STUDY OF HALO FORMATION IN J-PARC MR

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Abstract

J-PARC is a high intensity proton facility which is constructing as a joint project JAEA-KEK in Japan. J-PARC equips two proton ring accelerators, Rapid Cycle Synchrotron (RCS) and Main Ring (MR). We discuss the space charge effect of MR in this paper. The proton beam with the population of $4.15 \times 10^{13} \times 8$ bunches is accelerated from 3 GeV to 50 GeV and extracted with 0.3 Hz in MR. Beam loss during the acceleration is caused by an incoherent emittance growth due to the space charge force. We discuss the emittance growth and halo formation using a computer simulation based on the particle in cell method.

INTRODUCTION

In high intensity proton machine, radioactivity due to beam loss is serious issue. The beam loss is limited around 1 kW in general. It is only 0.1% for proton machine with the total power of 1 MW. The beam loss is caused by coherent instabilities and/or incoherent emittance growth from the viewpoint of the beam dynamics. Particle distribution in a beam changes turn by turn for a coherent instability, while it evolves slowly for the incoherent emittance growth. The beam loss of 0.1% has to be cared: that is, a small number of particles with a large amplitude, which is regarded as a halo of the beam, have an effect on the beam loss. We discuss emittance growth and halo formation due to the incoherent space charge effect in the J-PARC main ring in this paper.

J-PARC is a high intensity proton facility, which equips H^- linac and two proton synchrotron, rapid cycle synchrotron (RCS) and main ring (MR), with the top energy of 3 GeV and 50 GeV, respectively. The beam loss is limited to be less than 450 W at collimators for the total beam power of 1 MW in J-PARC MR. The main ring is operated 0.3 Hz. Bunch poplation is $4.15 \times 10^{13} \times 8$ bunches is stored 1567.5 m circumference.

The incoherent emittance growth has been studied for strongly nonlinear system in the beam-beam, space charge and beam-electron cloud interactions for a long time [1, 2, 3]. The emittance growth has been discussed with relation to diffusion due to the chaos and/or resonances, and have been also studied with computer simulations. Studies have been done for analizing the beam-particle motion in a potentil given by Gaussian charge distribution analytically.

For an arbitrary charge distribution, we have to rely a numerical simulation. Multiparticle tracking simulation with the particle in cell (PIC) method is typical tool to study the emittance growth. Simulations based on the PIC method have been widely used to study the space charge, beam-beam and beam electron cloud interactions. Macro-particles, which represents the beam, are mapped on a grid space and give a charge distribution, The potential on the mesh points is given by solving Poisson equation for the charge distribution. Motion of the beam particles are integrated in the potential.

The PIC simulation, exactly speaking a self-consistent simulation with macro-particles, is problematic in the estimation of the emittance growth, especially for a system with strongly nonlinearity; the simulation sometimes gives an artificial emittance growth. It is difficult to distinguish the artifact from a physical emittance growth. The artificial emittance growth is caused by the statistical error of macro-particles. The statistical error of the particle distribution is mapped on the mesh, and then it is taken over the potential.

The emittance growth considered here is quite slower compare than revolution. Since we have to care to the loss of $\sim 0.1\%$, it is not necessary to study a disaster situation with a fast emittance growth. For J-PARC-MR, the growth time should be slower than 100,000 turn ($\sim$sec). The beam distribution little changes in one revolution. Hamiltonian including space charge potential is approximately periodic; $\Phi(s + L) = \Phi(s)$. The beam also experiences slow adiabatic damping due to acceleration and slow emittance growth due to the space charge force, which are not periodic: the potential has a small and slow nonperiodic term, $\delta\Phi(s)$.

The potential actually change for an evolution of the distribution. However the potential can have a fluctuation of the potential due to the statistical noise of the particle distribution. A periodic system with a fluctuation has completely different characteristic from that without fluctuation for strong nonlinear system. Particle experience a transition between near integrable orbit to stochastic one and vice versa. In the word of frequency domain, resonance streaming arises.

PIC simulation has been also used in an electron-positron beam. The electron beam contains intrinsic fluctuation due to the radiation excitation. The fluctuation amplitude is $\sigma/2\sqrt{\tau}$, where σ and τ is the beam size and radiation damping time in unit of turn. Numerical noise less than the radiation fluctuation is not serious; number of macro-particles $N \gg 1/\tau$.

We have to treasure the characteristics of the potential. Especially, it should be careful to estimate of the halo formation. We show simulations of halo formation for J-

PARC main ring.

