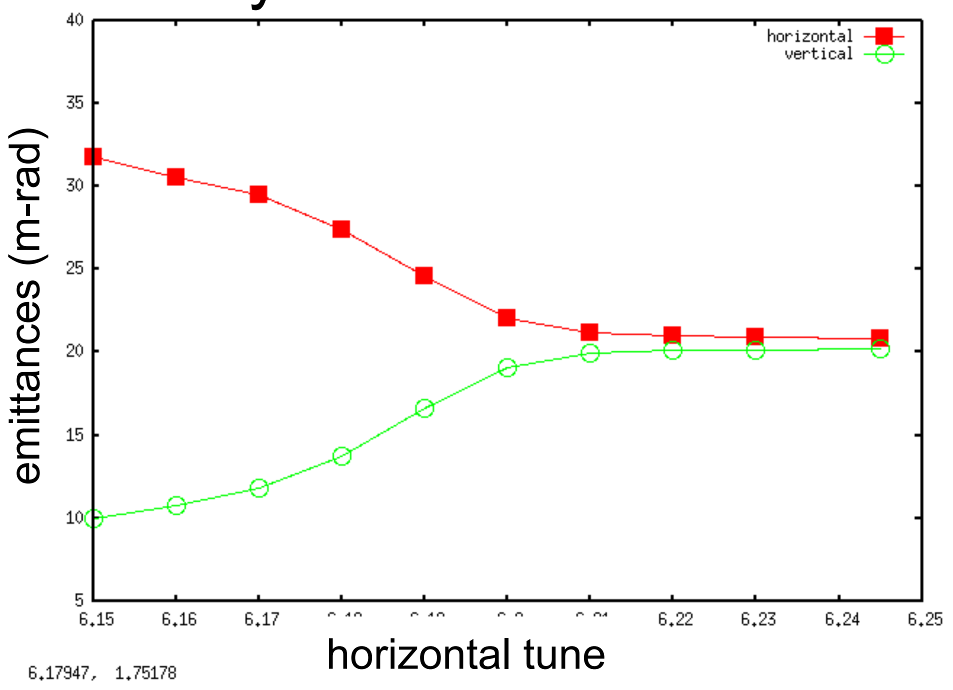


100 ms dynamic Crossing

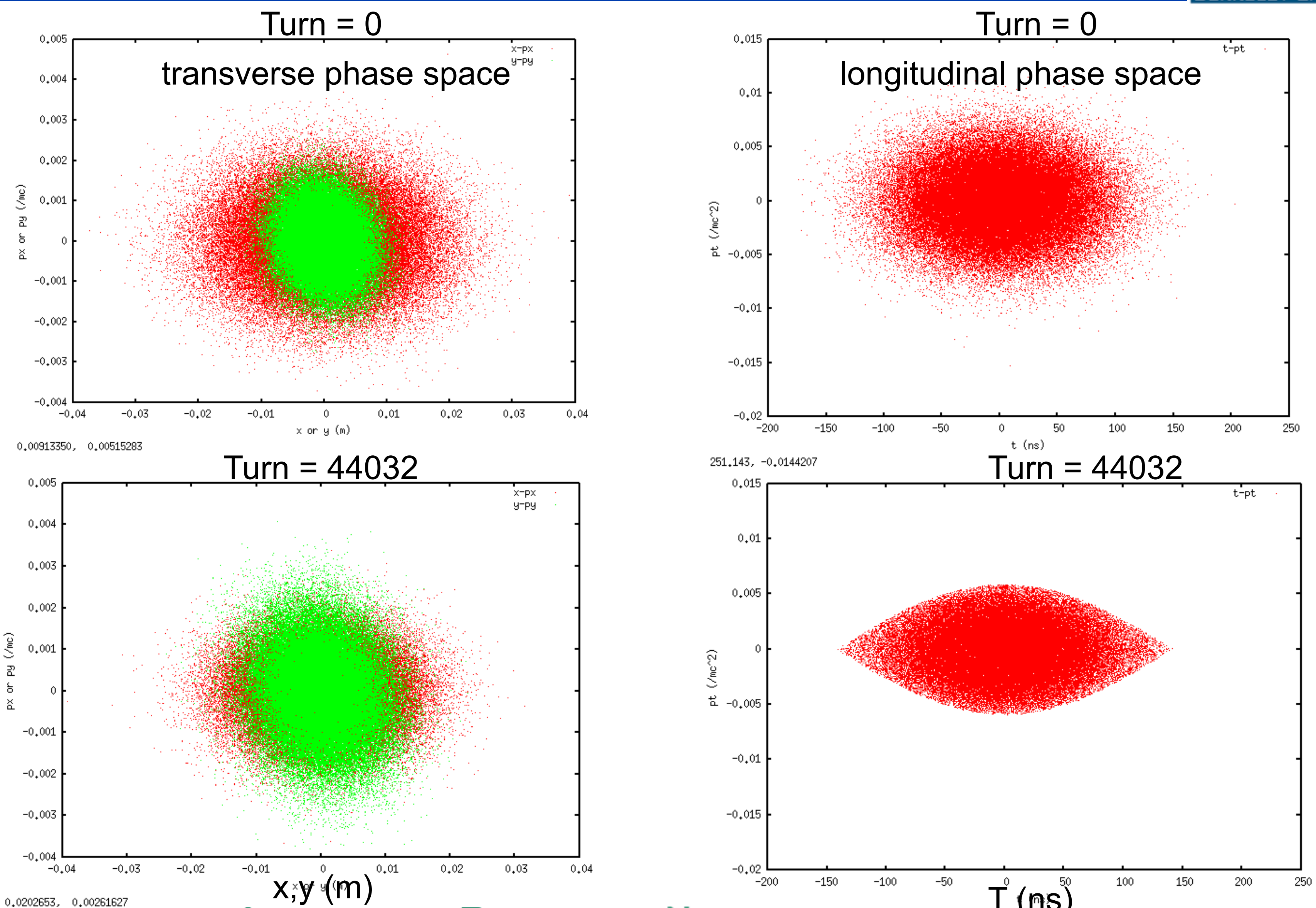
measurements



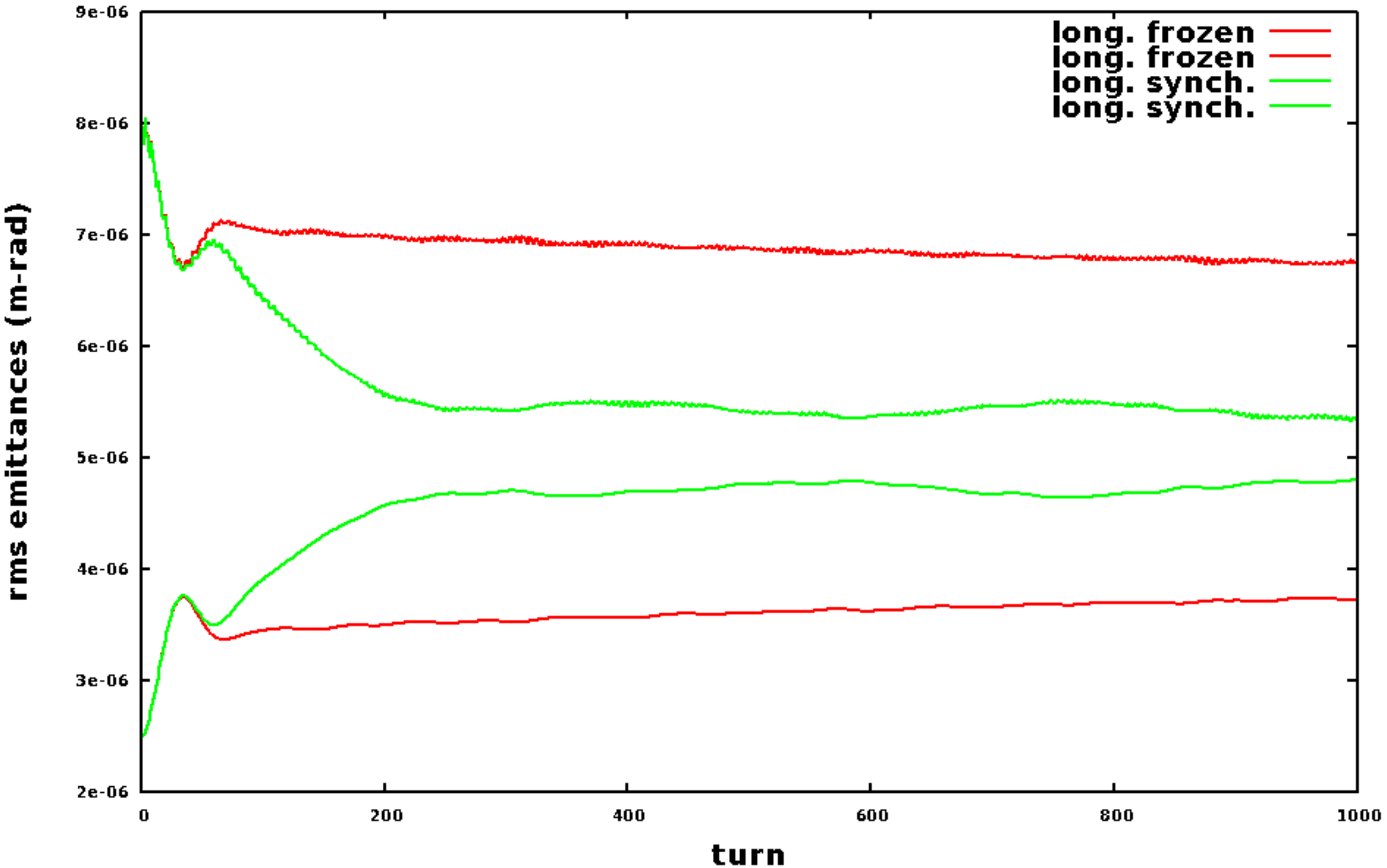
IMPACT simulation:
fully 3D+nonlinear lattice



