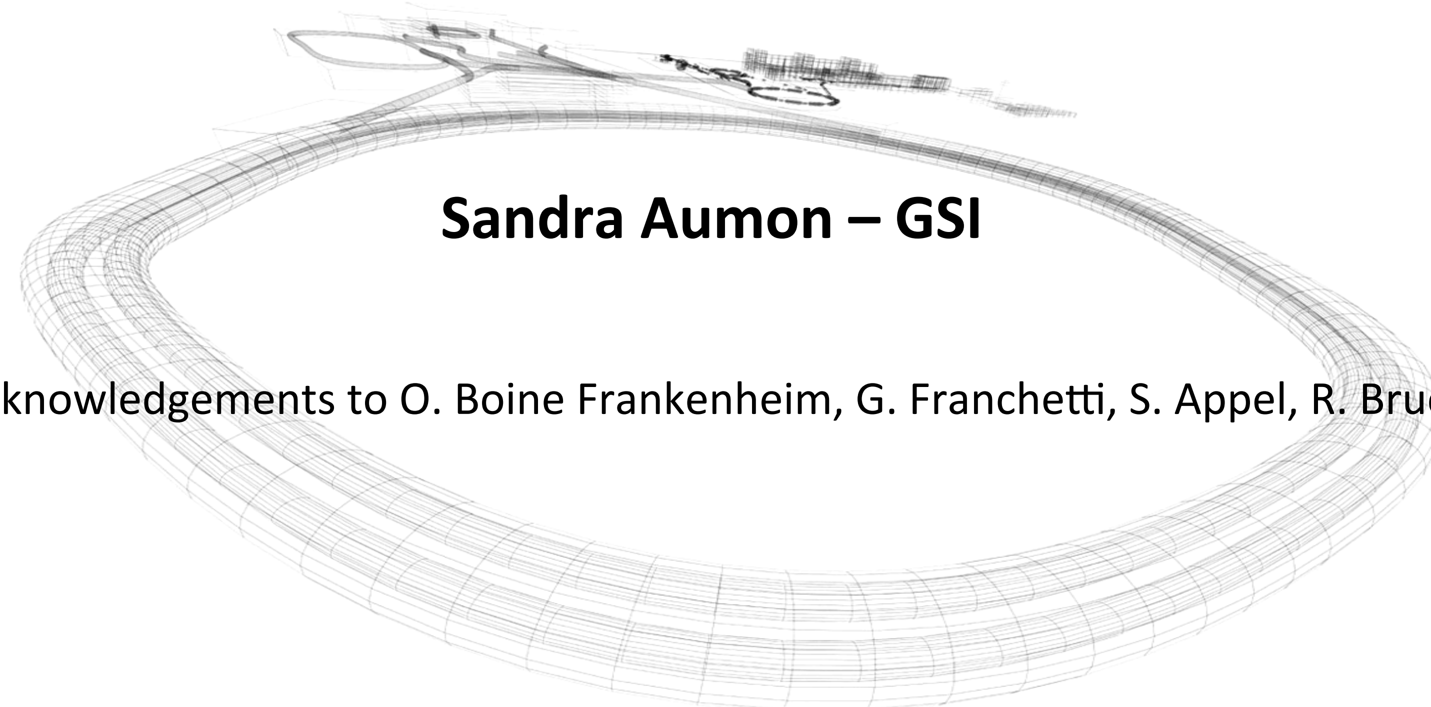
Status of Space Charge Effects Studies during Bunch Compression in the future FAIR SIS-100



Sandra Aumon – GSI

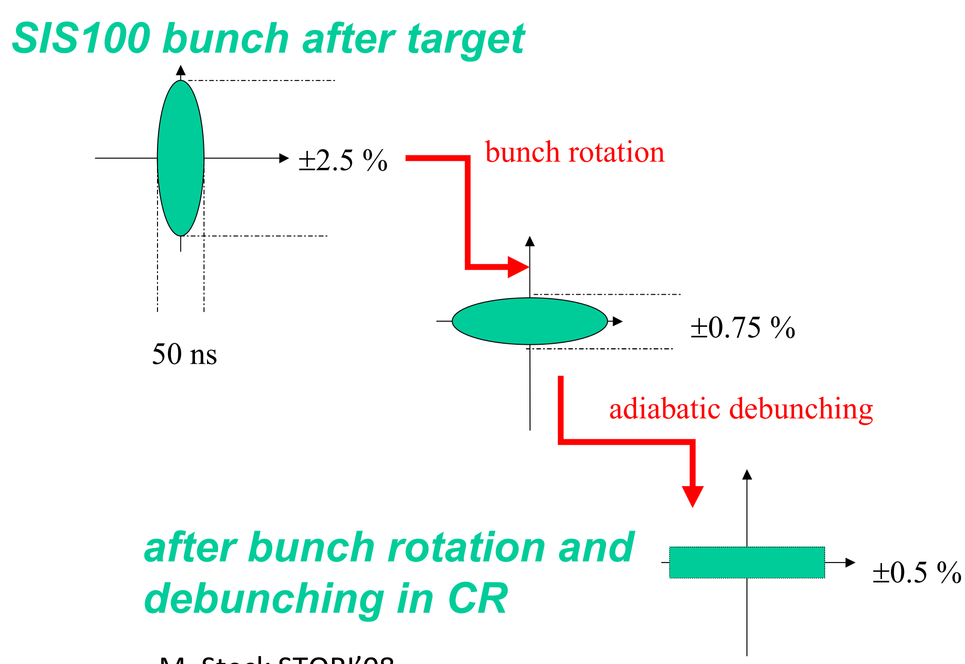
Acknowledgements to O. Boine Frankenheim, G. Franchetti, S. Appel, R. Bruce

Contents

Final Bunch Compression in SIS-100

- Intense short ions beam required by experiments for **plasma physics** and **exotic elements productions**
- 50ns ions beams after final bunch rotation. Why short beams ? Example:

This is why the bunch should be as short as possible



M. Steck STORI'08

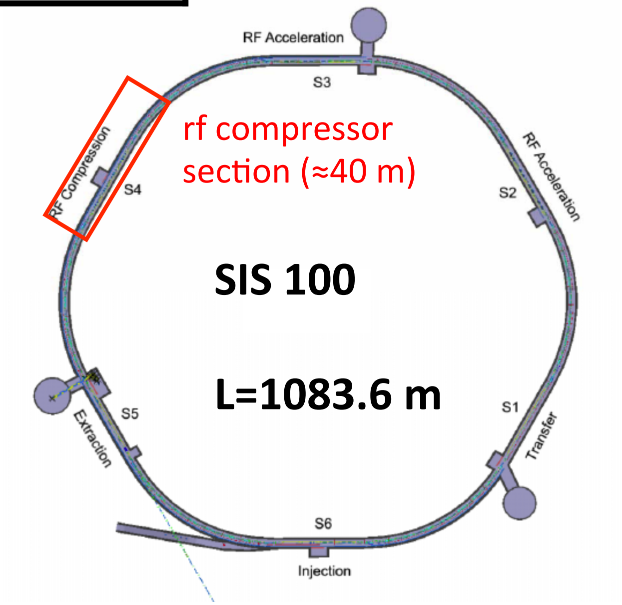
Final Bunch Compression in SIS-100

RF cavity systems in SIS 100:

	#cavities	Voltage [kV]	Frequency [MHz]	Concept
Acceleration	20	400	1.1-2.7 (h=10)	Ferrite
Compression	16	600 (later 1MV)	0.4-0.5 (h=2)	MA (low duty cycle)

	Particles/bunch	bunch length
1.5 GeV/u U^{28+}	5×10^{11}	50 ns
29 GeV/u p	$2\text{-}4 \times 10^{13}$	25 ns

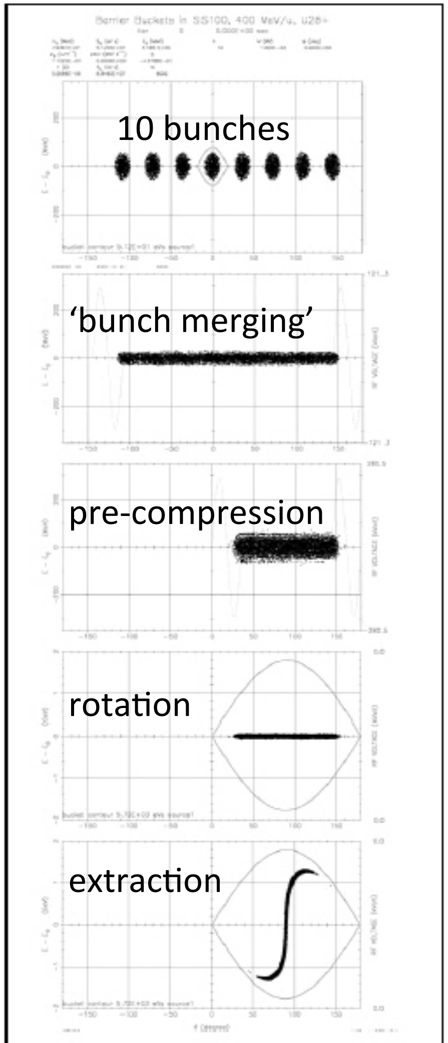
Magnetic alloy RF cavity for bunch compression



