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# Head-Tail Instability and Beam Break-Up Instability with Strong Space Charge

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# Head-Tail Instability

**PSB at CERN**  
 **$U^{28+}$  operation in SIS100 at GSI**

# HEAD-TAIL IN THE PS BOOSTER

once the transverse feedback system is switched off:  
strong transverse oscillations and losses (within ms)

R=25m

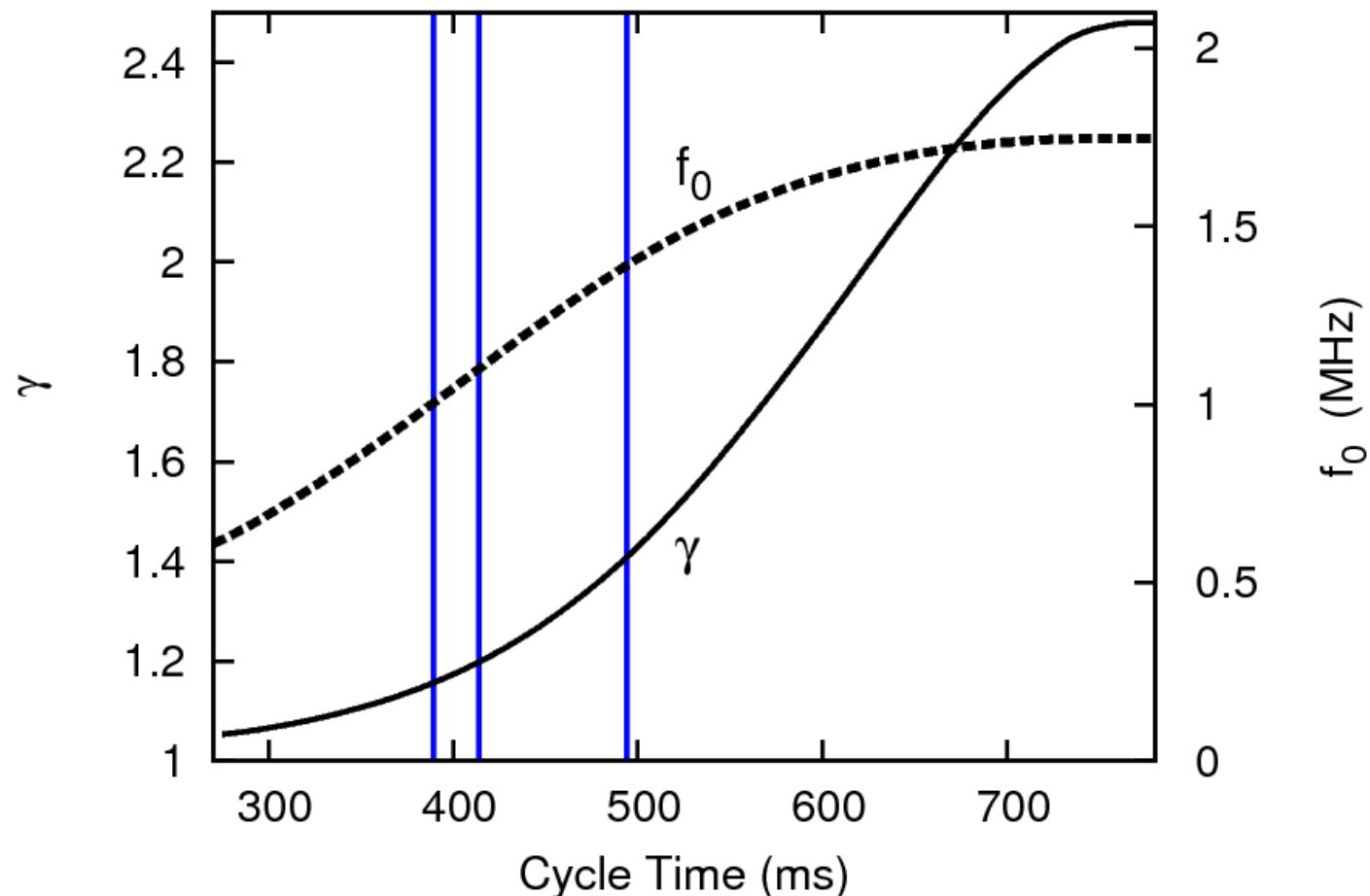
Kin. energy

50 MeV-1.4 GeV

$Q_h = 4.27 - 4.17$

$Q_v = 4.65 - 4.20$

$\xi_h = -0.95$   $\xi_v = -2.1$



Measurement campaign at the PS Booster in June 2012.  
Help of A.Findley, S.Aumon, B.Mikulec, G.Rumolo

The classical head-tail instability:

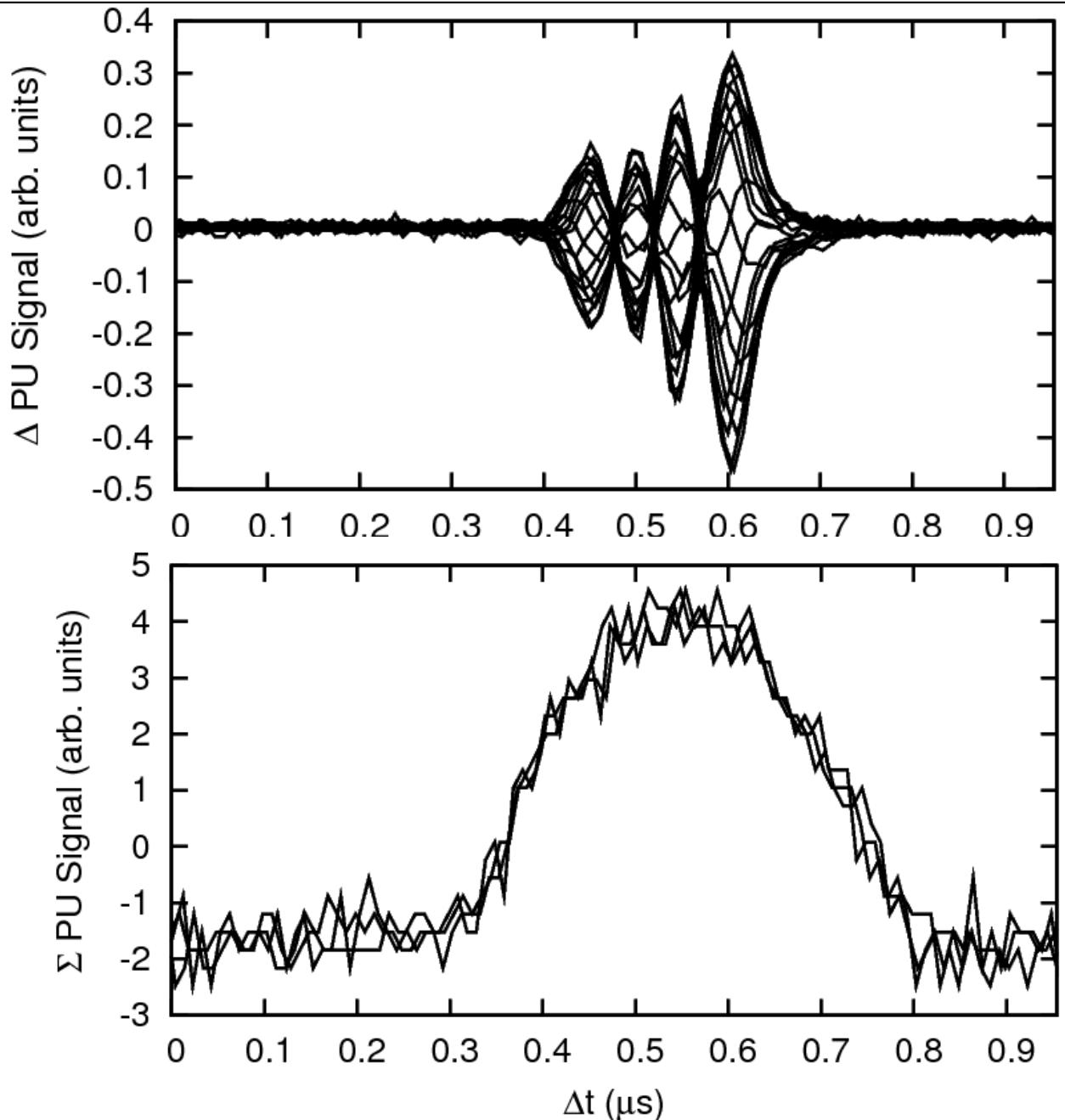
a standing-wave structure  
with the  $\xi$ -wiggles within;  
here 3 knots, the  $k=3$  mode

nice exponential growth

$\Delta Q/Q_s < 0.3$

Puzzling:

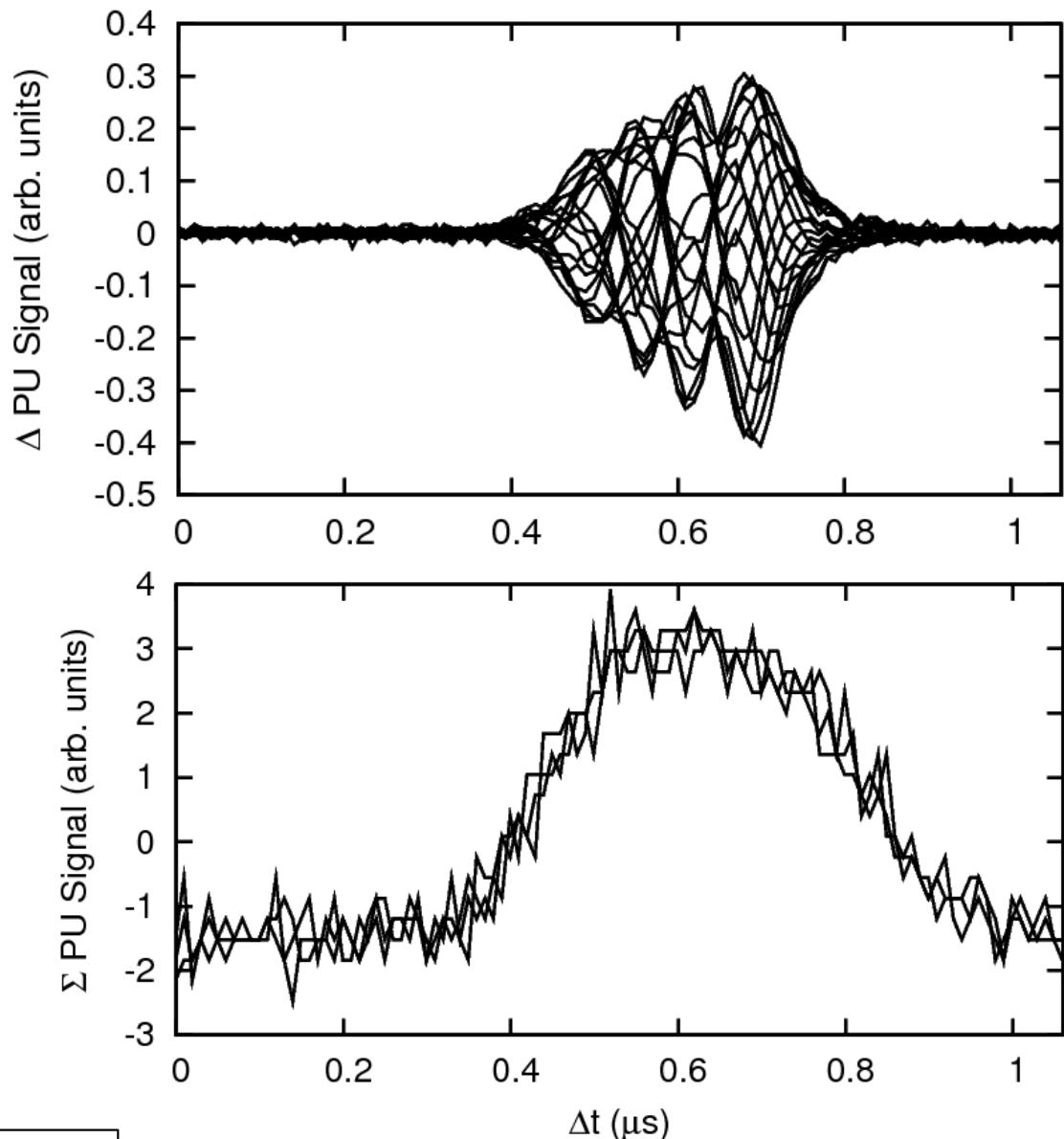
- horizontal plane only
- fixed Ctimes, regardless the bunch distribution



Instability at C386ms  
single rf

$N_p = 370e10$   
 $\Delta Q = 2.3e-4$ ,  
 $\Delta Q/Q_s = 0.13$

the mode  $k=3$   
the mode structure is not  
an ideal head-tail:  
modified by the  
impedance



more details in the upcoming CERN Report

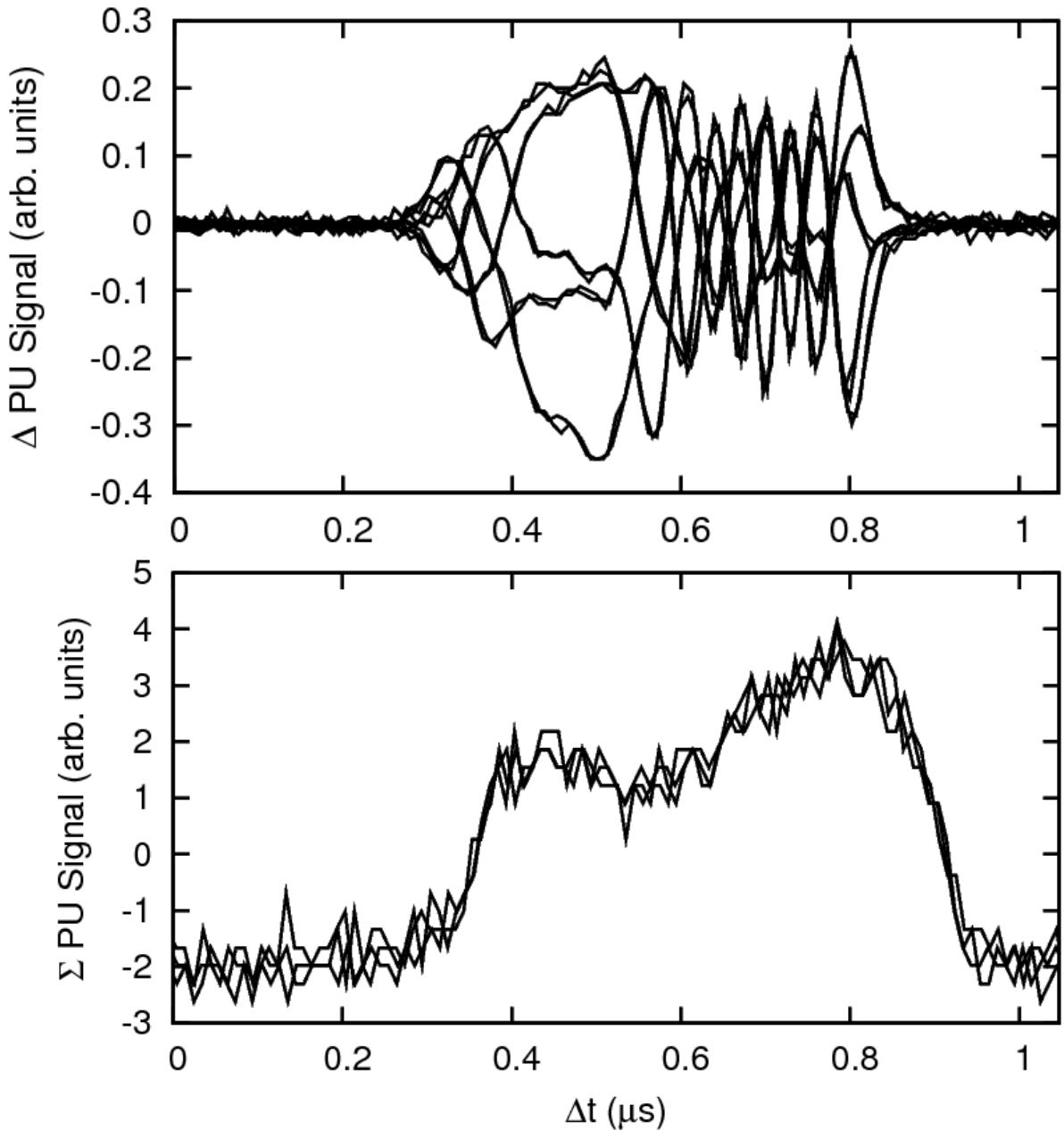
Instability at C394ms  
double rf, PSB standard