Initial and the Final Phase Space Distribution of the Dynamic Resonance Crossing



Emittance Evolution w/o Longitudinal Synchrotron Motion (6.197,6.21)



Synchrotron motion enhances the emittance exchange !

Space-Charge Simulation of a Proposed Synchrotron: PS2

➢PS2 was proposed for LHC upgrade with higher injection energy (4 GeV) to mitigate the space-charge effects to reach higher number of protons per bunch (4×10^{11}).

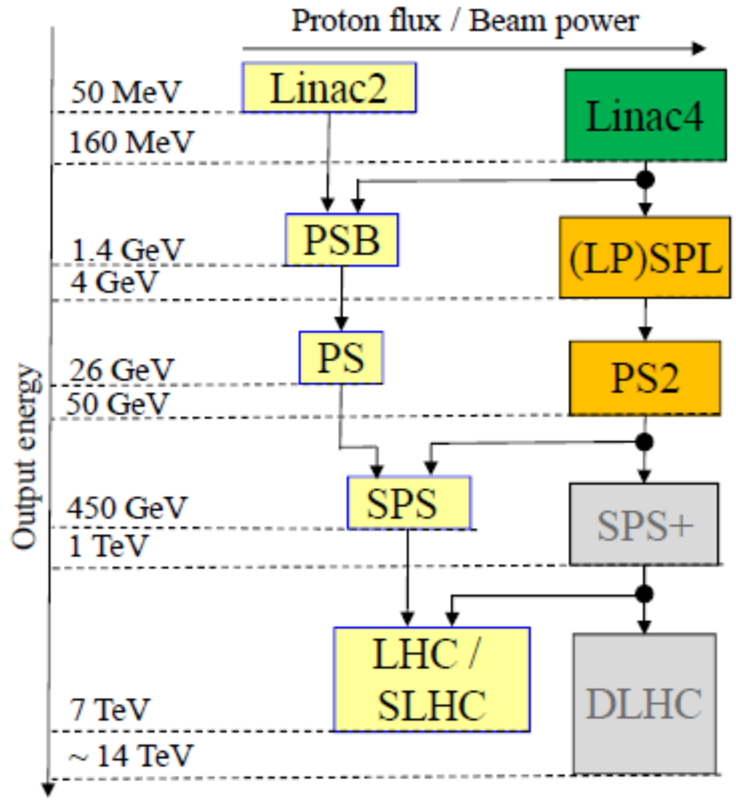


Figure 1: Overview on the CERN injector complex upgrade programme: stage 1 (green), stage 2 (orange).

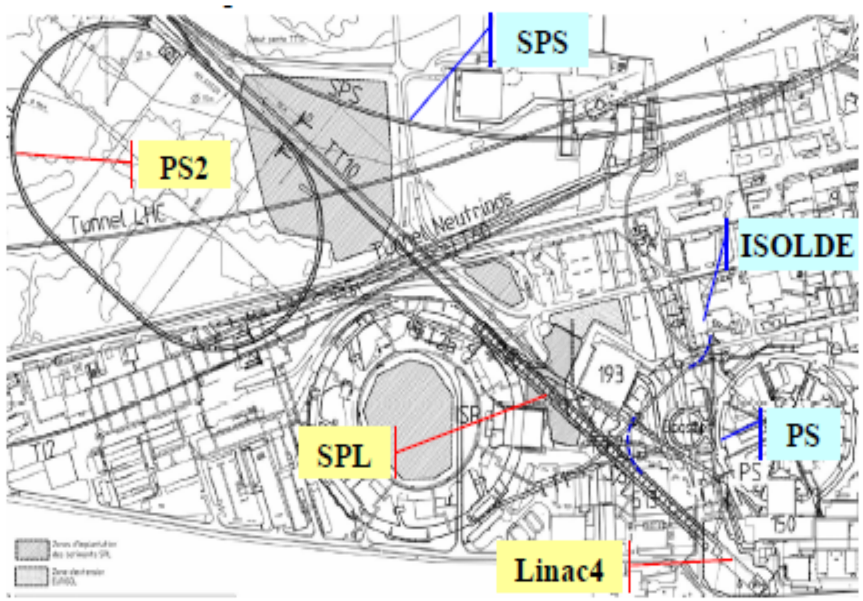


Figure 2: Integration of PS2 within the existing and future CERN accelerator complex.

M. Benedikt, et al., PAC09, WE1GRI03

Goals of PS2 Space-Charge Simulation

- Space-charge effects limit the performance of PS2 by degrading the beam quality, causing halo formation and particle losses
- Fully 3D self-consistent simulation help evaluate the potential dangers of the space-charge effects
- Identify the intensity limit of the space-charge dominated beam
- Optimize the PS2 accelerator design to minimize the space-charge causing beam quality degradation and particle losses

Physical Parameters for PS2 Simulations

Vrf = ramping with f = 39.3 MHz

Ek = 4 GeV

Emit_x = Emit_y = 3 mm-mrad

Emit_z = .098 eV-sec

Half Aperture = 6.3cm x 3.25cm

I = 4.0×10^{11}

Numerical Parameters:

