Fossil Primordial gravitational waves with a Generalized initial state

Suddhasattwa Brahma

Institute for Gravitation and the Cosmos
Pennsylvania State University

Work with Elliot Nelson and Sarah Shandera
arXiv: 13xx.xxxx

May 6, 2013

Table of contents

Why look for fossils? Are they even observable?

- Observation of primordial gravitational waves would be extremely important for the inflationary scenario $\implies$ Hubble scale of inflation, other parameters.
- Directly measuring primordial gravitational waves seems out of reach of current surveys. However...
- Tensor fluctuations sourced during inflation have self interactions as well as interactions with the scalar fluctuations.
- Three-point correlation between two scalar modes and one tensor mode manifests itself as local anisotropic contribution to the scalar power spectrum.
- One problem is that if the tensor mode is the primordial gravitational fluctuation, then this is still a small signal in typical inflationary scenarios.
- This "fossil" signature can be significantly enhanced for some models with non-Bunch Davies initial states.

Why look for fossils? Are they even observable?

- Observation of primordial gravitational waves would be extremely important for the inflationary scenario $\Longrightarrow$ Hubble scale of inflation, other parameters.
- Directly measuring primordial gravitational waves seems out of reach of current surveys. However...
- Tensor fluctuations sourced during inflation have self interactions as well as interactions with the scalar fluctuations.
- Three-point correlation between two scalar modes and one tensor mode manifests itself as local anisotropic contribution to the scalar power spectrum.
- One problem is that if the tensor mode is the primordial gravitational fluctuation, then this is still a small signal in typical inflationary scenarios.
- This “fossil” signature can be significantly enhanced for some models with non-Bunch Davies initial states.

Why look for fossils? Are they even observable?

- Observation of primordial gravitational waves would be extremely important for the inflationary scenario $\implies$ Hubble scale of inflation, other parameters.
- Directly measuring primordial gravitational waves seems out of reach of current surveys. However...
- Tensor fluctuations sourced during inflation have self interactions as well as interactions with the scalar fluctuations.
- Three-point correlation between two scalar modes and one tensor mode manifests itself as local anisotropic contribution to the scalar power spectrum.
- One problem is that if the tensor mode is the primordial gravitational fluctuation, then this is still a small signal in typical inflationary scenarios.
- This "fossil" signature can be significantly enhanced for some models with non-Bunch Davies initial states.

Anisotropic Scalar Power Spectrum due to fossil field

[D. Jeong & M. Kamionkowski, 2012]

- Primordial scalar perturbations are statistically isotropic and Gaussian: $\langle \zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\rangle = V\delta^3(\sum \mathbf{k}_i)P(k)$.
- Scalar curvature coupled to an unobservable fossil field: $\langle \zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\gamma^p(\mathbf{k}_3)\rangle \equiv (2\pi)^3\delta^3(\sum \mathbf{k}_i)B_p(k_1, k_2, k_3)$.
- Recall: $\mathrm{P}(A \cap B \cap C) = \mathrm{P}(A \cap B|C)\mathrm{P}(C)$
  $\implies$ Define function $f_p(\mathbf{k}_1, \mathbf{k}_2)$ in terms of bispectrum and fossil field power spectrum $P_p(K)$ through $B_p(k_1, k_2, K) \equiv P_p(K)f_p(\mathbf{k}_1, \mathbf{k}_2)\epsilon^p_{ij}k_1^i k_2^j$. (This factorization only holds for local interactions).
- Local power spectrum has off-diagonal components:

$$\langle \zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\rangle_{\gamma^p(\mathbf{K})} = f_p(\mathbf{k}_1, \mathbf{k}_2)\gamma^{p*}(\mathbf{K})\epsilon^p_{ij}k_1^i k_2^j \delta^D_{\mathbf{k}_{123}}, \qquad (2.1)$$

where $\delta^D_{\mathbf{k}_{123}}$ denotes a Kronecker delta function that sets $\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{K} = 0$, and ϵ^p_{ij} is the polarization tensor for the fossil field.

