



Space charge simulation toward accurate modelling

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Contents

- Define the scope.
- What we learned from the J-Parc simulation.
- Toward more accurate simulation.
- Single particle trajectory in PIC.

Define the scope

Two cases

Self-consistent simulation for short term modelling

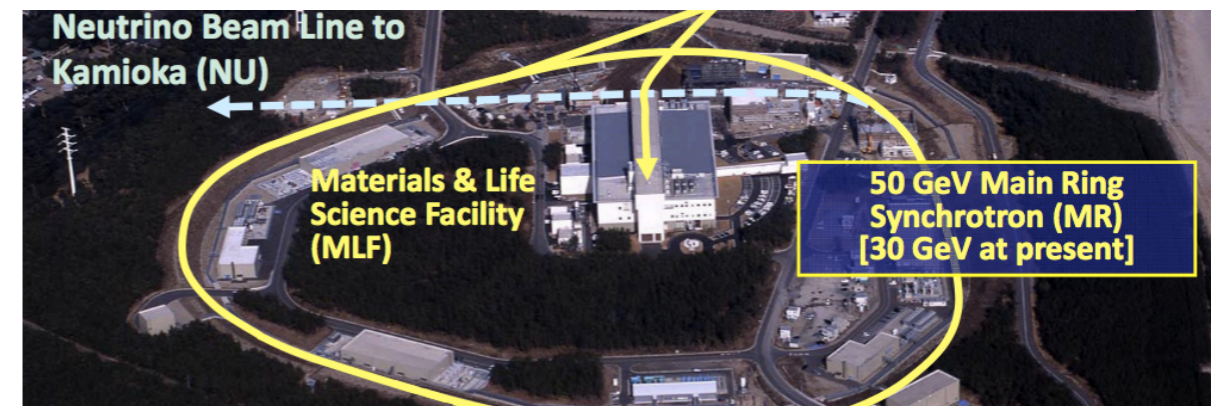
- Up to a few 10 ms or 100 ms time scale.
- Below 10^4 turns.



- PSB, ISIS, J-Parc RCS, ...

Frozen model for long term simulation

- 1 s or longer time tracking.
- Above 10^5 turns.



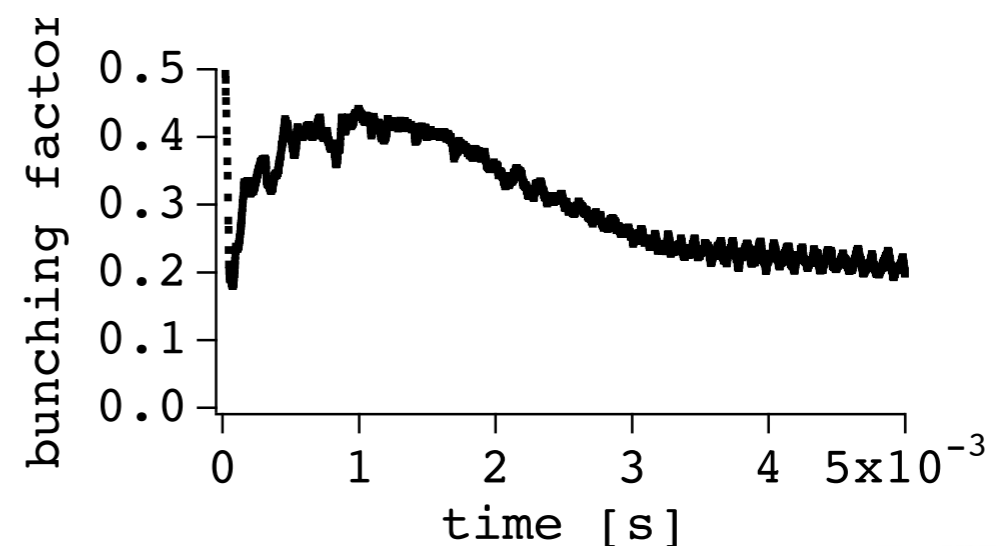
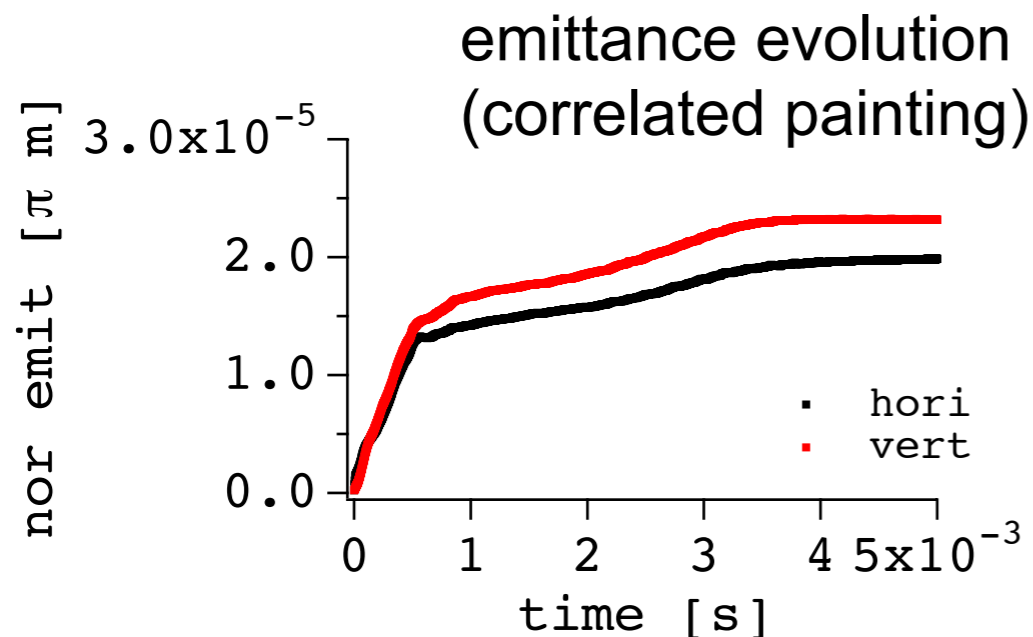
- PS, SPS, J-Parc MR, SIS100, ...

My talk is about the machine in the left.

Why do we need self-consistent model?

Space charge effects are critical during and right after injection where beam characteristics are transient.

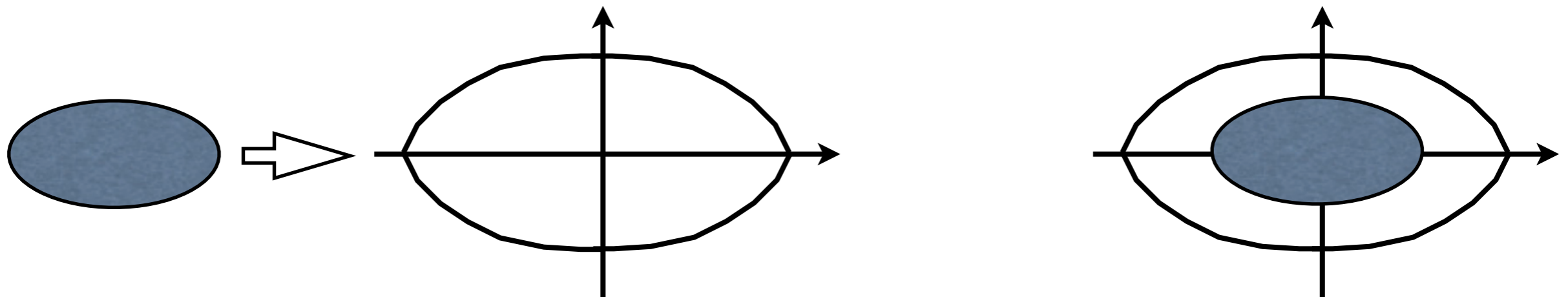
- Number of particles.
- Beam emittance.
- Fundamental (and higher harmonic) rf voltage and phase.
- Beam energy.
- External nonlinearities may have time dependence.



Space charge potential cannot be formulated a priori.

Just to compare ...

- Space charge effects in the second or downstream rings where
 - The whole beam is injected once by bunch to bucket transfer.
 - Good matching in transverse and longitudinal phase space suppresses transient coherent oscillations.
 - A beam is stored for long time without energy ramping.



- Frozen space charge potential model may be applied.

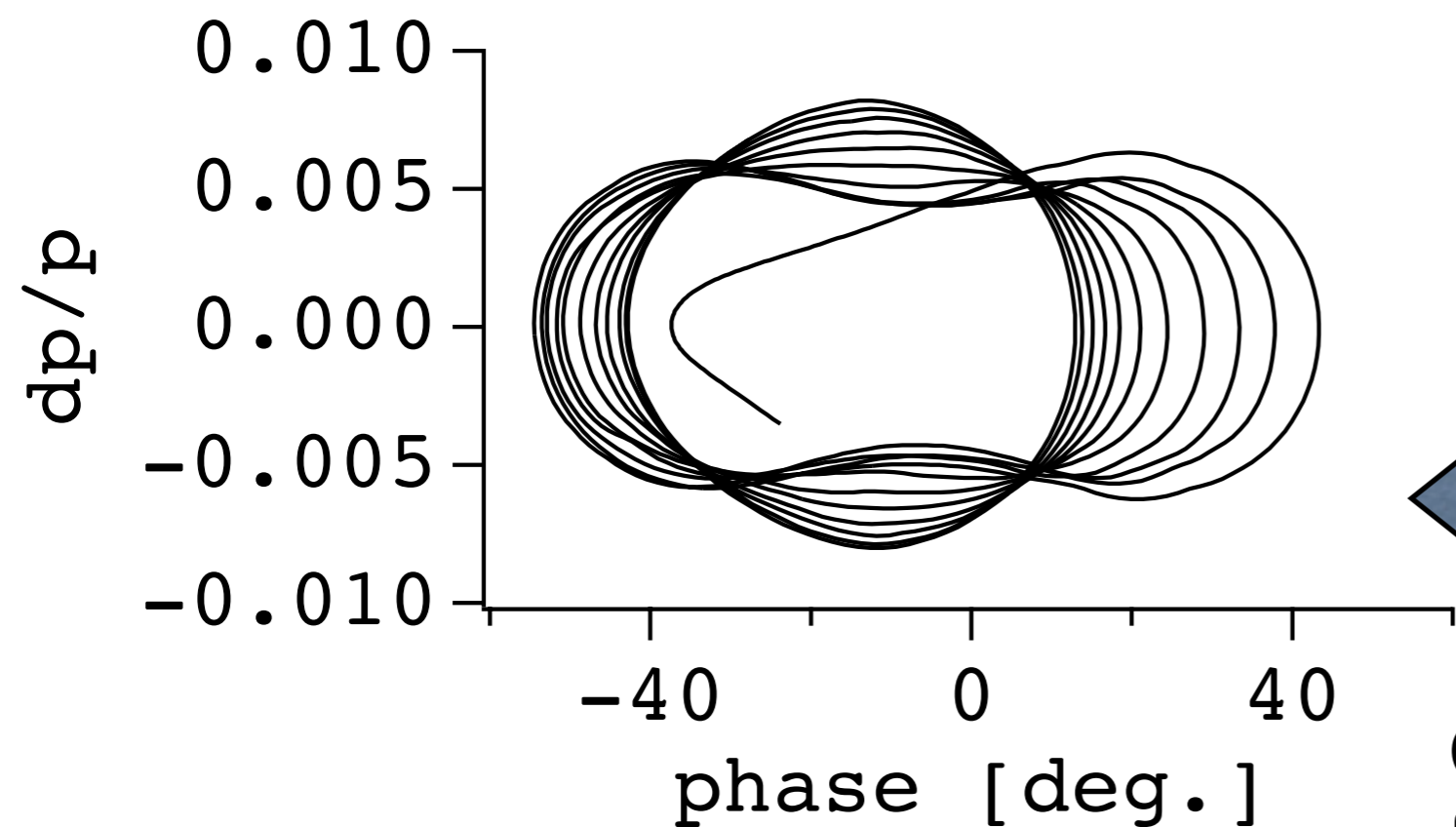
Learned from J-Parc modelling

What has to be included?