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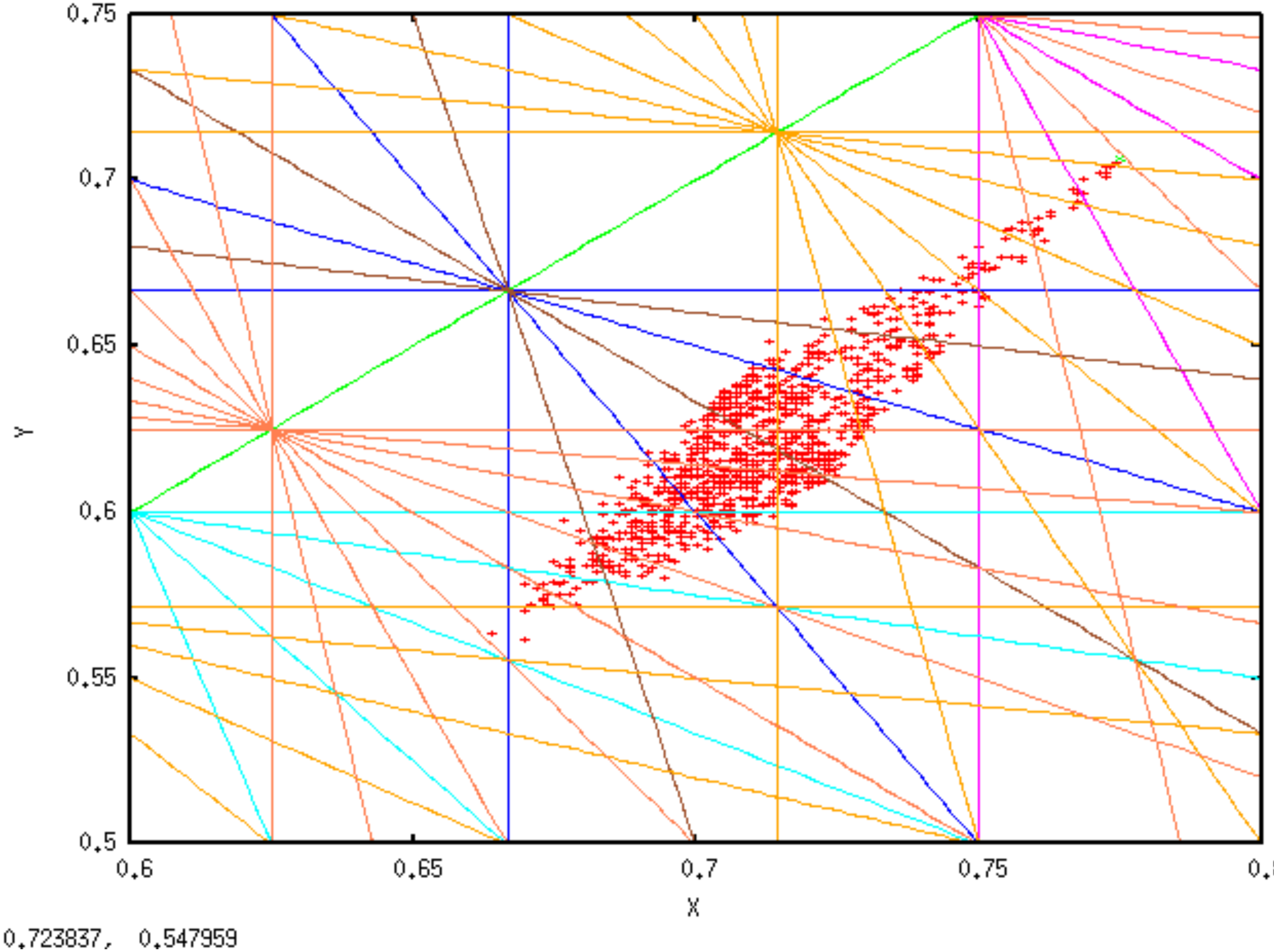
70 SC per turn

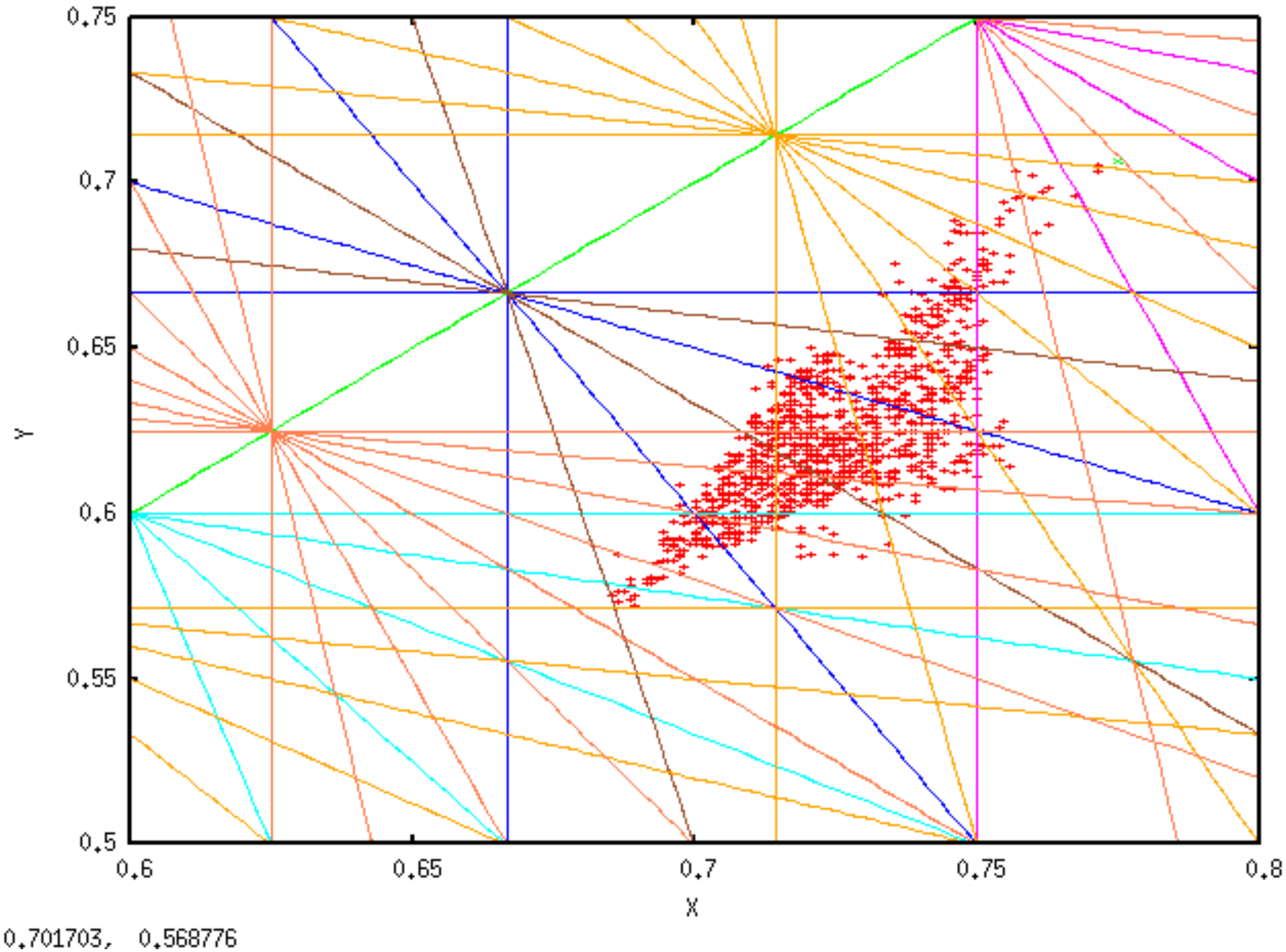
65x65x128 grid points

939,000 macroparticles

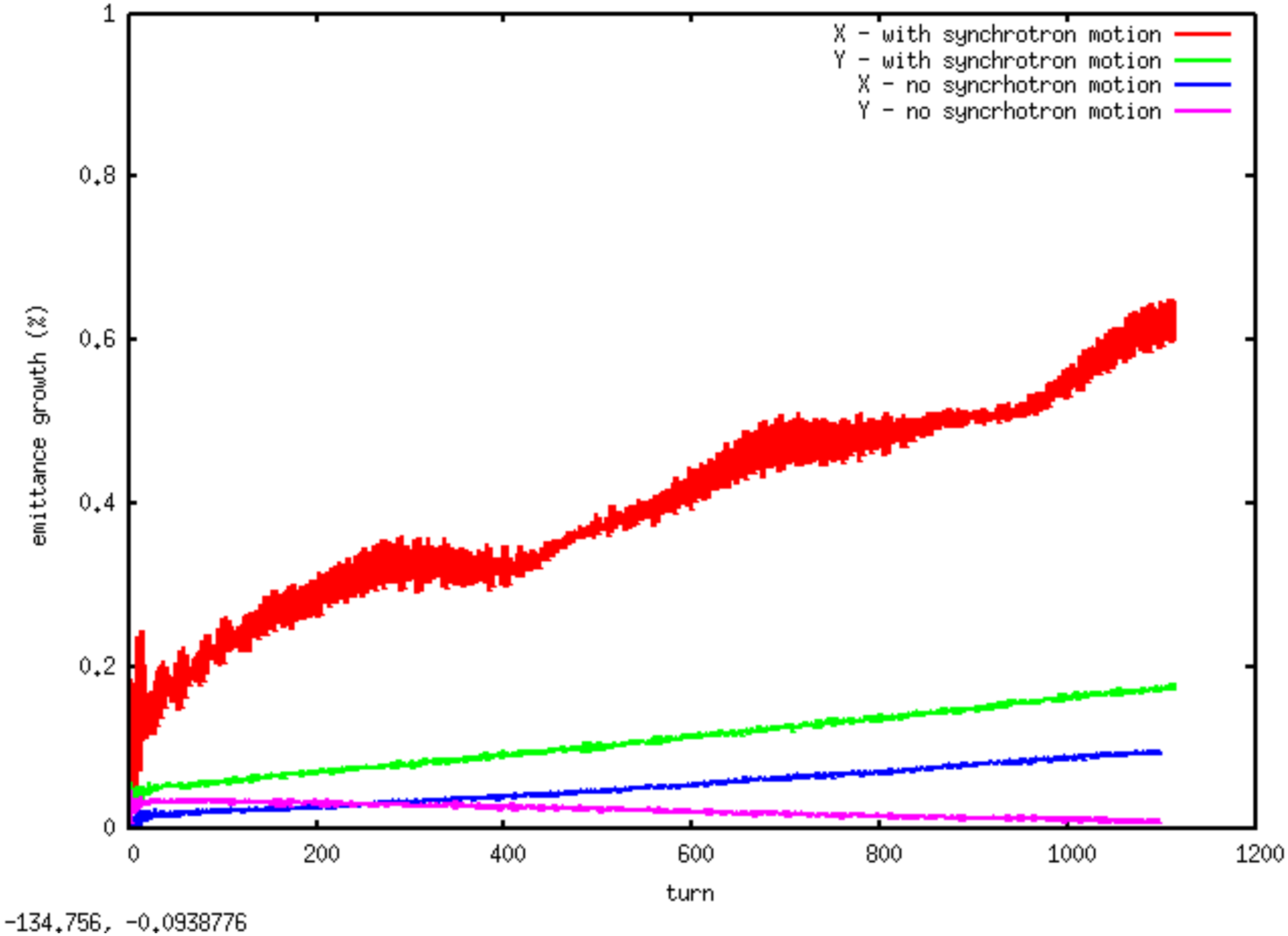
Betatron Tune Footprint with 0 Current and with SC but no Synchroton Motion