Anisotropic Scalar Power Spectrum due to fossil field

[D. Jeong & M. Kamionkowski, 2012]

- Primordial scalar perturbations are statistically isotropic and Gaussian: $\langle\zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\rangle = V\delta^3(\sum \mathbf{k}_i)P(k)$.
- Scalar curvature coupled to an unobservable fossil field: $\langle\zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\gamma^p(\mathbf{k}_3)\rangle \equiv (2\pi)^3\delta^3(\sum \mathbf{k}_i)B_p(k_1, k_2, k_3)$.
- Recall: $P(A \cap B \cap C) = P(A \cap B|C)P(C)$
  $\implies$ Define function $f_p(\mathbf{k}_1, \mathbf{k}_2)$ in terms of bispectrum and fossil field power spectrum $P_p(K)$ through $B_p(k_1, k_2, K) \equiv P_p(K)f_p(\mathbf{k}_1, \mathbf{k}_2)\epsilon^p_{ij}k^i_1 k^j_2$. (This factorization only holds for local interactions).
- Local power spectrum has off-diagonal components:

$$\langle\zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\rangle_{\gamma^p(\mathbf{K})} = f_p(\mathbf{k}_1, \mathbf{k}_2)\gamma^{p*}(\mathbf{K})\epsilon^p_{ij}k^i_1 k^j_2 \delta^D_{\mathbf{k}_{123}}, \tag{2.1}$$

where $\delta^D_{\mathbf{k}_{123}}$ denotes a Kronecker delta function that sets $\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{K} = 0$, and ϵ^p_{ij} is the polarization tensor for the fossil field.

Anisotropic Scalar Power Spectrum due to fossil field

[D. Jeong & M. Kamionkowski, 2012]

- Primordial scalar perturbations are statistically isotropic and Gaussian: $\langle\zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\rangle = V\delta^3(\sum \mathbf{k}_i)P(k)$.
- Scalar curvature coupled to an unobservable fossil field: $\langle\zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\gamma^p(\mathbf{k}_3)\rangle \equiv (2\pi)^3\delta^3(\sum \mathbf{k}_i)B_p(k_1, k_2, k_3)$.
- Recall: $\mathrm{P}(A \cap B \cap C) = \mathrm{P}(A \cap B|C)\mathrm{P}(C)$
  $\Longrightarrow$ Define function $f_p(\mathbf{k}_1, \mathbf{k}_2)$ in terms of bispectrum and fossil field power spectrum $P_p(K)$ through $B_p(k_1, k_2, K) \equiv P_p(K)f_p(\mathbf{k}_1, \mathbf{k}_2)\epsilon^p_{ij}k_1^i k_2^j$. (This factorization only holds for local interactions).
- Local power spectrum has off-diagonal components:

$$\langle\zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\rangle_{\gamma^p(\mathbf{K})} = f_p(\mathbf{k}_1, \mathbf{k}_2)\gamma^{p*}(\mathbf{K})\epsilon^p_{ij}k_1^i k_2^j \delta^D_{\mathbf{k}_{123}}, \tag{2.1}$$

where $\delta^D_{\mathbf{k}_{123}}$ denotes a Kronecker delta function that sets $\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{K} = 0$, and ϵ^p_{ij} is the polarization tensor for the fossil field.

Anisotropic Scalar Power Spectrum due to fossil field

[D. Jeong & M. Kamionkowski, 2012]

- Primordial scalar perturbations are statistically isotropic and Gaussian: $\langle\zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\rangle = V\delta^3(\sum \mathbf{k}_i)P(k)$.
- Scalar curvature coupled to an unobservable fossil field: $\langle\zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\gamma^p(\mathbf{k}_3)\rangle \equiv (2\pi)^3\delta^3(\sum \mathbf{k}_i)B_p(k_1, k_2, k_3)$.
- Recall: $\mathrm{P}(A \cap B \cap C) = \mathrm{P}(A \cap B|C)\mathrm{P}(C)$
  $\Longrightarrow$ Define function $f_p(\mathbf{k}_1, \mathbf{k}_2)$ in terms of bispectrum and fossil field power spectrum $P_p(K)$ through $B_p(k_1, k_2, K) \equiv P_p(K)f_p(\mathbf{k}_1, \mathbf{k}_2)\epsilon^p_{ij}k_1^i k_2^j$. (This factorization only holds for local interactions).
- Local power spectrum has off-diagonal components:

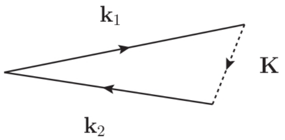
$$\langle\zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\rangle_{\gamma^p(\mathbf{K})} = f_p(\mathbf{k}_1, \mathbf{k}_2)\gamma^{p*}(\mathbf{K})\epsilon^p_{ij}k_1^i k_2^j \delta^D_{\mathbf{k}_{123}}, \tag{2.1}$$

where $\delta^D_{\mathbf{k}_{123}}$ denotes a Kronecker delta function that sets $\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{K} = 0$, and ϵ^p_{ij} is the polarization tensor for the fossil field.

