Status of Space Charge Effects Studies during Bunch Compression in the future FAIR SIS-100

Sandra Aumon – GSI

Acknowledgements to O. Boine Frankenheim, G. Franchetti, S. Appel, R. Bruce
Contents

1. Final Bunch Compression in the SIS-100 for the FAIR Project
2. Aim of the space charge studies during the bunch rotation
3. Longitudinal aspects of the SIS-100 bunch compression
4. Optics functions deformation due to space charge
   - Transverse envelop equation: Venturini equations
   - Constant focusing example
   - SIS-100 beam envelop with space charge
5. Simulations with PyOrbit: outlooks
6. Conclusions, Outlooks
Final Bunch Compression in SIS-100

- Intense short ions beam required by experiments for **plasma physics** and **exotic elements productions**
- 50ns ions beams after final bunch rotation. Why short beams?

Example:

This is why the bunch should be as short as possible

---

**SIS100 bunch after target**

- 50 ns
- ±2.5 %

**after bunch rotation and debunching in CR**

- ±0.75 %

M. Steck STORI’08

---


---

- Fast bunch rotation of SIS100 bunch to provide optimum initial parameters for stochastic cooling
- Total rf voltage 200 kV at h=1 reduces the momentum spread (2.5 ± 0.5 %) after passage of production target
- 50 ns ions beams after final bunch rotation. Why short beams?
- Example:

This is why the bunch should be as short as possible

---

**SIS100 bunch after target**

- 50 ns
- ±2.5 %

**after bunch rotation and debunching in CR**

- ±0.75 %

M. Steck STORI’08

---

Final Bunch Compression in SIS-100

RF cavity systems in SIS 100:

<table>
<thead>
<tr>
<th></th>
<th>#cavities</th>
<th>Voltage [kV]</th>
<th>Frequency [MHz]</th>
<th>Concept</th>
</tr>
</thead>
<tbody>
<tr>
<td>Acceleration</td>
<td>20</td>
<td>400</td>
<td>1.1-2.7 (h=10)</td>
<td>Ferrite</td>
</tr>
<tr>
<td>Compression</td>
<td>16</td>
<td>600 (later 1MV)</td>
<td>0.4-0.5 (h=2)</td>
<td>MA (low duty cycle)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Particles/bunch</th>
<th>bunch length</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.5 GeV/u U^{28+}</td>
<td>5\times10^{11}</td>
</tr>
<tr>
<td>29 GeV/u p</td>
<td>2-4\times10^{13}</td>
</tr>
</tbody>
</table>

Magnetic alloy RF cavity for bunch compression

SIS 100
L=1083.6 m

rf compressor section (≈40 m)

Courtesy O.Boine Frankenheim (HB2008)
Final Bunch Compression in SIS-100

Single bunch formation

10 bunches

‘bunch merging’

pre-compression

rotation

extraction

50 ns full bunch length after compression required.
Aim SC studies during the rotation

• **What can be wrong during the compression?**
  - Influence of longitudinal space charge
  - Transverse space charge tune shift
  - Resonance crossing?
  - **Effects of transverse space charge on the dispersion and beta functions.**

• **Skeleton of the study**
  - Longitudinal studies (Need the simulations)
  - Analytical study with Venturini transverse envelop equations
  
  *Apply to the SIS100 and the beam transfer*
  
  *Effect of transverse space charge on the optics and beam spot at the target (Still on going)*
  
  - Should be supported by simulations (for instance with PyOrbit, 3D)
    
    *(preliminary convergence studies on going)*

• **SIS-100 has a tiny loss budget (See Giuliano’s talk)**
Longitudinal envelop equation

\[ z''_m + k^2_{z_0} z_m - \frac{K_l}{z^2_m} - \frac{\epsilon_l^2}{z^3_m} = 0 \]

- **RF Potential term**

\[ k^2_{z_0} = \frac{eZVh|\eta|}{2\pi R^2\gamma\beta^2 Amc^2} \]

- **Longitudinal space charge term, Coulomb energy, Perveance**

\[ K_l = \frac{-3gN(Z^2/A)rp\eta}{2\beta^2\gamma^3} \]

\[ g = 0.5 + 2\log(R_p/R_b) \]