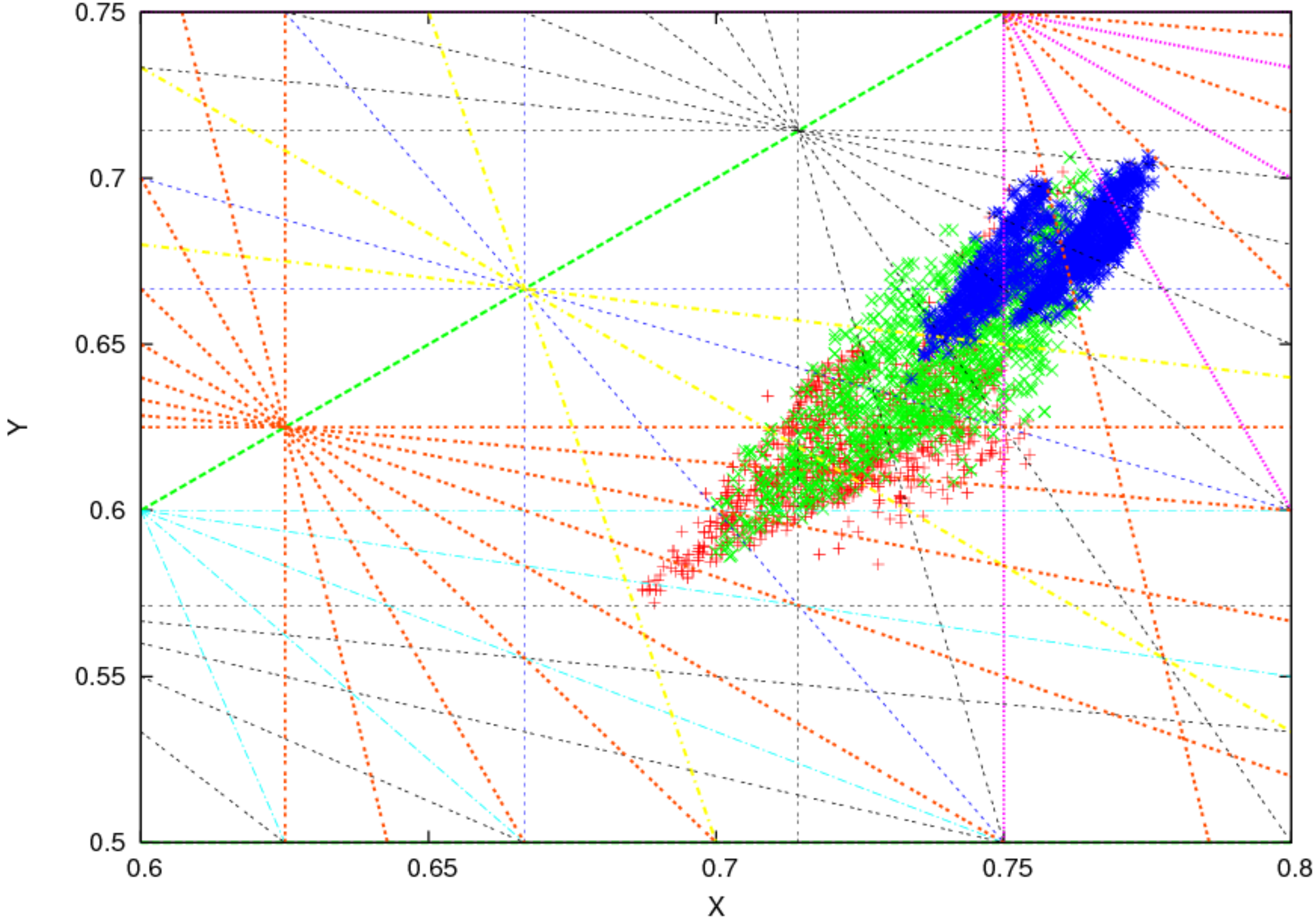
Betatron Tune Footprint with 0 Current and with SC and Synchroton Motion



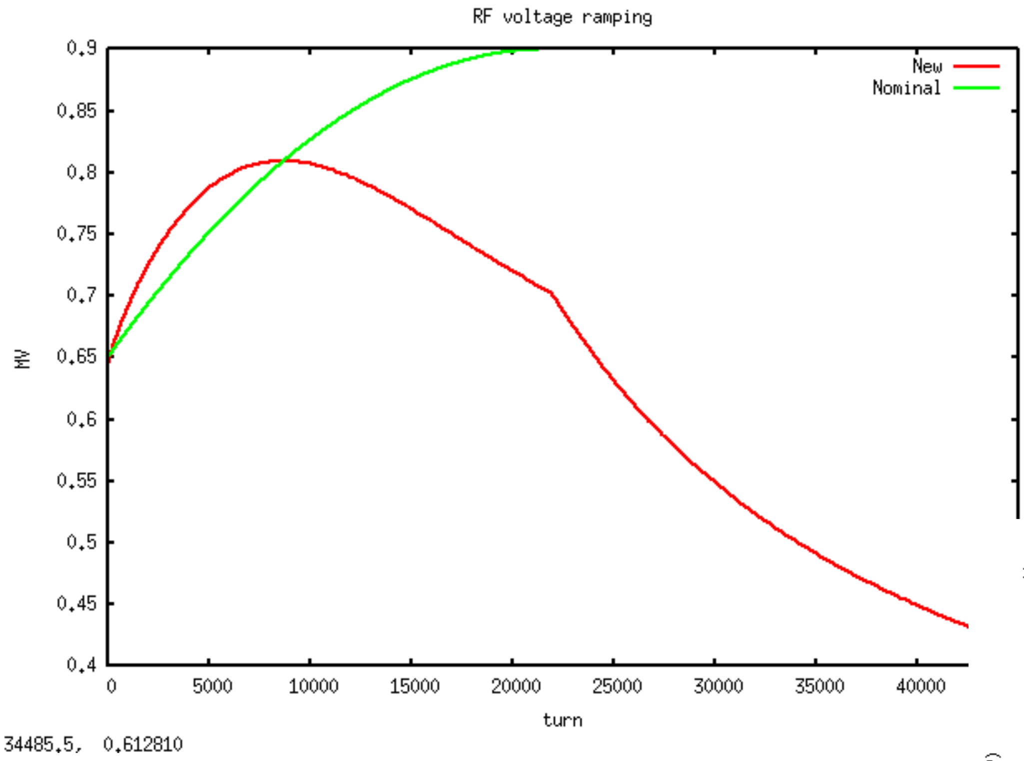
Transverse Emittance Growth with/without Synchrotron Motion