Physical Interpretation

Along each mode of the fossil field (primordial gravitational wave), space expands/contracts differently. This breaks the traslational invariance $\Longrightarrow$ Delta function ($\mathbf{k}_1 + \mathbf{k}_2 = 0$) in front of Power Spectrum not applicable any more. Thus the three-point function with an unseen fossil field gives rise to local anisotropy in the scalar power spectrum. This correlation could be detected by measuring anisotropy in the power spectrum between different regions in the sky.

Figure 1: Scalar mode correlations in presence of a fossil field[1]



[1] Figure from **[L. Dai, D. Jeong & M. Kamionkowski, 2013]**

The Bunch Davies Initial State

- The Power Spectrum of the primordial scalar fluctuations comes from the quadratic (in fluctuations) part of the action. It is just like evaluating the 2-point function in fourier space, from a Harmonic Oscillator hamiltonian.
- In flat space time, the initial state is chosen as the (unique) vacuum state which is annihilated by the lowering operator. However, for inflation, the background is de-Sitter. There is no longer any lowest energy eigenstate available at all times, to unambiguosly define the physical vacuum.
- Nevertheless, given a particular instant of time (η_0), we can still define the instantaneous vacuum as the lowest energy state (subject to certain restrictions). This is the the Bunch Davies vacuum, where co-moving observers see no particles.

The Bunch Davies Initial State

- The Power Spectrum of the primordial scalar fluctuations comes from the quadratic (in fluctuations) part of the action. It is just like evaluating the 2-point function in fourier space, from a Harmonic Oscillator hamiltonian.
- In flat space time, the initial state is chosen as the (unique) vacuum state which is annihilated by the lowering operator. However, for inflation, the background is de-Sitter. There is no longer any lowest energy eigenstate available at all times, to unambiguously define the physical vacuum.
- Nevertheless, given a particular instant of time (η_0), we can still define the instantaneous vacuum as the lowest energy state (subject to certain restrictions). This is the the Bunch Davies vacuum, where co-moving observers see no particles.

The Bunch Davies Initial State

- The Power Spectrum of the primordial scalar fluctuations comes from the quadratic (in fluctuations) part of the action. It is just like evaluating the 2-point function in fourier space, from a Harmonic Oscillator hamiltonian.
- In flat space time, the initial state is chosen as the (unique) vacuum state which is annihilated by the lowering operator. However, for inflation, the background is de-Sitter. There is no longer any lowest energy eigenstate available at all times, to unambiguously define the physical vacuum.
- Nevertheless, given a particular instant of time (η_0), we can still define the instantaneous vacuum as the lowest energy state (subject to certain restrictions). This is the the Bunch Davies vacuum, where co-moving observers see no particles.

Why deviate from Bunch Davies?

Let us consider a more general initial state, at some intial (conformal) time (η_0). For such a state, co-moving observers see inflaton excitations, for each mode. However, we demand that the modes are well within the horizon at η_0, i.e. $|k\eta_0| \ll 1$.

But why should we choose such a state?

In a scenario where inflation starts at some finite time, the state of perturbations at the onset of inflation may deviate from the vacuum state as a consequence of a non-trivial pre-inflationary evolution. We parametrize our ignorance about how inflation began and what preceded it through this generalized initial state.

Generically, pre-inationary evolution could produce a state for the perturbations with some number of excitations compared to the Bunch-Davies state, and which need not be Gaussian or pure. For simplicity let us consider a Gaussian, pure initial states which can be described by Bogoliubov transformations of the vacuum. [I. Agullo & S. Shandera, 2011]

Why deviate from Bunch Davies?

Let us consider a more general initial state, at some intial (conformal) time (η_0). For such a state, co-moving observers see inflaton excitations, for each mode. However, we demand that the modes are well within the horizon at η_0, i.e. $|k\eta_0| \ll 1$.