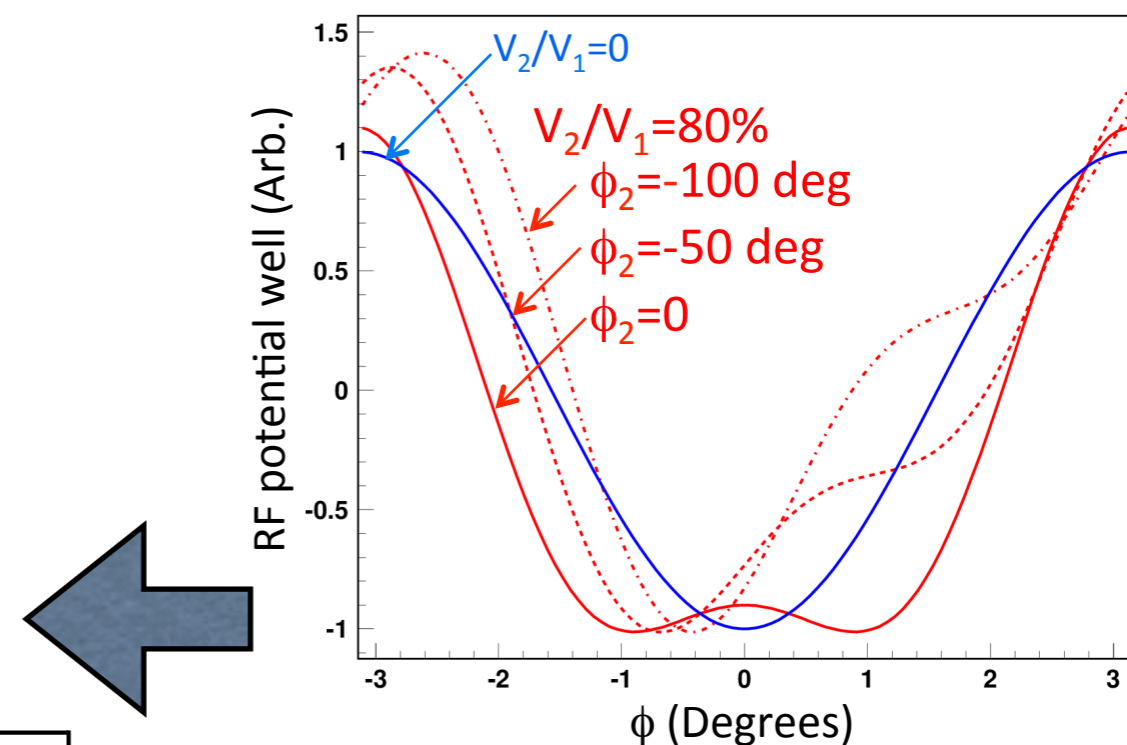
J-Parc RCS modelling tells

- Fully 6D dynamics with synchrotron oscillations and acceleration.
- Time dependent space charge tune spread
 - Bunching process with time-varying rf parameters.
 - Multi-turn injection process to gradually form the transverse emittance.
 - Acceleration gradually reduces tune spread.
- Inclusive list of resonance sources and its implementation.
 - Some of them are time dependent.
- Precious estimate of aperture limit.
 - Modern machines have well defined collimators.

Single particle trajectory in PIC



Manipulation of rf bucket



Change relative phase of the fundamental and 2nd harmonic rf.

$\phi_2=-100 \Rightarrow 0$ deg

What has to be included?

J-Parc RCS modelling tells

- Fully 6D dynamics with synchrotron oscillations and acceleration.
- Time dependent space charge potential or tune spread
 - Multi-turn injection process to gradually form the transverse emittance.
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A way to simulate multi-turn injection in PIC

1st turn: Inject N particles.



2nd turn: Inject another N particles.

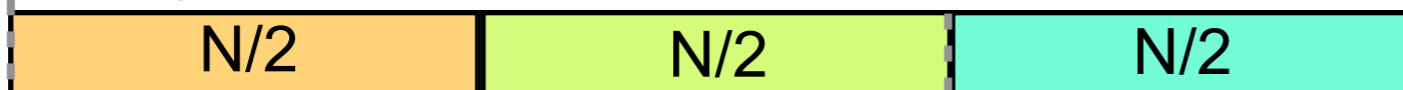


Throw away randomly selected N particles.

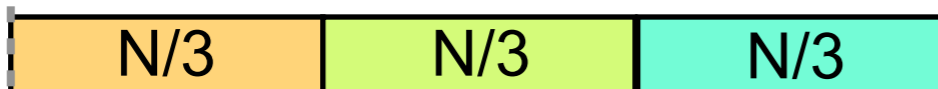


Increase charge per particle by 2.

3rd turn: Inject $N/2$ particles.

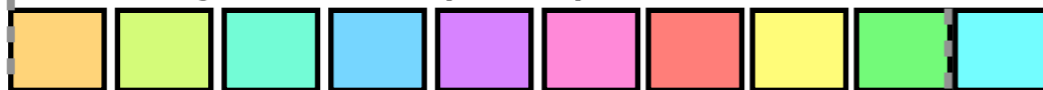


Throw away randomly selected $N/2$ particles.

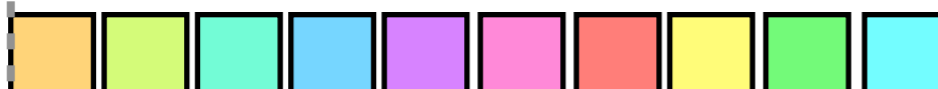


Increase charge per particle by $3/2$.

n -th turn: Inject $N/(n-1)$ particles.



Throw away randomly selected $N/(n-1)$ particles.

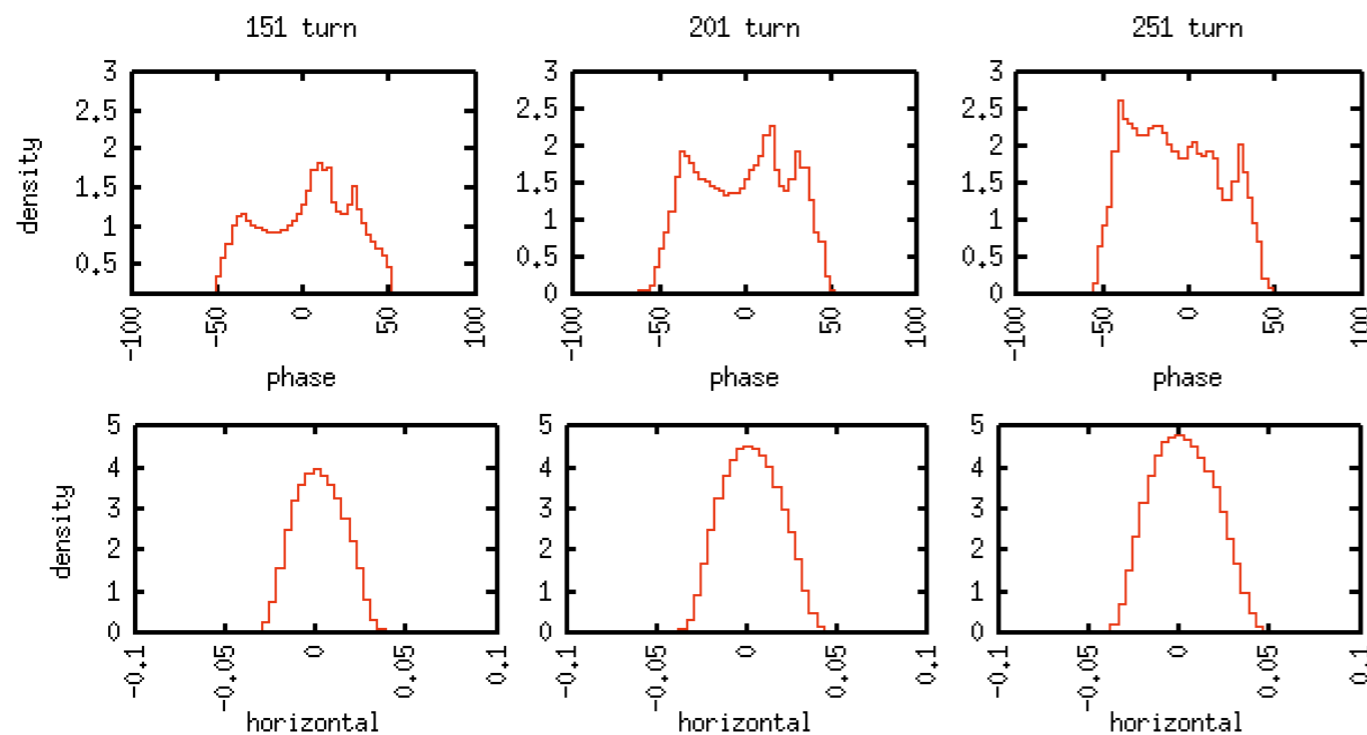


Increase charge per particle by $n/(n-1)$.

The same number of macro particles N are used during injection.

Initial charge distribution in a ring simulation

- Multi-turn injection painting process gives a big advantage to a ring simulation.
- **Charge distribution from linac is not crucial.**
- Quite different from simulation in a linac, where uncertainty of charge distribution from ion source (or RFQ) introduces large variations of the results.



profile during multi-turn injection

longitudinal

transverse

What has to be included?

