Relating CKM and MNS Mixing & Predicting the ν CP Phases

S.M. Barr and Heng-Yu Chen **(JHEP 1211 (2012) 092)**

I will describe a model of quark and lepton mixing that is based on two assumptions:

- **SU(5) symmetry**
- **All mixing *among* the families comes from the mixing of the "usual" 3 families with "extra" vectorlike fermions in $5 + \overline{5}$ multiplets.**

Specifically, the fermion content of the model is

$$\underbrace{10_A + \overline{5}_A}_{\text{"usual"}} \quad + \quad \underbrace{5'_I + \overline{5}'_I}_{\text{"extra"}}$$

where A = 1,2,3 is the family index and I = 1,2, …, N where N>1

We assume that an abelian family symmetry prevents *direct* mixing among the 3 families.

But the usual $\overline{5}_A$ mix with the extra $\overline{5}'_I$ in a way that breaks this family symmetry and (indirectly) mixes the families. This mixing of $\overline{5}_A$ with $\overline{5}'_I$ respects SU(5), however, and so the quark and lepton mixing are tightly connected.

Let's see how this works:

$$L_{yukawa} = \;\; Y_A(\mathbf{10}_A\overline{\mathbf{5}}_A)\langle\overline{\mathbf{5}}_H\rangle + Y'_A(\mathbf{10}_A\mathbf{10}_A)\langle\mathbf{5}_H\rangle \;\; + \lambda_A(\overline{\mathbf{5}}_A\overline{\mathbf{5}}_A)\langle\mathbf{5}_H\rangle\langle\mathbf{5}_H\rangle$$

$$+\, y_A(\mathbf{10}_A\,\overline{\mathbf{5}}_A)\langle\overline{\mathbf{45}}_H\rangle + y'_A(\,\mathbf{10}_A\mathbf{10}_A)\langle\mathbf{45}_H\rangle$$

$$+\, \tilde{Y}_{AI}\,(\overline{\mathbf{5}}_A\,\mathbf{5}'_I)\langle\mathbf{1}_{AH}\rangle \;\; + \;\; M_{IJ}\,(\overline{\mathbf{5}}'_I\,\mathbf{5}'_J)$$

Suppose a **discrete symmetry** K_1 under which **the 1st family** $\mathbf{10}_1 + \overline{\mathbf{5}}_1$ **and** $\mathbf{1}_{1H}$ **are odd** and everything else is even. **This does not allow the 1st family to mix *directly* with the 2nd and 3rd families**, but it does allow $\overline{\mathbf{5}}_1$ to couple to mix with the extra vector-like fermions.

And **similarly** one can have discrete symmetries $(e.g. K_2, K_3)$ that forbid direct interfamily mixing of **the 2nd and 3rd families.** So, one has

$$(\Delta\,\overline{\mathbf{5}} + M\,\overline{\mathbf{5}}')\mathbf{5}' \;\; = \sqrt{M^2+\Delta^2}\left(\frac{\Delta}{\sqrt{M^2+\Delta^2}}\overline{\mathbf{5}} + \frac{M}{\sqrt{M^2+\Delta^2}}\overline{\mathbf{5}}'\right)\mathbf{5}' \; = \; \sqrt{M^2+\Delta^2}\;(\overline{\mathbf{5}}_{heavy}\;\mathbf{5}')$$

$$\overline{\mathbf{5}}_{heavy} = \left(\frac{\Delta}{\sqrt{M^2+\Delta^2}}\overline{\mathbf{5}} + \frac{M}{\sqrt{M^2+\Delta^2}}\overline{\mathbf{5}}'\right) \qquad\qquad \overline{\mathbf{5}} = \left(\frac{M}{\sqrt{M^2+\Delta^2}}\overline{\mathbf{5}}_{light} + \frac{\Delta}{\sqrt{M^2+\Delta^2}}\overline{\mathbf{5}}_{heavy}\right)$$

invert

$$\overline{\mathbf{5}}_{light} = \left(\frac{M}{\sqrt{M^2+\Delta^2}}\overline{\mathbf{5}} - \frac{\Delta}{\sqrt{M^2+\Delta^2}}\overline{\mathbf{5}}'\right) \qquad\qquad \frac{M}{\sqrt{M^2+\Delta^2}} \equiv \left[I + \Delta^\dagger M^{-1\dagger}M^{-1}\Delta\right]^{-\frac{1}{2}} \equiv A$$