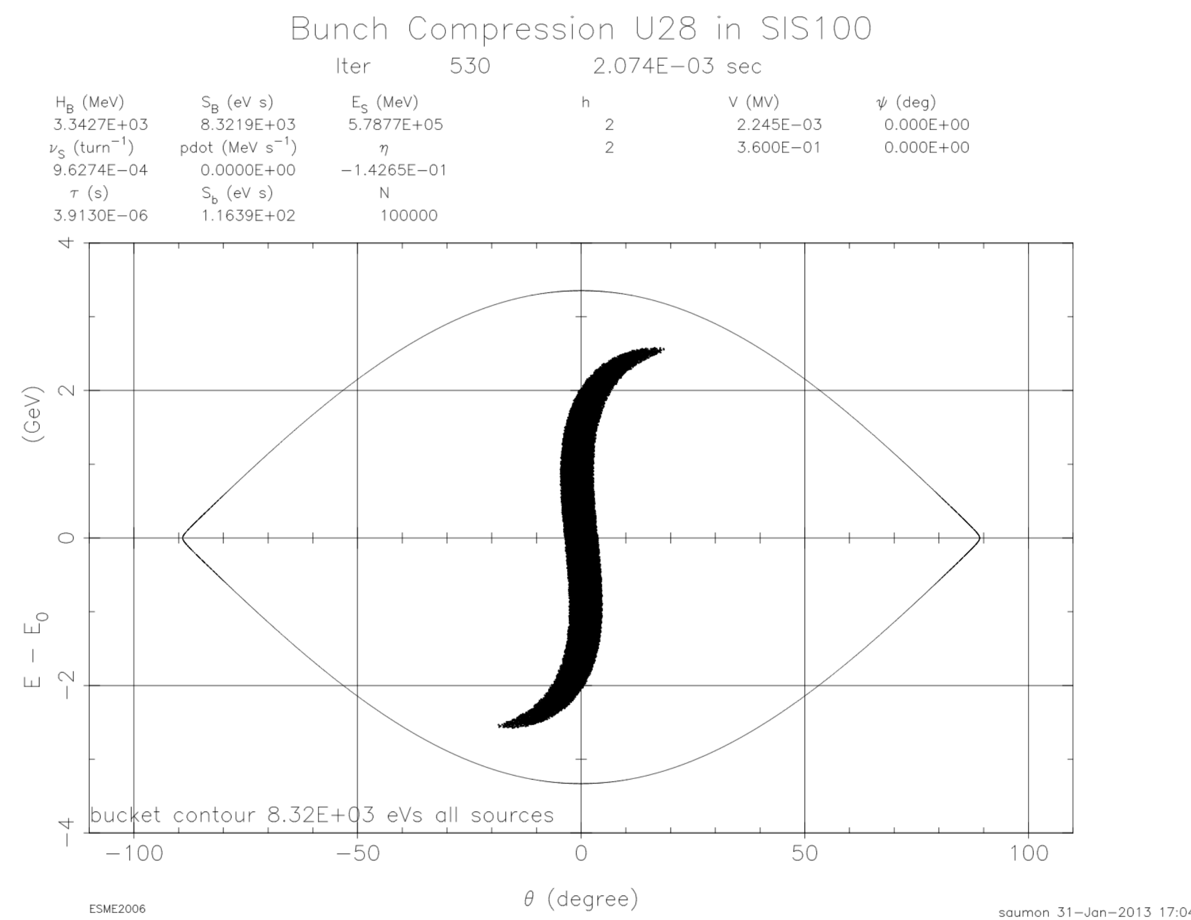
Courtesy O.Boine Frankenheim (HB2008)

Final Bunch Compression in SIS-100

Single bunch formation



50 ns full bunch length after compression required.



Aim SC studies during the rotation

FAIR

- **What can be wrong during the compression ?**
  - Influence of longitudinal space charge
  - Transverse space charge tune shift
  - Resonance crossing ?
  - **Effects of transverse space charge on the dispersion and beta functions.**
- **Squeleton of the study**
  - Longitudinal studies (Need the simulations)
  - Analytical study with Venturini transverse envelop equations

    Apply to the SIS100 and the beam transfer

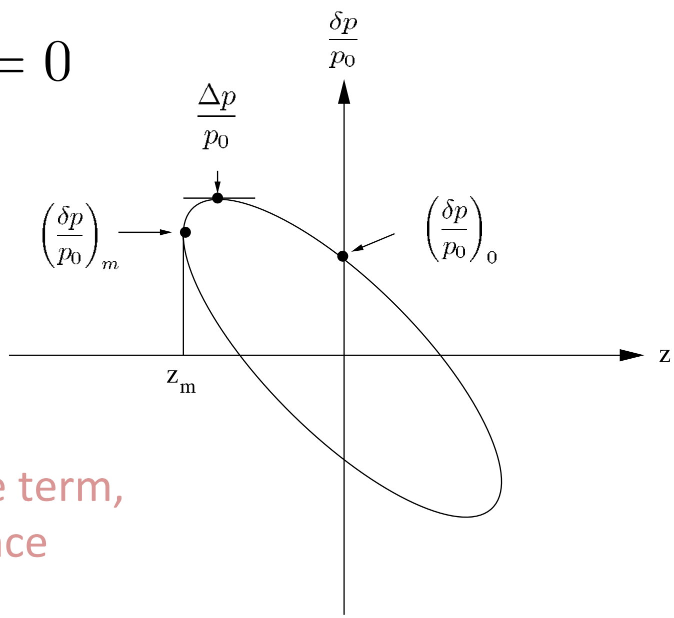
    Effect of transverse space charge on the optics and beam spot at the target (Still on going)
  - Should be supported by simulations (for instance with PyOrbit, 3D)

    (preliminary convergence studies on going)
- **SIS-100 has a tiny loss budget (See Giuliano's talk)**

GSI

Longitudinal envelop equation

$$z_m'' + k_{z_0}^2 z_m - \frac{K_l}{z_m^2} - \frac{\epsilon_l^2}{z_m^3} = 0$$



$$k_{z_0}^2 = \frac{eZVh|\eta|}{2\pi R^2 \gamma \beta^2 A m c^2}$$

RF Potential term

$$K_l = \frac{-3gN(Z^2/A)r_p\eta}{2\beta^2\gamma^3}$$

$$g = 0.5 + 2\log(R_p/R_b)$$

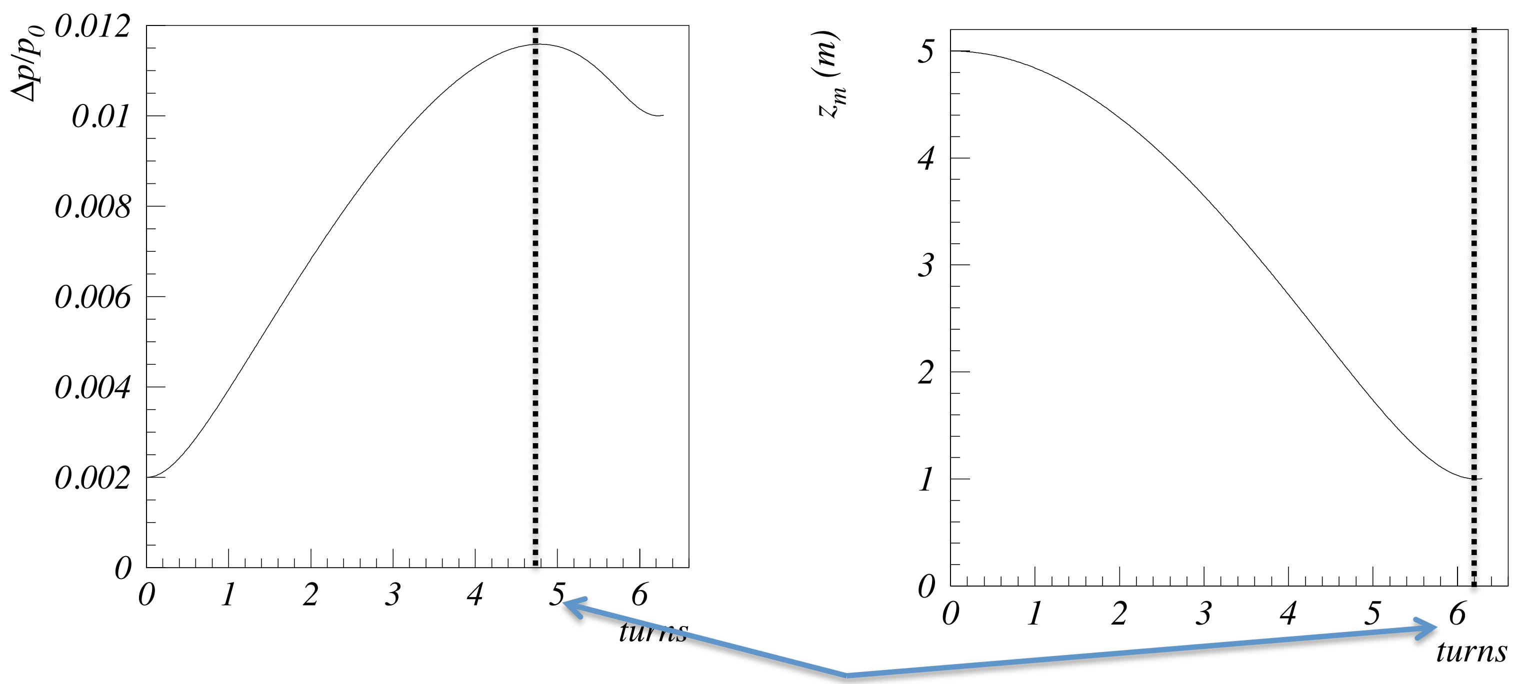
Longitudinal space charge term, Coulomb energy, Perveance

$$\epsilon_l = |\eta| z_m (\delta p/p_0)_0$$

Kinetic energy, or emittance term, not constant with energy

Ref[1]: M. Reiser, "Theory and Design of Charged Particle Beams".
Ref[2]: G. Franchetti, I. Hofmann, G. Rumolo, ***PhysRevSTAB.3.084201***

Effect of longitudinal space charge during the bunch compression

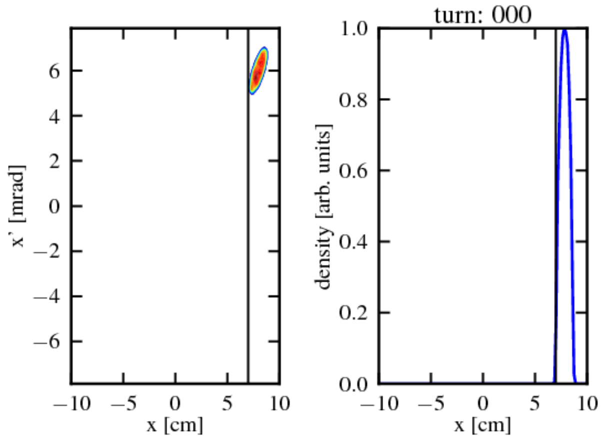


The maximum momentum spread is reached before the end of the compression

Examples from G. Franchetti, I. Hofmann, G. Rumolo, ***PhysRevSTAB.3.084201***

Beam parameters at SIS-18 injection

Beam Parameters	
Kinetic Energy (MeV/u)	11.4
Particle	U $^{28+}$
Total Energy (MeV)	224 415
beta/gamma	0.15/1.01
Momentum Compaction factor η	-0.94
Harmonic number h	2
Revolution number (μs)	4.68



Parabolic distribution in momentum
<u>Δp/p=1e-3</u>
Courtesy S. Appel

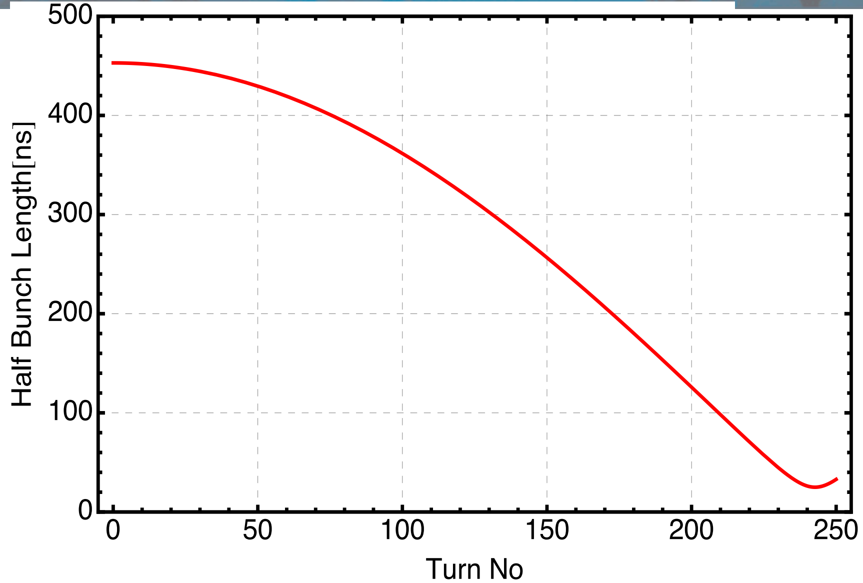
Longitudinal bunch area is computed from coasting beam SIS-18 injection parameters.