SIMULATION CODE

A simulation code based on the particle in cell method was developed to study the space charge effect in J-PARC. Each particle in the beam moves with experience of space charge force given by mean field of all of them. Longitudinal profile of the bunch length is further smooth than transverse profile, therefore the mean potential is satisfied to two dimensional Poisson equation for the transverse distribution integrated along z, $\rho(x, y; s)$. A local line density is a function of the relative position in a bunch (z), $\lambda(z)$. The potential, which is product of the two dimensional potential and the line density, is expressed by

$$\Phi(x, y, z; s) = \frac{Nr_p}{\beta\gamma^3}\lambda(z)\phi(x, y; s) \qquad \nabla_\perp\phi = \rho, \quad (1)$$

where the coefficient is the normalization factor for the space charge of the beam.

Hamiltonian for particles is expressed by

$$H = H_0 + \Phi(x, y, z; s), \qquad (2)$$

where H_0 is Hamiltonian for transformation of lattice. The equation of motion is integrated as follows,,

$$e^{-:H:ds} = e^{-:H_0:ds/2}e^{-:\Phi:ds}e^{-:H_0:ds/2}. \qquad (3)$$

The transverse kick given by the potential is well-known, $\bar{p}_{x(y)} = p_{x(y)} - Nr_p\lambda/\gamma^3\partial\phi(x, y)/\partial x(y)$. Hamiltonian also gives a longitudinal kick as follows,

$$\bar{p}_z == p_z - \frac{Nr_p}{\beta\gamma^3}\frac{d\lambda}{dz}\phi(x, y). \qquad (4)$$

The potential Φ should be a smooth function of x, y, and z. In the case of beam-beam interaction in e^+e^- colliders with a high tune shift, the beam size changes along the collision due to the so-called hour glass effect. The smooth treatment for z was essential to get a realistic emittance growth and luminosity; simulations gave less luminosity than experiments without this treatment [4]. In our model, ϕ and λ are given as smooth functions of $x-y$ and z with interpolations, respectively, therefore this requirement is satisfied.

The potential is solved by FACR (Fourier analysis and cyclic reduction) algorithm with a rectangular boundary condition. The boundary is chosen to be the actual boundary of the beam chamber. The potential is calculated at 1100 longitudinal positions of the ring with the circumference of 1567.5 m. The beam is kicked by the potential using Eq.(3) for $ds \approx 1.5$ m.

The potential given by PIC method contains a noise due to statistics of macro-particle. The statistical error for a realistic simulation is $1/\sqrt{N} \approx 3 \sim 1 \times 10^{-3}$ for 10^5 or 10^6 macro-particles. While the reduction of the beam size due to acceleration is about $\Delta\sigma/\sigma \approx 10^{-5}$, and enlarge

due to the emittance growth is considered to be similar. For the slow emittance growth, fixed potential is more realistic than turn by turn .

The emittance growth is estimated by some simulation methods; two methods for potential calculation, and two methods for acceleration.

Strong-strong and weak-strong simulation methods are used for potential calculation. In the weak-strong model, particles move in a fixed potential, which is given by a certain charge distribution; for example Gaussian or an arbitrary initial distribution. The potential can be given by PIC method with a fixed distribution. In the strong-strong model, the potential is basically estimated by PIC method turn by turn. In the weak-strong model, the potential given by PIC method is fixed. Though the potential deviates from true one due to the macro-particle statistics, particle trajectory has KAM curve or stochastic layer in the phase space. Though the resonance position and width are slightly different from true ones, the potential includes essentials of the physics: i.e., it equips the nature of the potential.

The frozen potential model is incompatible with acceleration, because of the adiabatic damping. The normalized emittance, $\beta\gamma\varepsilon$, is kept for acceleration. We use two methods to represent the acceleration in the simulation.

In the first method, energy is kept constant during a time interval, and then the beam is accelerated discretely with the energy change, E_0 to E_1, during the time interval. The potential is frozen during the interval. The transformation for transverse variable at the acceleration, which does not change Twiss parameters, is expressed by

$$\boldsymbol{x}_1 = \sqrt{\frac{\beta_0\gamma_0}{\beta_1\gamma_1}}\boldsymbol{x}_0 \qquad \boldsymbol{x} = (x, p_x, y, p_y). \qquad (5)$$

The longitudinal phase space change since the gradient of acceleration voltage is kept.

$$\beta_{z,1} = \sqrt{\frac{\beta_0\gamma_0}{\beta_1\gamma_1}}\beta_{z,0} \qquad (6)$$

In another method, acceleration is applied continuously as is done in actual accelerator; namely

$$p_{i,1} = \frac{\beta_0\gamma_0}{\beta_1\gamma_1}p_{i,0} \qquad i = x, y, z. \qquad (7)$$