So, for the term that gives the down-quark mass matrix, one obtains:

$$Y\,(10\,\overline{5})\langle\overline{5}_H\rangle \longrightarrow 10 m_d \overline{5} \longrightarrow 10\, m_d(A\overline{5}_{light} + \text{heavy}) \longrightarrow 10(m_d A)\overline{5}_{light}$$

diagonal, hierarchical

non-diagonal, non-hierarchical

In a similar way, one obtains factors of the same mixing matrix A in the other mass matrices:

$$(\underset{u_L}{10}\ \underset{u^c{}_L}{10})\langle 5_H\rangle \longrightarrow M_u = m_u$$

$$(\underset{d_L}{10}\ \underset{d^c{}_L}{\overline{5}})\langle\overline{5}_H\rangle \longrightarrow M_d = m_d A$$

$$(\underset{\ell_L}{\overline{5}}\ \underset{\ell^c{}_L}{10})\langle\overline{5}_H\rangle \longrightarrow M_\ell = A^T m_\ell$$

$$(\underset{\nu_L}{\overline{5}}\ \underset{\nu_L}{\overline{5}})\langle 5_H\rangle\langle 5_H\rangle \longrightarrow M_\nu = A^T m_\nu A$$

So, one matrix, A, controls all CKM and MNS mixing.

Moreover, this matrix A can be brought to a simple form by choice of bases and rescaling unknown parameters:

$$d'(m_d A)d^{c'} = (d',s',b')\mu_d \begin{pmatrix} \delta_d & 0 & 0 \\ 0 & \varepsilon_d & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{pmatrix} \begin{pmatrix} d^{c'} \\ s^{c'} \\ b^{c'} \end{pmatrix}$$

$$\longrightarrow (d',s',b')\bar{\mu}_d \begin{pmatrix} \bar{\delta}_d & 0 & 0 \\ 0 & \bar{\varepsilon}_d & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & b & ce^{-i\delta} \\ 0 & 1 & a \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} d^{c} \\ s^{c} \\ b^{c} \end{pmatrix}$$

$$= (d',s',b')\bar{\mu}_d \begin{pmatrix} \bar{\delta}_d & \bar{\delta}_d b & \bar{\delta}_d c e^{-i\delta} \\ 0 & \bar{\varepsilon}_d & \bar{\varepsilon}_d a \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} d^{c} \\ s^{c} \\ b^{c} \end{pmatrix} \qquad \bar{\delta}_d \ll \bar{\varepsilon}_d \ll 1$$

When we diagonalize this mass matrix , it tells us the elements of CKM matrix (M_u is already diagonal) . Therefore, we can solve for the 4 parameters (a, b, c, and δ) in the matrix A. And these parameters can be expressed in terms of CKM mixing angles and masses of quarks only.

$$\bar{\varepsilon}_d \cong \frac{m_s}{m_b} \quad and \quad \bar{\varepsilon}_d a \cong V_{cb} \quad \Longrightarrow \quad a \cong \frac{m_b}{m_s} V_{cb} \sim 2$$

$$\bar{\delta}_d \cong \frac{m_d}{m_b} \quad and \quad \bar{\delta}_d c e^{-i\delta} \cong |V_{ub}| e^{-i\delta_{KM}} \quad \Longrightarrow \quad ce^{-i\delta} \cong \frac{m_b}{m_d} V_{ub} e^{-i\delta_{CKM}} \sim 3e^{-i1.2}$$

$$\frac{\bar{\delta}_d}{\bar{\varepsilon}_d} \cong \frac{m_d}{m_s} \quad and \quad \frac{\bar{\delta}_d b}{\bar{\varepsilon}_d} \cong V_{us} \quad \Longrightarrow \quad b \cong \frac{m_s}{m_d} V_{us} \sim 4$$