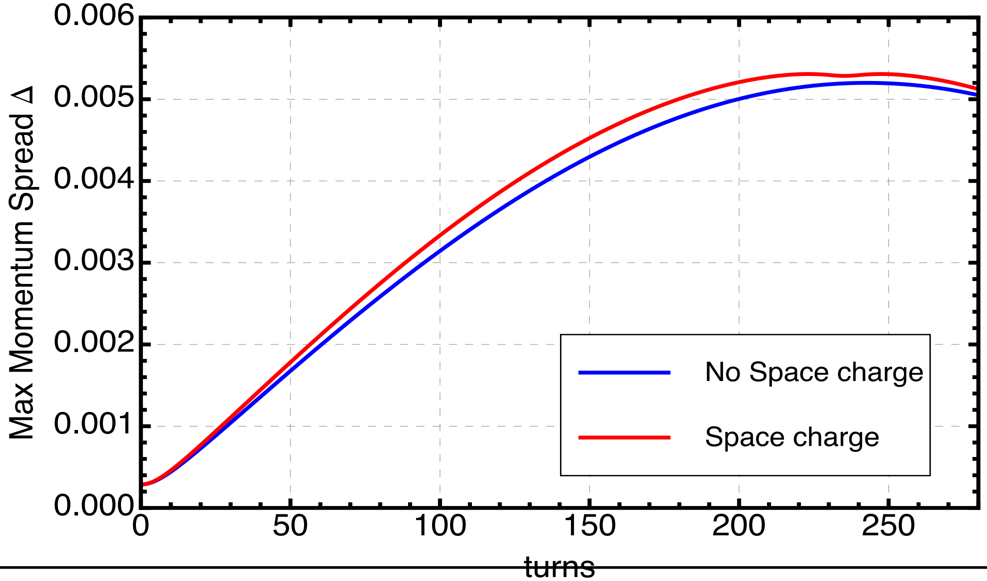
$$\mathcal{A} = 400.4 \text{ eVs}$$

Beam parameters before the final bunch compression in the SIS-100

Beam Parameters	
Radius (m)	172.5
Circumference (m)	1083.6
Extraction Kinetic Energy (GeV/u)	1.5
Total Energy (GeV)	552.553
Beta/gamma	0.91/2.49
Gamma transition	15.6
Harmonic number h	2
Revolution period (µs)	3.95
# bunches	1 (one empty bucket)
Momentum Compaction factor η	-0.15
RMS transverse emittance @ 1.5GeV/u	H 3.4, V 1.1 mm.mrad

Bunch Compression in SIS-100





The longitudinal space charge is a weak effect on the final momentum spread at the end of the bunch compression

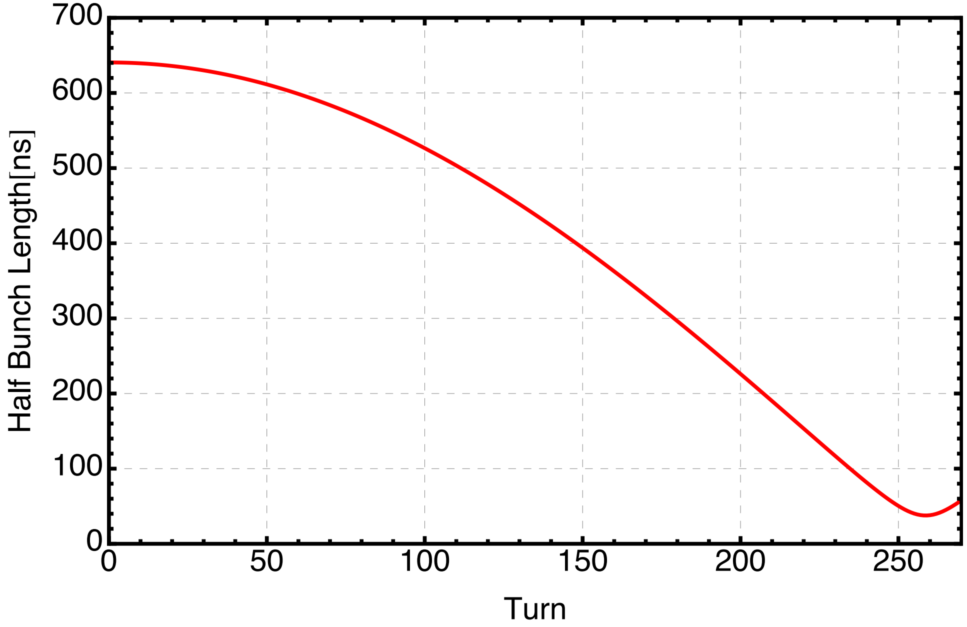
Will depend on the g factor.

$$g = 0.5 + 2\ln\left(\frac{R_p}{R_b}\right)$$

Rp Radius pipe
Rb Radius beam

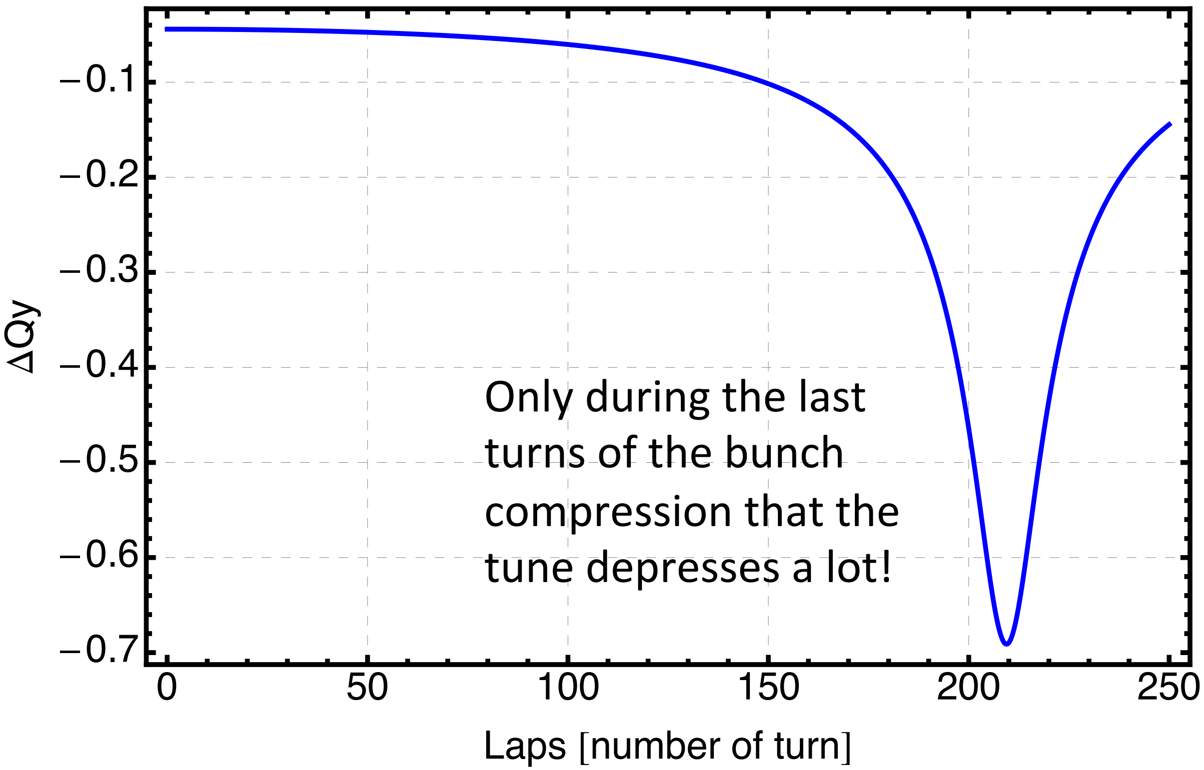
Present Situation

- If the bunch to bucket transfer from SIS-18 to SIS-100 is not improved, longitudinal dilution **by factor 2** is the present situation.
- **360 kV** available for compression voltage day 1,
  a full bunch length of **75 ns** is expected from longitudinal envelop equation.



Transverse Space Charge Tune Shift

$$\Delta Q_y = -\frac{r_p}{\pi}\frac{q^2}{A}\frac{N}{\beta^2\gamma^3}\frac{F_y G_y}{B_f}\frac{1}{\epsilon_y\left(1+\sqrt{\frac{\epsilon_x Q_y}{\epsilon_y Q_x}}\right)}$$



Only during the last turns of the bunch compression that the tune depresses a lot!