$$N_p = 500e10$$

$$\Delta Q = 1.3e-4$$

$$\Delta Q/Q_s = 0.071$$

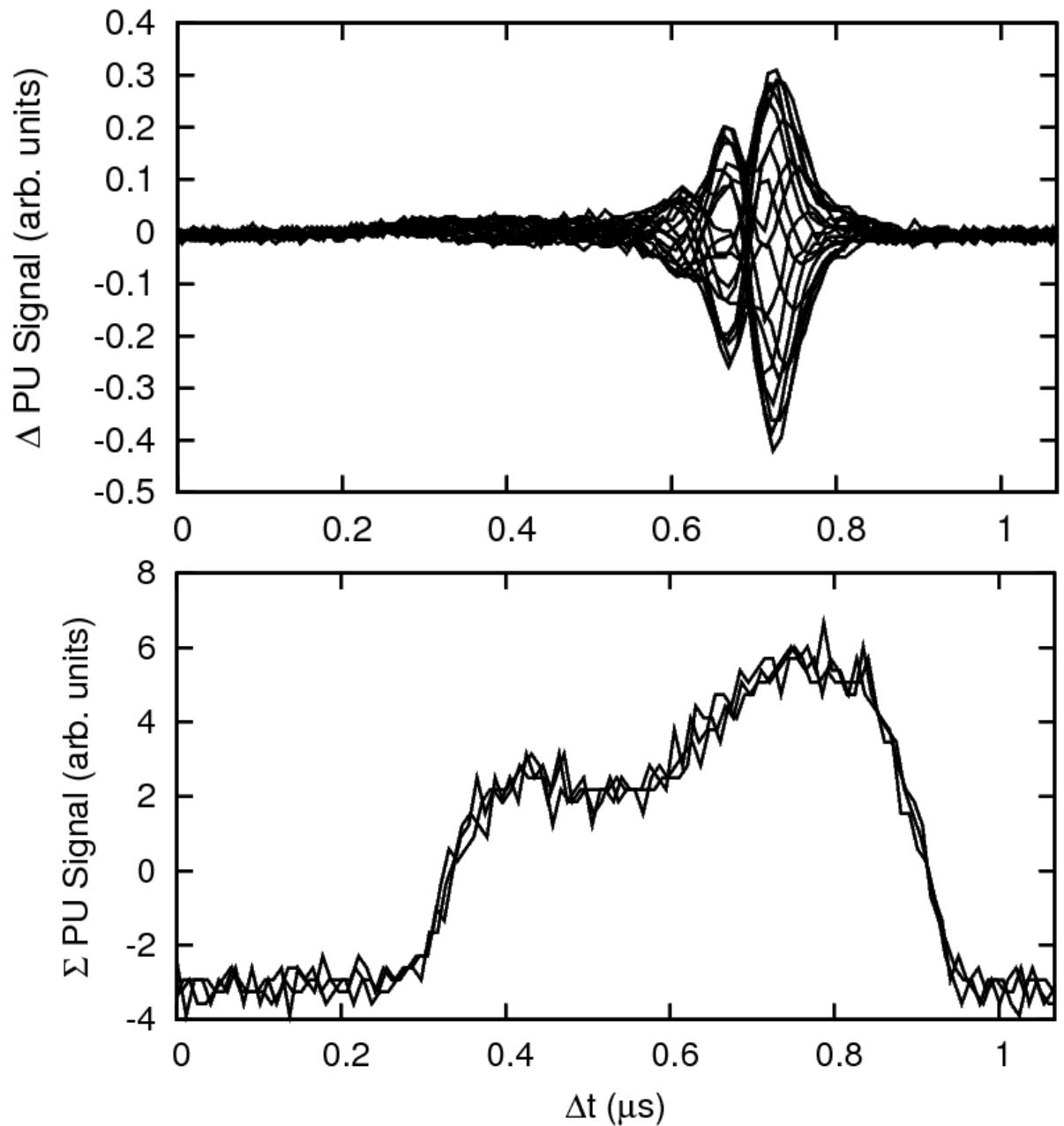
a complex mode structure  
in double rf



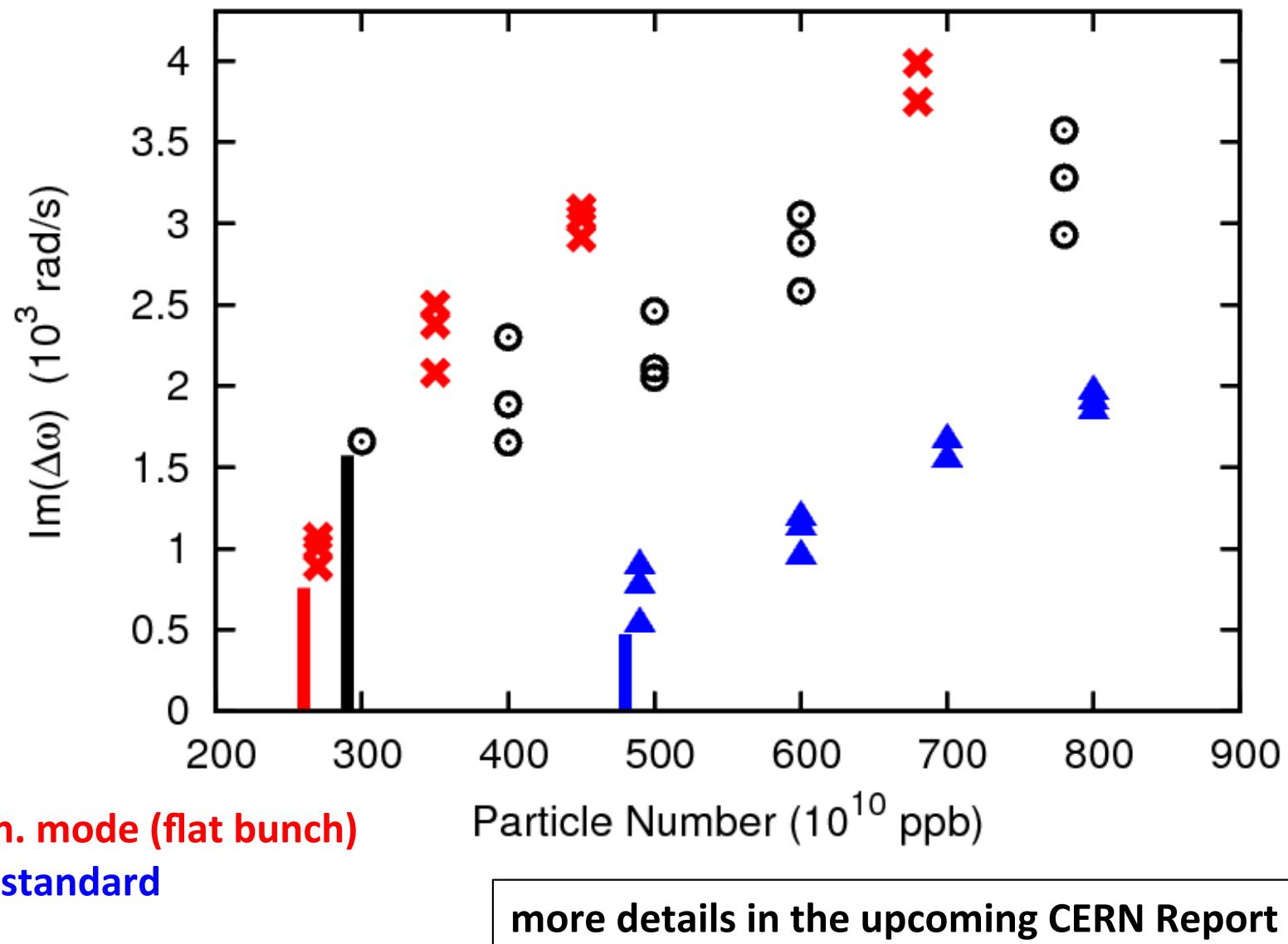
Instability at C394ms  
double rf, PSB standard  
higher intensity

$N_p = 950 \text{e}10$

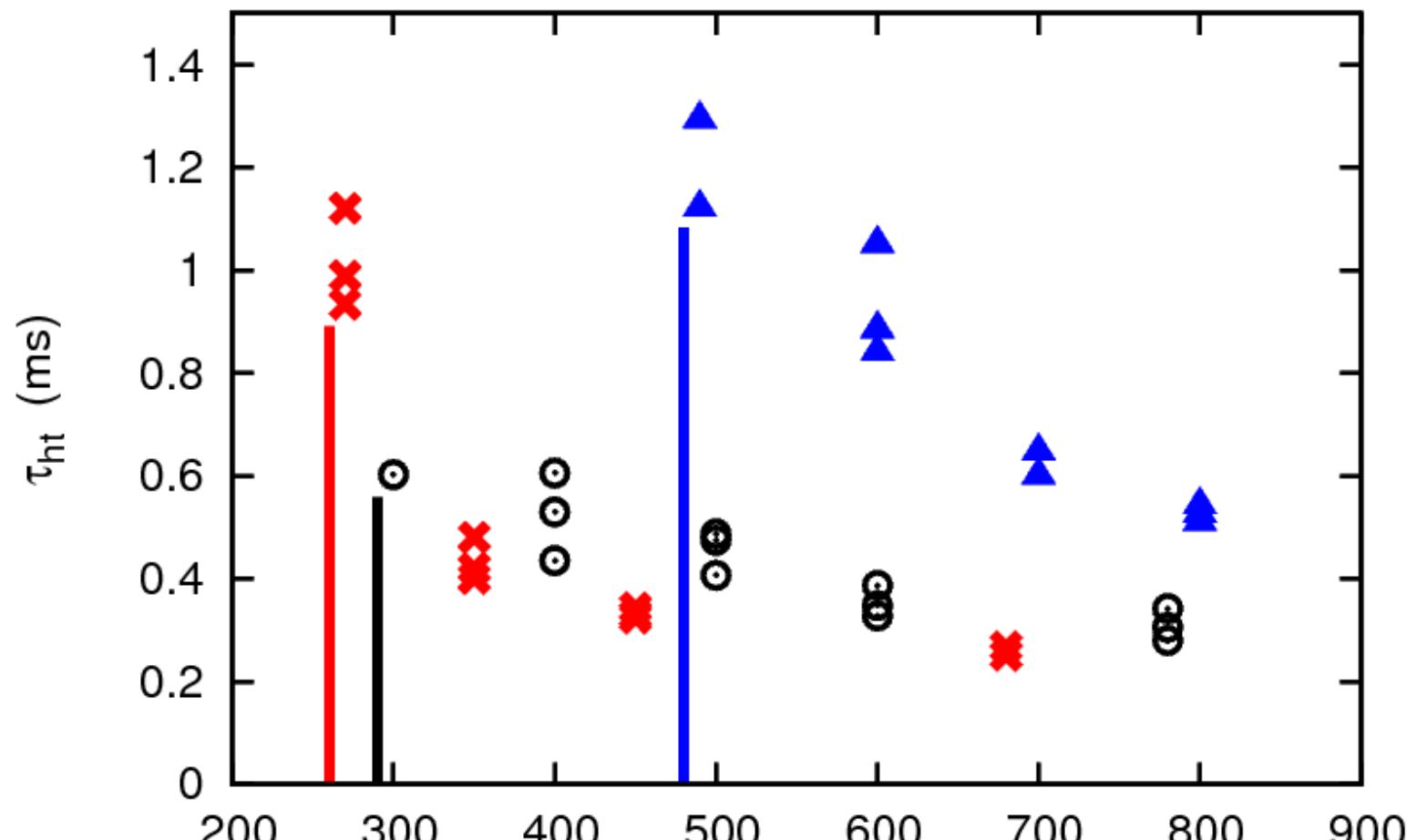
looks like the k=2 mode  
in the more dense, tail  
half of the bunch



Summary of  
the instability  
growth rates  
and thresholds



Summary of  
the instability  
growth times  
and thresholds



○ single rf

✖ double rf length. mode (flat bunch)

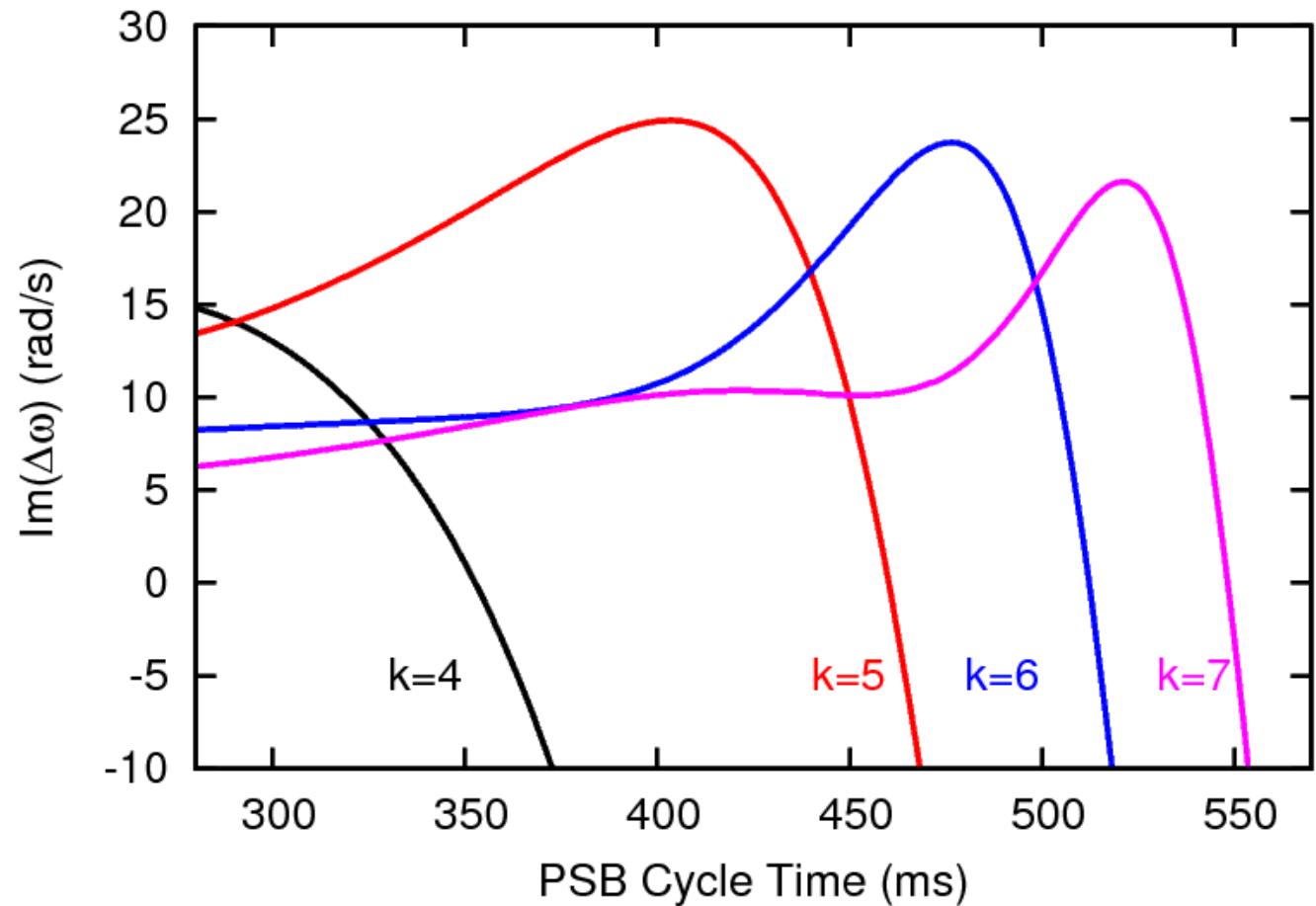
▲ double rf - PSB standard

Particle Number ( $10^{10}$  ppb)

more details in the upcoming CERN Report

The head-tail growth rate given by the Sacherer theory for each single ms along the PSB ramp.