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Numerical simulation setup

Simpsons (PIC particle tracking code developed by Dr. Shinji Machida)

Imperfections included:

◆ Time independent imperfections

- Multipole field components for all the main magnets:
BM ($K_{1\sim6}$), QM ($K_{5,9}$), and SM (K_8) obtained from field measurements
- Measured field and alignment errors

◆ Time dependent imperfections

- Static leakage fields from the extraction beam line:
 $K_{0,1}$ and $SK_{0,1}$ estimated from measured COD and optical functions
- Edge focus of the injection bump magnets:
 K_1 estimated from measured optical functions
- BM-QM field tracking errors
estimated from measured tune variation over acceleration
- 1-kHz BM ripple
estimated from measured orbit variation
- 100-kHz ripple induced by injection bump magnets
estimated from turn-by-turn BPM data

◆ Foil scattering:

Coulomb & nuclear scattering angle distribution calculated with GEANT

Time-dependent imperfections can be included easily, because “Simpsons” takes “time” as an independent variable.

From Hotchi's slides yesterday.

What has to be included?

J-Parc RCS modelling tells

- Fully 6D dynamics with synchrotron oscillations and acceleration.
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 - Some of them are time dependent.
- **Precious measure of aperture limit.**
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One last technical point of simulation

- Taking “time” as the independent variable make it easier to include time dependent parameters.
- Give tables which list parameters as a function of time.
- Reading of ~20 tables at the beginning in the case of J-Parc RCS simulation.

```
shinjis-mac-pro:dat machida$ head -20 qdlnoise.dat
0.00000000E+00 -2.56436815E-05
2.12679900E-06 -2.52122095E-05
4.25364800E-06 -2.47843650E-05
6.38054800E-06 -2.43601487E-05
8.50749700E-06 -2.39395621E-05
1.06344950E-05 -2.35226063E-05
1.27615420E-05 -2.31092822E-05
1.48886380E-05 -2.26995910E-05
1.70157810E-05 -2.22935339E-05
1.91429730E-05 -2.18911117E-05
2.12702110E-05 -2.14923258E-05
2.33974960E-05 -2.10971771E-05
2.55248270E-05 -2.07056667E-05
2.76522040E-05 -2.03177957E-05
2.97796260E-05 -1.99335651E-05
3.19070940E-05 -1.95529756E-05
3.40346060E-05 -1.91760288E-05
3.61621620E-05 -1.88027253E-05
3.82897620E-05 -1.84330662E-05
4.04174040E-05 -1.80670528E-05
```

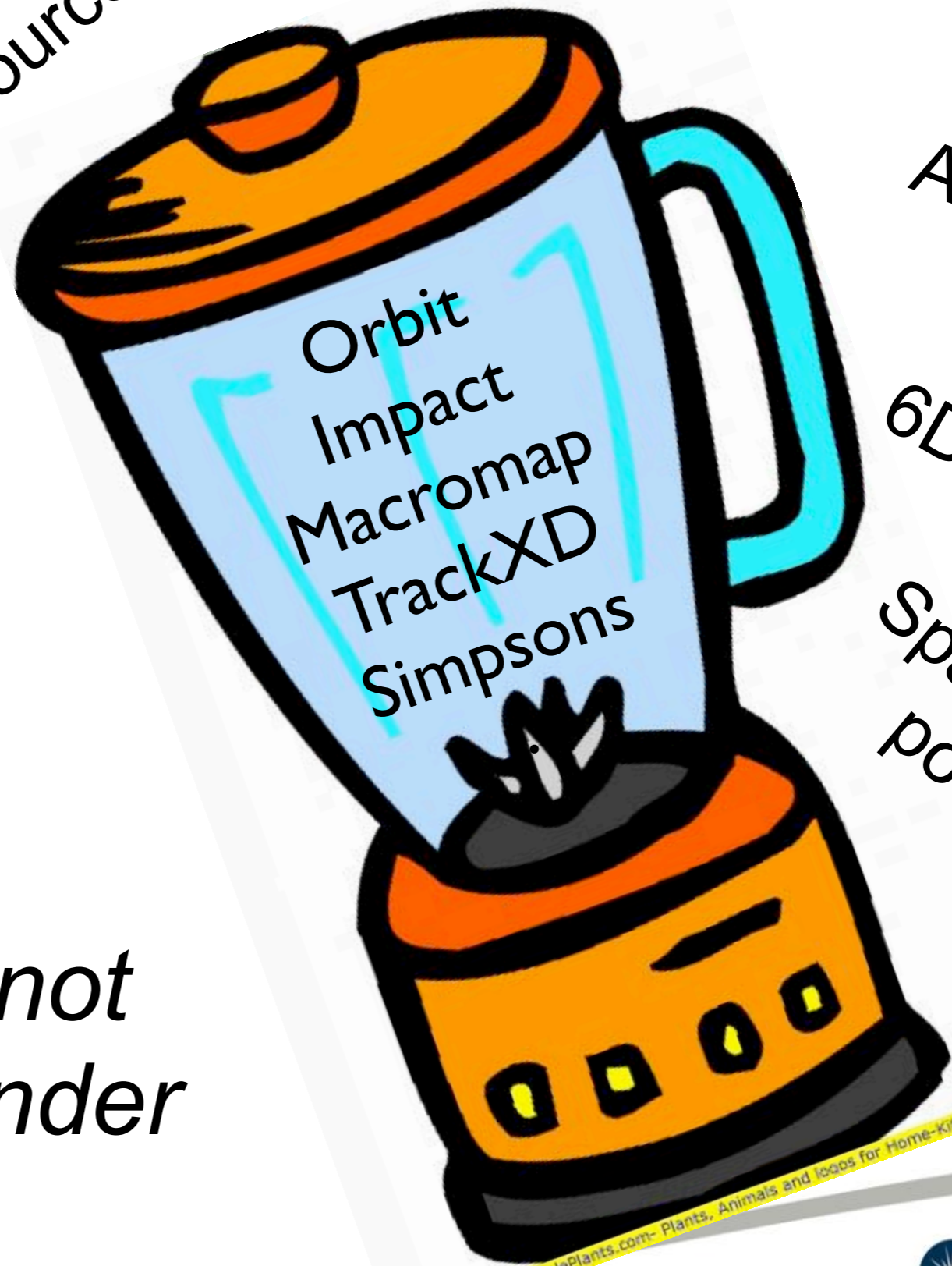
Table of BM-QM tracking error:
qdlnoise.dat

```
868 Mar 22 16:34 apr_coll.dat
761776 Mar 22 16:56 bmripple.dat
515620 Mar 21 15:44 brho.dat
18012 Mar 22 16:34 bump.dat
515620 Mar 22 16:34 dprf.dat
515620 Mar 21 15:44 elknoise.dat
6200062 Mar 22 16:34 exciterh.dat
6200062 Mar 22 16:56 exciterv.dat
60876 Mar 22 16:57 foilpos.dat
664599 Mar 22 16:34 fort.4
1052595 Mar 22 16:56 fort.77
136 Mar 21 16:01 inp
152076 Mar 21 15:44 phbump.dat
515585 Mar 21 15:44 qdlnoise.dat
515585 Mar 21 15:44 qdnnoise.dat
515585 Mar 21 15:44 qdxnoise.dat
515585 Mar 22 16:34 qflnoise.dat
515585 Mar 21 15:44 qfmnoise.dat
515585 Mar 21 15:44 qfnnoise.dat
515585 Mar 21 15:44 qfxnoise.dat
515585 Mar 21 15:44 qnoise.dat
984 Mar 21 15:44 run
766064 Mar 21 15:44 sbedge.dat
71000000 Mar 21 15:44 scat.dat
515620 Mar 21 15:44 snoise.dat
515620 Mar 21 15:44 vhrf.dat
515620 Mar 21 15:44 vrf.dat
```

aperture
bump ripple
brho
bump orbit
phase of 2nd harm rf
...

Good ingredients

Resonance sources



Aperture limit

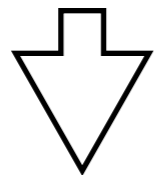
6D dynamics

Space charge potential

It probably does not matter which blender (code) is used.

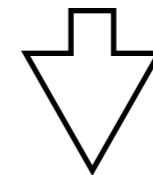
Beam loss budget and imperfection police !?

Impedance budget and
police



Check whether the new
equipment will disturb the
operation.

In the same way, beam loss
budget and police



Check whether the new
equipment will cause
unacceptable beam loss.

Power of modelling of beam loss in a synchrotron
is much improved in the last few years combined
with results from new experimental results.

Accuracy

Toward more accurate simulation

Is there any rational criteria to fix parameters in simulation?

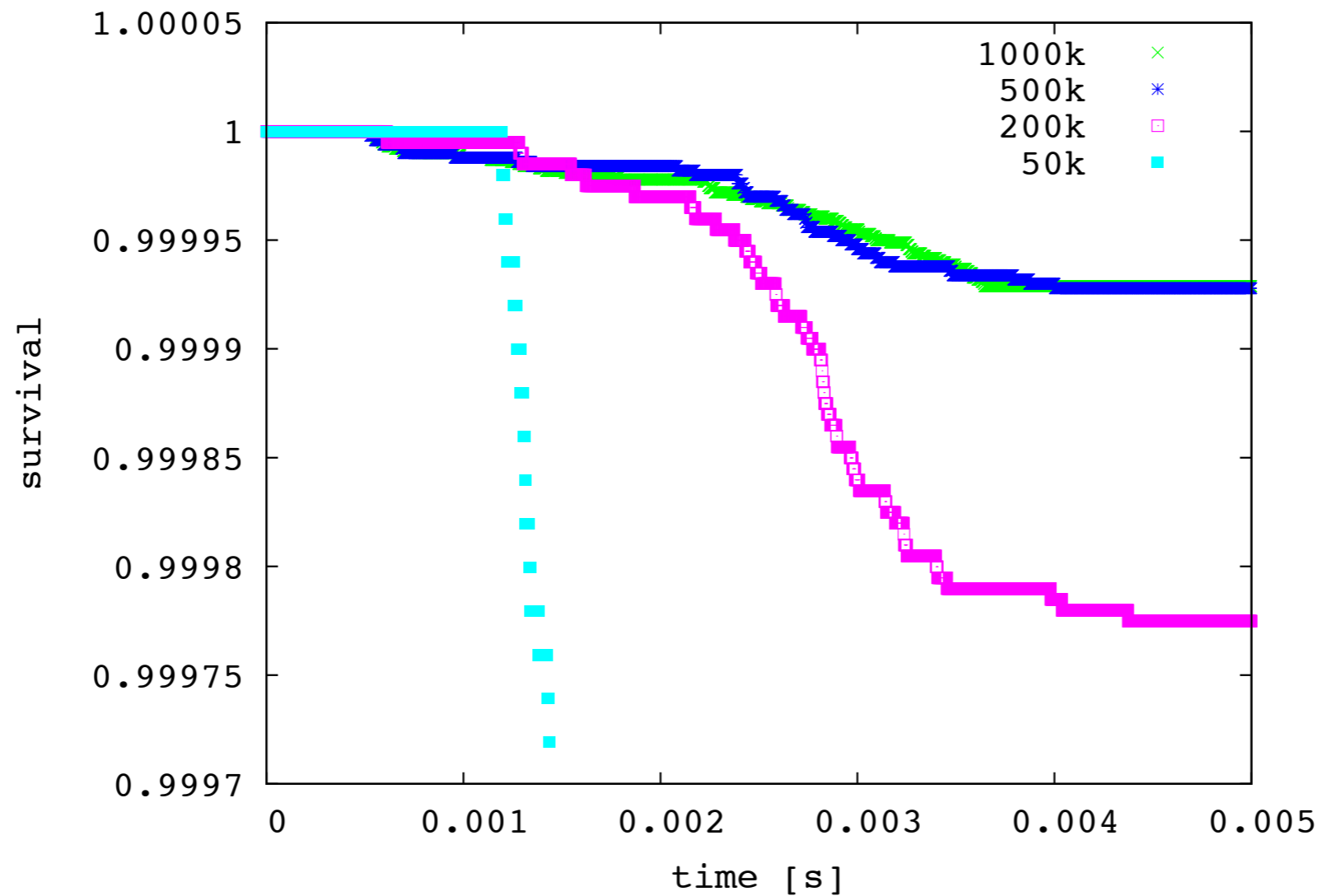
- Number of macro particles.
- Number of grid points in PIC.