But why should we choose such a state?
In a scenario where inflation starts at some finite time, the state of perturbations at the onset of inflation may deviate from the vacuum state as a consequence of a non-trivial pre-inflationary evolution. We parametrize our ignorance about how inflation began and what preceded it through this generalized initial state.

Generically, pre-inationary evolution could produce a state for the perturbations with some number of excitations compared to the Bunch-Davies state, and which need not be Gaussian or pure. For simplicity let us consider a Gaussian, pure initial states which can be described by Bogoliubov transformations of the vacuum. [I. Agullo & S. Shandera, 2011]

Why deviate from Bunch Davies?

Let us consider a more general initial state, at some intial (conformal) time (η_0). For such a state, co-moving observers see inflaton excitations, for each mode. However, we demand that the modes are well within the horizon at η_0, i.e. $|k\eta_0| \ll 1$.

But why should we choose such a state?
In a scenario where inflation starts at some finite time, the state of perturbations at the onset of inflation may deviate from the vacuum state as a consequence of a non-trivial pre-inflationary evolution. We parametrize our ignorance about how inflation began and what preceded it through this generalized initial state.

Generically, pre-inationary evolution could produce a state for the perturbations with some number of excitations compared to the Bunch-Davies state, and which need not be Gaussian or pure. For simplicity let us consider a Gaussian, pure initial states which can be described by Bogoliubov transformations of the vacuum. [I. Agullo & S. Shandera, 2011]

The non-Bunch Davies Initial State

Initial state for scalar fluctuations ζ as Bogoliubov tranforms of Bunch-Davies mode functions:

$$u_k(\eta) = \alpha_k^{(s)} u_k^{\mathrm{BD}}(\eta) + \beta_k^{(s)} u_k^{*\mathrm{BD}}(\eta) \tag{3.1}$$

where $u_k^{\mathrm{BD}}(\eta) = \frac{H^2}{\dot{\phi}} \frac{1}{\sqrt{2k^3}}(1 + ik\eta)e^{-ik\eta}$. Here $|(\beta_k^{(s)})|^2 = N_k^{(s)}$ is the number of particles in mode k, and, the relative phase between $\alpha_k^{(s)}$ and $\beta_k^{(s)}$ is $\Theta^{(s)}$.

The simple Bogoliubov rotation may not capture a typical initial state. Nonetheless, the possibility of deviations from Bunch-Davies are an important conceptual point about the inflationary paradigm.

In this particular case, as a side-effect of these non-BD initial state, primordial gravitational waves seem to become more easily observable.

The non-Bunch Davies Initial State

Initial state for scalar fluctuations ζ as Bogoliubov tranforms of Bunch-Davies mode functions:

$$u_k(\eta) = \alpha_k^{(s)} u_k^{\rm BD}(\eta) + \beta_k^{(s)} u_k^{*\rm BD}(\eta) \tag{3.1}$$

where $u_k^{\rm BD}(\eta) = \frac{H^2}{\dot\phi}\frac{1}{\sqrt{2k^3}}(1+ik\eta)e^{-ik\eta}$. Here $|(\beta_k^{(s)})|^2 = N_k^{(s)}$ is the number of particles in mode k, and, the relative phase between $\alpha_k^{(s)}$ and $\beta_k^{(s)}$ is $\Theta^{(s)}$.

The simple Bogoliubov rotation may not capture a typical initial state. Nonetheless, the possibility of deviations from Bunch-Davies are an important conceptual point about the inflationary paradigm.

In this particular case, as a side-effect of these non-BD initial state, primordial gravitational waves seem to become more easily observable.

The non-Bunch Davies Initial State

Initial state for scalar fluctuations ζ as Bogoliubov tranforms of Bunch-Davies mode functions:

$$u_k(\eta) = \alpha_k^{(s)} u_k^{\mathrm{BD}}(\eta) + \beta_k^{(s)} u_k^{*\mathrm{BD}}(\eta) \tag{3.1}$$

where $u_k^{\mathrm{BD}}(\eta) = \frac{H^2}{\dot\phi}\frac{1}{\sqrt{2k^3}}(1+ik\eta)e^{-ik\eta}$. Here $|(\beta_k^{(s)})|^2 = N_k^{(s)}$ is the number of particles in mode k, and, the relative phase between $\alpha_k^{(s)}$ and $\beta_k^{(s)}$ is $\Theta^{(s)}$.

