Some (not only historical) remarks on **noise with space charge**

1980's: Can we transport a space charge dominated beam through a long periodic system? (Heavy Ions for Inertial Fusion)

$n \cdot k + \Delta\omega_{coh} = 180$ or 360

$n = 2,3,4,5, ...$

$\Delta\omega_{coh}$ = complex → instabilities

- all coherent
- space charge driven
- infinitely many
- no Landau damping (KV)
- relief came from first PIC simulation (>1979 with $10^4 ... 10^5$ particles on Cray 1 in Munich) → nearly all "damp" with non-KV

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STABILITY OF THE KAPCHINSKIJ-VLADIMIRSKIJ (K-V) DISTRIBUTION IN LONG PERIODIC TRANSPORT SYSTEMS†

I. HOFMANN,‡ L. J. LASLETT, L. SMITH, AND I. HABER§

Lawrence Berkeley Laboratory, University of California, Berkeley, California 94720 U.S.A.

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Transport of intense beams of heavy ions over long distances may be restricted by space-charge induced transverse instabilities. The stability of the microcanonical, or K-V, distribution is analyzed with the help of the Vlasov equation, and reduced to a study of the characteristics of solutions for a set of ordinary differential equations with periodic coefficients. Numerical solutions for various periodic solenoid and quadrupole focusing channels are derived and provide information concerning stable regions of propagation in terms of betatron tune depression. The results are compared with computer simulation examples of beams in solenoid and quadrupole focusing channels to check linear growth rates and establish nonlinear saturation levels of instabilities. Conclusions are drawn for the design of a quadrupole lattice providing stable transport.

>1990: Can we find a matched distribution with space charge in periodic focusing and **no emittance growth** to benchmark our PIC codes?

- strong space charge $k_0 = 60^0$ → $k = 15^0$ (60^0 "safe" from internal resonances)
- exactly matched waterbag (profile!)
- periodic solenoid = const
- periodic quad = slow growth
- **why?** noise – resonances?

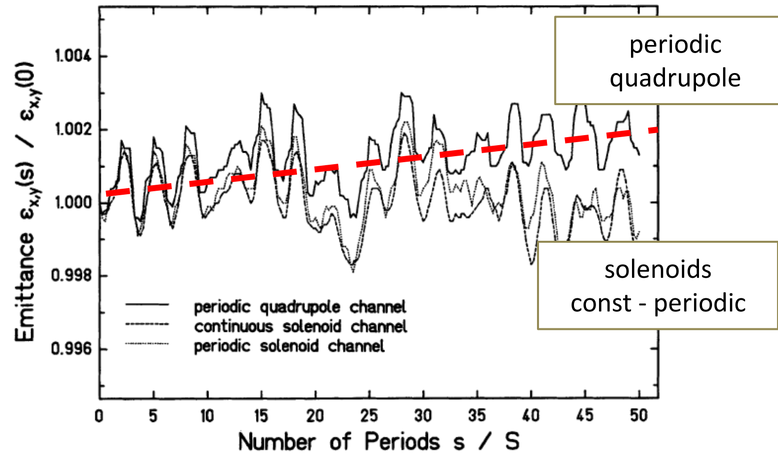

FIGURE 7: Emittance growth factors versus the number of periods for a stationary 'water bag' distribution at $\sigma_0 = 60°$, $\sigma = 15°$.

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THE PROBLEM OF SELF-CONSISTENT PARTICLE PHASE SPACE DISTRIBUTIONS FOR PERIODIC FOCUSING CHANNELS

JÜRGEN STRUCKMEIER and INGO HOFMANN

Gesellschaft für Schwerionenforschung (GSI), Postfach 11 05 52, 6100 Darmstadt, Germany

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Charged particle beams that remain stationary while passing through a transport channel are represented by 'self-consistent' phase space distributions. As the starting point, we assume the external focusing forces to act *continuously* on the beam. If Liouville's theorem applies, an infinite variety of self-consistent particle phase space distributions exists then. The method is reviewed how to determine the Hamiltonian of the focusing system for a given phase space density function. Subsequently, this Hamiltonian is transformed canonically to yield the appropriate Hamiltonian that pertains to a beam passing through a *non-continuous* transport system. It is shown that the total transverse beam energy is a conserved quantity, if the beam stays rotationally symmetric along the channel. It can be concluded that charged particle beams can be transmitted through periodic solenoid channels without loss of quality. Our computer simulations, presented in the second part of the paper, confirm this result. In contrast, the simulation for a periodic quadrupole channel yields a small but constant growth rate of the rms-emittance.