Turning to the leptons, one finds that the charged lepton mass matrix can be brought to the form (with the same matrix $\boldsymbol{A}$):

$$M_\ell = A^T m_\ell \approx \bar{\mu}_\ell \begin{pmatrix} 1 & 0 & 0 \\ \frac{m_s}{m_d} V_{us} & 1 & 0 \\ \frac{m_b}{m_d} V_{ub} e^{-i\delta} & \frac{m_b}{m_s} V_{cb} & 1 \end{pmatrix} \begin{pmatrix} \bar{\delta}_\ell & 0 & 0 \\ 0 & \bar{\varepsilon}_\ell & 0 \\ 0 & 0 & 1 \end{pmatrix} \approx \bar{\mu}_\ell \begin{pmatrix} \bar{\delta}_\ell & 0 & 0 \\ 4\bar{\delta}_\ell & \bar{\varepsilon}_\ell & 0 \\ 3\bar{\delta}_\ell e^{-i\delta} & 2\bar{\varepsilon}_\ell & 1 \end{pmatrix}$$

Diagonalizing this requires rotations of the right-handed leptons, but **negligible rotations of the left-handed leptons**. **So the MNS mixing effectively comes entirely from the neutrino mass matrix, which is of the form**

$$M_\nu = A^T m_\nu A \approx \bar{\mu}_\nu \begin{pmatrix} 1 & 0 & 0 \\ \frac{m_s}{m_d} V_{us} & 1 & 0 \\ \frac{m_b}{m_d} V_{ub} e^{-i\delta} & \frac{m_b}{m_s} V_{cb} & 1 \end{pmatrix} \begin{pmatrix} q e^{i\beta} & 0 & 0 \\ 0 & p e^{i\alpha} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & \frac{m_s}{m_d} V_{us} & \frac{m_b}{m_d} V_{ub} e^{-i\delta} \\ 0 & 1 & \frac{m_b}{m_s} V_{cb} \\ 0 & 0 & 1 \end{pmatrix}$$

THIS IS OUR MAIN RESULT!

NOTE: There are 5 unknown model parameters here. In terms of this 9 neutrino "observables" are predicted: 3 neutrino masses, 3 MNS angles, 1 Dirac CP phase, and 2 Majorana CP phases.

Before looking at the predictions, **notice an important feature**.

Because the family mixing comes from the mixing of $\overline{5}$s rather than 10s one expects the mixing of Left-handed leptons (which are in the $\overline{5}$s) to be bigger than the mixing of the Left-handed quarks (which are in 10s).