The simple Bogoliubov rotation may not capture a typical initial state. Nonetheless, the possibility of deviations from Bunch-Davies are an important conceptual point about the inflationary paradigm.

In this particular case, as a side-effect of these non-BD initial state, primordial gravitational waves seem to become more easily observable.

Constraints on the new parameters

[R. Flauger, D. Green & R. Porto, 2013]

There are some constraints on $N_k^{(s)}$ coming from backreaction, near scale-invariance, sub-horizon nature of modes etc. These impose the condition that when the relative phase $\Theta^{(s)} \neq \pi$ we have $\beta_k^{(s)} \leq 0.1$. However, for $\Theta^{(s)} = \pi$, there seems to be more freedom for the range of $\beta_k^{(s)}$.

Modified Power Spectrum

For this modified initial state, the scalar and tensor power spectra are, respectively, [J. Ganc, 2011]

$$P_\zeta(k) = P_\zeta^{\mathrm{BD}}(k)|\alpha_k^{(s)} + \beta_k^{(s)}|^2, \tag{3.2}$$

and

$$P_\gamma(k) = P_\gamma^{\mathrm{BD}}(k)|\alpha_k^{(t)} + \beta_k^{(t)}|^2. \tag{3.3}$$

With the normalization condition $|(\alpha_k^{(s,t)})|^2 - |(\beta_k^{(s,t)})|^2 = 1$, we have

$$|\alpha_k^{(s,t)} + \beta_k^{(s,t)}|^2 = 1 + 2|\beta_k^{(s,t)}|^2 + 2\sqrt{|\beta_k^{(s,t)}|^2(|\beta_k^{(s,t)}|^2+1)}\cos\Theta^{(s,t)}.$$

Modified $\langle\zeta\zeta\gamma^p\rangle$ Bispectrum

Three point funcion: $\langle\gamma^p(\mathbf{k}_1)\zeta(\mathbf{k}_2)\zeta(\mathbf{k}_3)\rangle \equiv (2\pi)^3\delta^3(\sum \mathbf{k}_i)B_p(k_1,k_2,k_3)$. For modified intial states,

$$\begin{aligned}\langle\gamma^p(\mathbf{k}_1)\zeta(\mathbf{k}_2)\zeta(\mathbf{k}_3)\rangle &= (2\pi)^3\delta^3(\sum \mathbf{k}_i)2\frac{H^4}{M_p^4}\frac{H^2}{\dot{\phi}^2}\frac{1}{\prod 2k_i^3}\epsilon^p_{ij}k_2^i k_3^j \\ &\quad (\alpha^{(t)}_{k_1}+\beta^{(t)}_{k_1})(\alpha^{(s)}_{k_2}+\beta^{(s)}_{k_2})(\alpha^{(s)}_{k_3}+\beta^{(s)}_{k_3})\times F \ + \text{c.c.} \quad (3.4)\end{aligned}$$

where

$$\begin{aligned} F &= \left(\alpha^{(t)*}_{k_1}\alpha^{(s)*}_{k_2}\alpha^{(s)*}_{k_3} - \beta^{(t)*}_{k_1}\beta^{(s)*}_{k_2}\beta^{(s)*}_{k_3}\right)\times \mathrm{I}(k_1,k_2,k_3) \\ &+ \left(\beta^{(t)*}_{k_1}\alpha^{(s)*}_{k_2}\alpha^{(s)*}_{k_3} - \alpha^{(t)*}_{k_1}\beta^{(s)*}_{k_2}\beta^{(s)*}_{k_3}\right)\times \mathrm{I}(-k_1,k_2,k_3) \\ &+ \ 2 \text{ perms.}, \qquad (3.5)\end{aligned}$$

with

$$\mathrm{I}(k_1,k_2,k_3) = \left(-k_t + \sum_{i>j}\frac{k_ik_j}{k_t} + \frac{k_1k_2k_3}{k_t^2}\right) \qquad (3.6)$$

and $k_t = k_1 + k_2 + k_3$.

Amplitude for Fossil Primordial Gravitational Wave

The modified bispectrum is largest in the collinear, squeezed limit, characterized by an unusually strong k_1^{-4} dependence. This goes in calcuation for the estimator for each fourier mode of the fossil field.