Since the beam distribution shrink turn y turn, the potential can not be frozen. The beam distribution (ρ_1) at an energy of E_1 is estimated due to acceleration turn by turn as follows,

$$\rho_1(x_i, y_j) = \frac{1}{a^2}\rho_0(x_i/a, y_i/a) \qquad a = \sqrt{\frac{\beta_0\gamma_0}{\beta_1\gamma_1}}. \qquad (8)$$

where $\rho_0(x, y)$ is the initial distribution. The potential at an energy E_1 is given by solving Poisson equation for ρ_1. The noise included in ρ_1 has the same nature as that of

ρ_0, therefore the turn by turn fluctuations of ρ and ϕ are suppressed.

Figure 1 shows growth of rms emittance for the weak-strong and strong-strong models. Two plots (a) and (b) are depicted for two types of acceleration described above. In the plot (a), the beam is accelerated every 4000 turns. The beam is enlarged during tracking with keeping the energy and is reduced at the discrete acceleration every 4000 turn. Emittance growth during the tracking is no more than 1-2% in 4000 turns for the injection energy. The growth of the weak-strong model is faster than that of strong-strong model at the early stage, while the difference of the emittance shrinks gradually after 10000 turns.
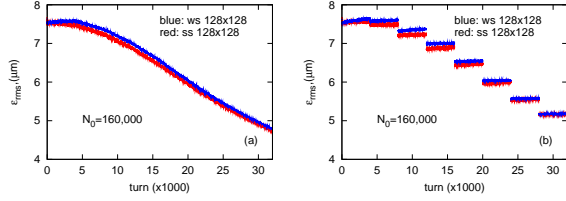

Figure 1: Evolution of rms emittance, $\sqrt{\langle x^2 \rangle \langle p_x^2 \rangle - \langle x p_x \rangle}$.

Figure 2 shows the beam loss for the weak-strong and strong-strong models given by various simulation conditions. Plot (a) depicts the beam loss for the 4 methods, weak-strong or strong-strong models, and discrete or continuous accelerations. Loss for continuous acceleration is better than that for discrete acceleration. It is due to that the beam particles are accelerated with a delay of 4000 turns for the discrete acceleration. The loss given by the weak-strong and strong-strong model is quite different. The loss for weak-strong model is remarkably less than that of the strong-strong model. It is contrast that the rms emittance for strong-strong model is better than weak-strong in Figure 1.

Plot (b) depicts the beam loss for different mesh sizes and the numbers of macro-particles, where the covered area is kept and the mesh size (granularity) of 64×64 is twice rough. In weak-strong model, the loss does not depend on the mesh size and number of macro-particles. In strong-strong, the loss does not depend on the mesh size, but strongly depend on the number of macro-particles.

CONCLUSION

We have studied halo formation in J-PARC-MR using a simulation based on the particle in cell method. The halo estimated by the weak-strong and strong-strong model is quite different. The beam loss for weak-strong model is remarkably less than that of the strong-strong model. It is contrast that the rms emittance for strong-strong model is better than weak-strong. This difference seems to arise from the fact that the potential is quasi-static or fluctuative. The qualitative features of the potential, quasi-periodic and static is rather important than self-consistence and accuracy.
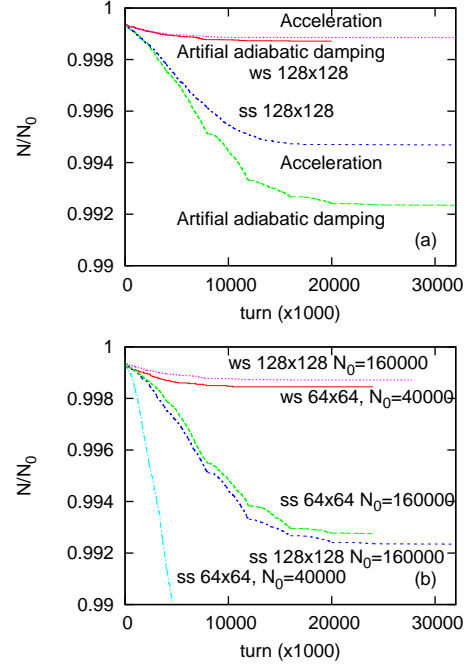

Figure 2: Beam loss for the collimator aperture 70π mm·mrad.

We know similar situation in the simulation of the beam. A rough symplectic integral is better than a precise non-symplectic integral in circular accelerator: i.e., symplecticity is more important than accuracy.

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