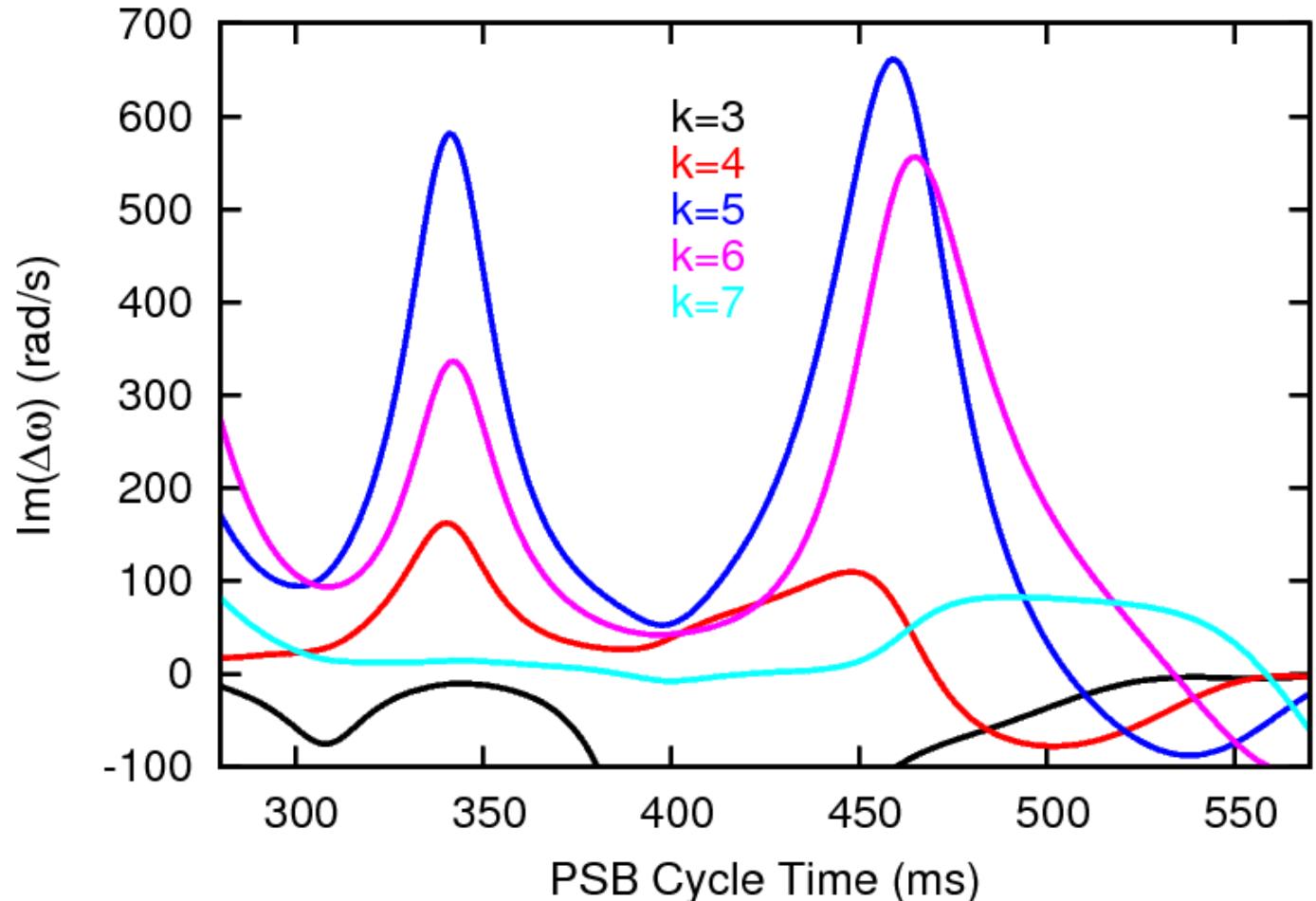
The Resistive Wall Impedance: PSB overestimated



Growth rates orders of magnitude smaller than observed

The head-tail growth rate given by the Sacherer theory for each single ms along the PSB ramp.

**an example:**  
A Narrow-Band Impedance  
 $f_r = 2.2\text{MHz}$ ,  $Q_T = 8$ ,  
 $R_T = 3\text{M}\Omega/\text{m}$



horizontal  
 $\xi_h = -0.95$

Corresponds to the observations:

- a “resonant” behavior as the bunch frequencies hit the impedance
- independent from the distribution details

The head-tail growth rate given by the Sacherer theory for each single ms along the PSB ramp.

**an example:**

A Narrow-Band

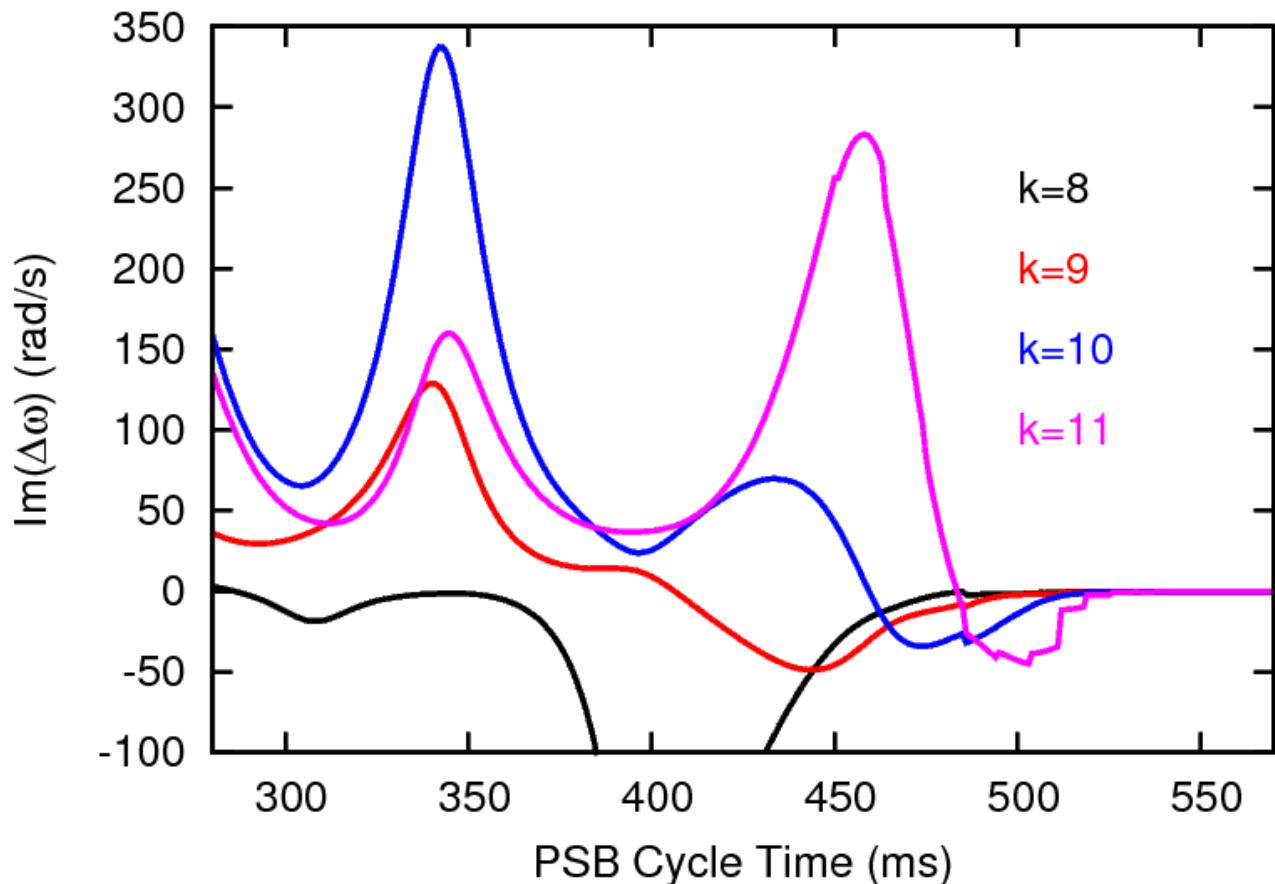
Impedance

$f_r = 2.2\text{MHz}$ ,  $Q_T = 8$ ,

$R_T = 3\text{M}\Omega/\text{m}$

vertical

$\xi_v = -2.1$



Corresponds to the observations:  
Vertical plane instabilities are weaker due to  
higher-order modes provided by higher chromaticity

In the PSB experiment, by changing the lattice tunes a systematic shift of the instability Ctime has been observed:

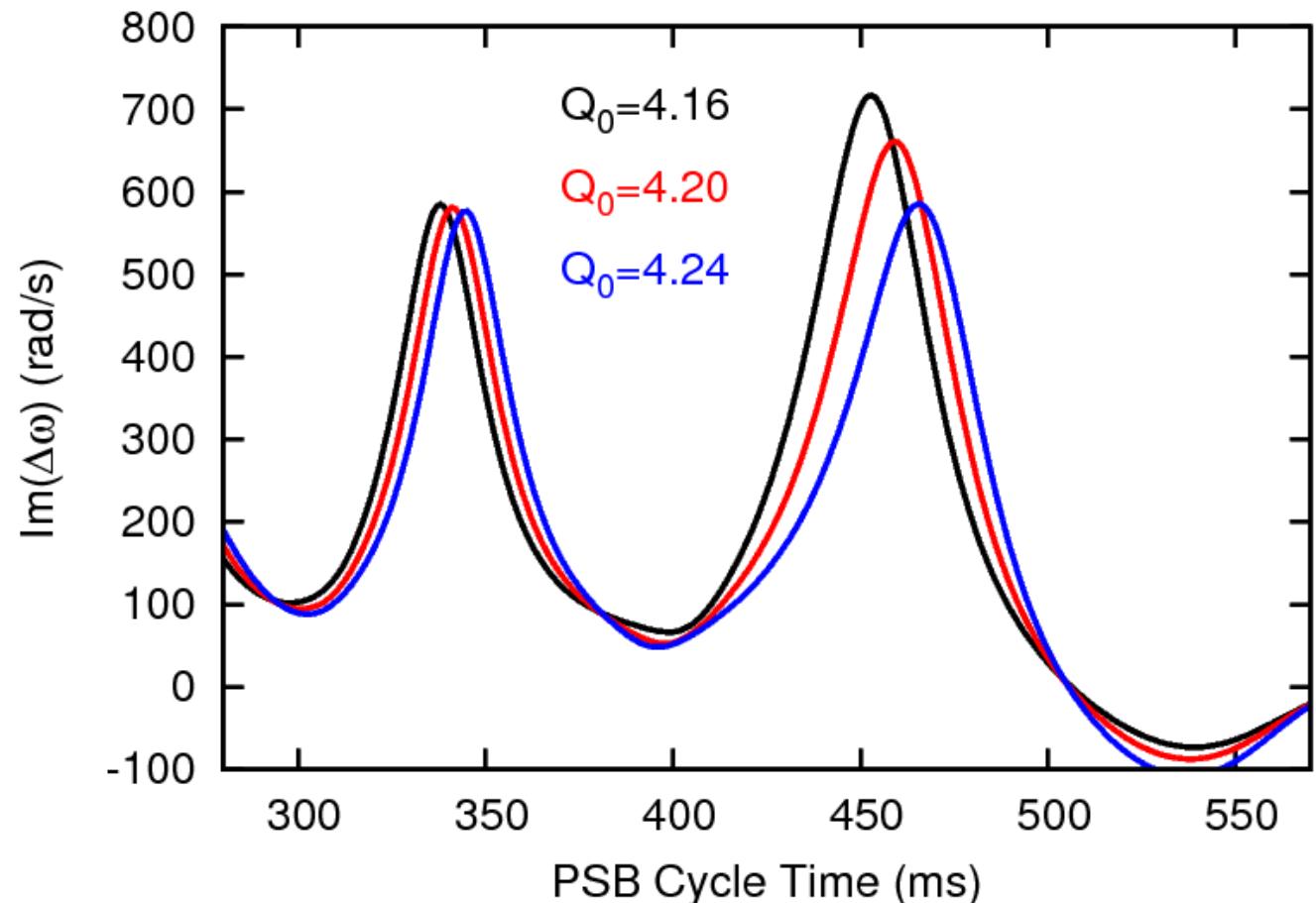
$Q_h=4.19$ : around C384

$Q_h=4.20$ : around C386

$Q_h=4.23$ : around C389

$Q_h=4.25$ : around C392

The head-tail growth rate given by the Sacherer theory for the narrow-band imp.



Another nice agreement with the observations at the PS Booster

the space-charge tune shift  
(rms-equiv. K-V beam)

$$\Delta Q_{\text{sc}} = \frac{\lambda_0 r_p R}{\gamma^3 \beta^2 \epsilon_{\perp}}$$

the space-charge  
parameter

$$q = \frac{\Delta Q_{\text{sc}}}{Q_s}$$

Elliptic cross-section:  
( $\epsilon_x, \epsilon_y$  rms emittances,  
 $\epsilon_{\perp}$  total for the rms-equivalent K-V)

$$\epsilon_{\perp} = 2 \left( \epsilon_x + \sqrt{\epsilon_x \epsilon_y \frac{Q_{0x}}{Q_{0y}}} \right)$$

Gaussian profile:  $\Delta Q_{\text{sc}}^{\max} = 2\Delta Q_{\text{sc}}$

Space-charge tune spread:

- different transverse amplitudes
- density variation along the bunch

# SPACE CHARGE

Space Charge in bunches:

- shifts the head-tail line. for an airbag bunch:

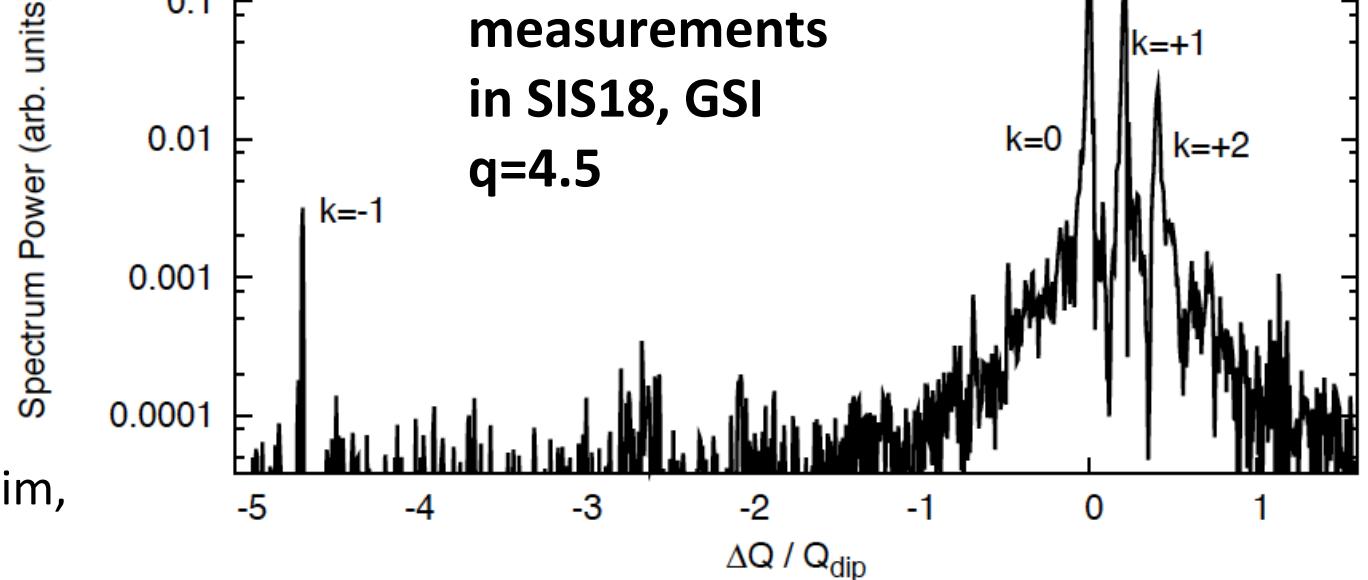
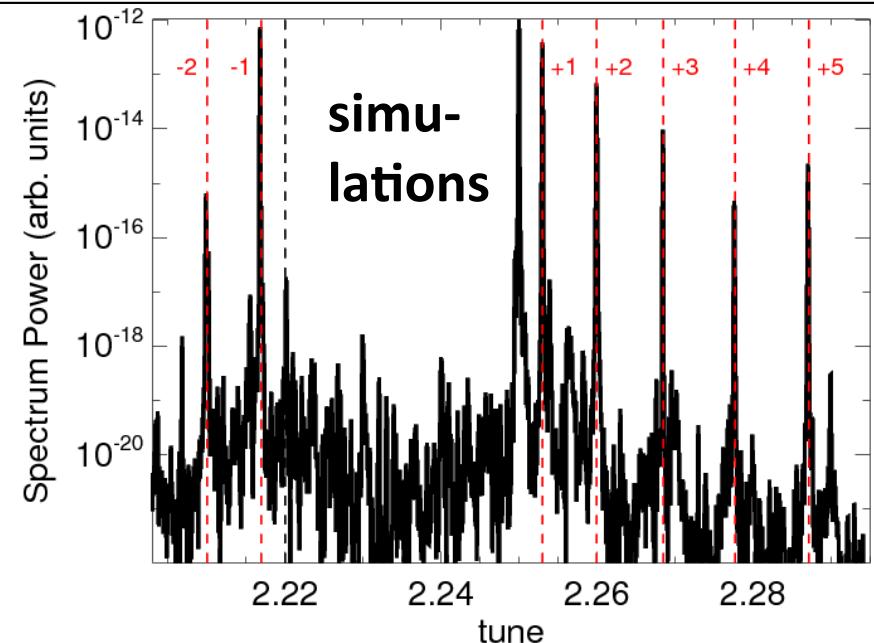
$$\Delta Q_k = -\frac{\Delta Q_{sc}}{2} \pm \sqrt{\frac{\Delta Q_{sc}^2}{4} + k^2 Q_s^2}$$