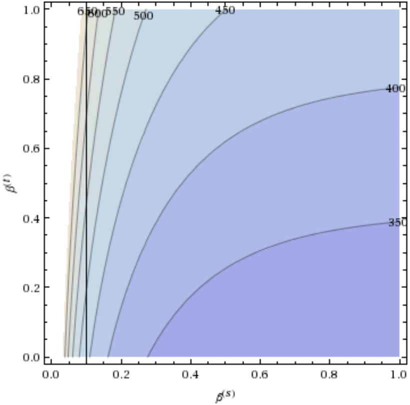
The estimates for each mode $\gamma^p(\mathbf{K})$ can then be combined to give a minimum variance estimator for the power in tensor fluctuations A_t, but with some uncertainty σ_t. For a 3σ detection, the minimum detectable amplitude for the tensor power spectrum is

$$A_t > 3\sigma_t = 30\pi\sqrt{3\pi}\left(\frac{k_{\rm max}}{k_{\rm min}}\right)^{-5} F(\beta^{(s)}, \beta^{(t)}, \Theta^{(s)}, \Theta^{(t)}). \qquad (4.1)$$

where $k_{\rm max}$ & $k_{\rm min}$ are determined by the survey size and $F(\beta^{(s)}, \beta^{(t)}, \Theta^{(s)}, \Theta^{(t)})$ is a function which has a small effect for the allowed values of $\beta^{(s,t)}$.

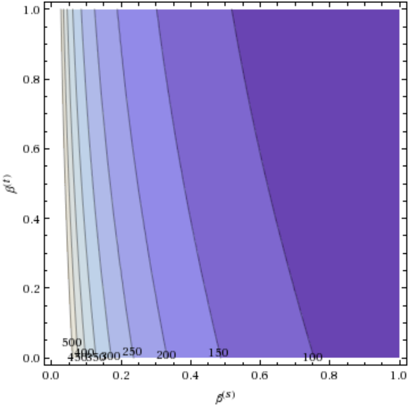
Results

Figure 2: Contour plot of $k_{\max}/k_{\min}$ for minimum survey size needed to detect primordial tensor fluctuations, for $A_t^{\mathrm{BD}} \equiv H^2/M_p^2 = 2 \times 10^{-9}$ [D. Jeong & M. Kamionkowski, 2012]. Here we set $\Theta^{(s)} = \Theta^{(t)} = 0$.



Results

Figure 3: Contour plot of $k_{\max}/k_{\min}$, with $\Theta^{(s)} = \Theta^{(t)} = \pi$.



Comparison with the BD case

The expression for the amplitude of the tensor power spectrum in the two cases are

$$A_t^{\text{non-BD}} > 30\pi\sqrt{3\pi}\left(\frac{k_{\max}}{k_{\min}}\right)^{-5} \times F(\beta^{(s)}, \beta^{(t)}, \Theta^{(s)}, \Theta^{(t)})$$

$$A_t^{\text{BD}} > 30\pi\sqrt{3\pi}\left(\frac{k_{\max}}{k_{\min}}\right)^{-3} \tag{4.2}$$

For sensitivity to tensor (primordial wave) amplitude ($r = .22$), the survey size required in the BD case is $(\frac{k_{\max}}{k_{\min}}) > 5200$. However, for the same amplitude, in the non-BD case (for the allowed range of $\beta^{(s)}$), we require $(\frac{k_{\max}}{k_{\min}}) > 550$ (Fig 2).

Improvement for the non-BD case

For sensitivity to the same amplitude, this an order of magnitude improvement over the BD case in terms of survey size $(\frac{k_{max}}{k_{min}})$

Conclusion

For a given realization of the fossil field, the three-point function with an unseen fossil field manifests itself as local anisotropy in the power spectrum.

Further, if there is a non-Bunch Davies component to the initial state of the scalar and tensor fluctuations during inflation, then

- The fossil relic of primordial gravitational waves in the anisotropic curvature two-point function can be significantly more observable.
- Insight into general condition for scalar-scalar-fossil correlation, provided the fossil signal comes primarily in the squeezed limit $K \ll k_1 \simeq k_2$ (work in progress to include models with a small speed of sound c_s).