Can we estimate error bars in simulation results?

- In particular, when beam loss is very small, e.g. 10^{-4} ?

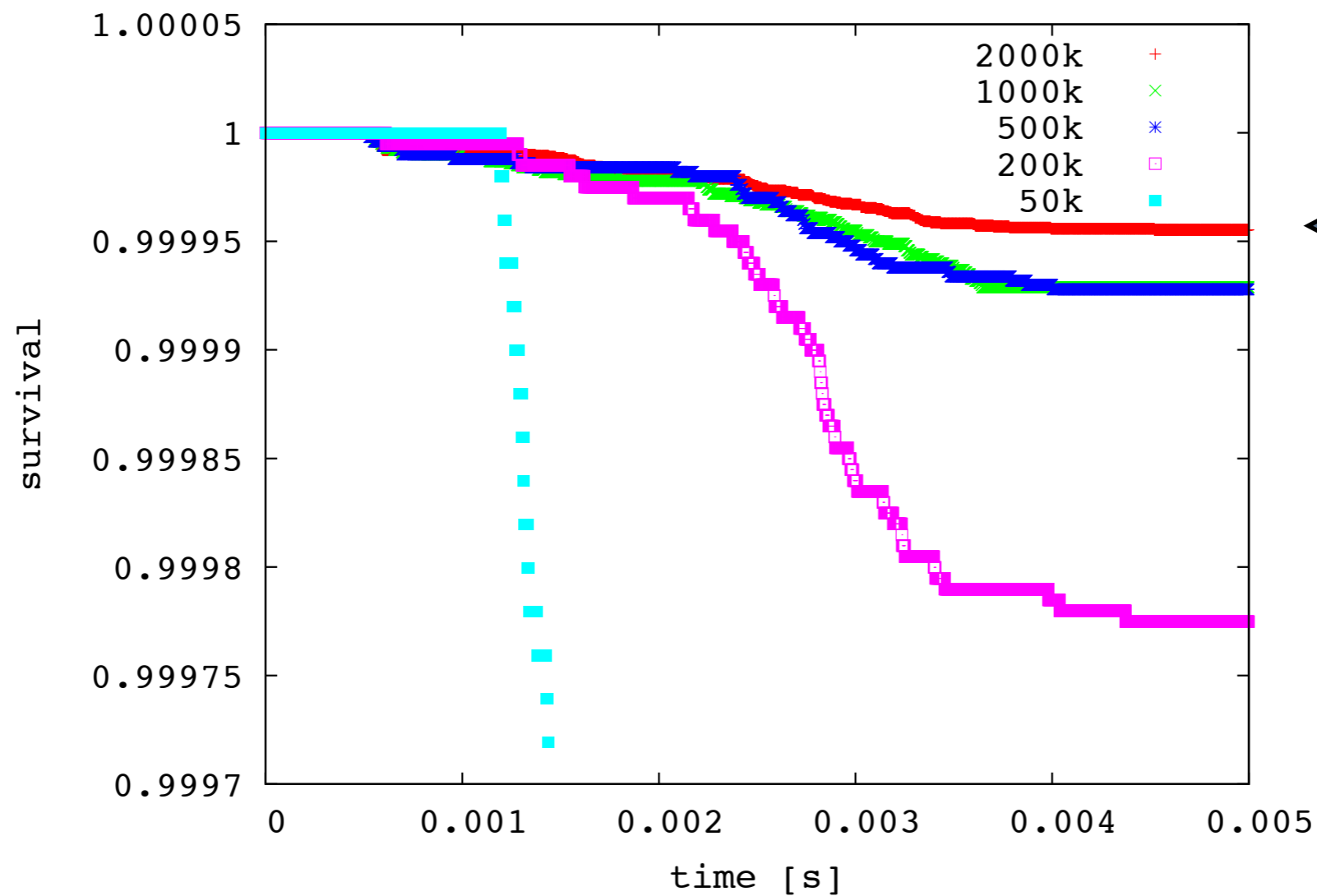
Example 1

Convergence of number of macro particles !



Example 2

Convergence of number of macro particles !?



← x 2

Number of macro particles

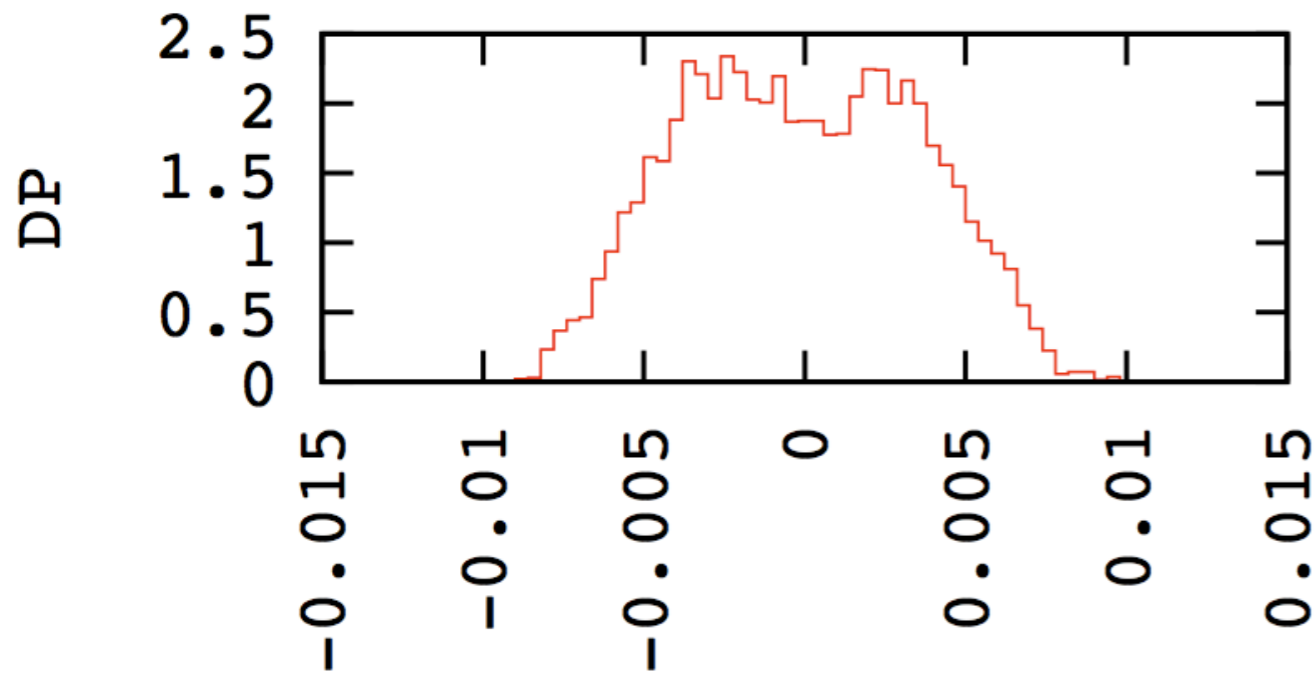
two different problems

If it is not enough,

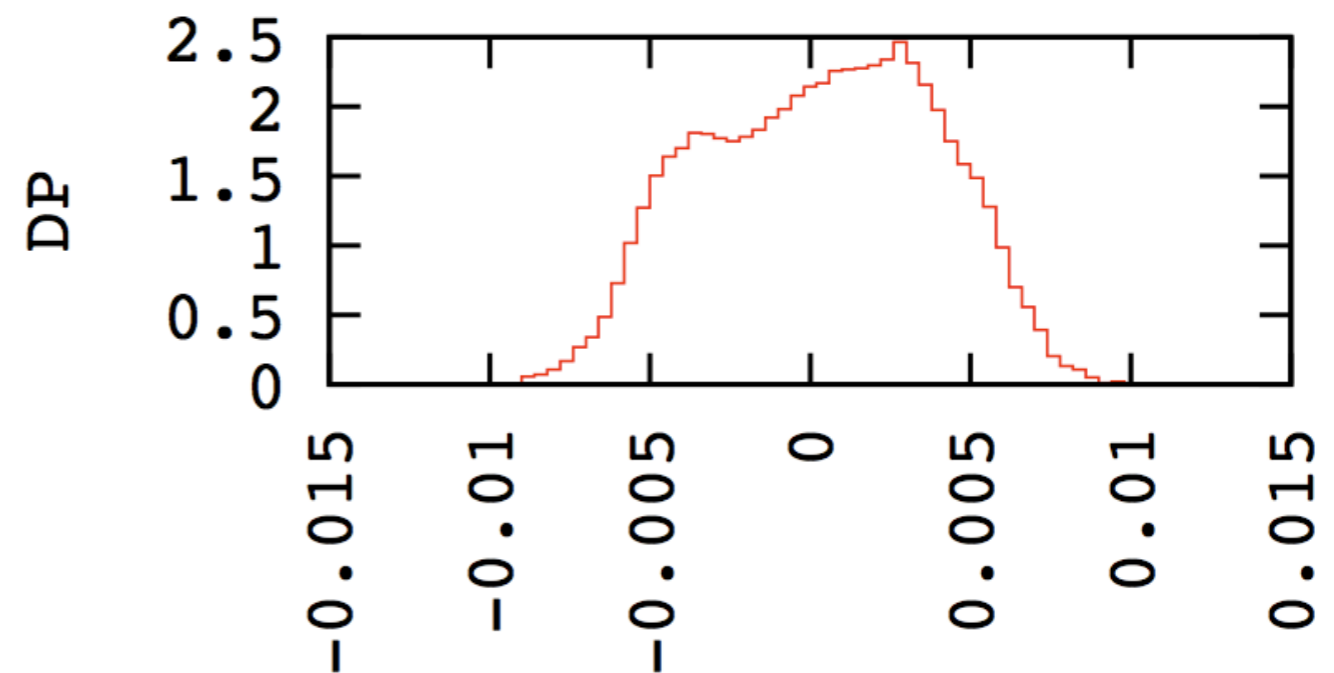
- Fluctuation in each cell (of PIC) region becomes large and causes noise.
 - Distortion of space charge potential.
- Does not cover phase space evenly so that beam loss ratio, for example, is not accurate.
 - Increase statistical error.