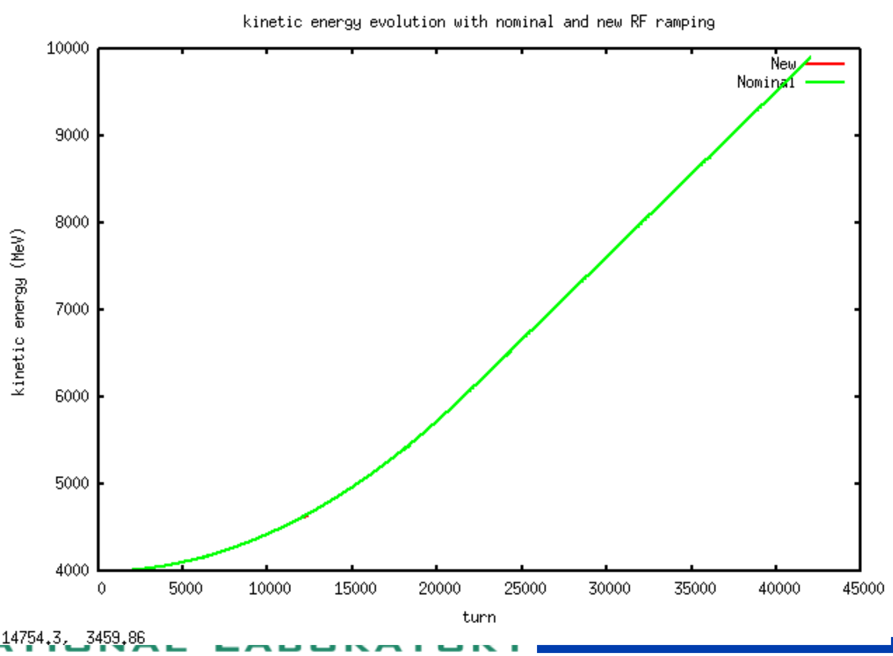


Tune Footprint around 4 GeV, 6 GeV and 8 GeV Energy

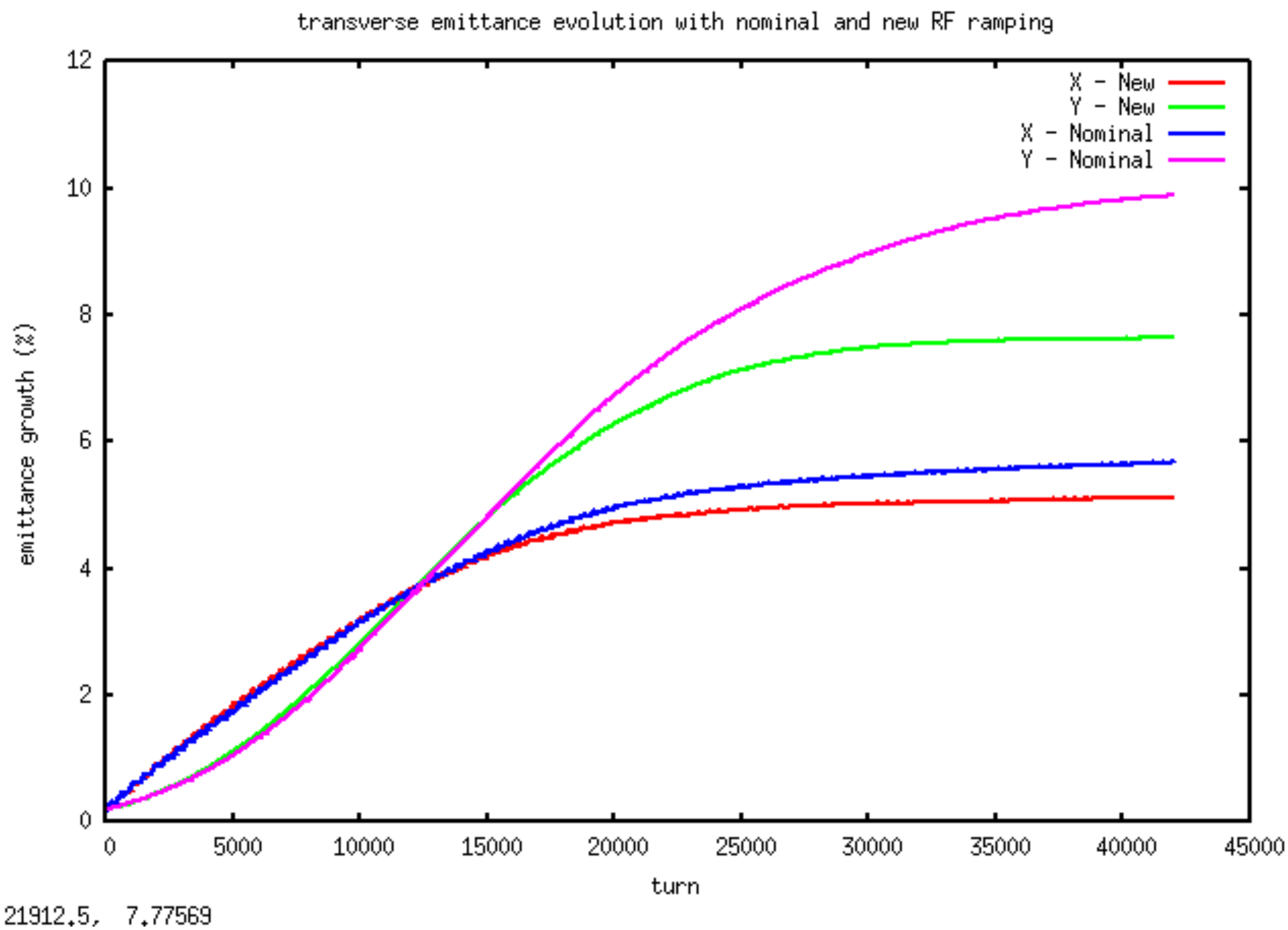


Nominal and New RF Voltage Ramping

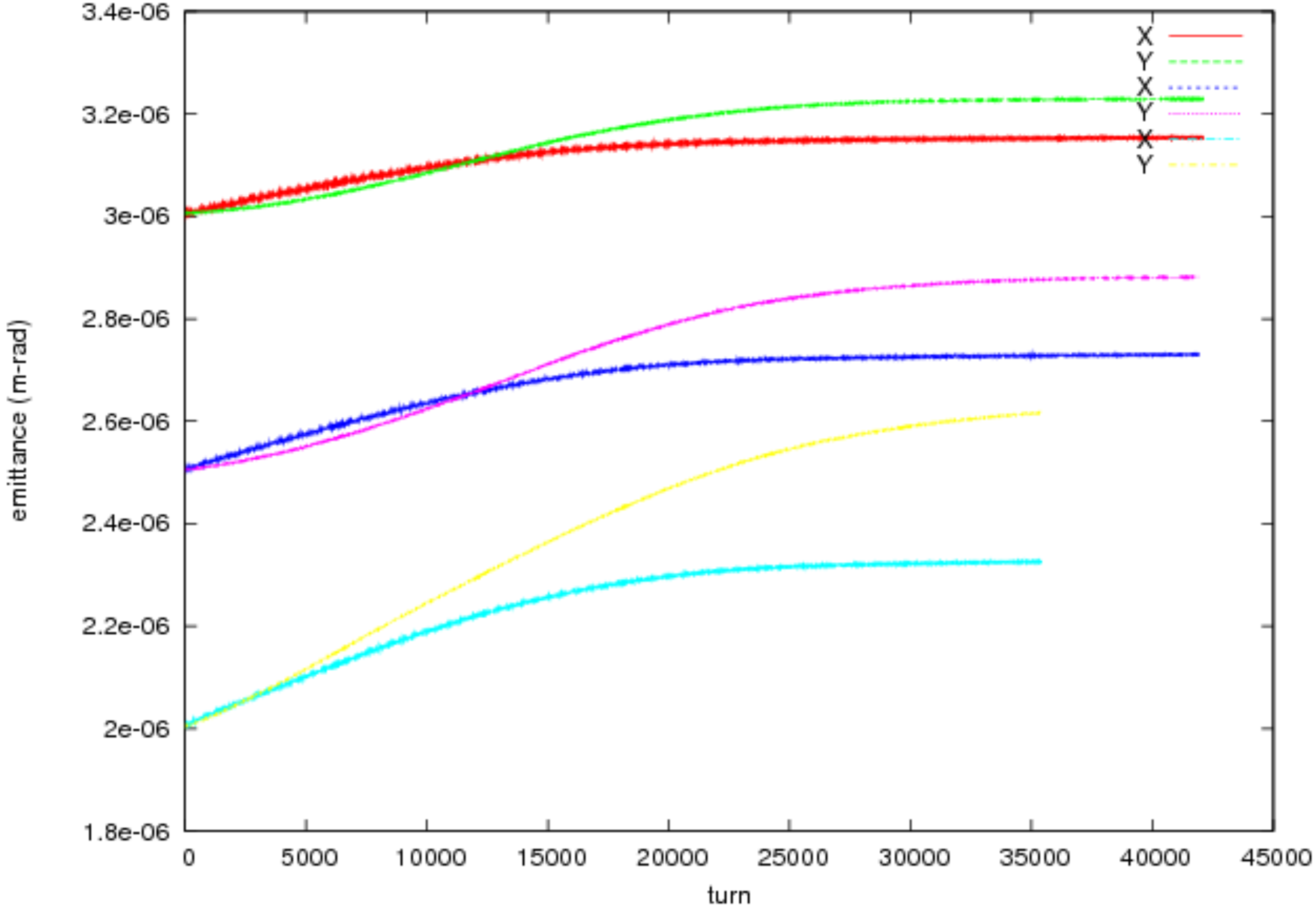




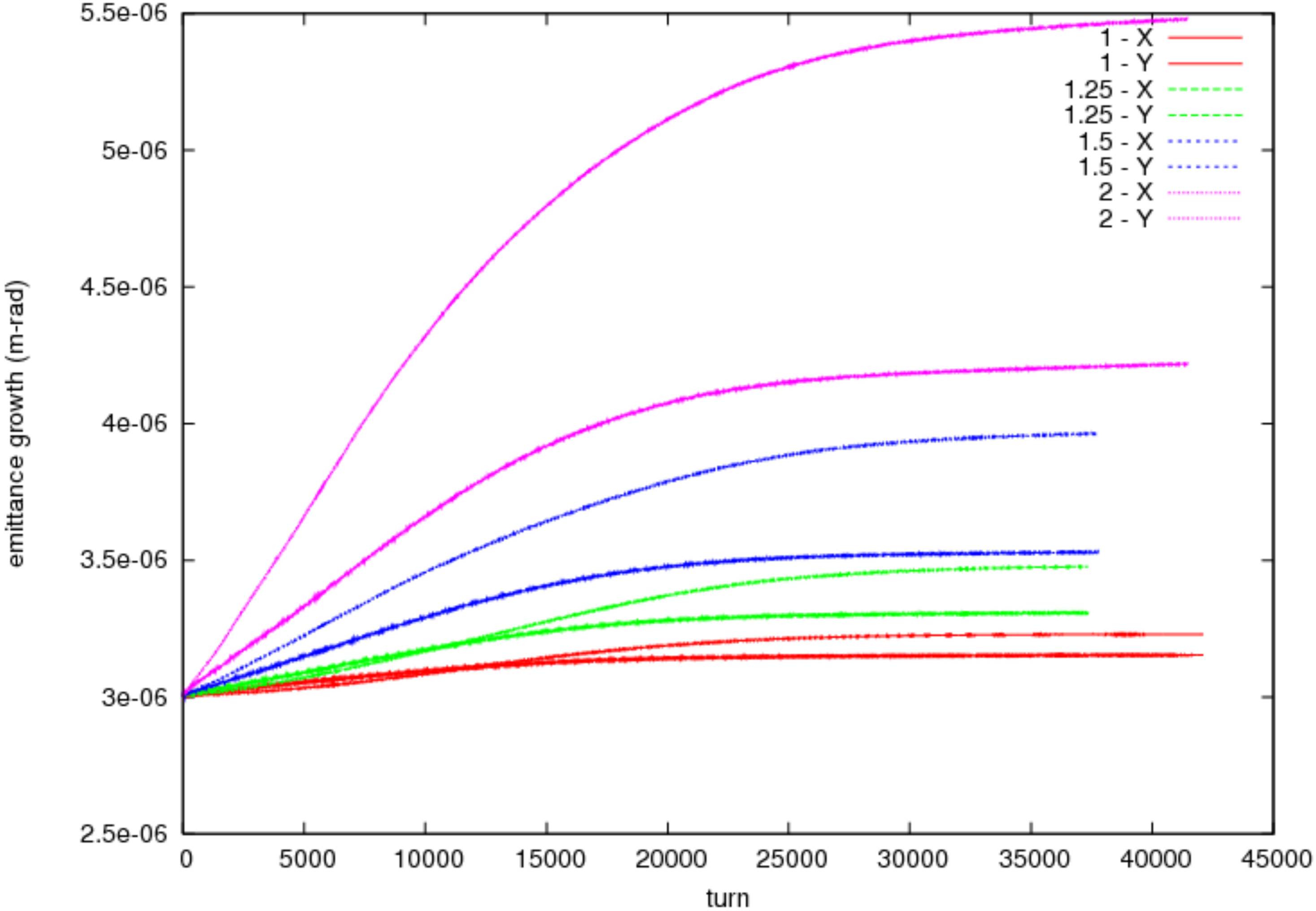
Transverse Emittance Growth with Nominal and New RF Ramping