- induces strong Landau damping

M.Błaskiewicz, PRSTAB **1**, 044202 (1998)

V.Kornilov, O.Boine-Frankenheim,  
PRSTAB **13**, 114201 (2010)

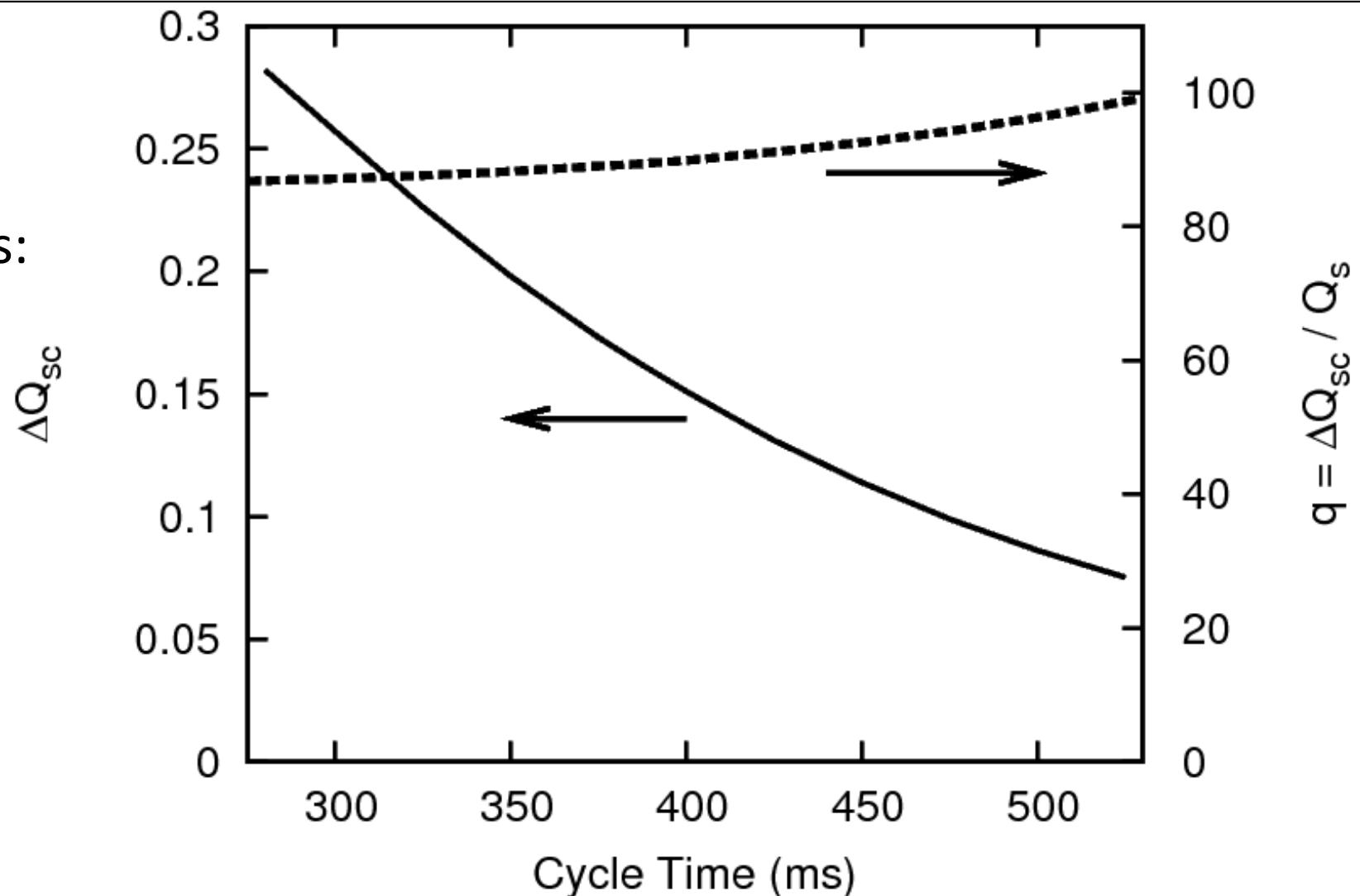
**Confirmed by simulations and experiment**



V.Kornilov, O.Boine-Frankenheim,  
PRSTAB **15**, 114201 (2012)

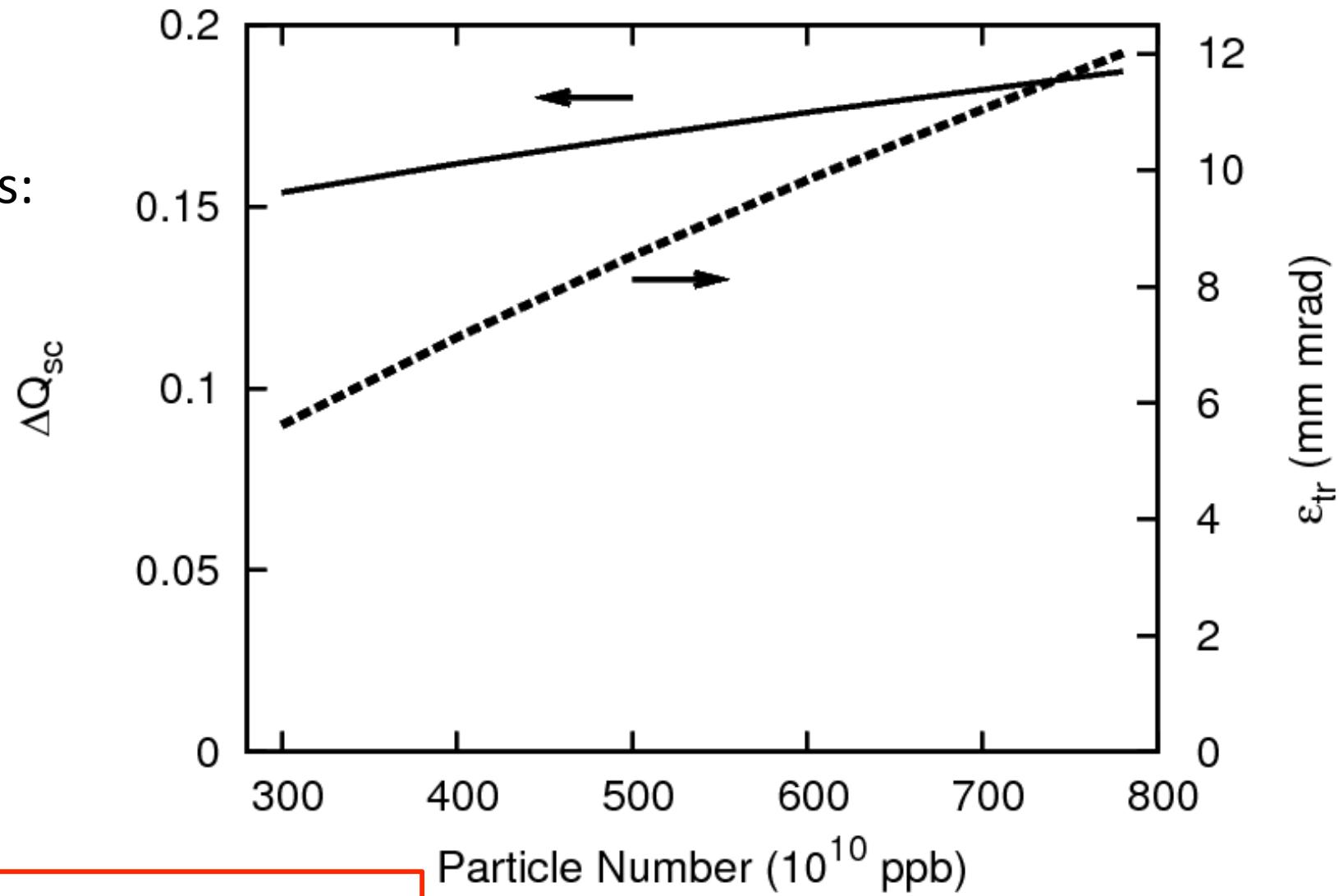
## HEAD-TAIL IN THE PS BOOSTER

from the measurements:  
space-charge  
tune shift for  
single rf,  
 $V_0=8\text{kV}$ ,  
 $300\text{e}10 \text{ ppb}$ ,  
horizontal



the space-charge parameter  $q$  stays const. during the ramp;  
very strong space charge regime;  
no direct-SC Landau damping for the relevant ( $k < 6$ ) head-tail modes

from the  
measurements:  
space-charge  
tune shift  
at C385,  
for single rf,  
 $V_0=8\text{kV}$ ,  
horizontal



the space-charge tune shift  
is a flat function of the intensity

## another aspect of Space Charge: The Pipe Image Charges

the eigenfrequencies of the bunch head-tail modes  
with arbitrary space-charge and coherent shift:

$$\Delta Q_k = -\frac{\Delta Q_{sc} + \Delta Q_{coh}}{2} \pm \sqrt{\left(\frac{\Delta Q_{sc} - \Delta Q_{coh}}{2}\right)^2 + k^2 Q_s^2}$$

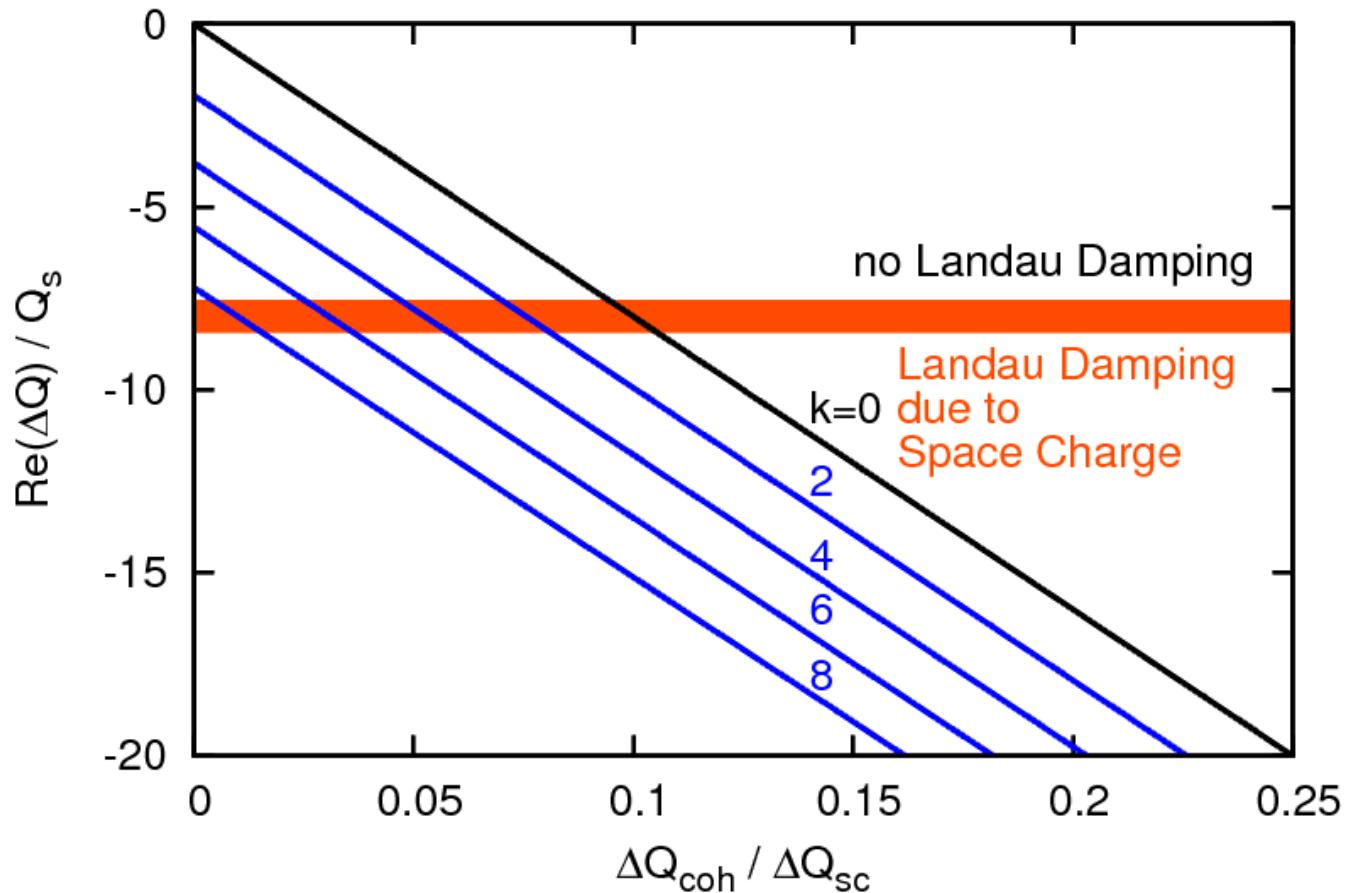
$$\frac{\Delta Q_k}{Q_s} = -\frac{q}{2} \left(1 + \frac{\Delta Q_{coh}}{\Delta Q_{sc}}\right) \pm \sqrt{\frac{q^2}{4} \left(1 - \frac{\Delta Q_{coh}}{\Delta Q_{sc}}\right)^2 + k^2}$$