Transverse Envelope Equations

FAIR

Transverse envelop model (see Reiser book)

$$a'' + K_x a - \frac{2K_{sc}}{a+b} - \frac{\epsilon_x^2}{a^3} = 0,$$

$$b'' + K_z b - \frac{2K_{sc}}{a+b} - \frac{\epsilon_z^2}{b^3} = 0.$$

a: hor. Beam size
b: vert. Beam size

$$K = \frac{I_p}{I_0 \beta_3 \gamma_3}$$

Interesting for beam with high momentum spread

Transverse envelop model Venturini et al.PhysRevLetter, Volume 81, number 1

$$\sigma_x'' = \frac{\epsilon_{dx}^2 + (\sigma_x \sigma_x' - DD'\langle\delta^2\rangle)^2}{\sigma_x(\sigma_x^2 - D^2\langle\delta^2\rangle)} - \frac{1}{\sigma_x}(\sigma_x')^2 - k_x\sigma_x + \frac{K}{2(\sigma_x + \sigma_y)} + \frac{\langle\delta^2\rangle}{\sigma_x}\left(\frac{D}{\rho} + D'^2\right), \quad (10)$$

$$\sigma_y'' = \frac{\epsilon_y^2}{\sigma_y^3} - k_y\sigma_y + \frac{K}{2(\sigma_x + \sigma_y)}. \quad (11)$$

$$D'' + \left[k_x(z) - \frac{K}{2\sigma_x(\sigma_x + \sigma_y)}\right]D = \frac{1}{\rho(z)}.$$

$$H = \frac{1}{2}p_x^2 + \frac{k_x(z)}{2}x^2 + \frac{m^2c^4}{E_o^2}\delta^2 - \frac{x}{\rho(z)}\delta.$$

$$\overline{H} = \frac{1}{2}\overline{p}_x^2 + \frac{k_x}{2}\overline{x}^2 + \frac{m^2c^4}{E_o^2}\delta^2 + \delta\overline{x}\left(D'' + k_xD - \frac{1}{\rho}\right) + \ldots.$$

$$\epsilon_{\overline{x}}^2 = \epsilon_{dx}^2 = (\langle x^2\rangle - 2D\langle x\delta\rangle + D^2\langle\delta^2\rangle) \times (\langle p_x^2\rangle - 2D'\langle p_x\delta\rangle + D'^2\langle\delta^2\rangle) - (\langle xp_x\rangle - D\langle p_x\delta\rangle - D'\langle x\delta\rangle + DD'\langle\delta^2\rangle)^2.$$

GSI

Constant focusing case

Circumference L	1083.6 m
Radius R	172.6 m
Transverse tune Q	18.8
Lorentz factor β/Υ	0.92/2.60
Bρ	63 T.m
Bending radius ρ	31.68 m
Smooth k	0.01188 m^{-2}
Smooth β function	9.17 m
Smooth dispersion D	2.66 m
Transverse RMS ε	H 3.4, V 1.1 mm.mrad
Final bunch length	50 ns (full)

Strength smooth focusing approximation

$$<k>=\left(\frac{2\pi Q}{L}\right)^2$$

Beta function smooth focusing approximation

$$<\beta>=\frac{R}{Q}$$

Dispersion smooth focusing approximation

No space charge

$$D'' + kD = \rho$$

$$D = \frac{\rho}{<k>}$$

With D''=0

Matched beam

Supposing no longitudinal dilution between SIS18 and SIS100 – **very optimistic**

Constant focusing case

- Stationary solutions for constant focusing D'' = σ_x'' = σ_y'' =σ_x' = σ_y' = D' = 0 (matched beam)

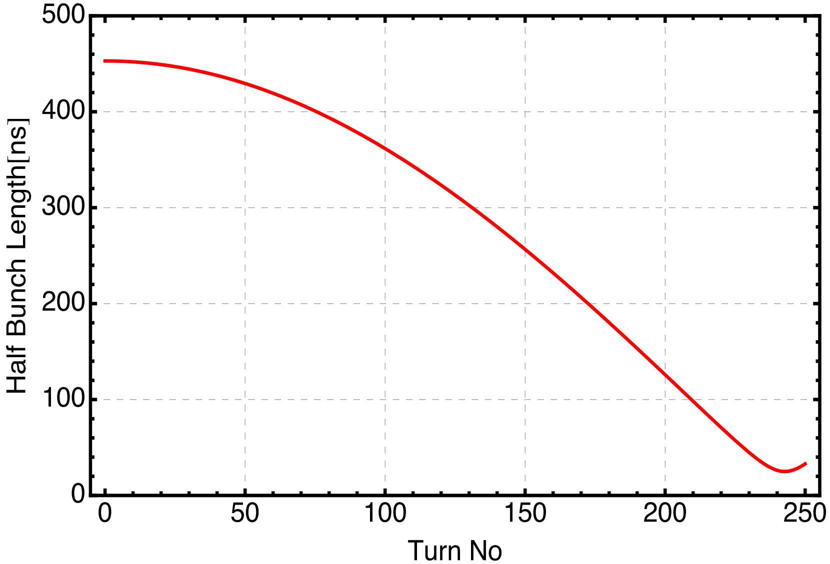
$$\frac{\epsilon_{dx}^2}{\sigma_x(\sigma_x^2-(D\delta)^2)} - <k> \sigma_x + \frac{K_{sc}}{2(\sigma_x+\sigma_y)} + \frac{\delta^2}{\sigma_x}\left(\frac{D}{\rho}\right) = 0$$

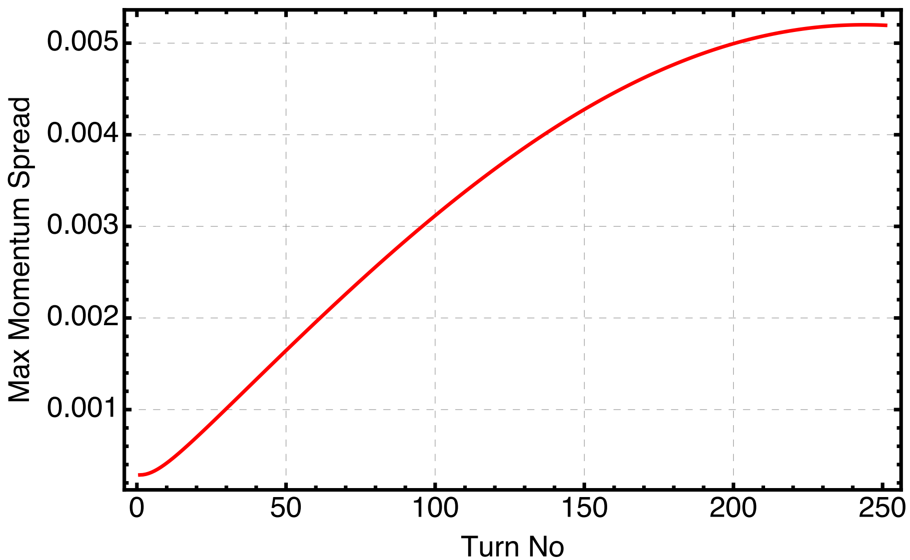
$$\frac{\epsilon_y^2}{\sigma_y^3} - <k> \sigma_y + \frac{K_{sc}}{2(\sigma_x+\sigma_y)} = 0$$

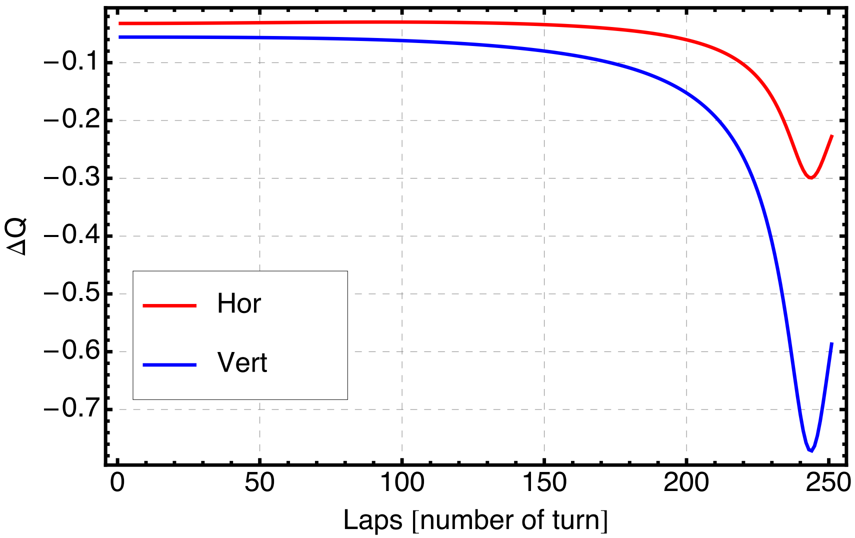
Simple (!) system of equations to solve

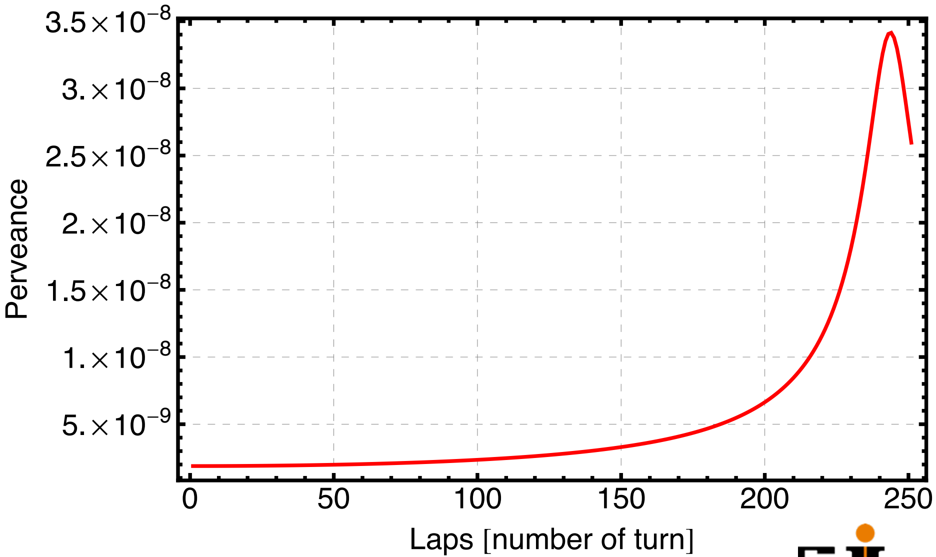
$$D\left(<k> - \frac{K_{sc}}{\sigma_x(\sigma_x+\sigma_y)}\right) = \frac{1}{\rho}$$