Transverse Emittance Growth with Different Initial Emittances



Transverse Emittance Evolution with Different Proton Intensity

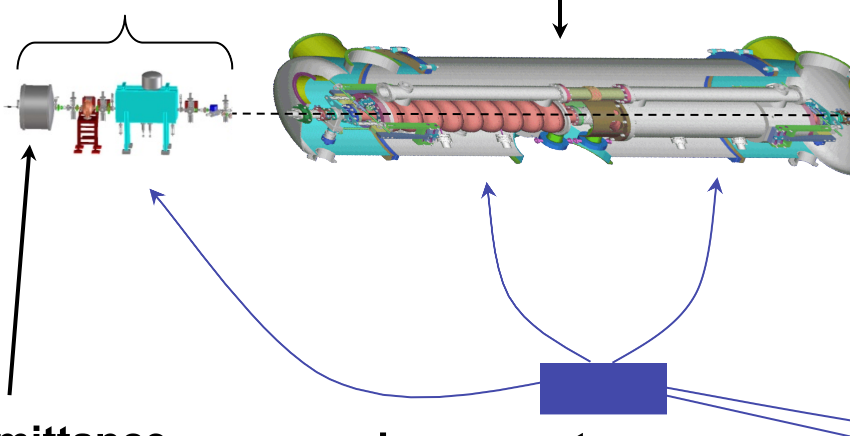


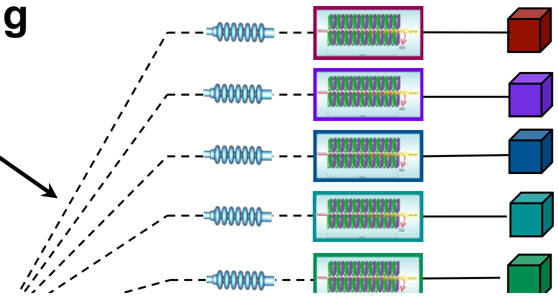
Space-Charge Driven Microbunching Instability in Electron Linac for Next Generation Light Sources