- **Kinetic energy, or emittance term, not constant with energy**

\[ \epsilon_l = |\eta|z_m(\delta p/p_0)_0 \]

Ref[1]: M. Reiser, “Theory and Design of Charged Particle Beams”.
Effect of longitudinal space charge during the bunch compression

# Beam parameters at SIS-18 injection

<table>
<thead>
<tr>
<th>Beam Parameters</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Kinetic Energy (MeV/u)</td>
<td>11.4</td>
</tr>
<tr>
<td>Particle</td>
<td>U^{28+}</td>
</tr>
<tr>
<td>Total Energy (MeV)</td>
<td>224 415</td>
</tr>
<tr>
<td>beta/gamma</td>
<td>0.15/1.01</td>
</tr>
<tr>
<td>Momentum Compaction factor $\eta$</td>
<td>-0.94</td>
</tr>
<tr>
<td>Harmonic number $h$</td>
<td>2</td>
</tr>
<tr>
<td>Revolution number ($\mu$s)</td>
<td>4.68</td>
</tr>
</tbody>
</table>

**Parabolic distribution** in momentum

$\Delta p/p = 1e^{-3}$

Courtesy S. Appel

Longitudinal bunch area is computed from coasting beam SIS-18 injection parameters.

$A = 400.4 \text{ eVs}$
Beam parameters before the final bunch compression in the SIS-100

<table>
<thead>
<tr>
<th>Beam Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radius (m)</td>
<td>172.5</td>
</tr>
<tr>
<td>Circumference (m)</td>
<td>1083.6</td>
</tr>
<tr>
<td>Extraction Kinetic Energy (GeV/u)</td>
<td>1.5</td>
</tr>
<tr>
<td>Total Energy (GeV)</td>
<td>552.553</td>
</tr>
<tr>
<td>Beta/gamma</td>
<td>0.91/2.49</td>
</tr>
<tr>
<td>Gamma transition</td>
<td>15.6</td>
</tr>
<tr>
<td>Harmonic number h</td>
<td>2</td>
</tr>
<tr>
<td>Revolution period (μs)</td>
<td>3.95</td>
</tr>
<tr>
<td># bunches</td>
<td>1 (one empty bucket)</td>
</tr>
<tr>
<td>Momentum Compaction factor η</td>
<td>-0.15</td>
</tr>
<tr>
<td>RMS transverse emittance @ 1.5GeV/u</td>
<td>H 3.4, V 1.1 mm.mrad</td>
</tr>
</tbody>
</table>
Bunch Compression in SIS-100

The longitudinal space charge is a weak effect on the final momentum spread at the end of the bunch compression.

Will depend on the $g$ factor.

$$
g = 0.5 + 2 \ln \left( \frac{R_p}{R_b} \right)
$$

$R_p$ Radius pipe
$R_b$ Radius beam

Sandra Aumon – ICFA Space Charge Workshop 2013
Present Situation

- If the bunch to bucket transfer from SIS-18 to SIS-100 is not improved, longitudinal dilution by factor 2 is the present situation.
- **360 kV** available for compression voltage day 1, a full bunch length of **75 ns** is expected from longitudinal envelop equation.
\[ \Delta Q_y = -\frac{r_p q^2}{\pi} \frac{N}{A} \frac{F_y G_y}{\beta^2 \gamma^3 B_f} \frac{1}{\epsilon_y \left(1 + \sqrt{\frac{\epsilon_x Q_y}{\epsilon_y Q_x}}\right)} \]

Only during the last turns of the bunch compression that the tune depresses a lot!
Transverse Envelope Equations

Transverse envelop model (see Reiser book)

\[ a'' + K_x a - \frac{2K_{sc}}{a + b} - \frac{\epsilon_x^2}{a^3} = 0, \]
\[ b'' + K_z b - \frac{2K_{sc}}{a + b} - \frac{\epsilon_x^2}{b^3} = 0. \]

a: hor. Beam size
b: vert. Beam size

Transverse envelop model Venturini et al. PhysRevLetter, Volume 81, number 1

\[ \sigma''_x = \frac{\epsilon_{dx}^2}{\sigma_x} + (\sigma_x \sigma'_x - DD'(\delta^2))^2 \frac{2}{\sigma_x (\sigma_x^2 - D^2)} \frac{\sigma_x^2}{D} - \frac{1}{\sigma_x} (\sigma'_x)^2 \]
\[ \quad - k_x \sigma_x + K \frac{2}{\sigma_x + \sigma_y} + \langle \delta^2 \rangle \frac{D}{\rho} + D'^2, \]
\[ \sigma''_y = \frac{\epsilon_{dz}^2}{\sigma_y^3} - k_y \sigma_y + K \frac{2}{\sigma_x + \sigma_y}. \]

\[ D'' + \left[ k_x(z) - \frac{K}{2\sigma_x (\sigma_x + \sigma_y)} \right] D = \frac{1}{\rho(z)}. \]