Fluctuation in each grid region

Profile at the end of the multi-turn injection



10k macro p



500k macro p

Number of macro particles

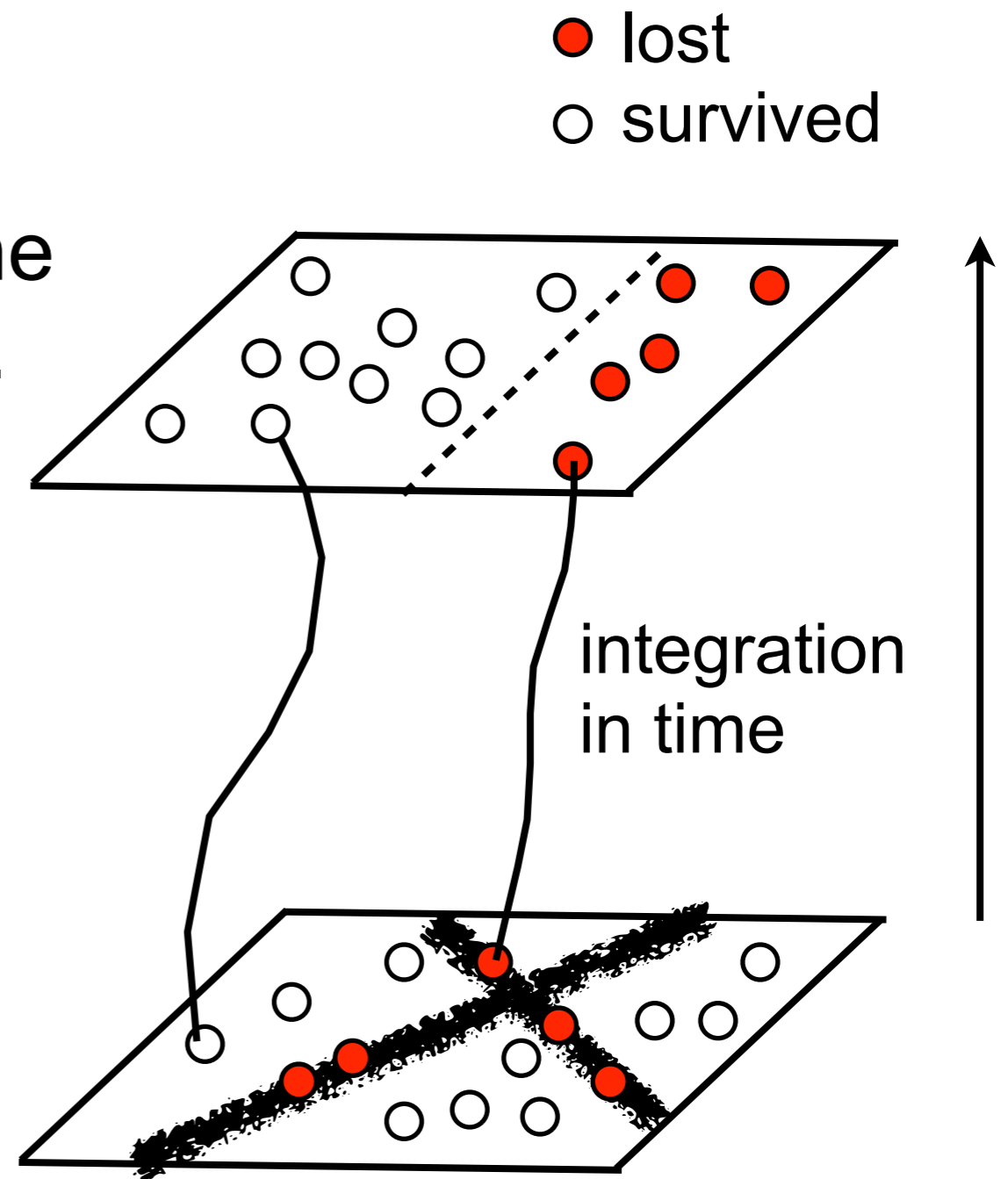
two different problems

If it is not enough,

- Fluctuation in each cell (of PIC) region becomes large and causes noise.
 - Distortion of space charge potential.
- They does not cover fully phase space so that beam loss ratio is not accurate.
 - Statistical error problem.

Not enough cover of initial phase space

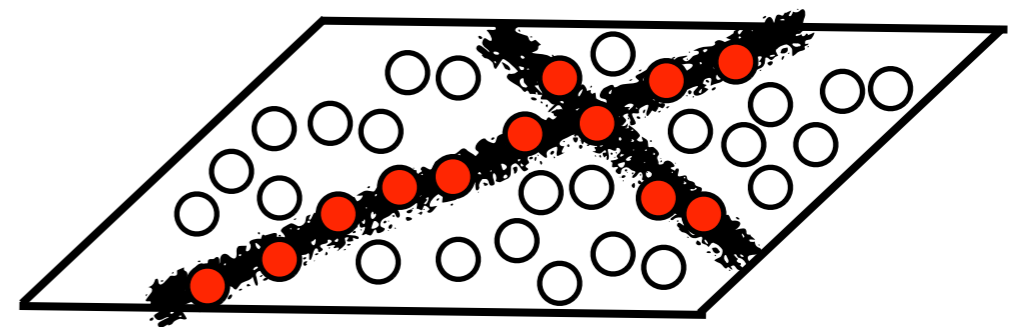
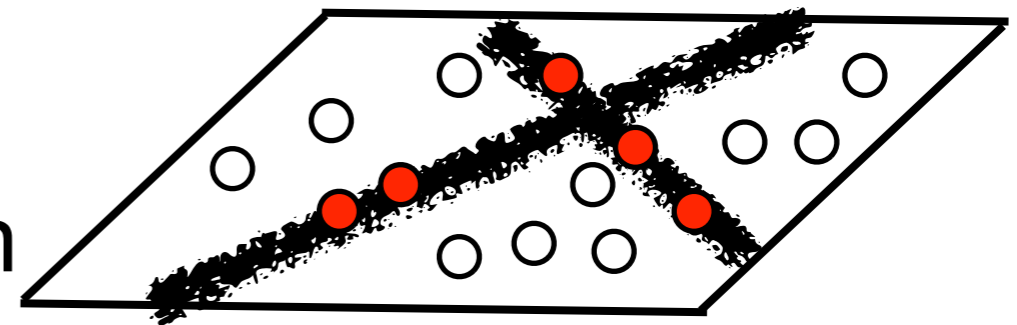
- Assuming there is no chaos, the particle motion is deterministic.
- Multi-particle simulation == Integration in time to find the weighted area in 6D phase space where particles end up outside of aperture.



Not enough cover of initial phase space

- Statistical argument can be applied.
- Estimate of the weighted area in 6D phase space becomes more accurate with more number of particles.
- Error should be $(n_lost)^{1/2}$.

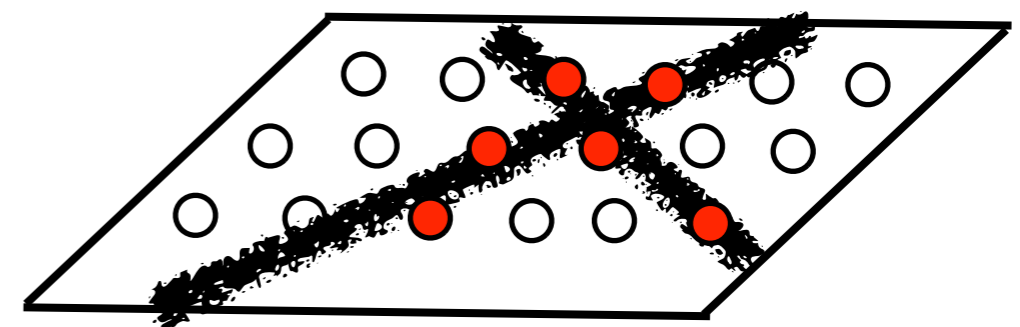
● lost
○ survived



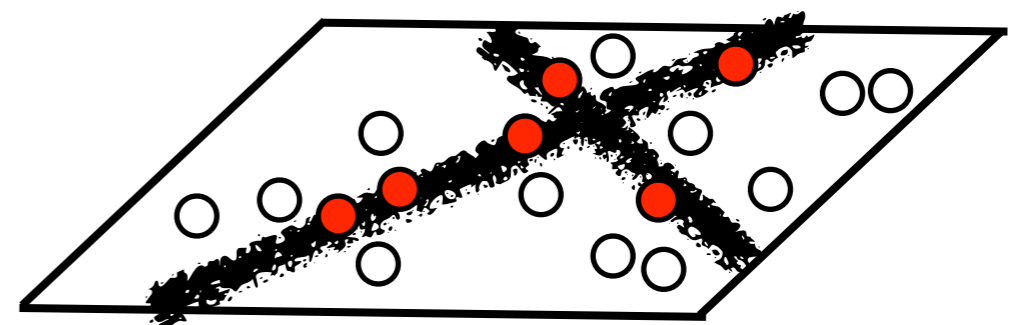
Not enough cover of initial phase space

- Does Quasi Monte Carlo method with low discrepancy sequences (e.g. Halton) help?
- Error could be smaller than $(n_lost)^{1/2}$.