Beam manipulation and conditioning

Beam distribution and individual beamline tuning

~2 GeV CW superconducting linac

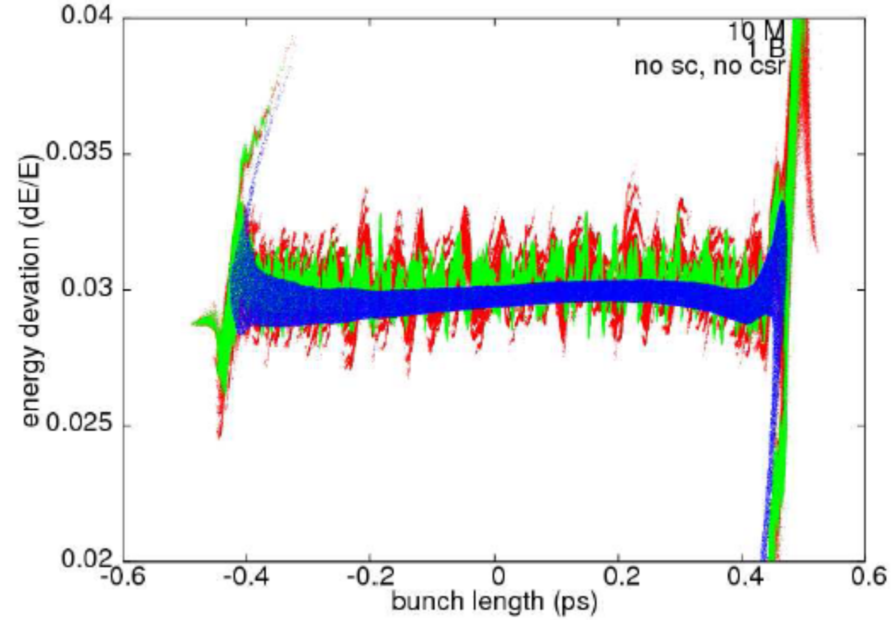




Low-emittance, high rep-rate electron gun

Laser systems, timing & synchronization

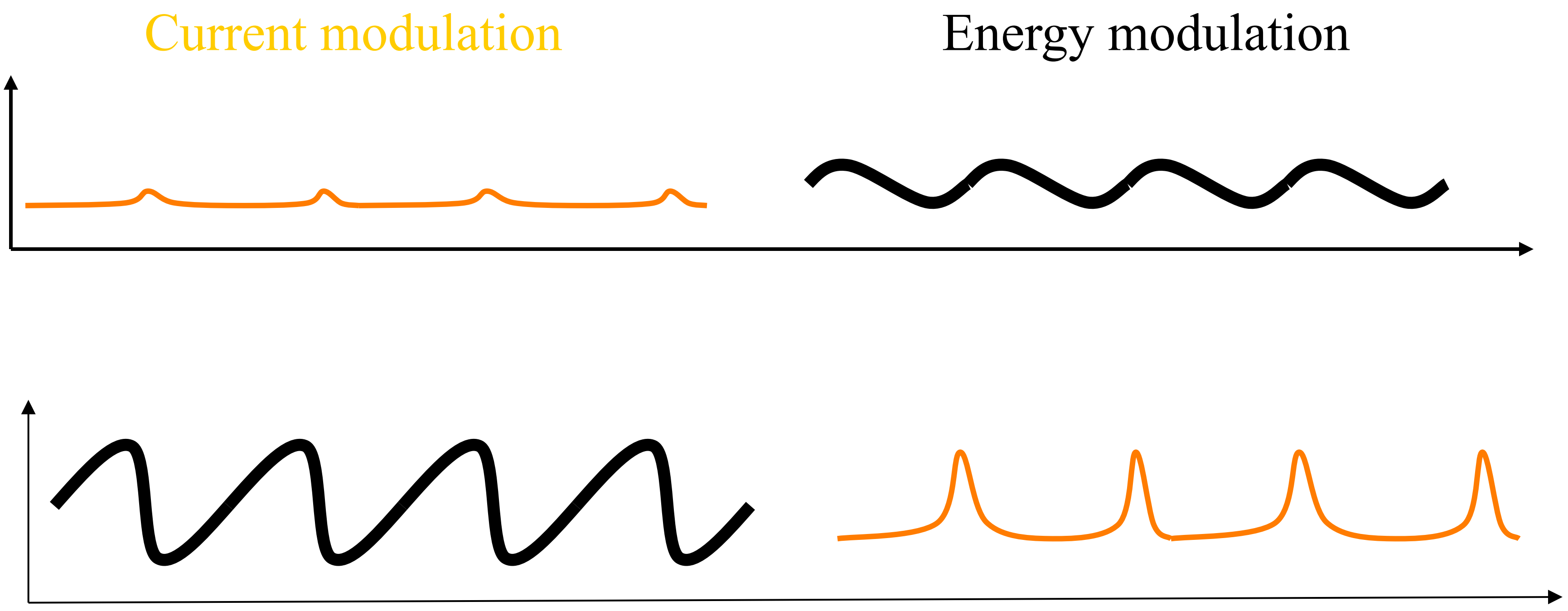
$$\text{FEL Power/undulator length} \sim f\left(I_{peak}, \frac{\varepsilon_n}{\gamma}, \sigma_E\right)$$



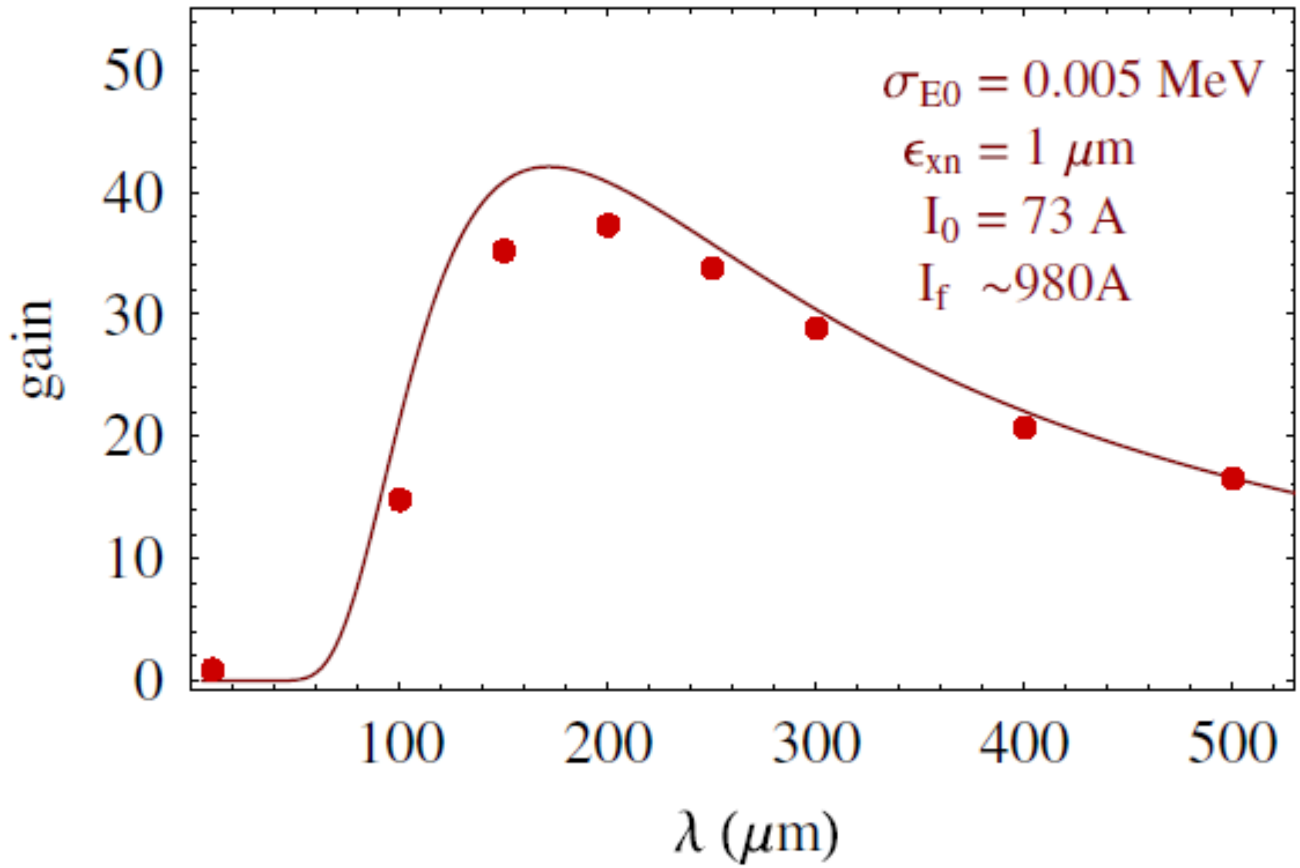
The longitudinal phase space of a beam at the exit of a linac