O.Boine-Frankenheim, V.Kornilov, PRSTAB **12**, 114201 (2009)  
V.Kornilov, O.Boine-Frankenheim, PRSTAB **13**, 114201 (2010)  
M.Błaskiewicz, PRSTAB **1**, 044202 (1998)

consider  
Space Charge typical  
for PSB:  
 $q=80$

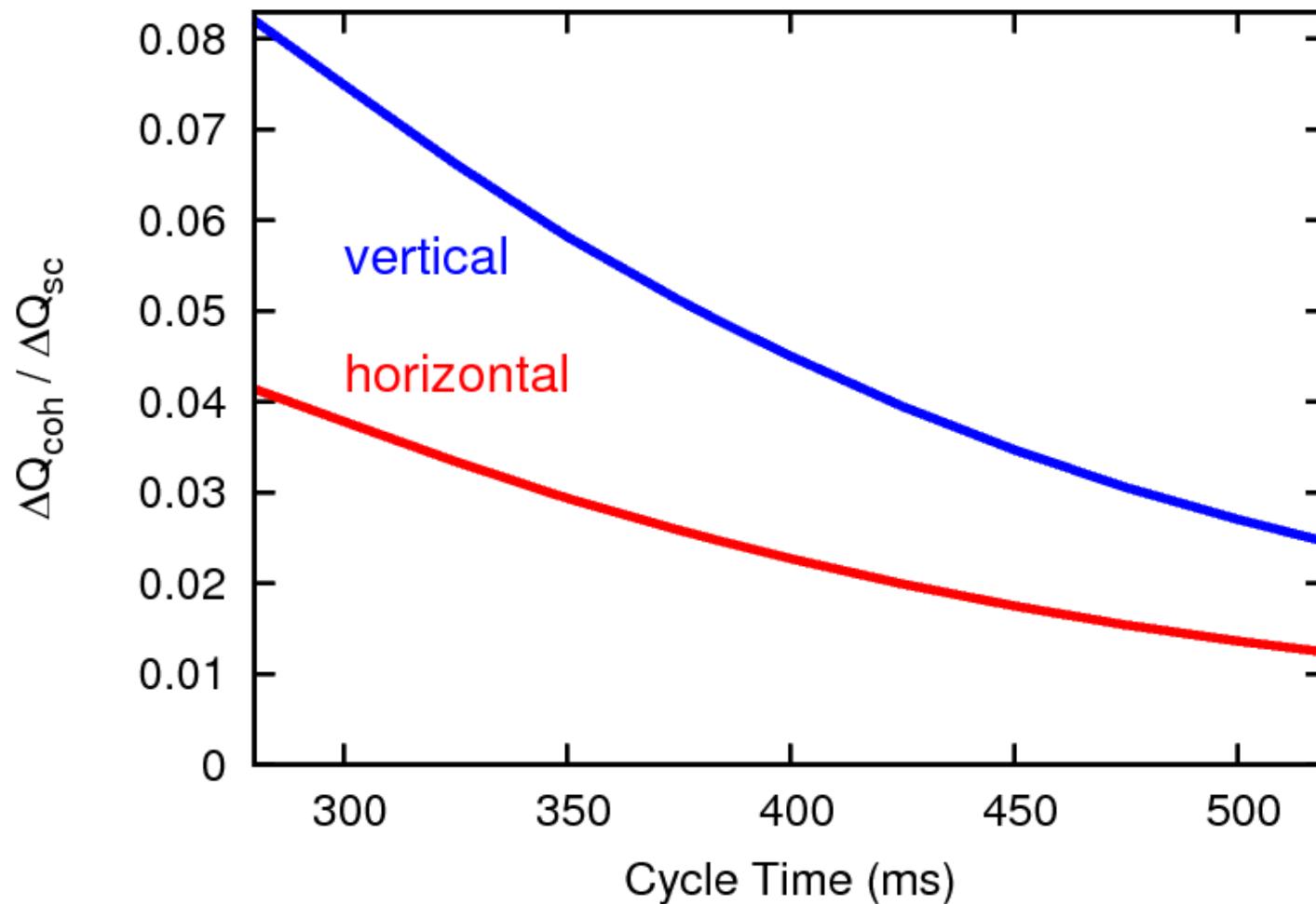
Landau damping  
effective boundary  
( $-0.1q_{sc}$ )

$$\frac{\Delta Q_{coh}}{\Delta Q_{sc}} = \left( \frac{a_{beam}}{b_{pipe}} \right)^2$$



might be important for machines like  
PSB, SIS100. Further simulation studies  
under progress

# HEAD-TAIL IN THE PS BOOSTER

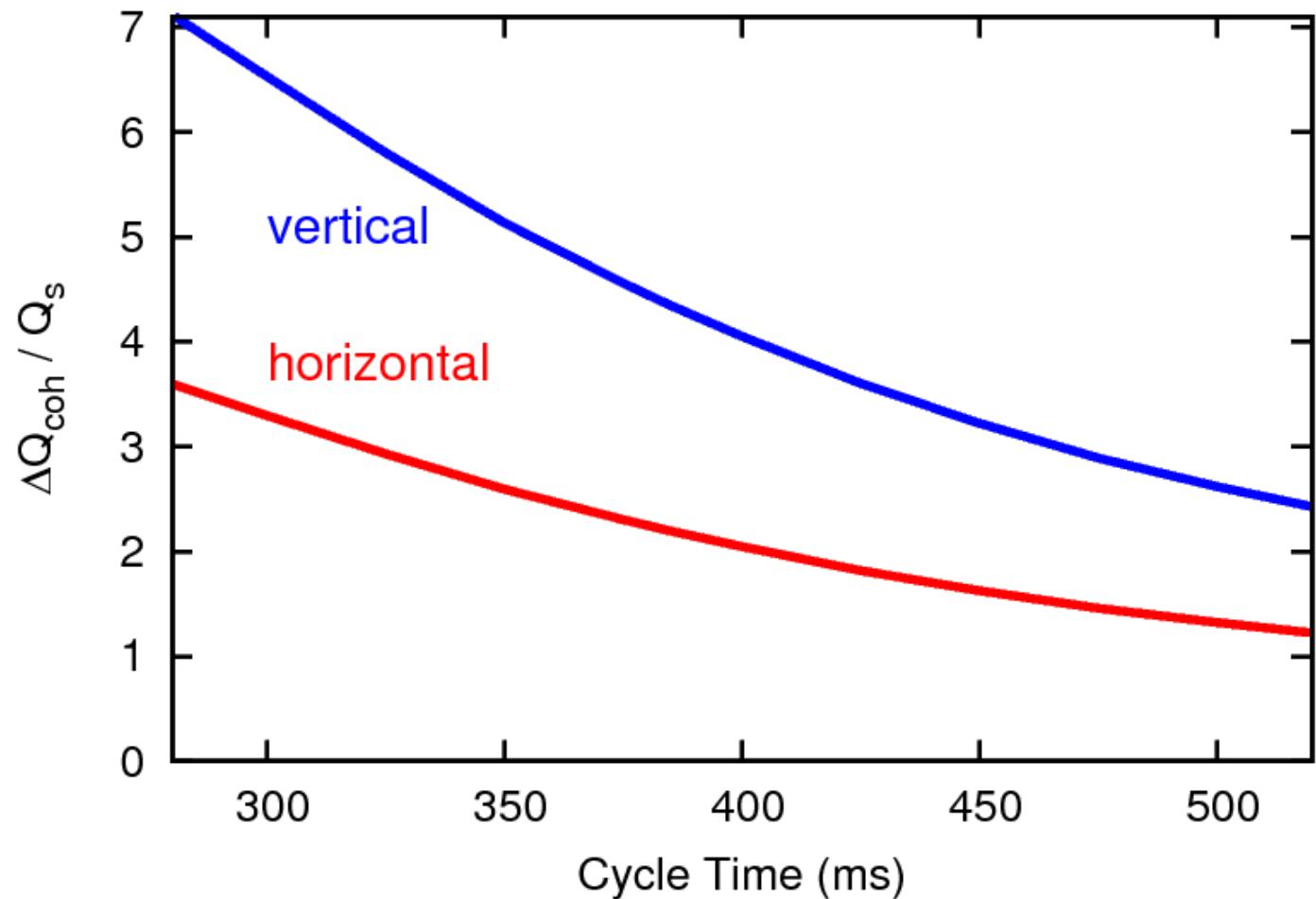


Landau damping is stronger in the vertical plane; the damping contribution decreases along the cycle. **This may contribute to the horizontal exclusiveness and to the later occurrence in CTime**

good news:

The real  $\Delta Q_{coh}$  is larger than the synchrotron tune.

During the cycle, the head-tail modes cross a wide frequency range



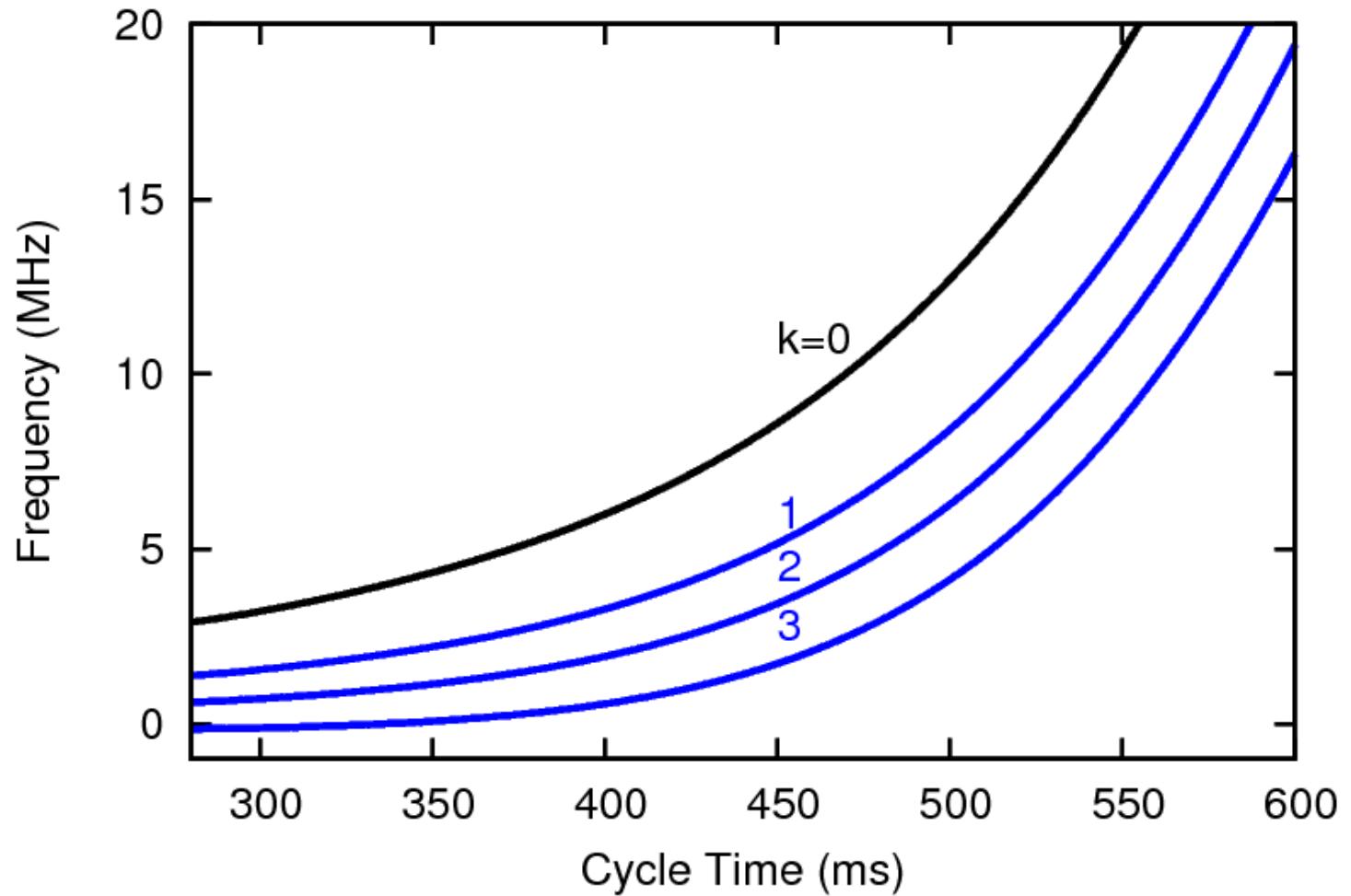
Still, no fast Transverse Mode Coupling Instability (TMCI) observed. Suppressed by space charge?

Theory predictions: Blaskiewicz prstab 1998; Burov prstab 2009

**good news:**

The real  $\Delta Q_{coh}$  is larger than the synchrotron tune.

During the cycle, the head-tail modes cross a wide frequency range



**Still, no fast Transverse Mode Coupling Instability (TMCI) observed. Suppressed by space charge?**

Theory predictions: Blaskiewicz prstab 1998; Burov prstab 2009

R=172.46m

Heavy ions:

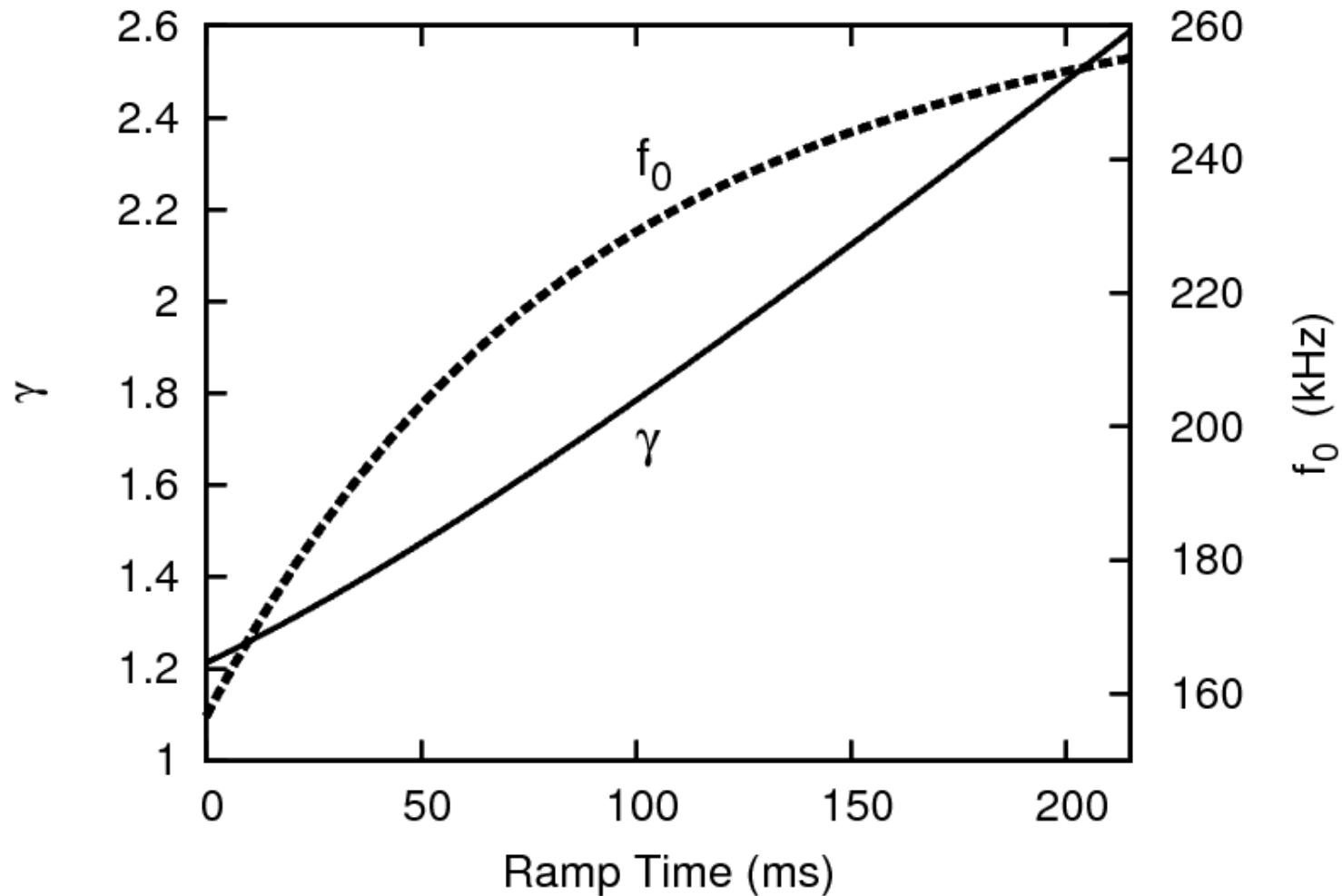
Kin. energy

200MeV-1.5GeV

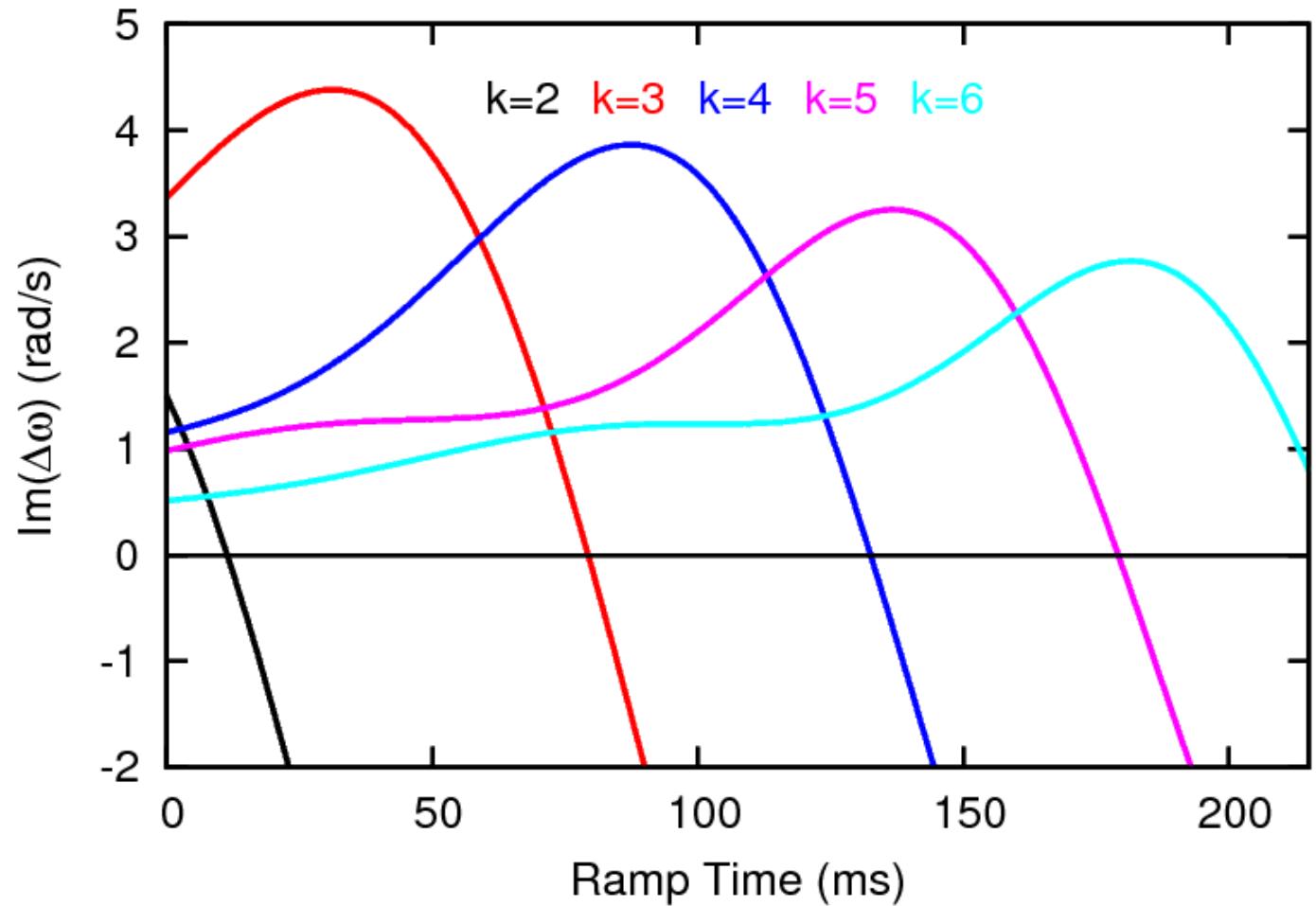
$Q_h = 18.84$

$Q_v = 18.73$

$\xi_v = -1.2$



The head-tail growth rate given by the Sacherer theory for the Resistive-Wall Impedance