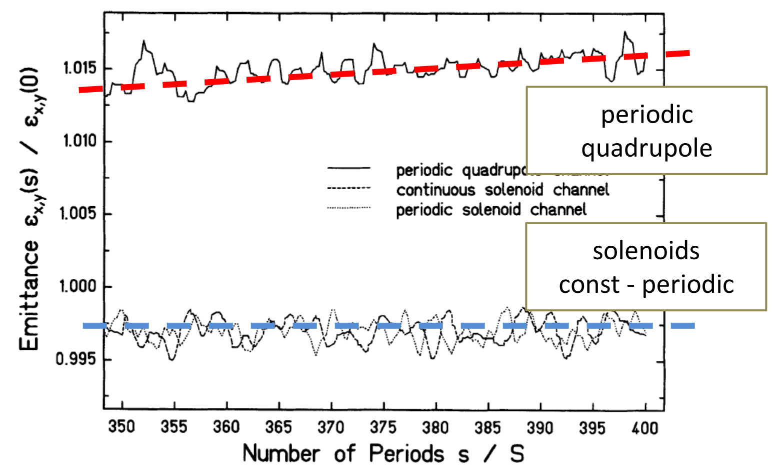

FIGURE 12: Long term emittance growth factors versus the number of periods for a stationary 'water bag' distribution at $\sigma_0 = 60°$, $\sigma = 15°$.

1990's: Struckmeier's stochastic approach with x – y temperature imbalance (FODO) → entropy + emittance growth

flow of heat = irreversible

$\beta_{friction}$=friction coefficient

- fluctuations of fields and particles

T_x-T_y = given by lattice

$$\frac{dS}{dt} = \frac{1}{2} k_B \beta_f \frac{(T_x - T_y)^2}{T_x T_y}$$

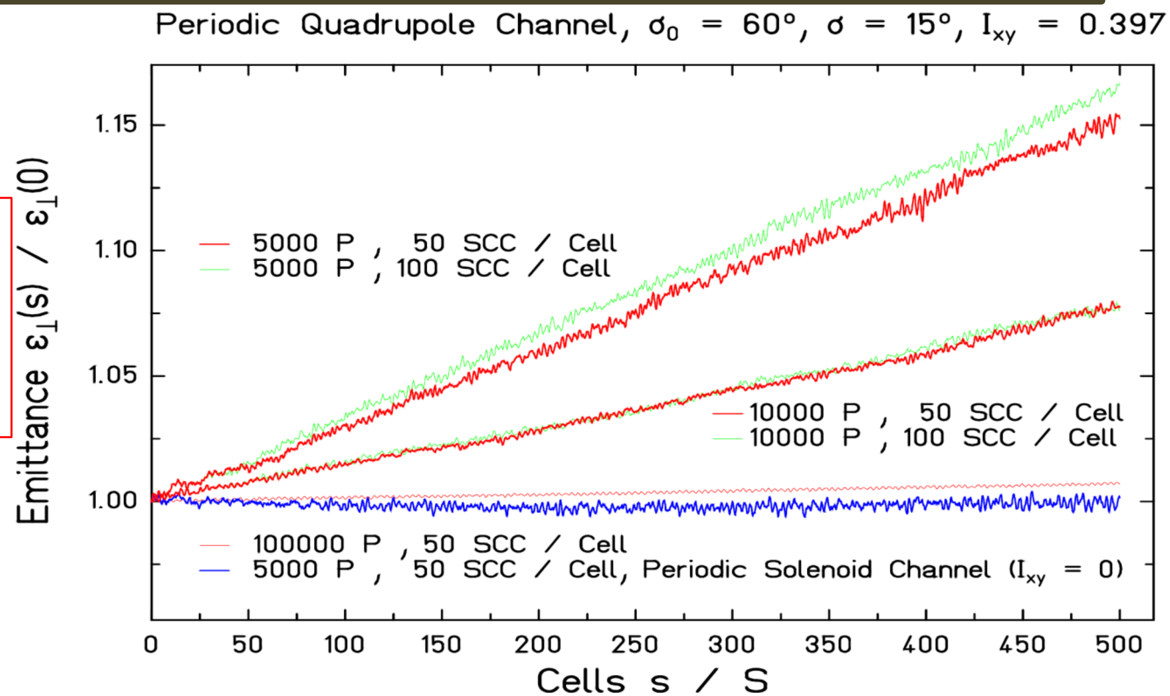

FIGURE 3: Emittance growth functions obtained by 'particle-in-cell' simulations of matched beams passing through different types of beam transport channels at $\sigma_0 = 60°$, $\sigma = 15°$. The underlying beam parameters are listed in Tab. 1.

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IMPROVED ENVELOPE AND EMITTANCE DESCRIPTION OF PARTICLE BEAMS USING THE FOKKER-PLANCK APPROACH

JÜRGEN STRUCKMEIER

Gesellschaft für Schwerionenforschung (GSI), Postfach 11 05 52, 64220 Darmstadt, Germany

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Beam dynamics calculations that are based on the Vlasov equation do not permit the treatment of stochastic phenomena such as intra-beam scattering. If the nature of the stochastic process can be regarded as a Markov process, we are allowed to use the Fokker-Planck equation to describe the change of the phase space volume the beam occupies. From the Fokker-Planck equation we derive equations of motion for the beam envelopes and for the rms-emittances. Compared to previous approaches based on Liouville's theorem, these equations contain additional terms that describe the temperature balancing within the beam. Our formalism is applied to the effect of intra-beam scattering relevant for beams circulating in storage rings near thermodynamical equilibrium. In this case, the Fokker-Planck coefficients can be treated as adiabatic constants of motion. Due to the simplified analysis based on 'beam moments', we obtain fairly simple equations that allow us to estimate the growth rate of the beam emittance.