Interesting for beam with high momentum spread

\[ H = \frac{1}{2} p_x^2 + \frac{k_x(z)}{2} x^2 + \frac{m^2 c^4}{E_0^2} \delta^2 - \frac{x}{\rho(z)} \delta. \]
\[ \bar{H} = \frac{1}{2} p_x^2 + \frac{k_x(z)}{2} x^2 + \frac{m^2 c^4}{E_0^2} \delta^2 \]
\[ \quad + \delta x \left( D'' + k_x D - \frac{1}{\rho} \right) + \ldots. \]
\[ \epsilon_{x}^2 = \epsilon_{dx}^2 = \langle x^2 \rangle - 2D' \langle x \delta \rangle + D'^2 \langle \delta^2 \rangle. \]
\[ = \langle x^2 \rangle - 2D' \langle x \delta \rangle + D'^2 \langle \delta^2 \rangle \]
\[ \times \left( (p_x^2 - 2D' \langle p_x \delta \rangle + D'^2 \langle \delta^2 \rangle) \right) \]
\[ - (\langle x p_x \rangle - D \langle p_x \delta \rangle - D' \langle x \delta \rangle + DD' \langle \delta^2 \rangle)^2. \]
# Constant focusing case

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Circumference L</td>
<td>1083.6 m</td>
</tr>
<tr>
<td>Radius R</td>
<td>172.6 m</td>
</tr>
<tr>
<td>Transverse tune Q</td>
<td>18.8</td>
</tr>
<tr>
<td>Lorentz factor $\beta/Y$</td>
<td>0.92/2.60</td>
</tr>
<tr>
<td>$B\rho$</td>
<td>63 T.m</td>
</tr>
<tr>
<td>Bending radius $\rho$</td>
<td>31.68 m</td>
</tr>
<tr>
<td>Smooth $k$</td>
<td>0.01188 m$^{-2}$</td>
</tr>
<tr>
<td>Smooth $\beta$ function</td>
<td>9.17 m</td>
</tr>
<tr>
<td>Smooth dispersion $D$</td>
<td>2.66 m</td>
</tr>
<tr>
<td>Transverse RMS $\epsilon$</td>
<td>H 3.4, V 1.1 mm.mrad</td>
</tr>
<tr>
<td>Final bunch length</td>
<td>50 ns (full)</td>
</tr>
</tbody>
</table>

**Strength smooth focusing approximation**

\[ < k > = \left( \frac{2\pi Q}{L} \right)^2 \]

**Beta function smooth focusing approximation**

\[ < \beta > = \frac{R}{Q} \]

**Dispersion smooth focusing approximation**

No space charge

\[ D'' + kD = \rho \]

With $D''=0$

Matched beam

\[ D = \frac{\rho}{< k >} \]

Supposing no longitudinal dilution between SIS18 and SIS100 – **very optimistic**
• Stationary solutions for constant focusing  
  \( D'' = \sigma_x''' = \sigma_y''' = \sigma_x' = \sigma_y' = D' = 0 \) 
  (matched beam)

\[
\frac{e_{dx}^2}{\sigma_x (\sigma_x^2 - (D\delta)^2)} - k > \sigma_x + \frac{K_{sc}}{2(\sigma_x + \sigma_y)} + \frac{\delta^2}{\sigma_x} \left( \frac{D}{\rho} \right) = 0
\]

\[
\frac{e_y^2}{\sigma_y^3} = k > \sigma_y + \frac{K_{sc}}{2(\sigma_x + \sigma_y)} = 0
\]

Simple (!) system of equations to solve

\[
D \left( k > - \frac{K_{sc}}{\sigma_x (\sigma_x + \sigma_y)} \right) = \frac{1}{\rho}
\]
Constant focusing case