$\Longrightarrow$ MNS angle >> CKM angles

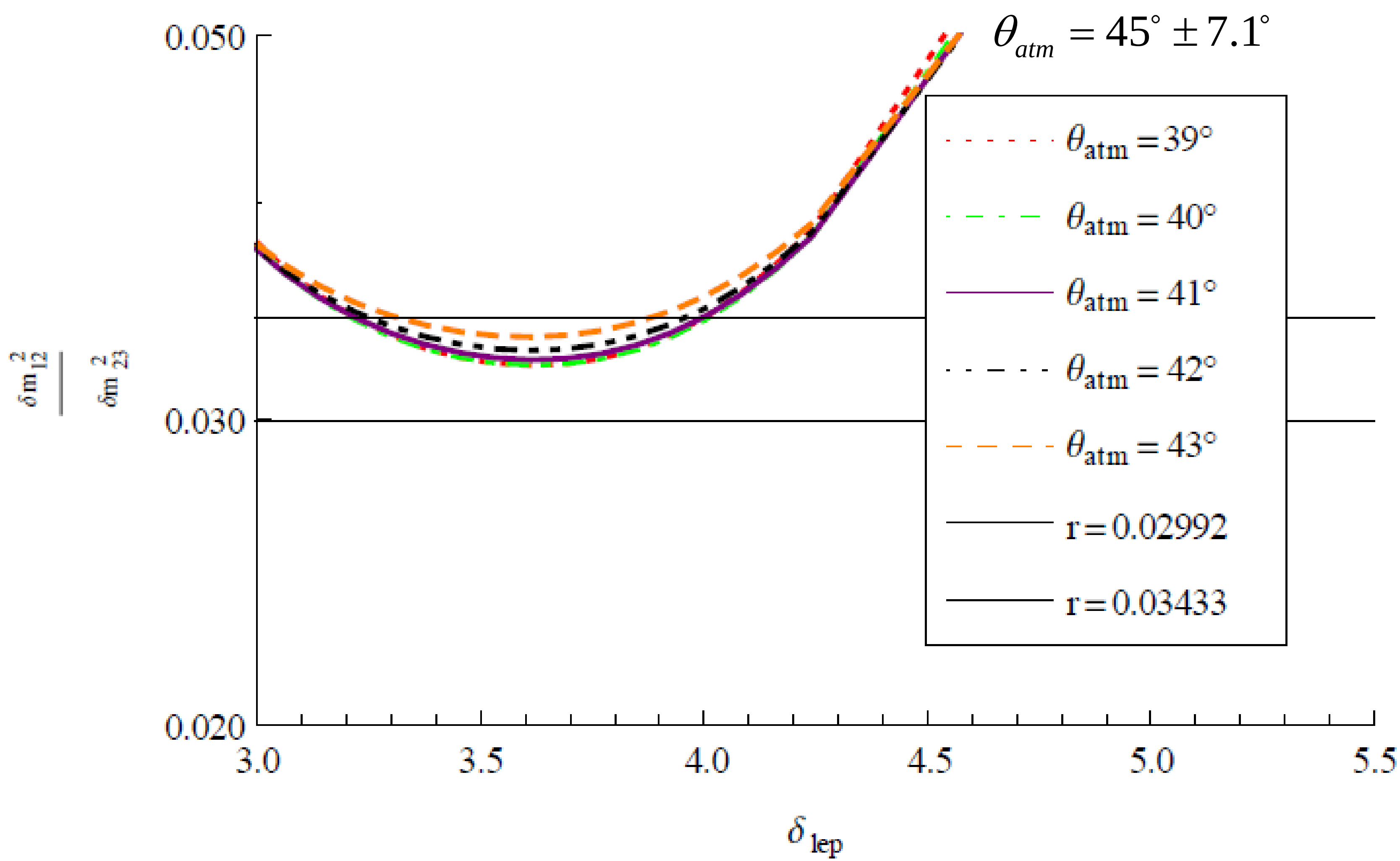
One sees this in the formula

$$\boldsymbol{M_\nu = A^T\, m_\nu A} \approx \boldsymbol{\overline{\mu}_\nu} \begin{pmatrix} 1 & 0 & 0 \\ \frac{m_s}{m_d} V_{us} & 1 & 0 \\ \frac{m_b}{m_d} V_{ub} e^{-i\delta} & \frac{m_b}{m_s} V_{cb} & 1 \end{pmatrix} \begin{pmatrix} \boldsymbol{q e^{i\beta}} & 0 & 0 \\ 0 & \boldsymbol{p e^{i\alpha}} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & \frac{m_s}{m_d} V_{us} & \frac{m_b}{m_d} V_{ub} e^{-i\delta} \\ 0 & 1 & \frac{m_b}{m_s} V_{cb} \\ 0 & 0 & 1 \end{pmatrix}$$