Constant focusing case

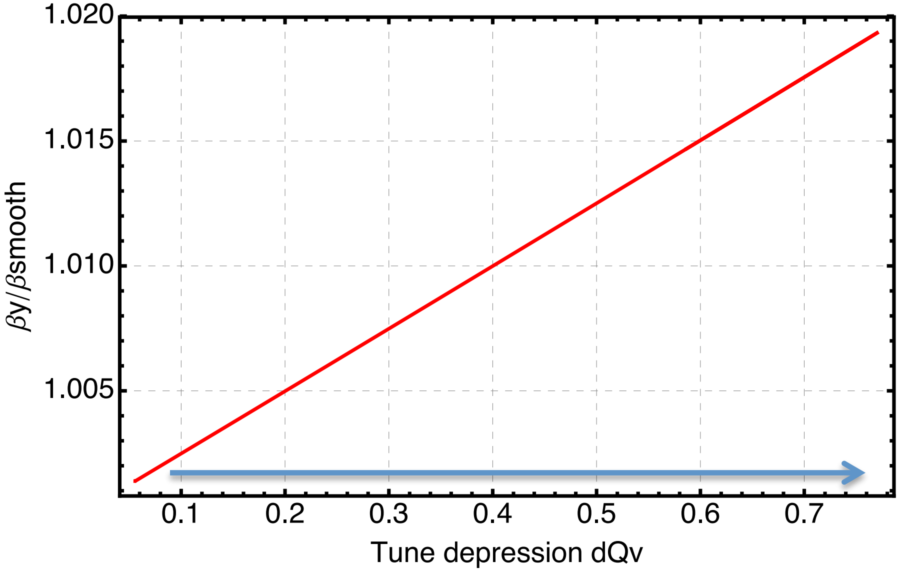


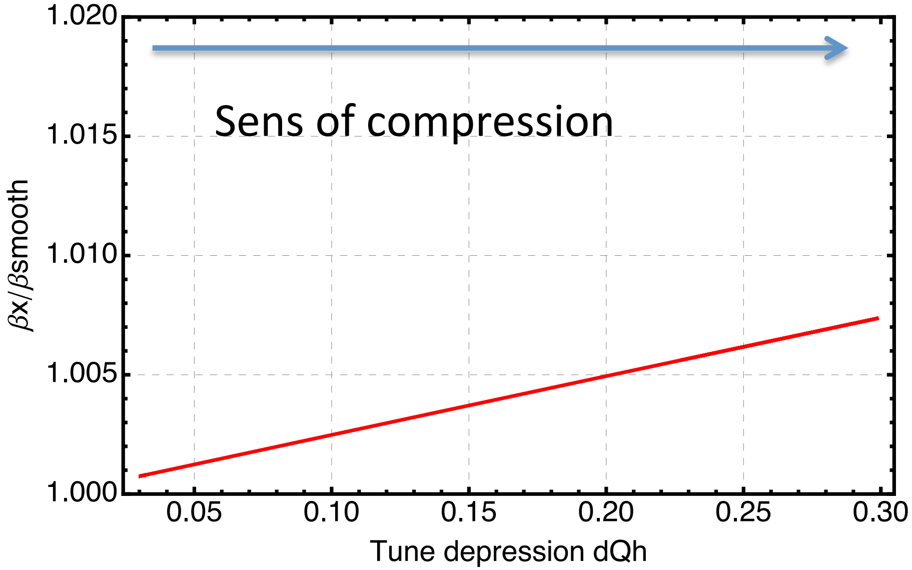


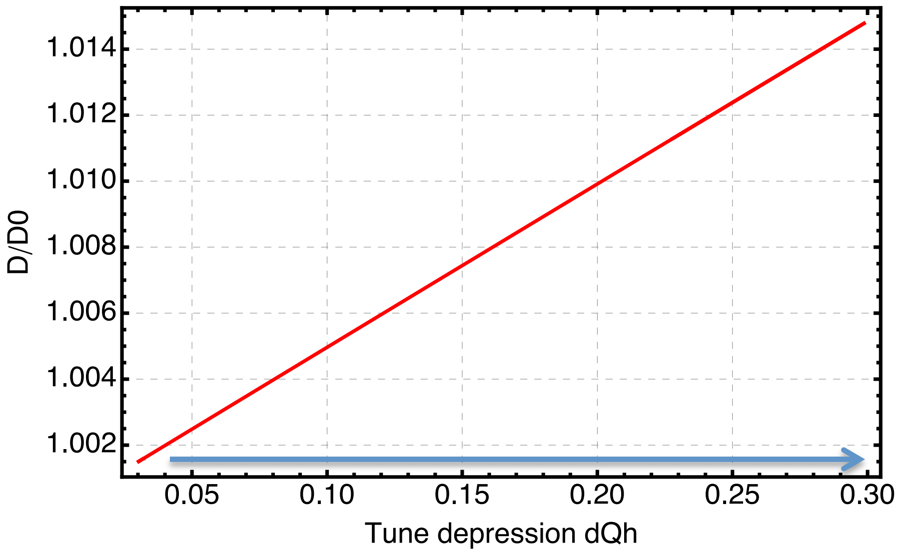




Constant focusing case

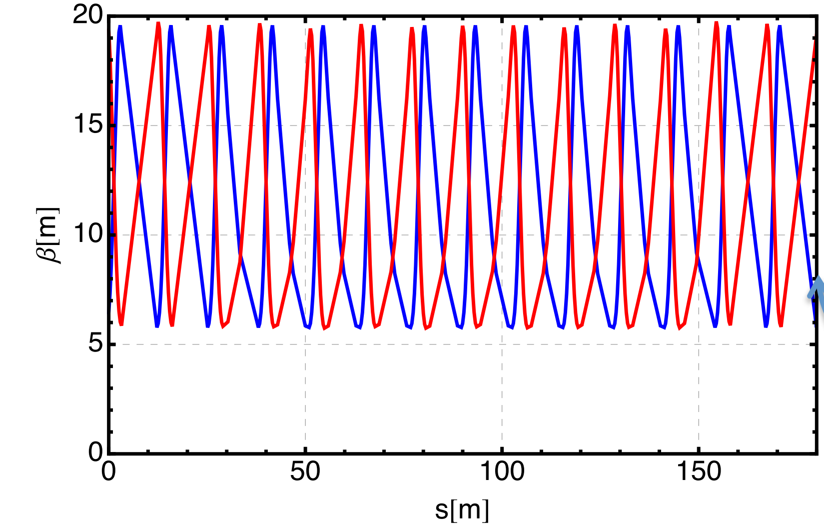




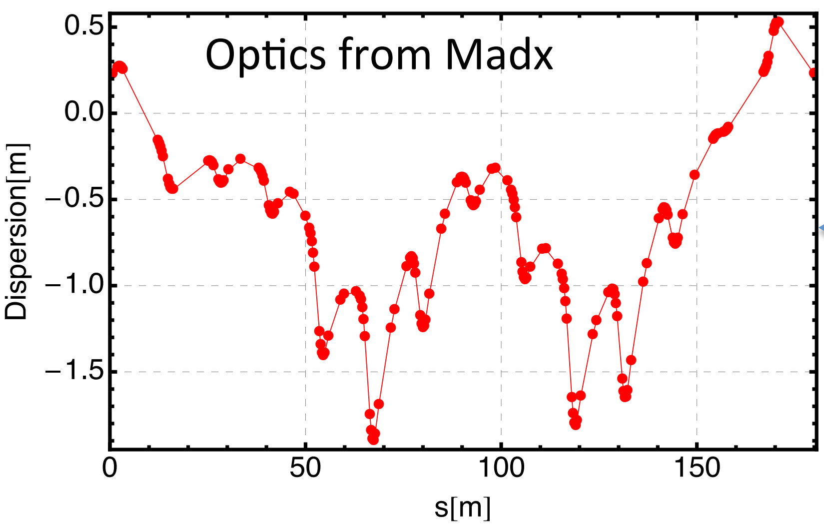


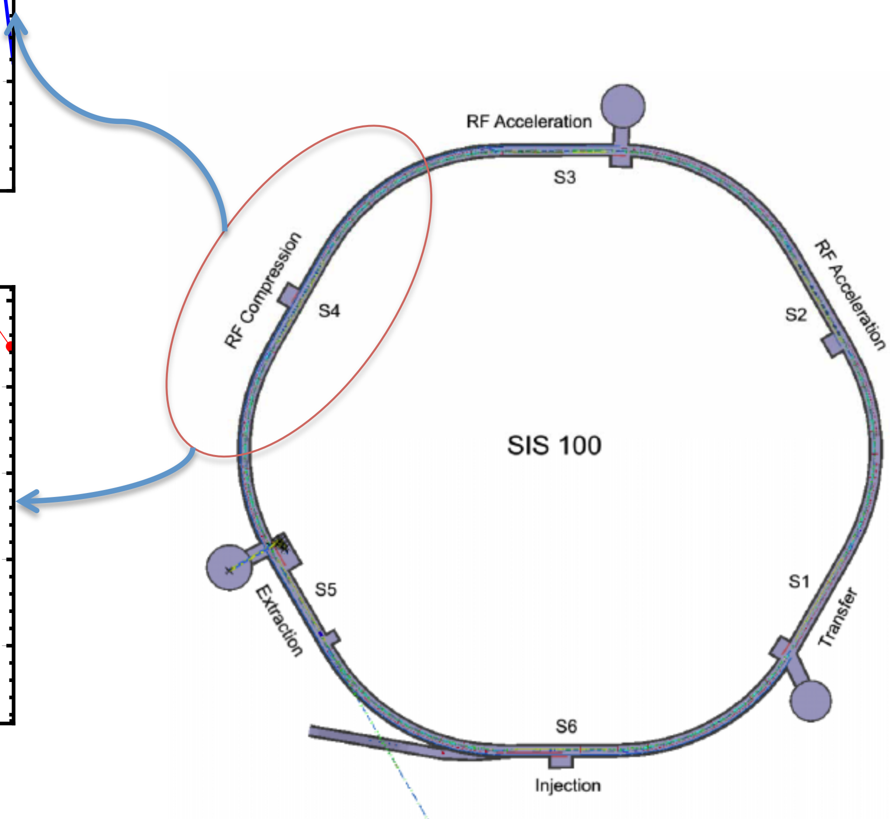
Moderate effect
In constant focusing

Transverse Equations in SIS-100



- SIS-100: 6 sectors of ~ 180 m
- Dispersion suppression





Transverse Equations in SIS-100

- Read a Madx file with :
  s location, Kxy (strenght of quad), l (length of each element), bending angle