Conclusion

For a given realization of the fossil field, the three-point function with an unseen fossil field manifests itself as local anisotropy in the power spectrum.

Further, if there is a non-Bunch Davies component to the initial state of the scalar and tensor fluctuations during inflation, then

- The fossil relic of primordial gravitational waves in the anisotropic curvature two-point function can be significantly more observable.
- Insight into general condition for scalar-scalar-fossil correlation, provided the fossil signal comes primarily in the squeezed limit $K \ll k_1 \simeq k_2$ (work in progress to include models with a small speed of sound c_s).

Conclusion

For a given realization of the fossil field, the three-point function with an unseen fossil field manifests itself as local anisotropy in the power spectrum.

Further, if there is a non-Bunch Davies component to the initial state of the scalar and tensor fluctuations during inflation, then

- The fossil relic of primordial gravitational waves in the anisotropic curvature two-point function can be significantly more observable.
- Insight into general condition for scalar-scalar-fossil correlation, provided the fossil signal comes primarily in the squeezed limit $K \ll k_1 \simeq k_2$ (work in progress to include models with a small speed of sound c_s).

Conclusion

For a given realization of the fossil field, the three-point function with an unseen fossil field manifests itself as local anisotropy in the power spectrum.

Further, if there is a non-Bunch Davies component to the initial state of the scalar and tensor fluctuations during inflation, then

- The fossil relic of primordial gravitational waves in the anisotropic curvature two-point function can be significantly more observable.
- Insight into general condition for scalar-scalar-fossil correlation, provided the fossil signal comes primarily in the squeezed limit $K \ll k_1 \simeq k_2$ (work in progress to include models with a small speed of sound c_s).

Minimum Variance Estimator

Each pair of scalar modes whose momenta add to **K** provides an estimator

$$\widehat{\gamma^p(\mathbf{K})} := \zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\left[f_p(\mathbf{k}_1,\mathbf{k}_2)\epsilon^p_{ij}k_1^i k_2^j\right]^{-1} \quad (6.1)$$

The minimum variance estimator obtained by summing over all such pairs (with an inverse-variance weighting):

$$\widehat{\gamma^p(\mathbf{K})} = \quad P_p^n(\mathbf{K})\sum_{\mathbf{k}} \frac{f_p^*(\mathbf{k},\mathbf{K}-\mathbf{k})\epsilon^p_{ij}k^i(K-k)^j}{2VP_{tot}(k)P_{tot}(|\mathbf{K}-\mathbf{k}|)} \times \zeta(\mathbf{k})\zeta(\mathbf{K}-\mathbf{k}), \quad (6.2)$$

where the noise power spectrum is part of the measured power spectrum $P_{tot}(k) = P(k) + P_n(k)$, given by

$$\left[P_p^n(K)\right]^{-1} = \sum_{\mathbf{k}} \frac{B_p^2(K,k,|\mathbf{K}-\mathbf{k}|)}{2VP_\gamma^2(K)P_\zeta^{\rm tot}(k)P_\zeta^{\rm tot}(|\mathbf{K}-\mathbf{k}|)}, \quad (6.3)$$

Noise Power Spectrum

Taking the continuous limit $\sum_{\mathbf{k}} \to V \int d^3k/(2\pi)^3$ and noting $V \equiv (2\pi/k_{\rm min})^3$ is the volume of the survey,

$$[P_p^n(K)]^{-1} = \frac{1}{20\pi^2} k_{\rm max}^3 \left(\frac{k_{\rm max}}{k_{\rm min}}\right)^2 F^{-1}(\beta^{(s)}, \beta^{(t)}, \Theta^{(s)}, \Theta^{(t)}), \tag{6.4}$$

where

$$\begin{aligned} & F^{-1}(\beta^{(s)}, \beta^{(t)}, \Theta^{(s)}, \Theta^{(t)}) \\ = \; & \beta^{(s)2}(1+\beta^{(s)2}) \left[\left(1 + 2\beta^{(t)2} + 2\sqrt{\beta^{(t)2}(\beta^{(t)2}+1)} \cos\Theta^{(t)}\right) \left(1 + 2\beta^{(s)2} + 2\sqrt{\beta^{(s)2}(\beta^{(s)2}+1)} \cos\Theta^{(s)}\right) \right]^{-2} \\ \times \; & \left[\left(\sqrt{1+\beta^{(s)2}} e^{i\Theta^{(s)}} + \beta^{(s)}\right)^2 \left(\sqrt{1+\beta^{(t)2}} e^{i\Theta^{(t)}} + \beta^{(t)}\right) \left(\sqrt{1+\beta^{(t)2}} e^{-i\Theta^{(t)}} - \beta^{(t)}\right) e^{-i\Theta^{(s)}} + \text{c.c.} \right]^2 \end{aligned} \tag{6.5}$$