- Half Bunch Length [ns]
  - Turn No: 0 to 250
  - Values: 400 to 0

- Max Momentum Spread
  - Turn No: 0 to 250
  - Values: 0.001 to 0.005

- ΔQ
  - Laps [number of turn]: 0 to 250
  - Values: -0.7 to -0.1

- Pervance
  - Laps [number of turn]: 0 to 250
  - Values: 3.5 × 10⁻⁸ to 5 × 10⁻⁹

Sandra Aumon – ICFA Space Charge Workshop 2013
Constant focusing case

- Sens of compression
- Moderate effect
In constant focusing

- Tuning depression dQv
- Tuning depression dQh

Sandra Aumon – ICFA Space Charge Workshop 2013
Transverse Equations in SIS-100

- SIS-100: 6 sectors of ~ 180 m
- Dispersion suppression
- Read a Madx file with: s location, Kxy (strength of quad), I (length of each element), bending angle

- Integration of the equations element by element.
- Method used for integration Runge-Kutta with a maximum step size integration 1mm
- Small envelop tracking w/wo space charge.
- Any other suggestion for other integrator?
- This is not final, because extraction line is going up! **Vertical dispersion**! Will be add later.

\[
\begin{align*}
\sigma''_x &= \frac{\epsilon_x^2}{\sigma_x} + \frac{(\sigma_x \sigma_k^\prime - DD'(\delta^2))^2}{\sigma_x (\sigma_x^2 - D^2(\delta^2))} - \frac{1}{\sigma_x} (\sigma_k^\prime)^2 \\
&- k_x \sigma_x + \frac{K}{2(\sigma_x + \sigma_y)} + \frac{\langle \delta^2 \rangle}{\sigma_x} \left( \frac{D}{\rho} + D^2 \right), \\
\sigma''_y &= \frac{\epsilon_y^2}{\sigma_y^3} - k_y \sigma_y + \frac{K}{2(\sigma_x + \sigma_y)}.
\end{align*}
\]

\[
D'' + \left[ k_x(z) - \frac{K}{2\sigma_x(\sigma_x + \sigma_y)} \right] D = \frac{1}{\rho(z)}.
\]
Transverse Equations in SIS-100

Comparison of my small tracking code with the optics computing by Madx. No space charge and (delta p/p=0) Good agreement.

\[ Q_{x,y} = \frac{1}{2\pi} \int_{s}^{s+C} \frac{1}{\beta_{x,y}(s)} ds \]

Qx=18.84
Qy=18.63

From envelope equation
Transverse Equations in SIS-100

Vertical Beta change with space charge for the expected vertical tune shift, initial conditions in the tracker the same as the non-SC case.

Preliminary results show about 5% change in beta-function at some location.

dQx~0.39  
dQy~0.73

Zoom on half sector
Transverse Equations in SIS-100

Focus on one part of the SIS-100 sector
Transverse Equations in SIS-100

Focus on one part of the SIS-100 sector
Lattice matching

- Transverse space charge strong enough to change even slightly the optics functions.
- Consequences can be emittance blow up and/or beam size breathing during the transport of the compressed bunch to the target.
- New matching: find new matched solutions.
- Now, Newton Method, not robust yet.

\[
\begin{bmatrix}
X \\
X' \\
Y \\
Y'
\end{bmatrix}_0 \rightarrow \begin{bmatrix}
X \\
X' \\
Y \\
Y'
\end{bmatrix}_1
\]

Then delta are applied at each component of the vector

In one dimension, with \(x_{fp}\) is the fixed point

\[
x_{fp} = \frac{x_1 - Jm(x_0)x_0}{1 - Jm(x_0)}
\]

\[
Jm = \frac{f(x_0 + \Delta x) - x_1}{\Delta x}
\]

- This idea would be to go for Jacobian method like done in Madx (thanks R. De Maria, F. Schmidt)
Simulations: outlook

- Use PyOrbit (see the talk of J. Holmes and S. Appel for PyOrbit @ GSI)
- What is planned?
  - Purely longitudinal plan first for bunch compression
  - Longitudinal + Transverse through the full SIS-100 accelerator with space charge.
  - Then transport to the target to see any deformation of the beam spot.
  - Maybe comparison with MICROMAP from Giuliano.