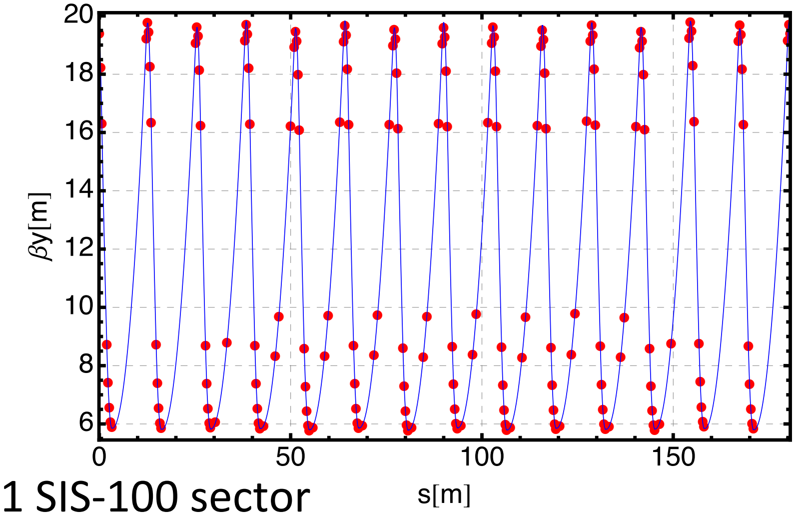
- Integration of the equations element by element.
- Method used for integration **Runge-Kutta** with a maximum step size integration **1mm**
- Small envelop tracking w/wo space charge.
- Any other suggestion for other integrator ?
- This is not final, because extraction line is going up ! **<u>Vertical dispersion</u>**! Will be add later.

$$\sigma_x'' = \frac{\epsilon_{dx}^2 + (\sigma_x \sigma_x' - DD'\langle\delta^2\rangle)^2}{\sigma_x(\sigma_x^2 - D^2\langle\delta^2\rangle)} - \frac{1}{\sigma_x}(\sigma_x')^2 - k_x\sigma_x + \frac{K}{2(\sigma_x + \sigma_y)} + \frac{\langle\delta^2\rangle}{\sigma_x}\left(\frac{D}{\rho} + D'^2\right), \tag{10}$$

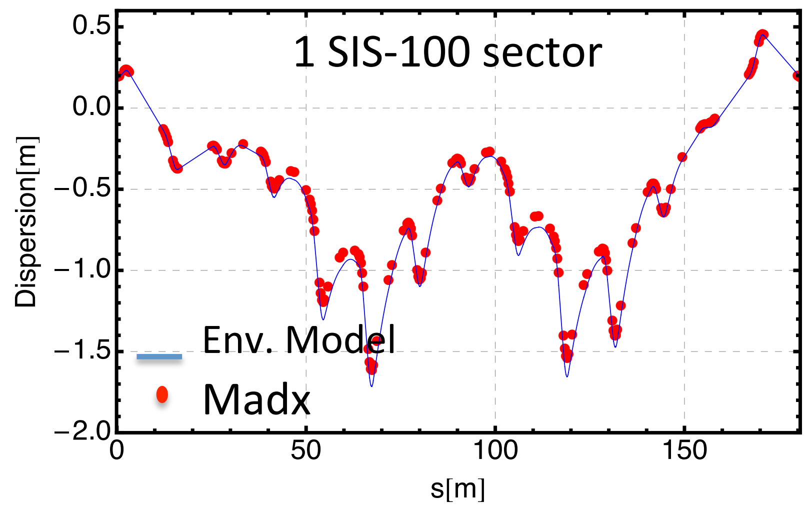
$$\sigma_y'' = \frac{\epsilon_y^2}{\sigma_y^3} - k_y\sigma_y + \frac{K}{2(\sigma_x + \sigma_y)}. \tag{11}$$

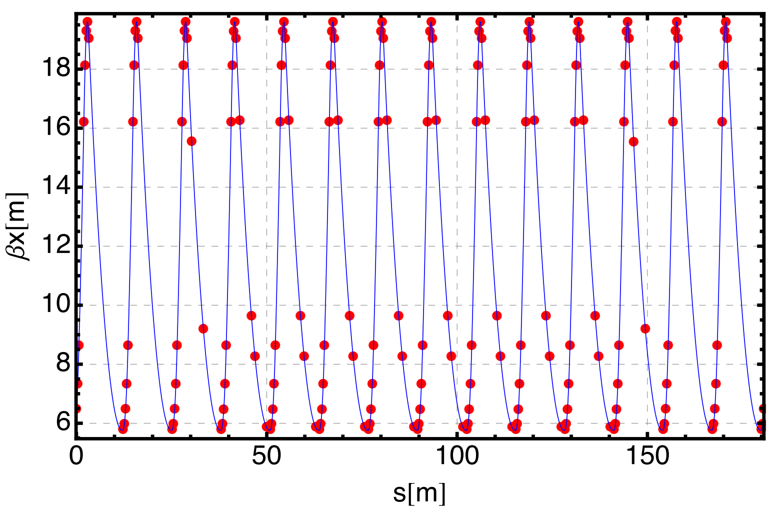
$$D'' + \left[k_x(z) - \frac{K}{2\sigma_x(\sigma_x + \sigma_y)}\right]D = \frac{1}{\rho(z)}.$$

Transverse Equations in SIS-100



1 SIS-100 sector





Comparison of my small tracking code with the optics computing by Madx.
No space charge and (delta p/p=0)
Good agreement.

$$Q_{x,y} = \frac{1}{2\pi}\int_s^{s+C} \frac{1}{\beta_{x,y}(s)} ds$$

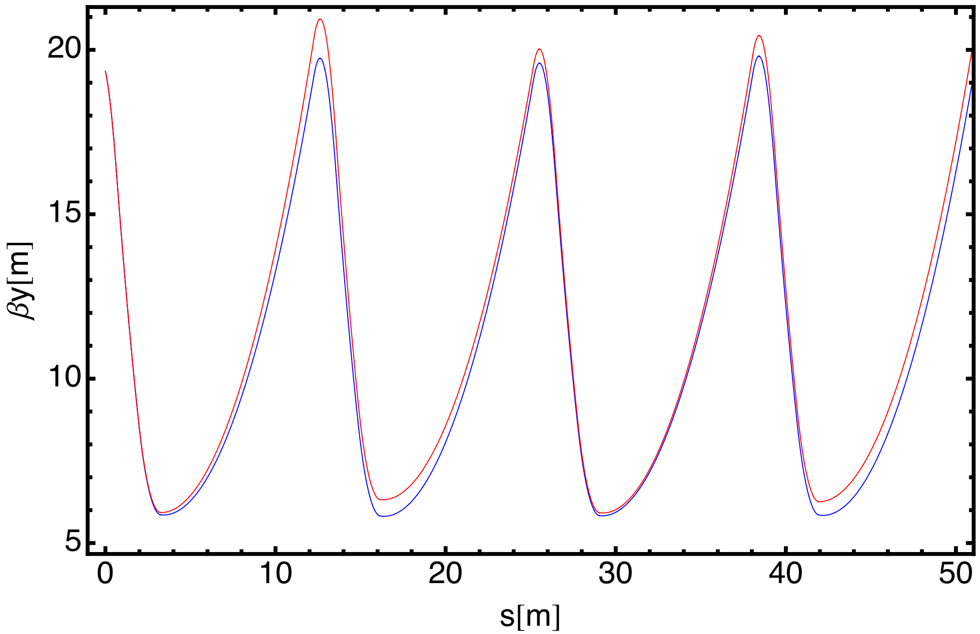
Qx=18.84
Qy=18.63

From envelope equation

18.72 with madx

Transverse Equations in SIS-100

Vertical Beta change with space charge for the expected vertical tune shift, initial conditions in the tracker the same as the non-SC case.