Microbunching Instability

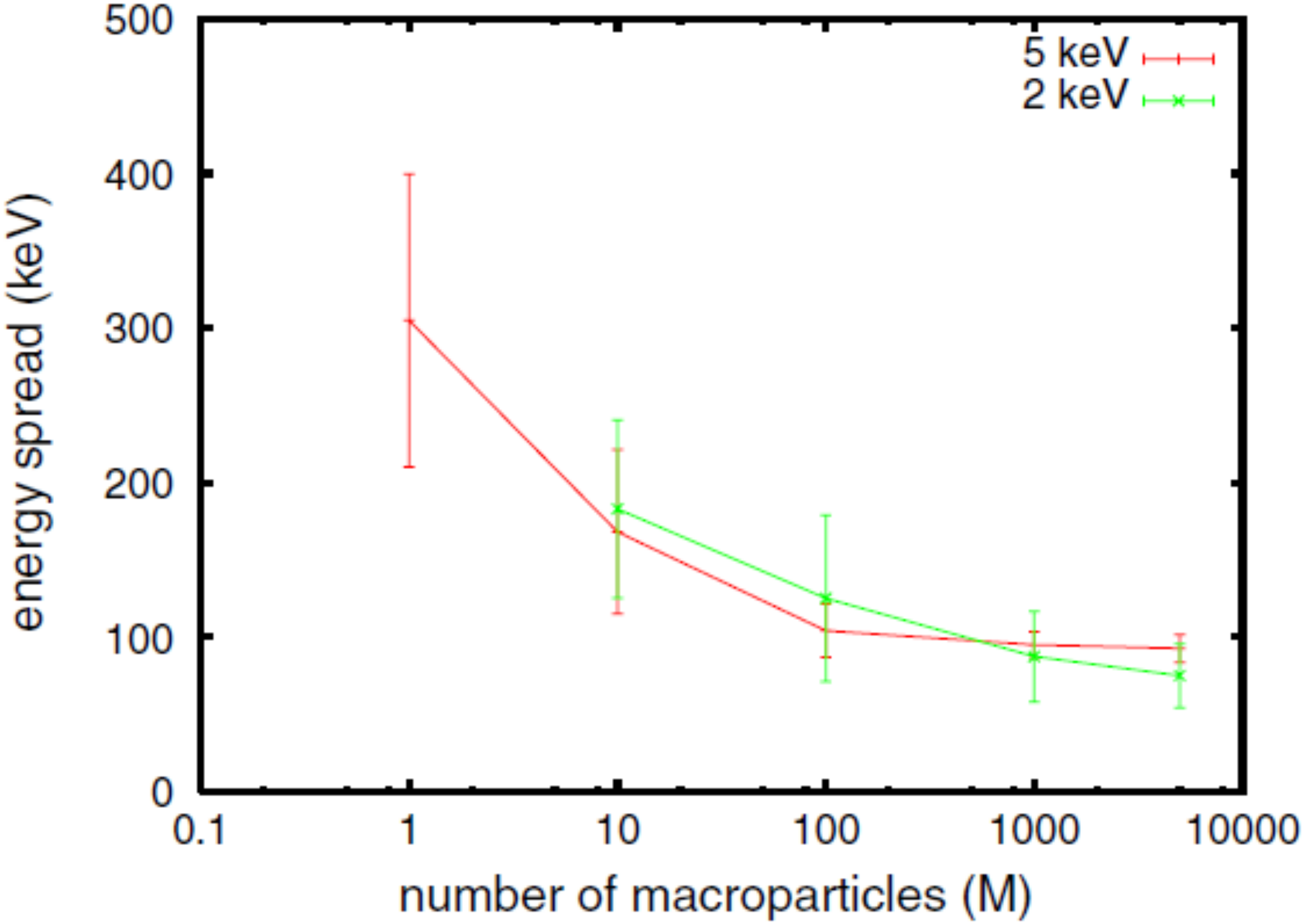
• Initial current modulation (even shot-noise) induces energy modulation through space-charge impedance Z(k), converted to more current modulation by a chicane ➔ growth of slice energy spread / emittance!



Microbunching Instability Gain Curve

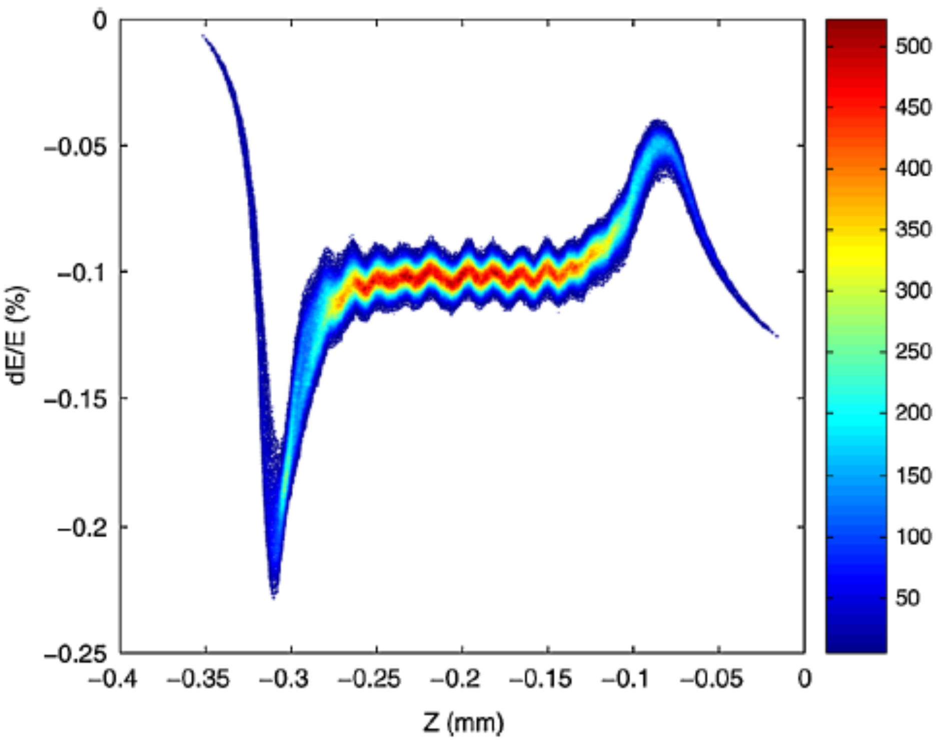


Final Uncorrelated Energy Spread vs. Number of Macroparticles

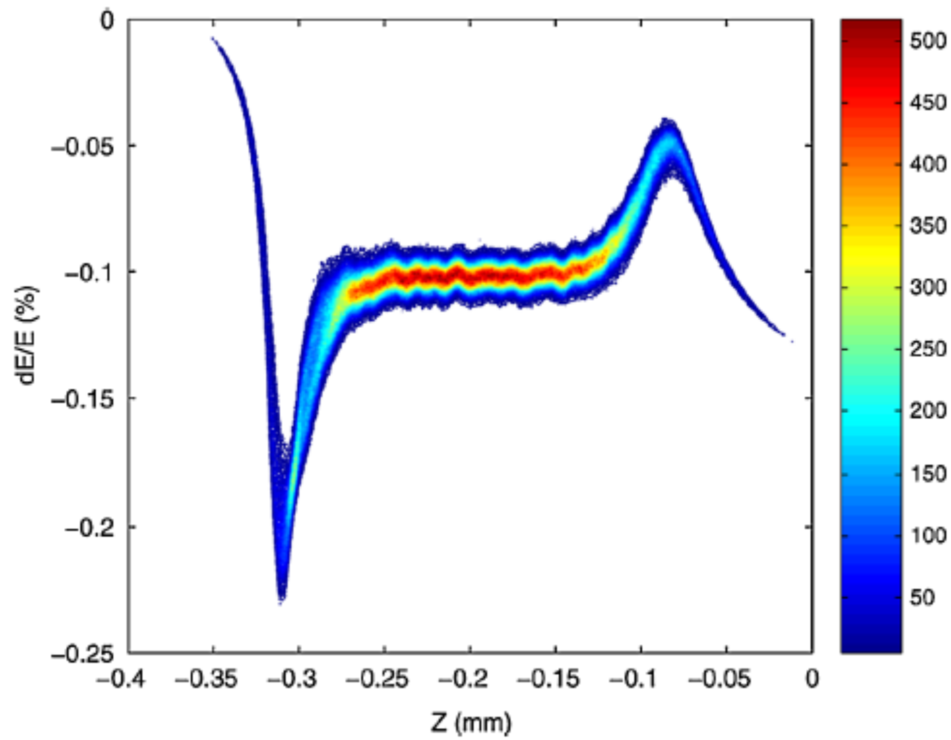


Final Longitudinal Phase Space Distribution with One and Five Billion Macroparticles

one billion



five billion



Acknowledgement:

H. Bartosik, C. Carli, J. Corlett,,G. Franchetti, S. Gilardoni, I. Hofmann, X. Li, E. Metral, Y. Papaphilippou, R. Ryne, M. Venturini, U. Wienands, A. Zholents, and many other collaborators.

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Thank you for your attention!