No growth:

1. $\beta_{friction}$=0: no stochastic source
   - frozen smooth space charge interaction
2. T_x=T_y: beam keeps round symmetry
   - no entropy is generated

I_e describing ellipticity→ $$\frac{\varepsilon_\perp(t)}{\varepsilon_\perp(0)} = \exp\left\{\tfrac{1}{4}\,\beta_f\, I_e \cdot t\right\}$$

New challenge:

- scattering and trapping of particles in nonlinear resonances may occur outside core
- noise issue and "temperatures" need to be refined – localized?
- what is noise effect on trapping / scattering?

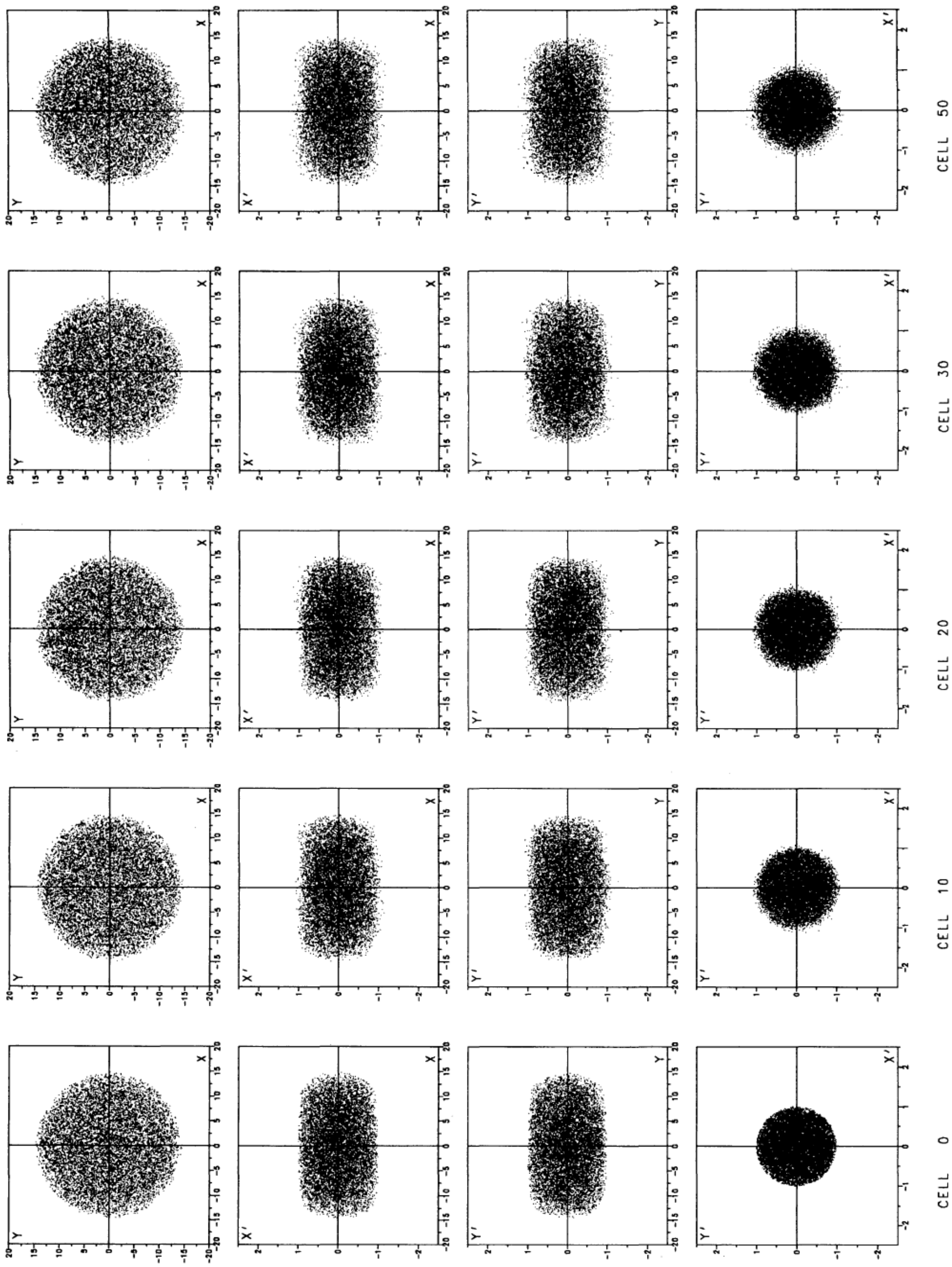

FIGURE 10: Stationary 'water bag' distribution transformed through a periodic quadrupole channel at $\sigma_0 = 60°$, $\sigma = 15°$. The x, x'- and y, y'-phase space projections have been transformed to main axes (units : mm and mrad).