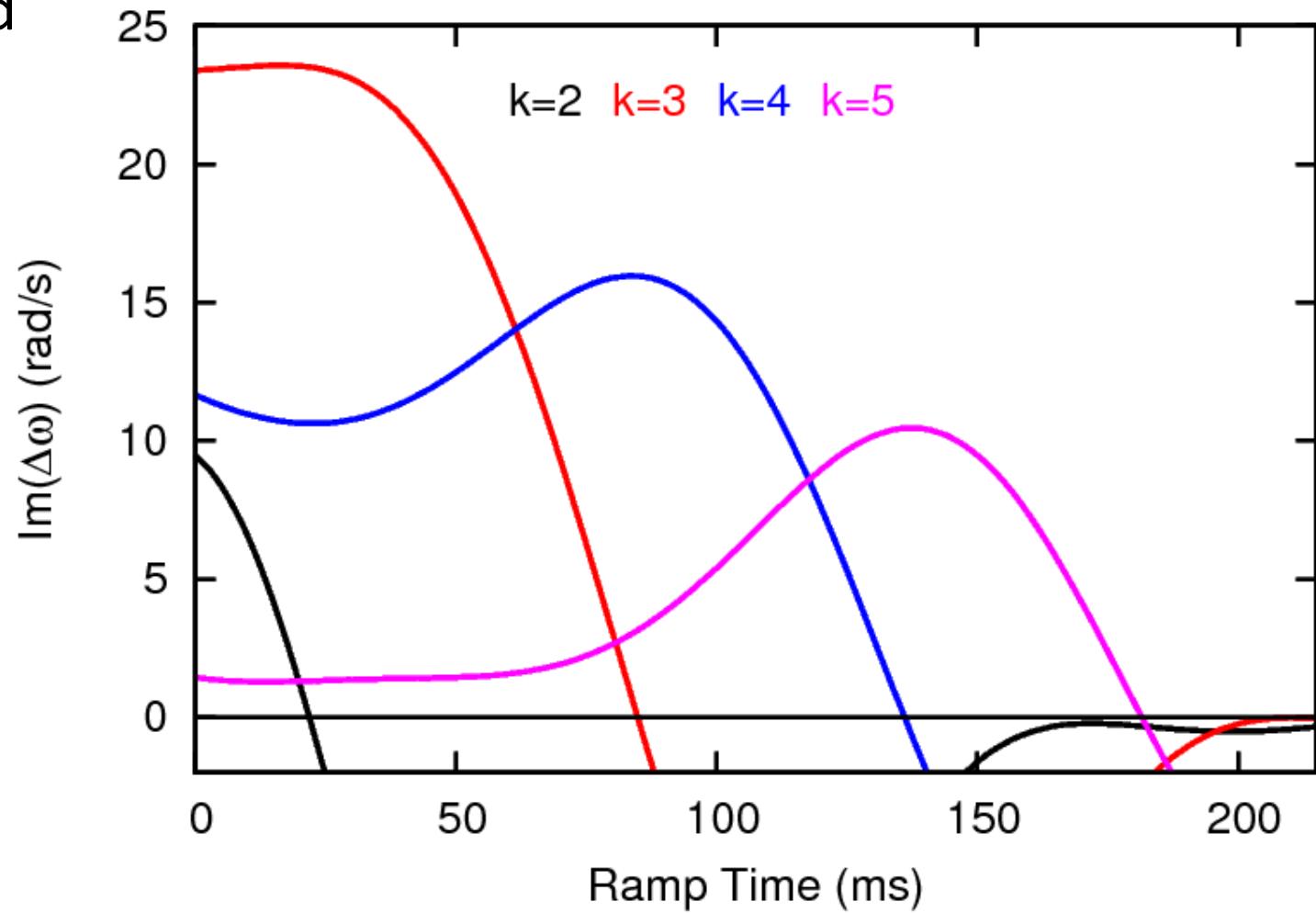
For a Narrow-Band

Impedance

$f_r = 3\text{MHz}$ ,

$R_T = 3\text{M}\Omega/\text{m}$ ,

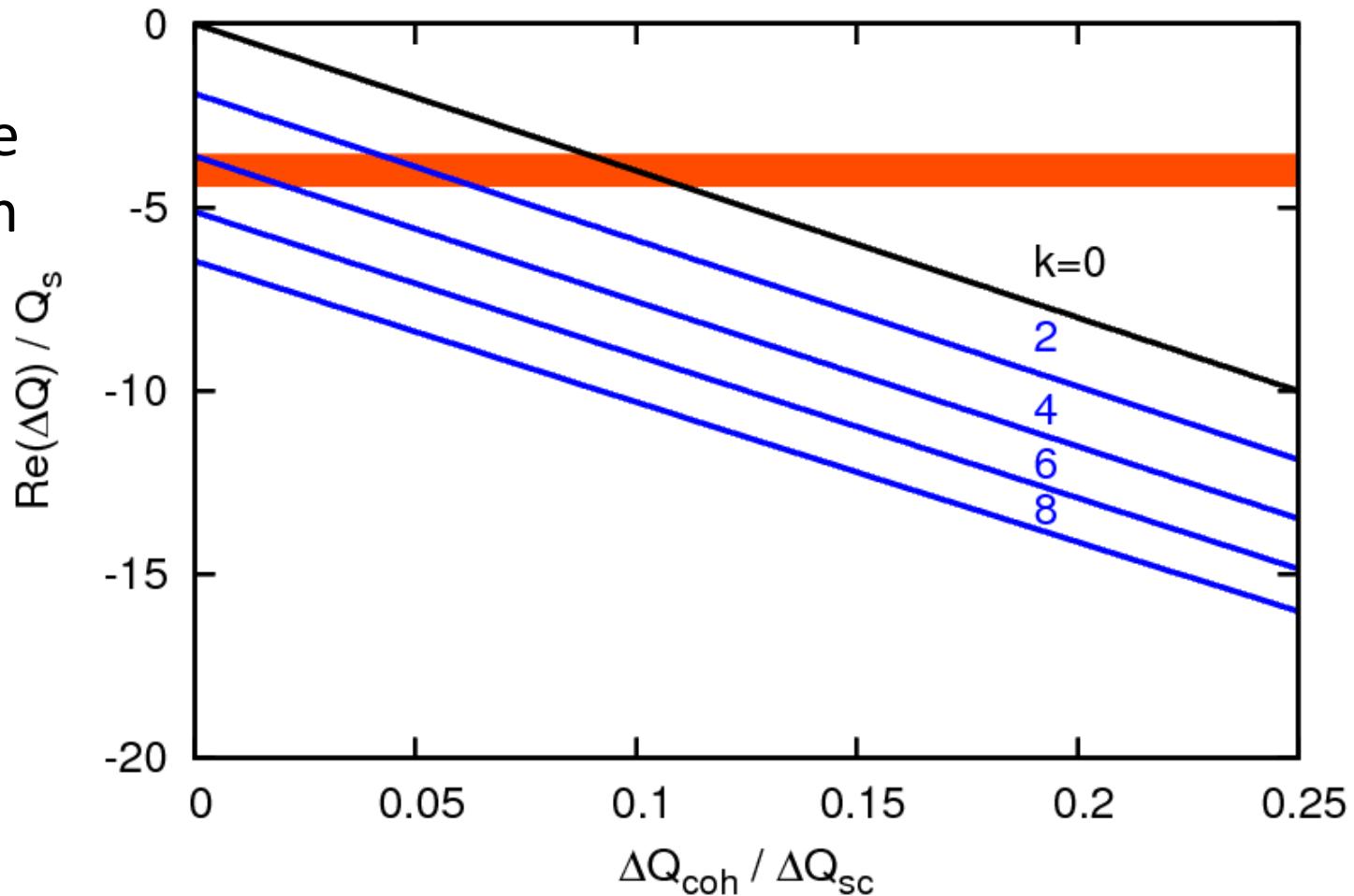
$Q_T = 2$



Landau Damping due  
to Space Charge with  
Image Charges

$U^{28+} : q=40$

$\Delta Q_{coh} / \Delta Q_{sc} \approx 0.12$



**Summarized:**  
with a transverse feedback and space-charge effects, the situation  
with the head-tail modes and TMCI in SIS100 looks encouraging

# Beam Break-Up Instability

Transition Crossing in PS at CERN  
 $p^+$  operation in SIS100 at GSI

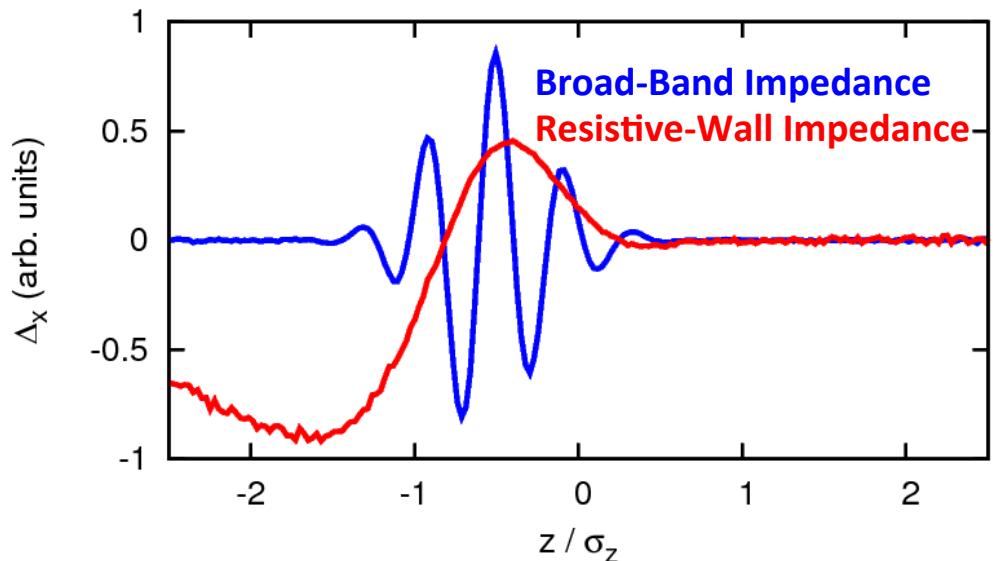
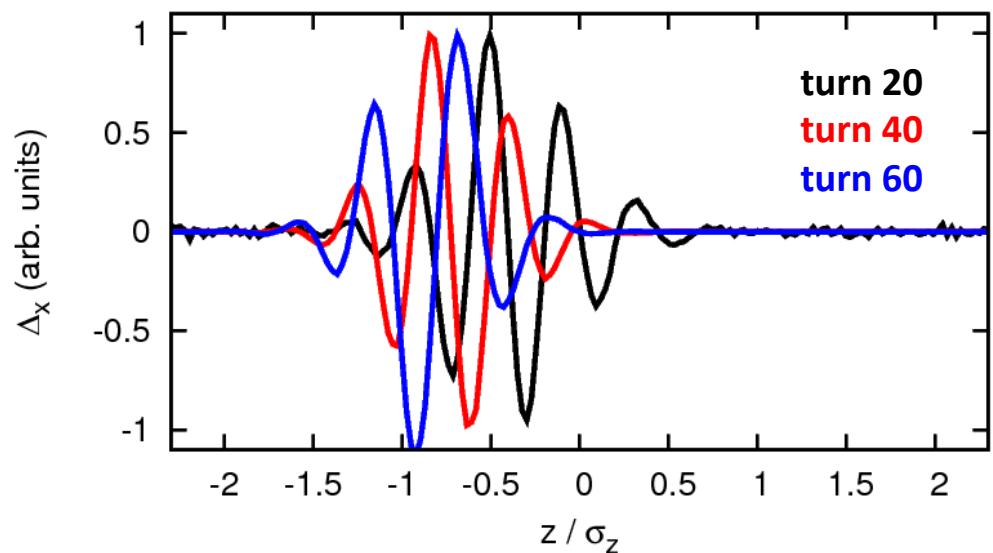
PS observation first reported:  
R.Cappi, E.Metral, G.Metral, 2000  
Significant progress since then,  
see S.Aumon, PhD Thesis, 2012.