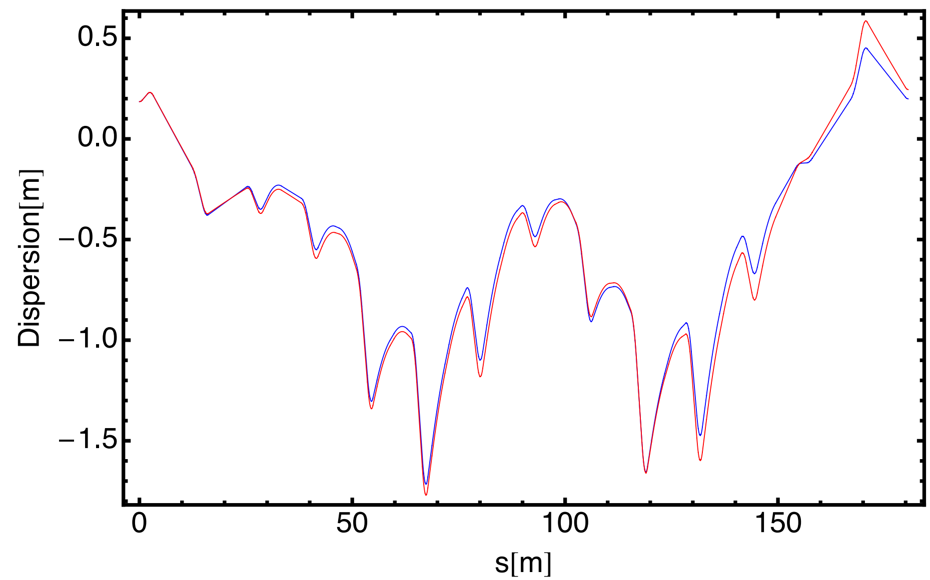


Zoom on half sector

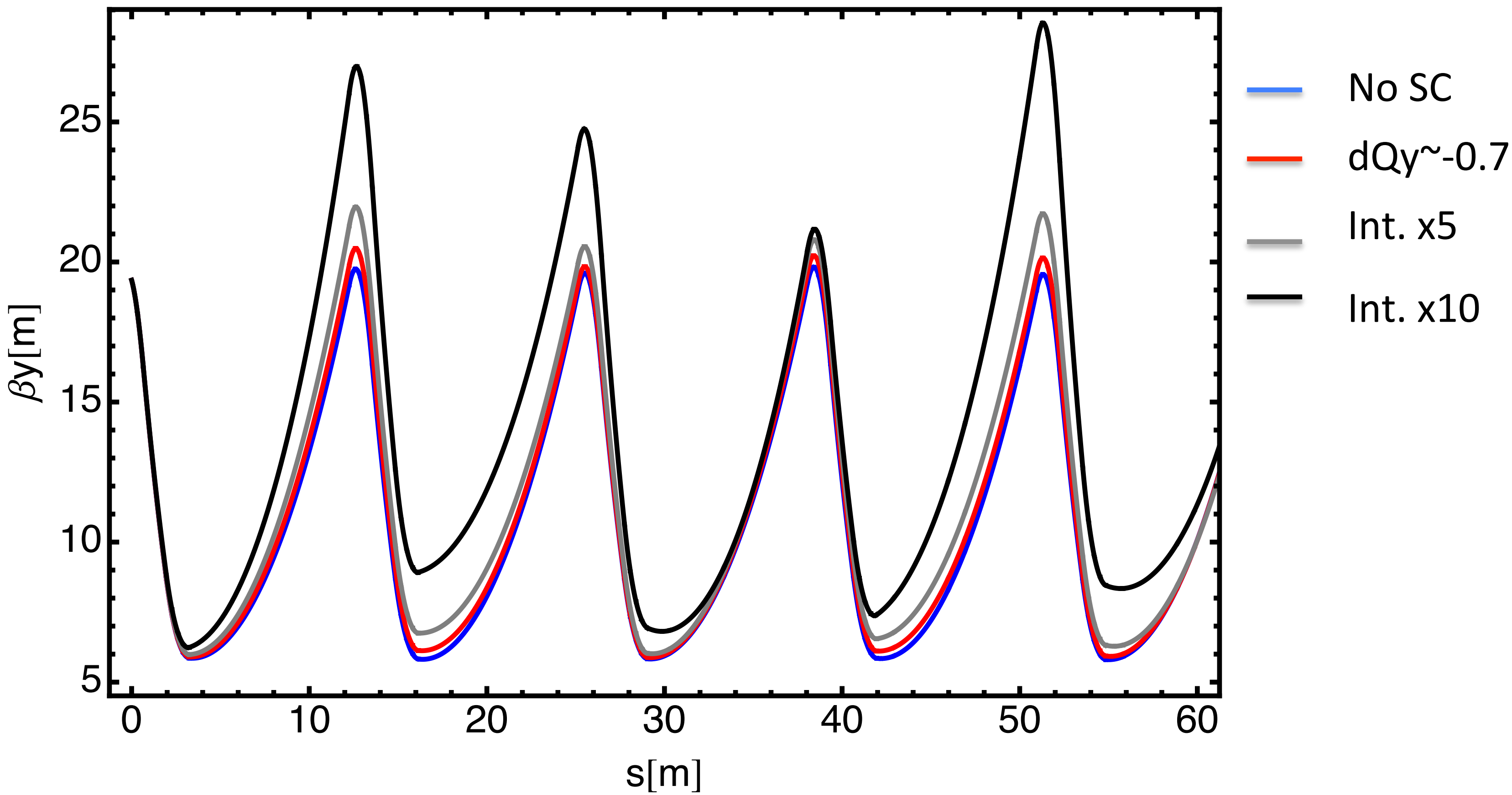
Preliminary results shows about 5% change in beta-function at some location

No space charge
Space charge

dQx~-0.39
dQy~-0.73

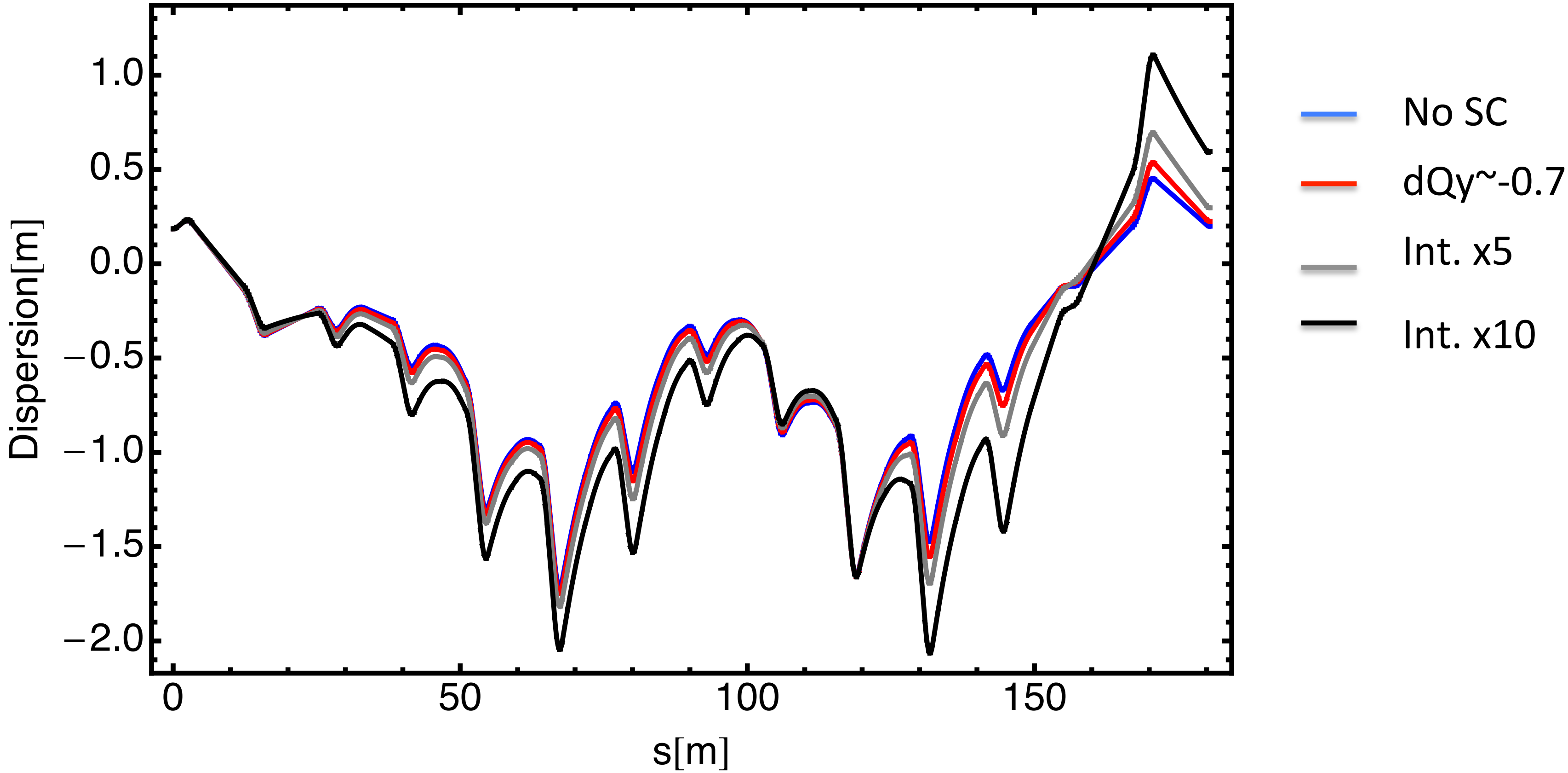


Transverse Equations in SIS-100



Focus on one part of the SIS-100 sector

Transverse Equations in SIS-100



Focus on one part of the SIS-100 sector

Lattice matching

- Transverse space charge strong enough to change even slightly the optics functions.
- Consequences can be emittance blow up and/or beam size breathing during the transport of the compressed bunch to the target.
- New matching: find new matched solutions.
- Now, Newton Method, not robust yet.

$$\begin{bmatrix} X \\ X' \\ Y \\ Y' \end{bmatrix}_0 \longrightarrow \begin{bmatrix} X \\ X' \\ Y \\ Y' \end{bmatrix}_1$$

Then delta are applied at each component of the vector

$$\begin{bmatrix} X + \Delta X \\ X' \\ Y \\ Y' \end{bmatrix}$$

In one dimension, with xfp is the fixed point

$$x_{fp} = \frac{x_1 - Jm(x_0)x_0}{1 - Jm(x_0)}$$

$$Jm = \frac{f(x_0 + \Delta x) - x_1}{\Delta x}$$

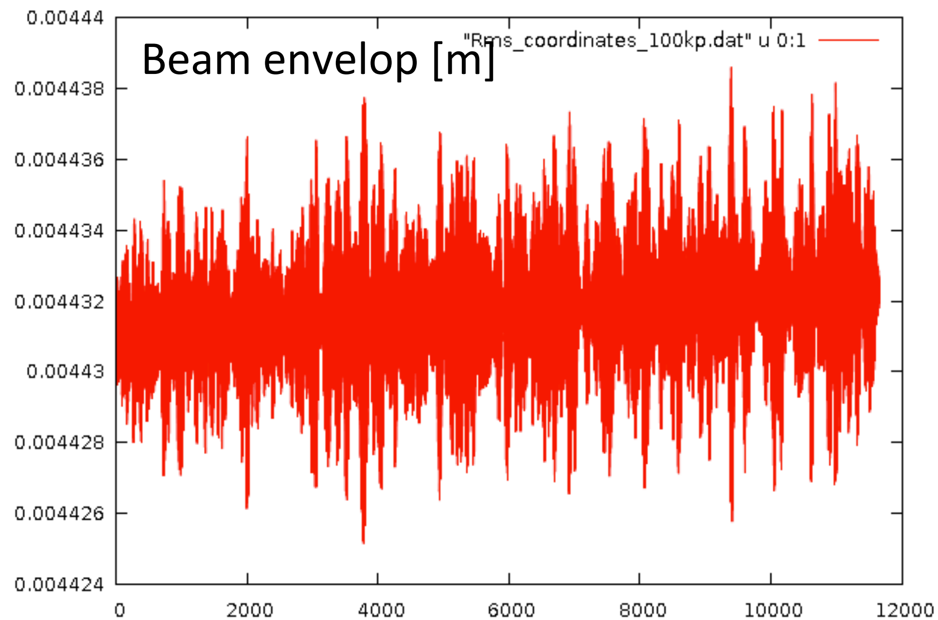
- This idea would be to go for Jacobian method like done in Madx (thanks R. De Maria, F. Schmidt)

Simulations: outlook

- Use PyOrbit (see the talk of J. Holmes and S. Appel for PyOrbit @ GSI)
- What is planned ?
  - Purely longitudinal plan first for bunch compression
  - Longitudinal + Transverse through the full SIS-100 accelerator with space charge.
  - Then transport to the target to see any deformation of the beam spot.
  - Maybe comparison with MICROMAP from Giuliano.

Status: still convergence study of the code, testing.
- Transverse KV distribution from PyOrbit
- Longitudinal parabolic distribution.