● lost
○ survived



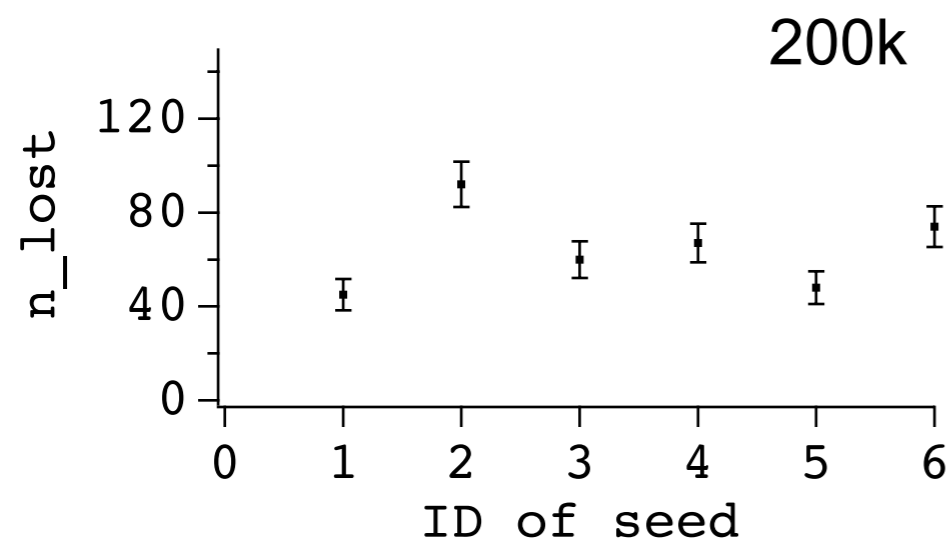
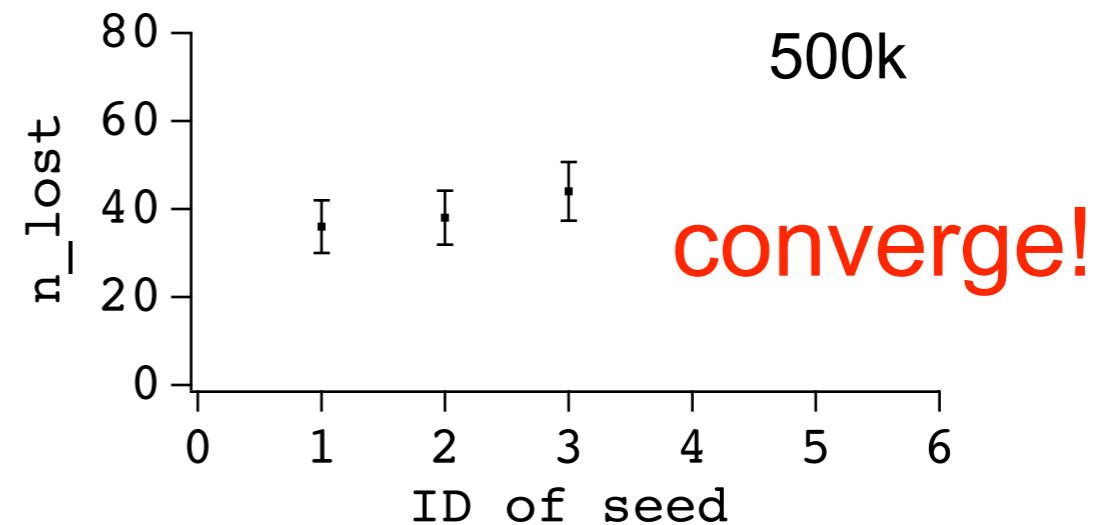
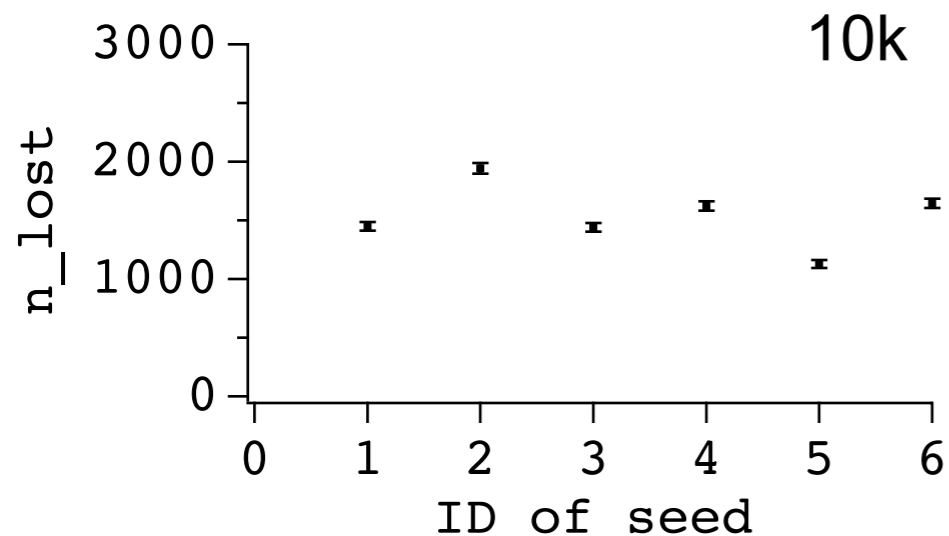
QMC



MC

Test of $(n_lost)^{1/2}$ error

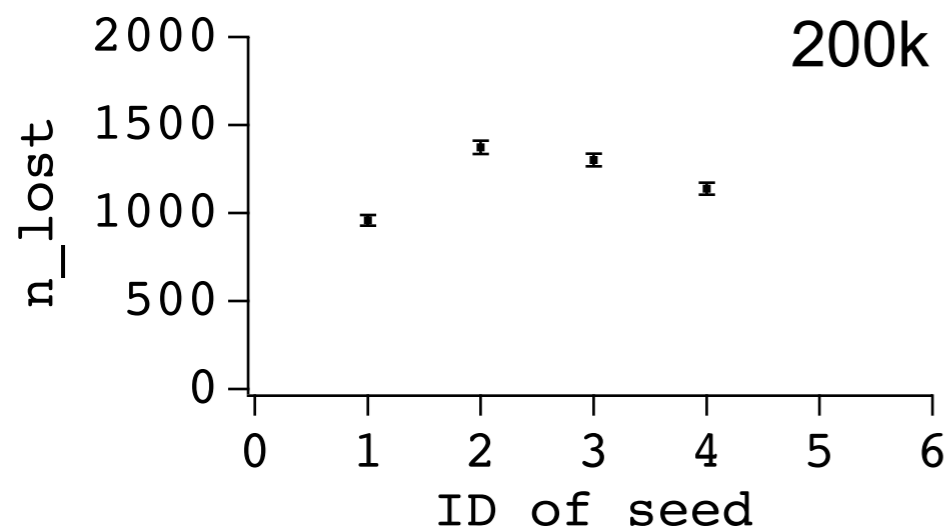
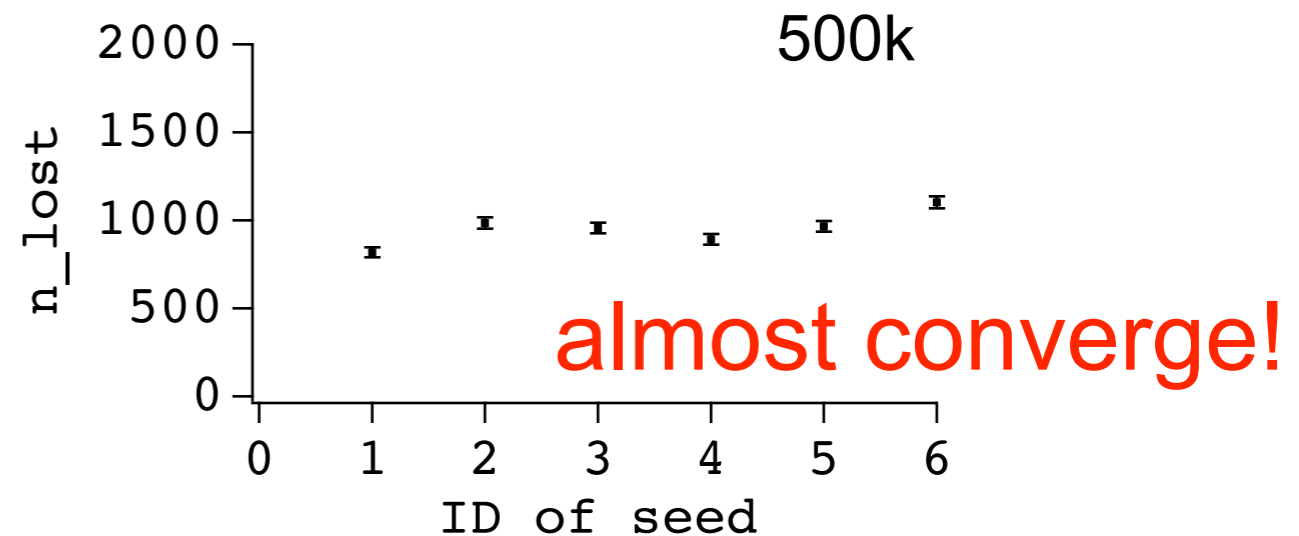
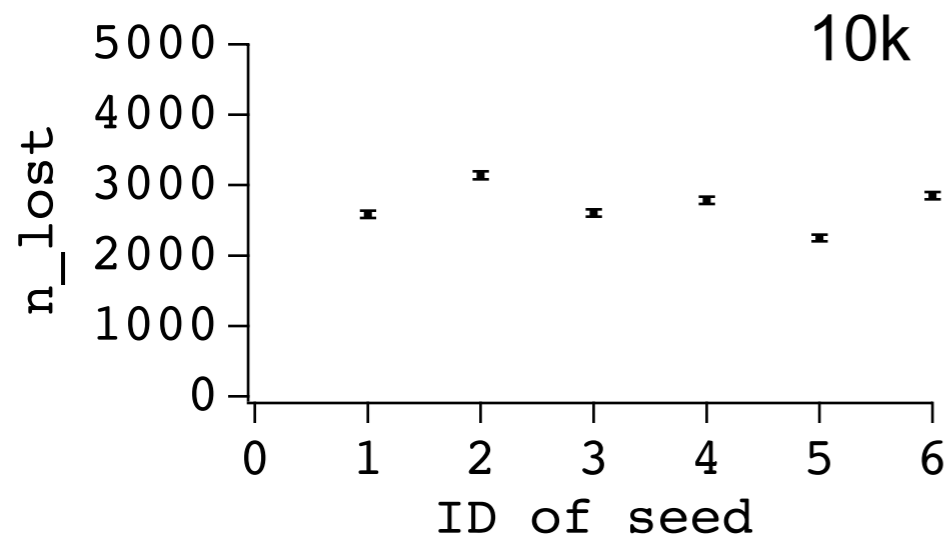
- See the simulation results with different random seed taking the number of macro particles as a parameter.



	n_average	$(n_av.)^{1/2}$	n_std
10k	1537	39	298
200k	64.3	8.0	19.1
500k	39.3	6.3 >	4.1