Strong impedance or  
slow synchrotron motion,  
 $\text{Im}(\Delta Q) \gg Q_s$



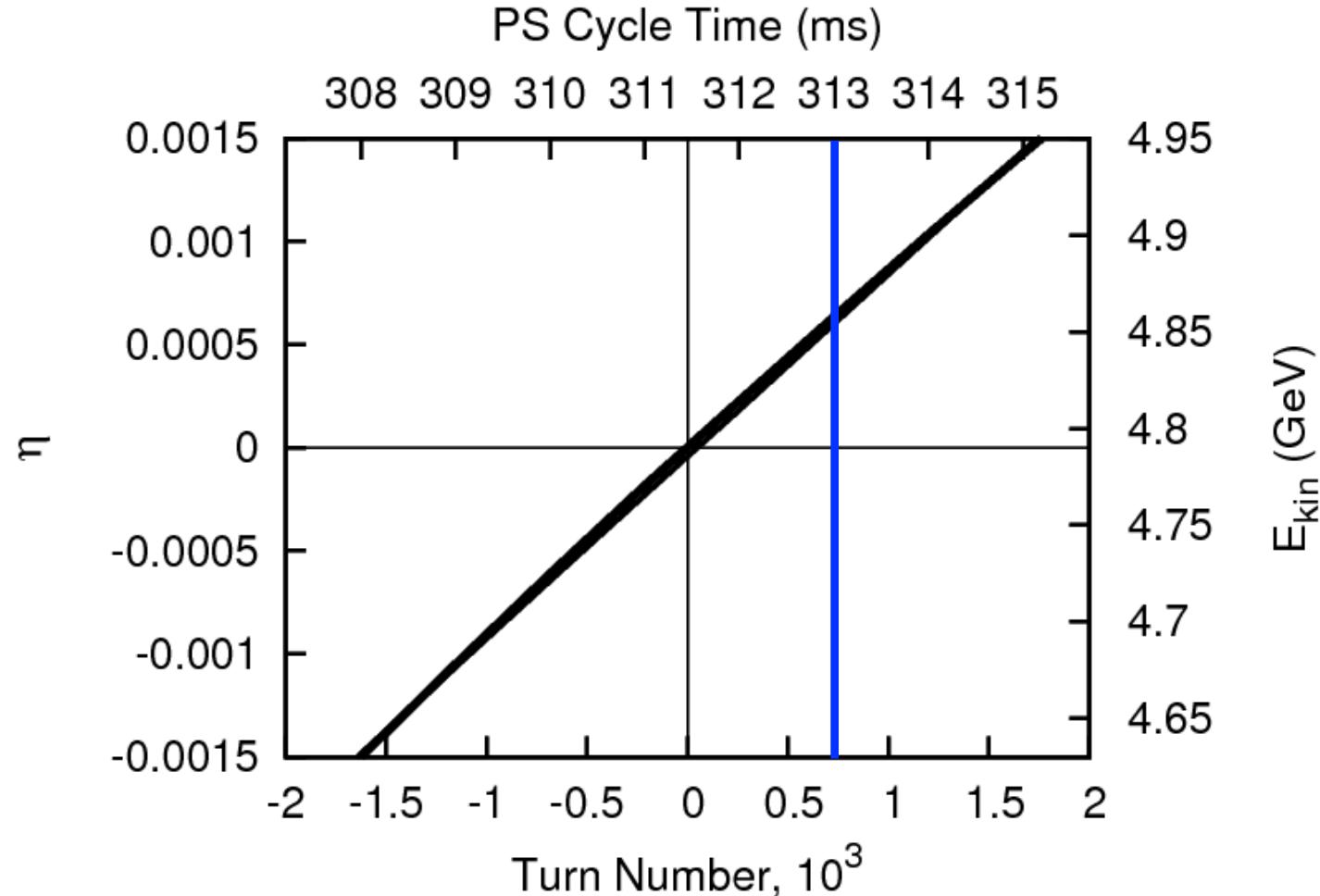
a fast instability with a  
convective character  
(oscillation migrates towards tail)

Driven at high-frequency (BB, GHz)  
but also lower-frequency  
(RW, bunch-length)



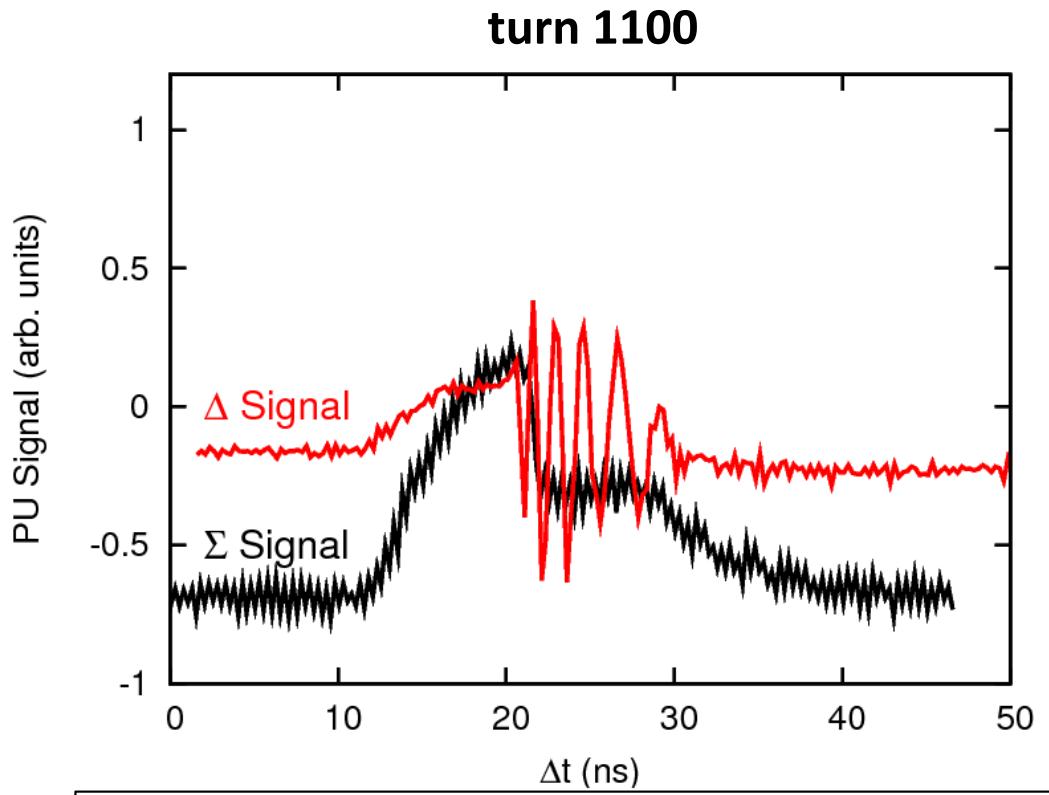
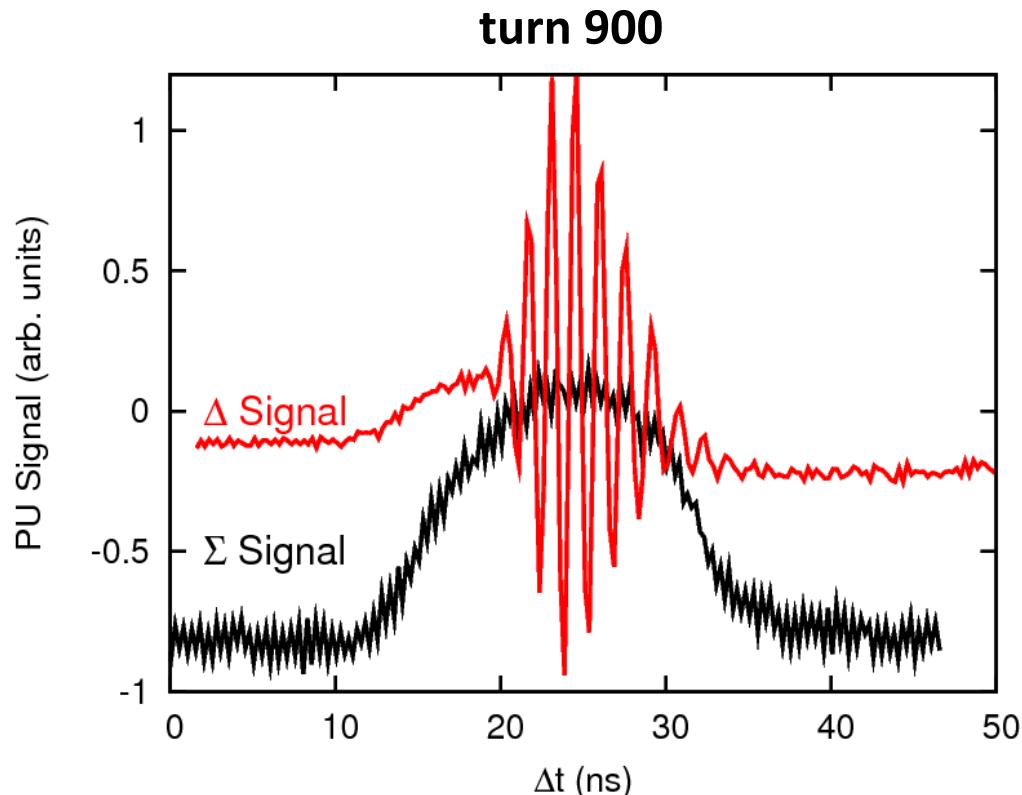
simulation examples

strong transverse oscillations and fast losses around C313ms



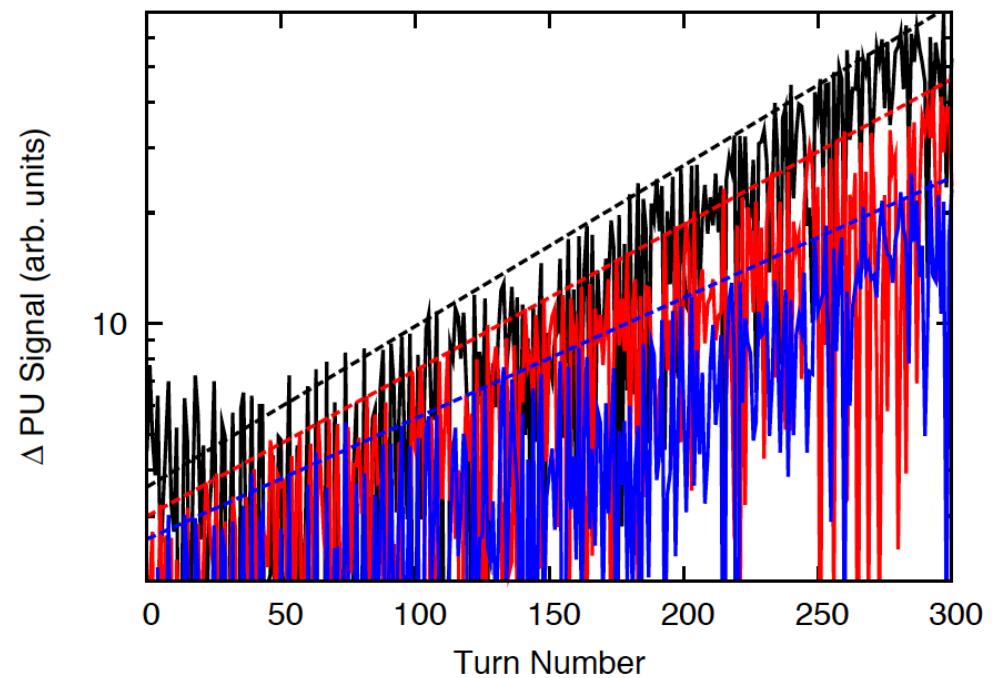
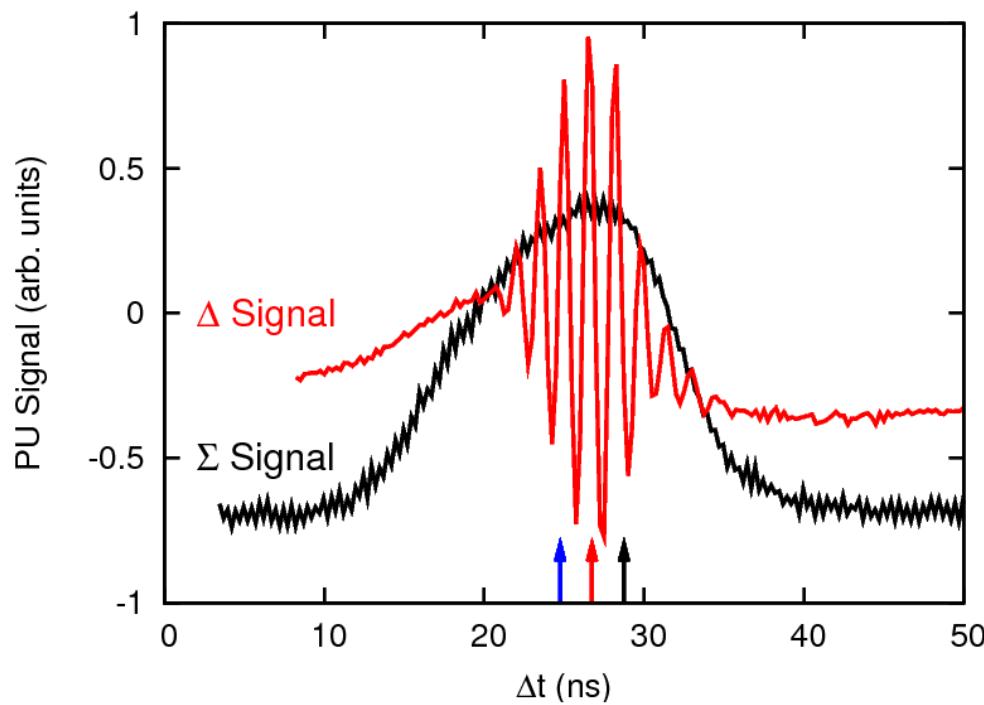
Measurement campaign at PS in June 2012.  
Help of S.Aumon, S.Gilardoni, PS Operation Group

- high-frequency (0.7GHz) oscillations
- fast losses
- this example:  $\Delta Q/Q_s = 11$



more details in the upcoming CERN Report

## BEAM BREAK-UP IN PS



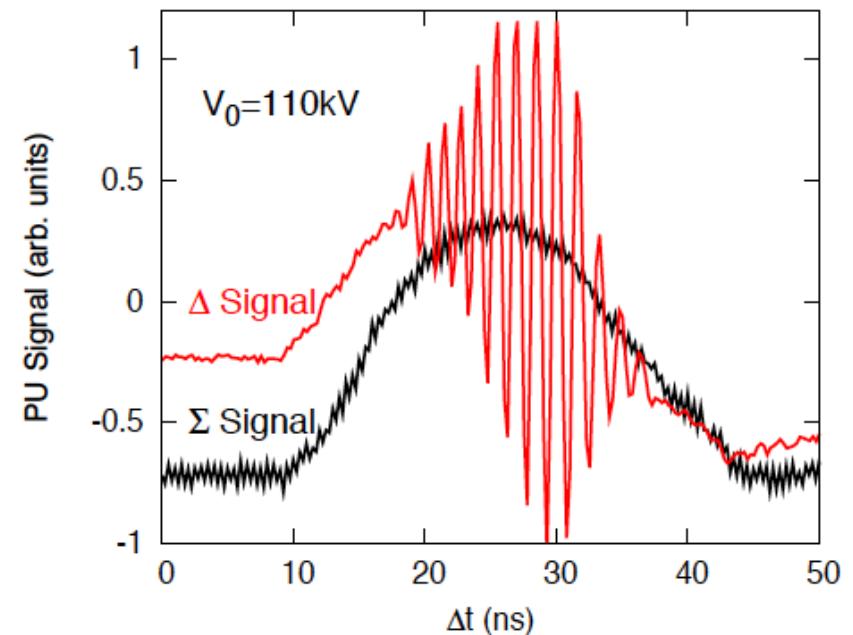
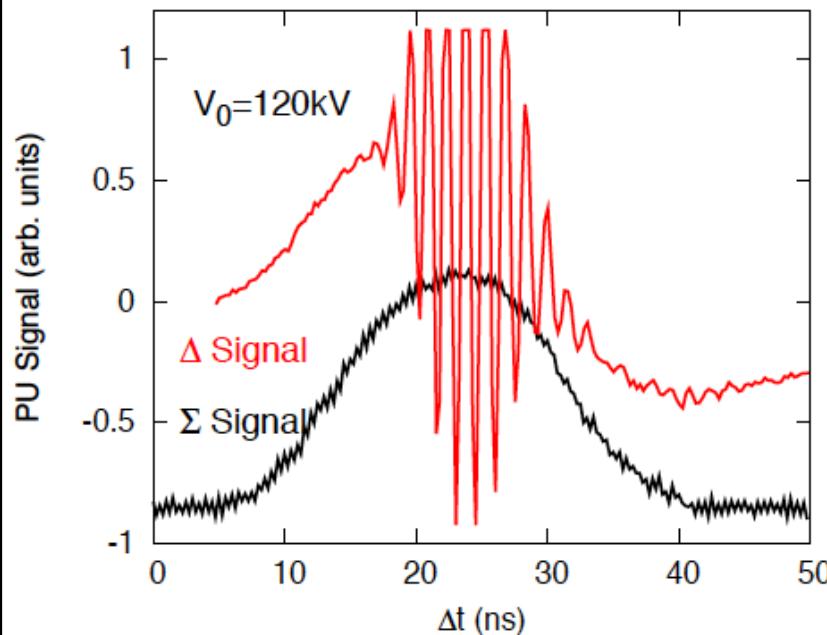
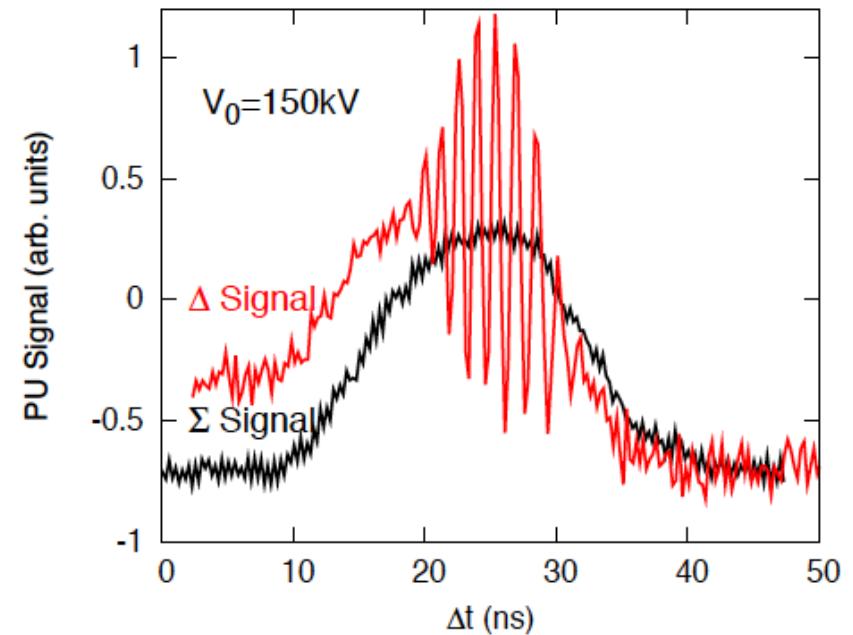
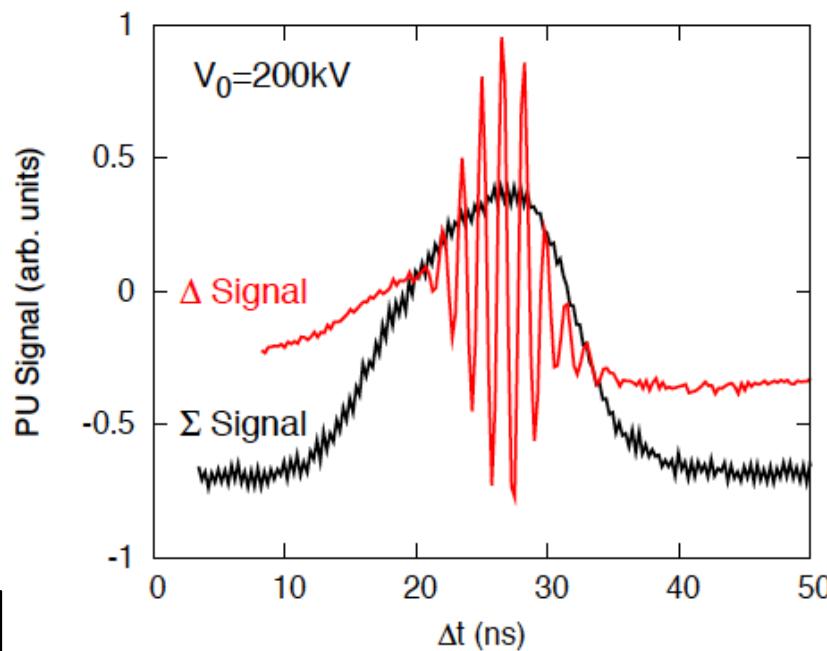
head:  $\Delta Q = 1.2 \text{e-}3$