with $\beta^{(s)} \equiv |\beta_k^{(s)}|$, $\beta^{(t)} \equiv |\beta_k^{(t)}|$, and $\Theta^{(s)}$ and $\Theta^{(t)}$ are the relative phases between $\alpha^{(s)}$ and $\beta^{(s)}$, and $\alpha^{(t)}$ and $\beta^{(t)}$, respectively.

- Caveat: Some assumptions regarding the behaviour of $\beta_k^{(s,t)}$ have been made for this derivation.

Noise Power Spectrum

Taking the continuous limit $\sum_{\mathbf{k}} \to V \int d^3k/(2\pi)^3$ and noting $V \equiv (2\pi/k_{\rm min})^3$ is the volume of the survey,

$$[P_p^n(K)]^{-1} = \frac{1}{20\pi^2} k_{\rm max}^3 \left(\frac{k_{\rm max}}{k_{\rm min}}\right)^2 F^{-1}(\beta^{(s)}, \beta^{(t)}, \Theta^{(s)}, \Theta^{(t)}), \tag{6.4}$$

where

$$\begin{aligned} & F^{-1}(\beta^{(s)}, \beta^{(t)}, \Theta^{(s)}, \Theta^{(t)}) \\ = \; & \beta^{(s)2}(1+\beta^{(s)2}) \left[\left(1 + 2\beta^{(t)2} + 2\sqrt{\beta^{(t)2}(\beta^{(t)2}+1)}\cos\Theta^{(t)}\right)\left(1 + 2\beta^{(s)2} + 2\sqrt{\beta^{(s)2}(\beta^{(s)2}+1)}\cos\Theta^{(s)}\right)\right]^{-2} \\ \times \; & \left[\left(\sqrt{1+\beta^{(s)2}}e^{i\Theta^{(s)}} + \beta^{(s)}\right)^2 \left(\sqrt{1+\beta^{(t)2}}e^{i\Theta^{(t)}} + \beta^{(t)}\right)\left(\sqrt{1+\beta^{(t)2}}e^{-i\Theta^{(t)}} - \beta^{(t)}\right)e^{-i\Theta^{(s)}} + \text{c.c.}\right]^2 \end{aligned} \tag{6.5}$$

with $\beta^{(s)} \equiv |\beta_k^{(s)}|$, $\beta^{(t)} \equiv |\beta_k^{(t)}|$, and $\Theta^{(s)}$ and $\Theta^{(t)}$ are the relative phases between $\alpha^{(s)}$ and $\beta^{(s)}$, and $\alpha^{(t)}$ and $\beta^{(t)}$, respectively.

- Caveat: Some assumptions regarding the behaviour of $\beta_k^{(s,t)}$ have been made for this derivation.

Amplitude for Fossil Primordial Gravitational Wave

The estimates for each mode $\gamma^p(\mathbf{K})$ can be combined to give a minimum variance estimator for the power in tensor fluctuations A_t, but again with some uncertainty

$$\sigma_t^{-2} \equiv \frac{1}{2} \sum_{\mathbf{K},p} \left(\frac{P_\gamma^f(K)}{P_p^n(K)} \right)^2 , \tag{6.6}$$

where $P_\gamma^f(K) \simeq K^{-3}$, assuming scale invariance. For a 3σ detection, the minimum detectable amplitude for the tensor power spectrum is

$$A_t > 3\sigma_t = 30\pi\sqrt{3\pi} \left(\frac{k_{\mathrm{max}}}{k_{\mathrm{min}}} \right)^{-5} F(\beta^{(s)}, \beta^{(t)}, \Theta^{(s)}, \Theta^{(t)}). \tag{6.7}$$

This has an additional factor of $(k_{max}/k_{min})^{-2}$ as compared to the the Bunch-Davoes Case.