Another test of $(n_lost)^{1/2}$ error

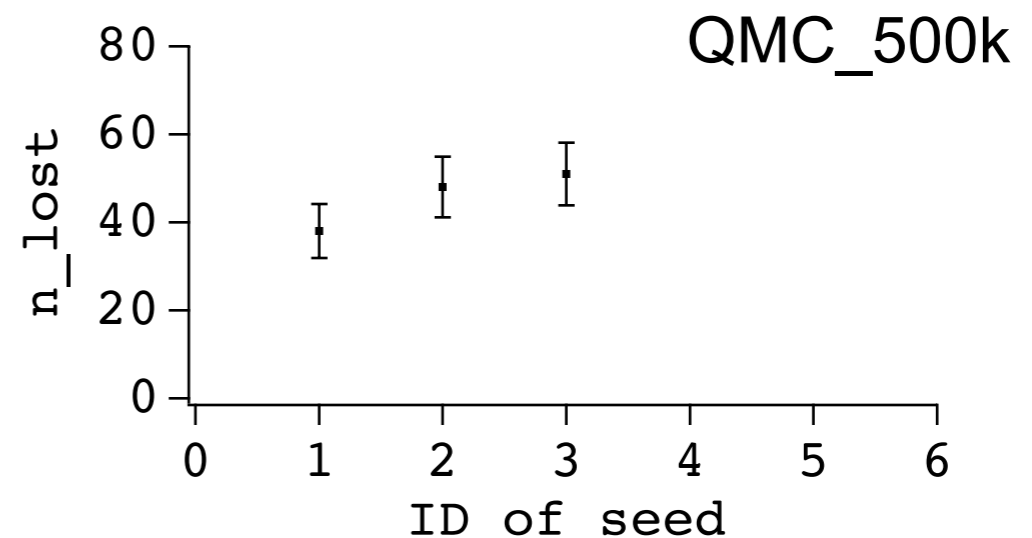
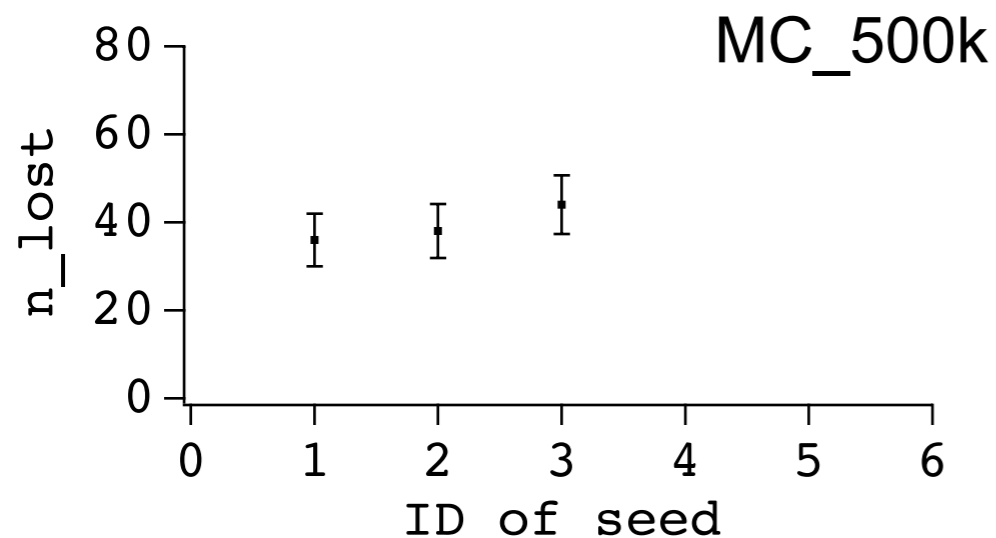
- Test with higher intensity and larger beam loss.



	n_average	$(n_av.)^{1/2}$	n_std
10k	2701	52	327
200k	1193	35	213
500k	953	31	104

MC vs. QMC

- Multi-turn injection destroys low discrepancy sequences.



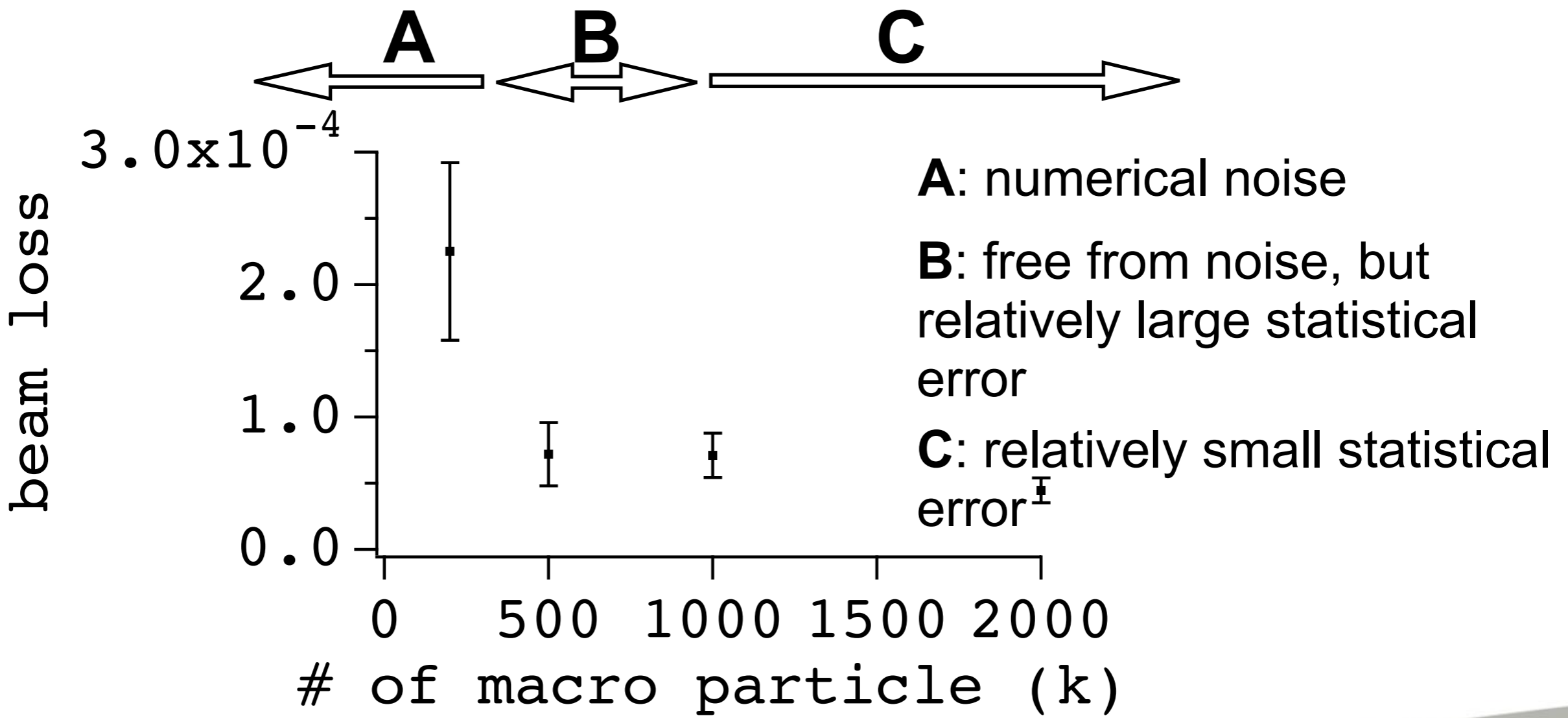
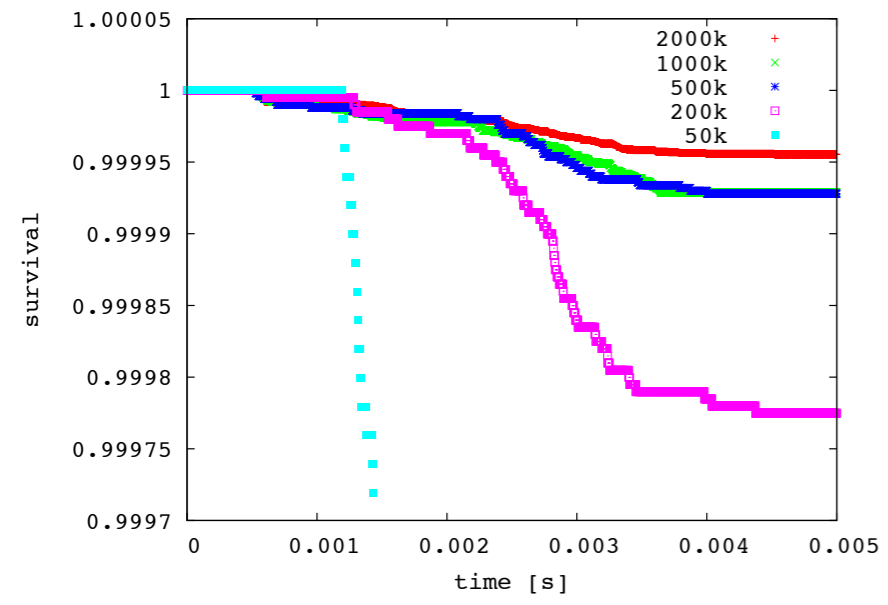
	n_average	$(n_{av.})^{1/2}$	n_std
MC	39.3	6.3	5.1
QMC	45.6	6.8	8.3

no difference!