middle:  $\Delta Q = 1.45 \text{e-}3$

tail:  $\Delta Q = 1.6 \text{e-}3$

no definite growth rate,  
the oscillation migrates towards the bunch tail:  
the convective character of the Beam Break-Up

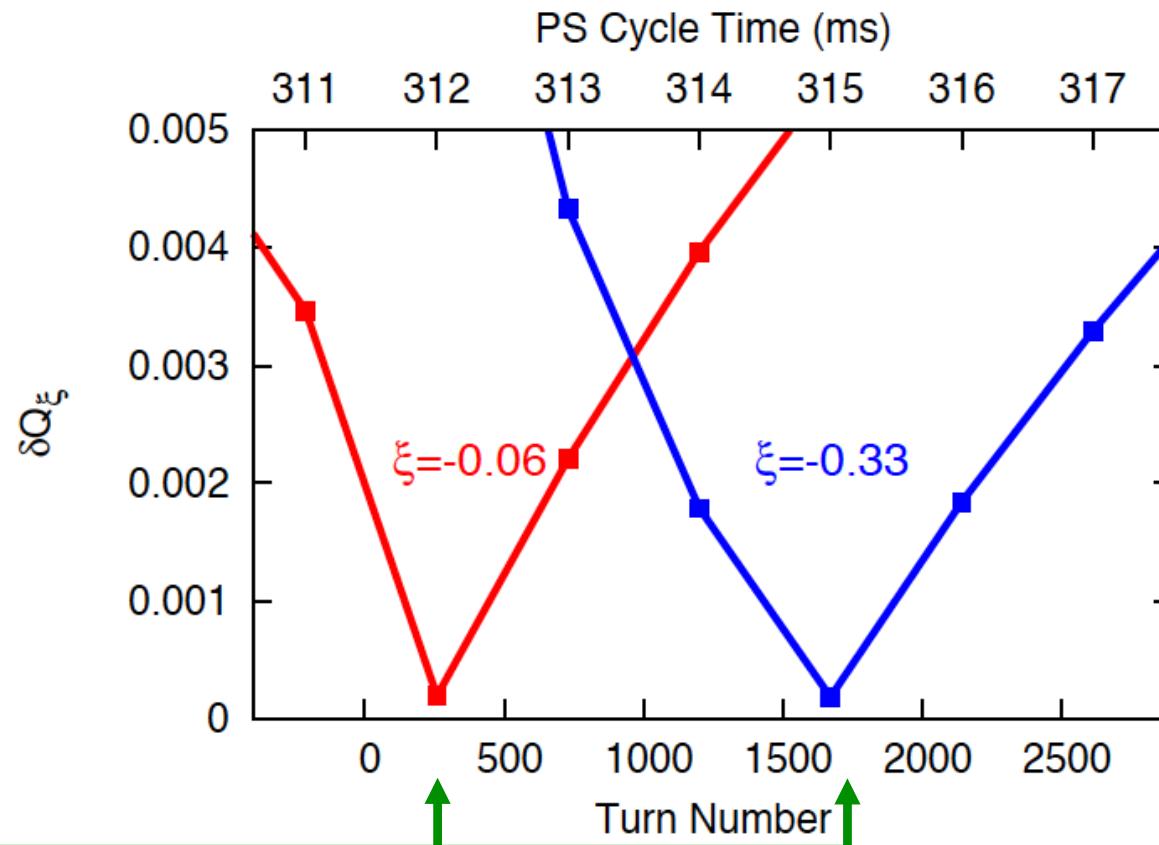
# BEAM BREAK-UP IN PS



**Instability thresholds for different rf voltage and the role of the bunch length: more details in the upcoming CERN Report**

Landau Damping for the  
Coasting-Beam Estimations:

$$\delta Q_\xi = |\eta(n - Q_0) + Q_0 \xi| \delta_p$$

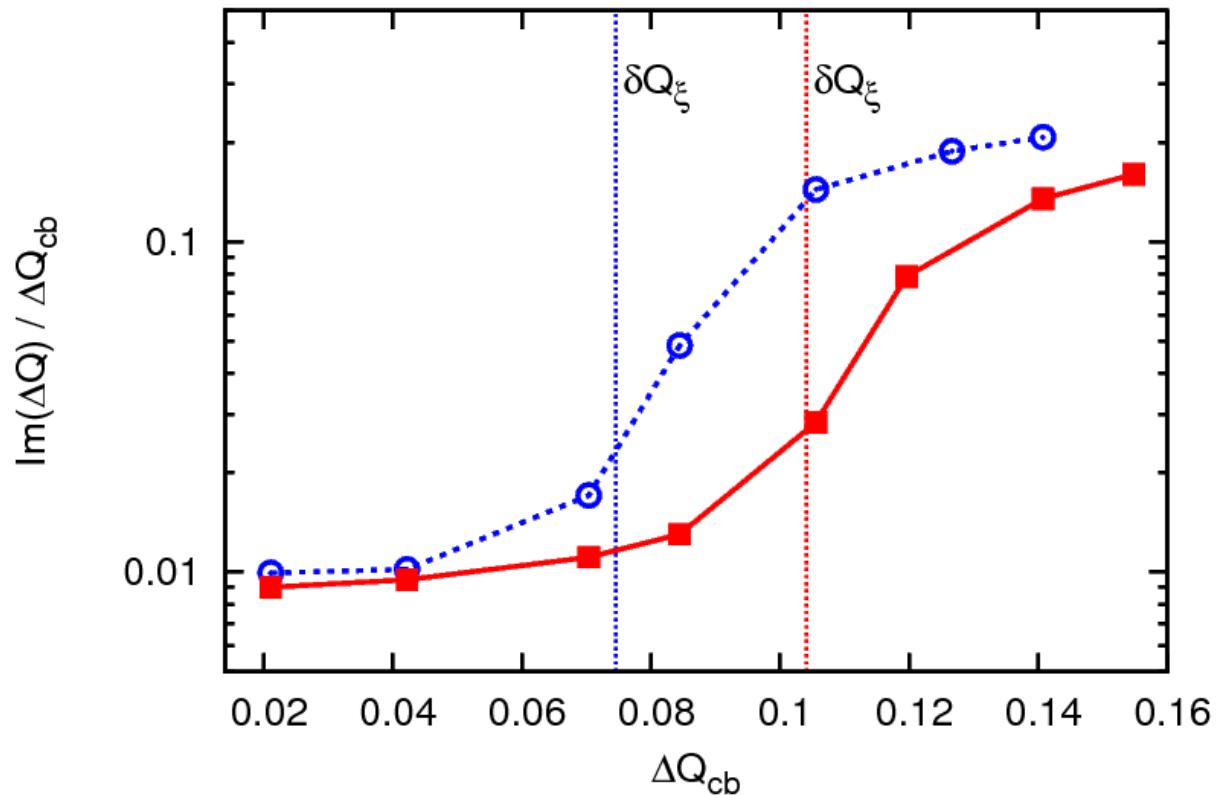


Observations in PS:  
at 313ms for the “zero- $\xi$ -lattice”,  
at 315ms for the “finite- $\xi$ -lattice”.

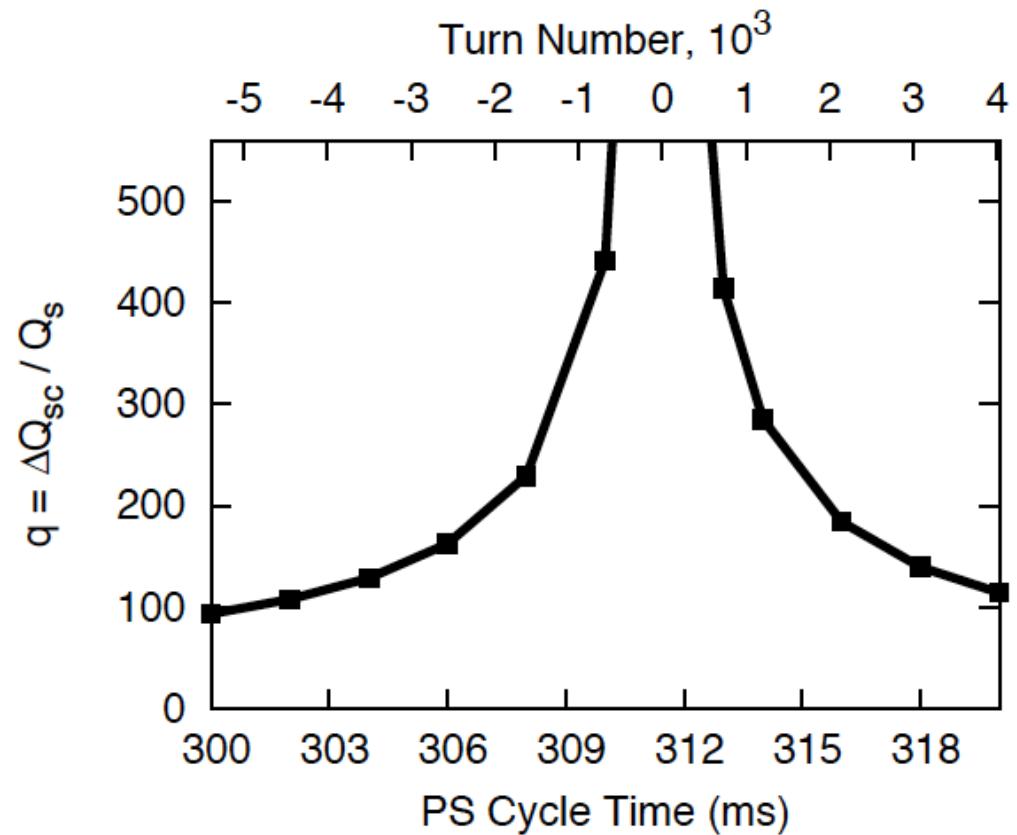
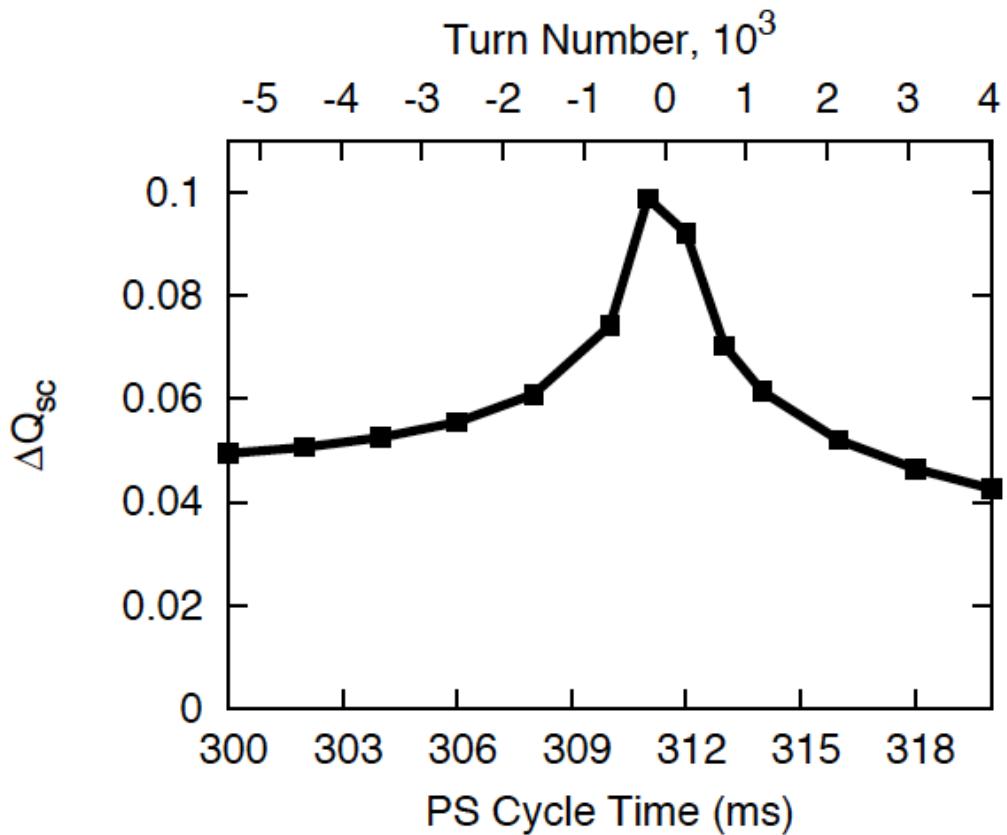
PIC Simulations with  
a Broad-Band Impedance

Beam Break-Up for  
an increasing impedance

$Q_s = 0.0025$   
 $Q_s = 0.0035$



Quasi-Coasting-Beam Type of the Threshold for the  
Beam Break-Up Instability



$N_p = 90e10$ ,  $t_b = 50\text{ns}$  at C300ms,  $V_0 = 200\text{kV}$

Very strong space-charge regime.  
Transverse beam blow-up showed no effects

Due to the Broad-Band Impedance (assuming the PS model):

$$\Delta Q/\delta Q_\xi < 0.15$$

Due to the Resistive-Wall Impedance:

$$\Delta Q/\delta Q_\xi < 0.5$$

but: not observed in PS

Additional damping by other mechanisms (bunch length,  $Q_s$ )

Strong Space Charge  $q > 130$

Well in the safe operation

# CONCLUSIONS

The classical head-tail instability during the ramp at PSB:

- ◆ thresholds for different bucket forms
- ◆ a low-frequency narrow-band could explain the puzzling questions

Theory, Simulations and Measurements at PS Booster and at SIS18 show that  
**Space Charge is our Best Friend:**

- ◆ SC induces a strong Landau Damping
- ◆ together with Image Charges, SC can stabilize some Head-Tail modes
- ◆ SC suppresses the TMCI

Experiments near transition at PS and PIC Simulations:

- ◆ Beam Break-Up with a convective character at high frequency (Broad-Band)
- ◆ the quasi-coasting-beam estimation gives a good threshold estimate
- ◆ other damping effects important
- ◆ Space Charge does not play an important role

**At SIS100 seems to be safe  
regarding the Head-Tail (feedback), TMCI and the Beam Break-Up**