Status: still convergence study of the code, testing.
- Transverse KV distribution from PyOrbit
- Longitudinal parabolic distribution.

Difference about 0.012mm
Conclusions - Outlooks

- Longitudinal space charge should not be a problem for SIS-100 operation.
- Large transverse space charge tune shift ($\Delta Qx=-0.3$, $\Delta Qy=-0.7$) during bunch compression.
- The optics is affected by space charge (up to this point, moderate effect, can be corrected), HOWEVER this has to be propagate through the full lattice and until the target. Vertical dispersion?
- New periodic solutions for mismatch beams wrt to the extraction line.
- 3D simulations to observe the beam spot at the target but also to compare envelope with analytical formula.
- Comparison with simple env. Model (Oliver)
- Other effects during bunch compression (quadrupolar error, resonances etc)

Courtesy Oliver
EPAC 2002
THANK YOU FOR YOUR ATTENTION
Spare Slides
Cross check with Madx

![Graph showing the relationship between Initial $\sigma_x$ and $|\Delta|$]
Comparison ESME - Analytical

- Bunch length as a function of pre-compressed voltage
\[
H = \frac{1}{2} p_x^2 + \frac{k_x(z)}{2} x^2 + \frac{m^2 c^4}{E_o^2} \delta^2 - \frac{x}{\rho(z)} \delta.
\]

One can easily verify that because of the coupling term \(\delta x/\rho\) the standard rms emittance \(\varepsilon_x^2 = \langle x^2 \rangle \langle p_x^2 \rangle - \langle xp_x \rangle^2\), is not an invariant for the system (1). Here \(\langle \cdot \rangle\) denotes the averaging over the phase space variables.

A suitable canonical transformation is generated by \(G_2(x, p_x, z) = p_x[x - \delta D(z)] + x \delta D'(z)\), where \(D(z)\) is a function that will eventually be identified with the dispersion function: \(\bar{x} = x - \delta D(z)\) and \(p_x = \bar{p}_x + \delta D'(z)\).

The transformed Hamiltonian reads

\[
\bar{H} = \frac{1}{2} \bar{p}_x^2 + \frac{k_x}{2} \bar{x}^2 + \frac{m^2 c^4}{E_o^2} \delta^2 \\
+ \delta \bar{x} \left( D'' + k_x D - \frac{1}{\rho} \right) + \ldots.
\]
Venturini Paper: PIC simulations

**Figure 3:** Vertical effective emittances (in units of mm-mrad) for the matched and mismatched case; $I=50$ mA.

**Figure 4:** Horizontal and vertical effective emittances and invariant of Eq. (2) (in units of mm-mrad) for the matched and mismatched case; $I=100$ mA.

Next, in Fig. 3 the evolution of the vertical emittances are reported. The rms $\epsilon_y$ emittance increases because of the nonlinear coupling with the horizontal motion induced by space charge. This effect is not captured by Eqs. (2). However, notice how a matching based on Eq. (2) nevertheless succeeds in reducing the amount of the $\epsilon_y$-emittance growth. The sharp growth that we can observe at extraction is due in part to the fact that at extraction the matching was done under the assumption that $\epsilon_y$ was the same as at injection. In Fig. 4 we show the evolution of the emittances for a case with larger current $I=100$ mA (corresponding to a detuning $\nu/\nu_0=0.15$). In this case the matching is less efficient although still significant. In the last picture (Fig. 5) we report the case in which the matching is done using the equations proposed by A. Garren [1]. One can see that under the regime we are considering that model would lead to an even more pronounced mismatch (A. Garren’s equations coincide with Eqs. (2) in the zero energy spread limit).

**CONCLUSIONS**

The results reported in this paper show that use of the generalized rms envelope equations appears to be effective in achieving acceptable matching conditions for space charge dominated beams in the presence of an energy spread. Moreover, if the tune depression is not extreme the rms emittance growth in the horizontal plane due to dispersion seems to be to a large extent reversible. A measure of the non reversibility is given by the growth of the generalized emittance defined in Eq. (3).

**ACKNOWLEDGMENTS**

We are grateful to A. Friedman and D. Grote for permission to use and assistance with the code WARP.

**REFERENCES**