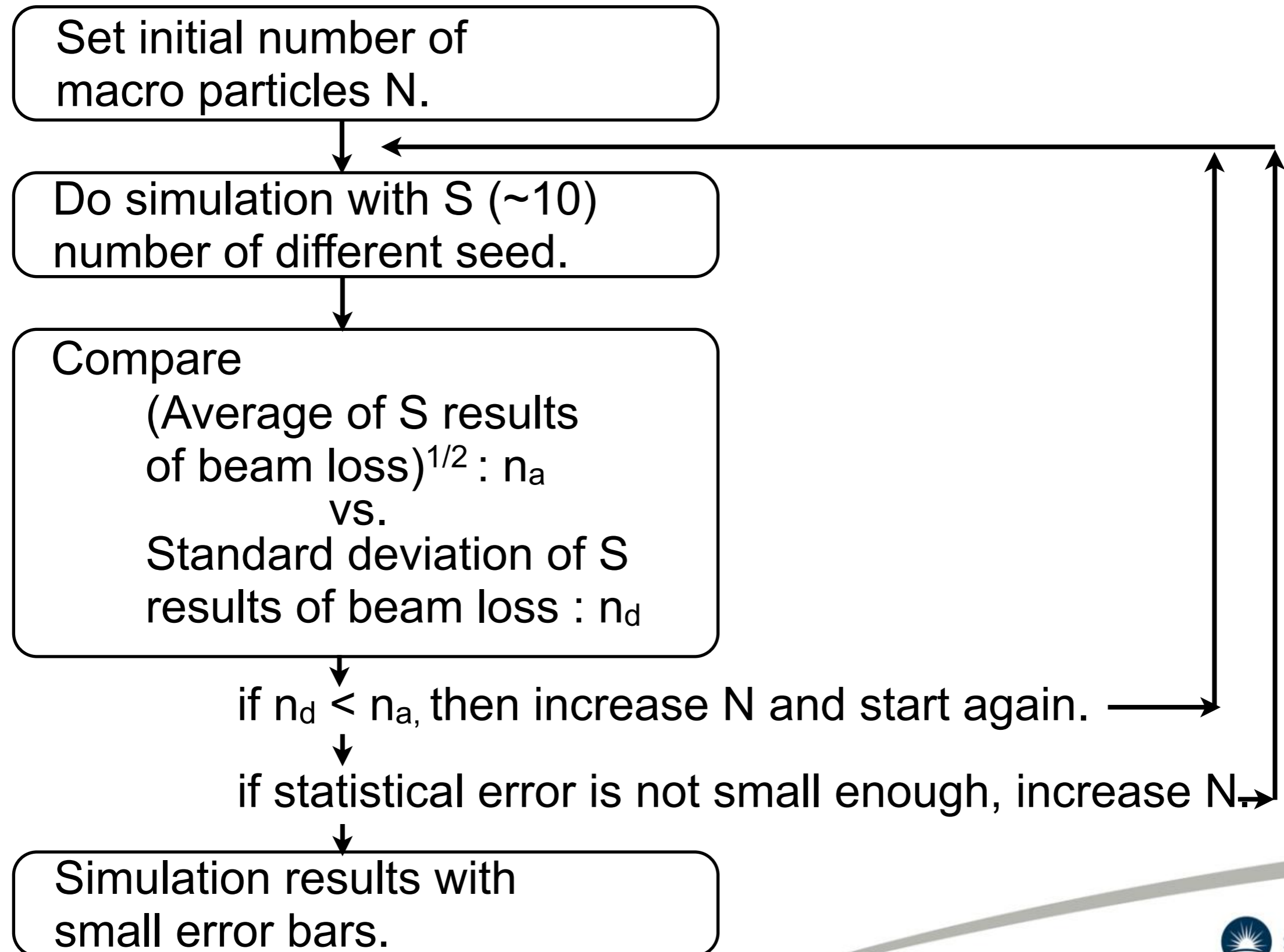
Observations

- When the number of macro particles are too small, fluctuation of the beam loss results with different initial seed is **more than $(n_lost)^{1/2}$. Particles are lost by noise**, not by statistics.
- When the number of macro particles are enough, fluctuation of the beam loss results converges **within $(n_lost)^{1/2}$. It is determined by statistical error**, not by the numerical noise.
- QMC does not show any improvements because the way of simulating multi-turn injection.

Applied to the previous result



Proposed convergence check and error bar



Single particle trajectory

Beam loss is basically single particle phenomena

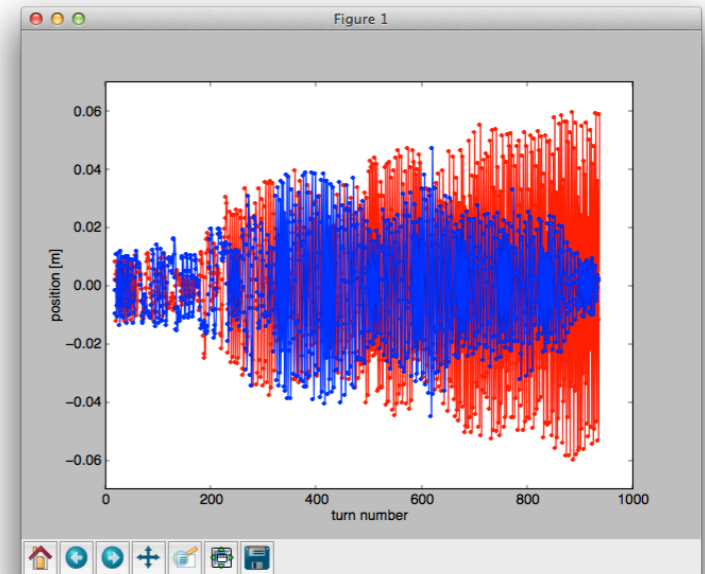
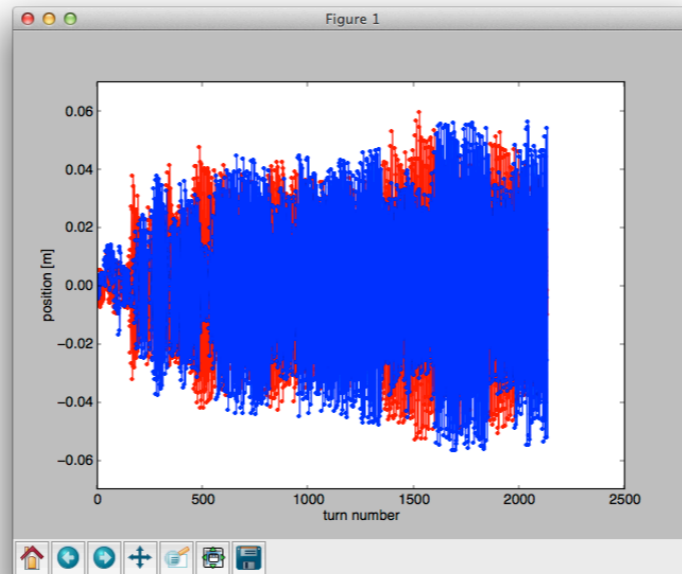
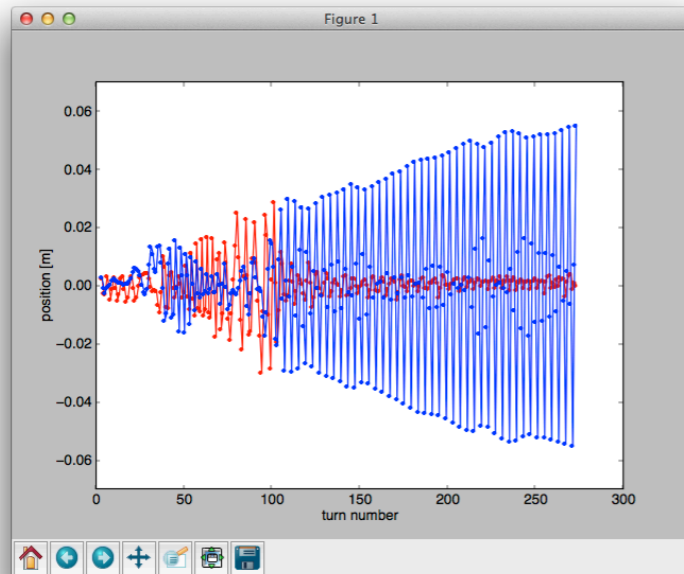
I did not see any presentation discussing a single particle trajectory in PIC simulation.

If we can simulate a small beam loss, single particle trajectory should be accurately modelled.

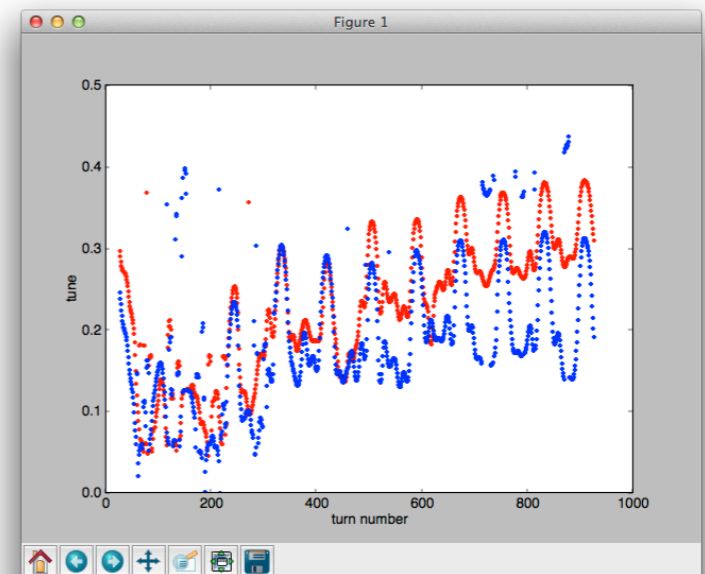
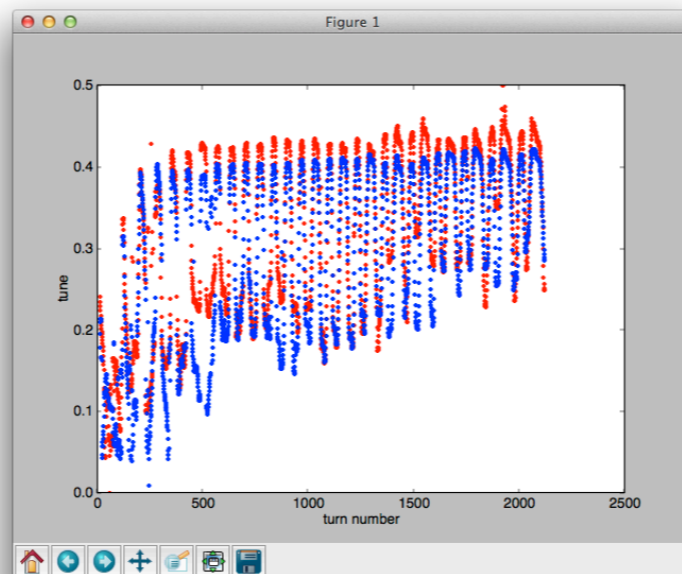
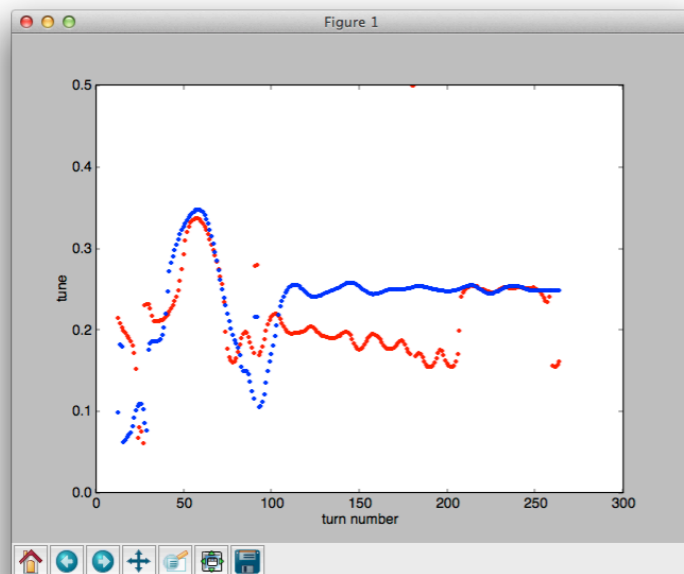
Single particle trajectory in PIC

red: hori, blue: vert

position



tune



turn number

turn number
37

turn number

$$Q_y=0.25$$

$$Q_x-Q_y=0$$

$$Q_x-2Q_y=0$$

Summary

- J-Parc modelling shows us the necessary ingredients. Power of beam loss simulation increases a lot for the last few years.
- A procedure to check the accuracy of simulation is proposed.
- Single particle trajectory in PIC gives useful information to see some physics.