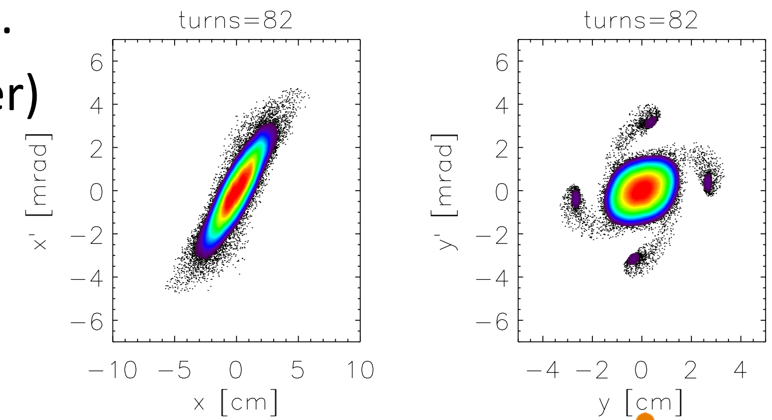
100k part



Difference about 0.012mm

Conclusions - Outlooks

- **Longitudinal space charge should not be a problem for SIS-100 operation**.
- Large transverse space charge tune shift (~dQx=-0.3, dQy=-0.7) during bunch compression.
- The optics is affected by space charge (**up to this point, moderate effect, can be corrected**), HOWEVER this has to be propagate through the full lattice and until the target. Vertical dispersion ?
- New periodic solutions for mismatch beams wrt to the extraction line.
- 3D simulations to observe the beam spot at the target but also to compare envelope with analytical formula.
- Comparison with simple env. Model (Oliver)
- Other effects during bunch compression (quadrupolar error, resonances etc..)

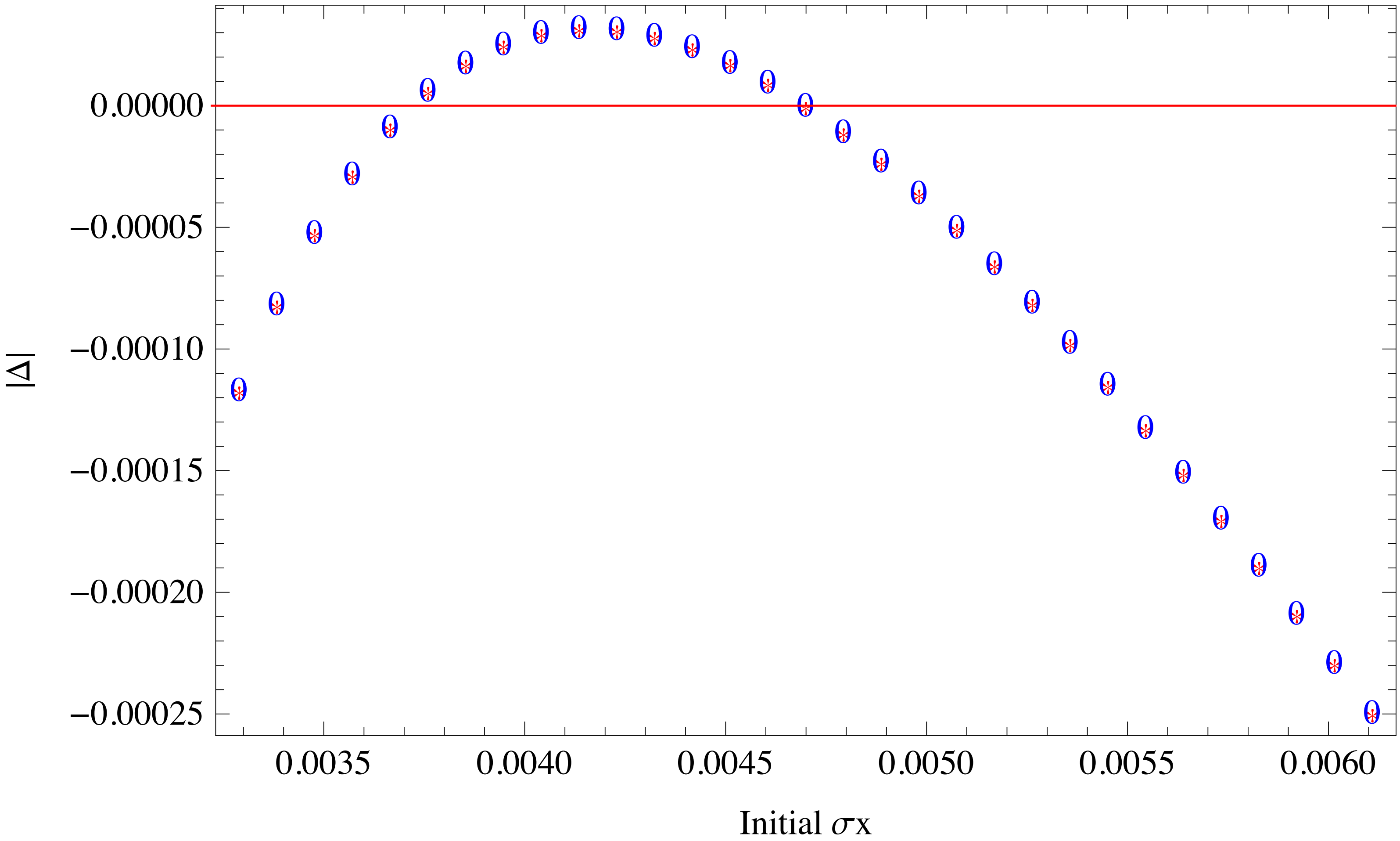


Courtesy Oliver
EPAC 2002

THANK YOU FOR YOUR ATTENTION

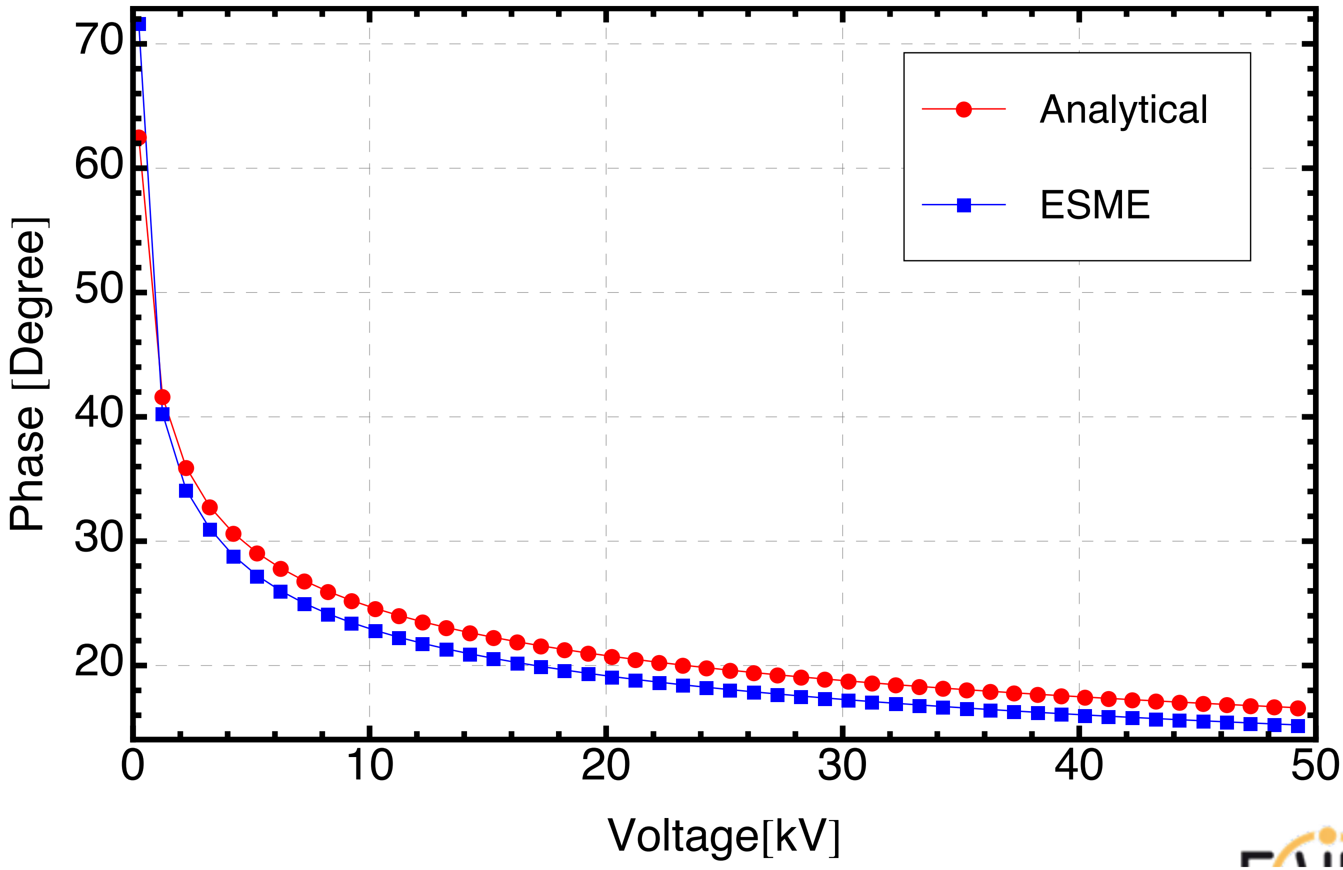
Spare Slides

Cross check with Madx



Comparison ESME - Analytical

- Bunch length as a function of pre-compressed voltage



8%

difference

Venturini Paper: Hamiltonian

$$H = \frac{1}{2} p_x^2 + \frac{k_x(z)}{2} x^2 + \frac{m^2c^4}{E_o^2} \delta^2 - \frac{x}{\rho(z)} \delta \,.$$

One can easily verify that because of the coupling term $\delta x/\rho$ the standard rms emittance $\epsilon_x^2 = (\langle x^2 \rangle \langle p_x^2 \rangle - \langle x p_x \rangle^2)$, is not an invariant for the system (1). Here $\langle \cdot \rangle$ denotes the averaging over the phase space variables.

A suitable canonical transformation is generated by $G_2(x, \overline{p}_x, z) = \overline{p}_x[x - \delta D(z)] + x\delta D'(z)$, where $D(z)$ is a function that will eventually be identified with the dispersion function: $\overline{x} = x - \delta D(z)$ and $p_x = \overline{p}_x + \delta D'(z)$. The transformed Hamiltonian reads

$$\overline{H} = \frac{1}{2} \overline{p}_x^2 + \frac{k_x}{2} \overline{x}^2 + \frac{m^2c^4}{E_o^2} \delta^2 + \delta\overline{x}\left(D'' + k_x D - \frac{1}{\rho} \right) + \ldots \,.$$

Venturini Paper: PIC simulations