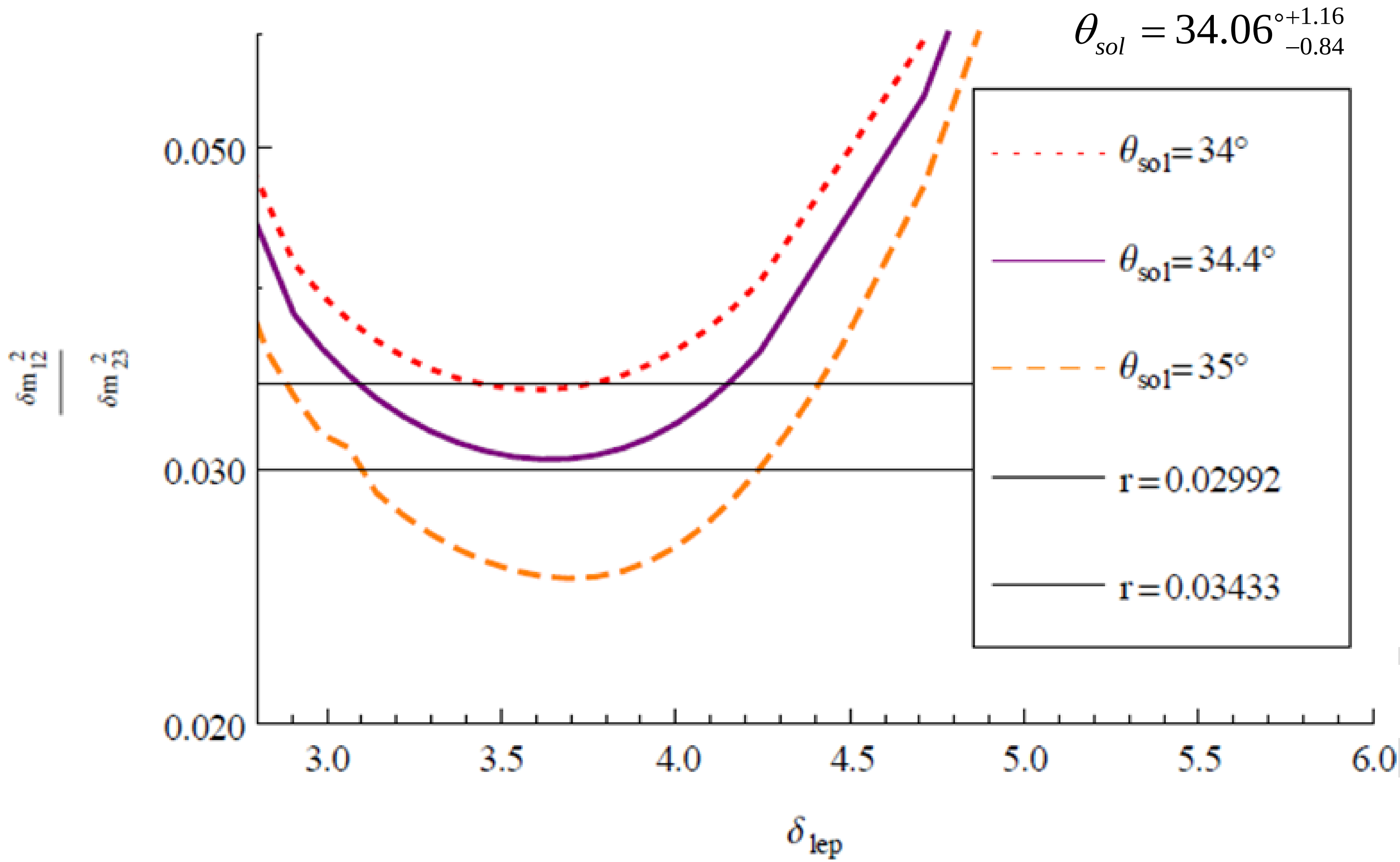
Note that the MNS mixing is proportional to the CKM elements, but multiplied by large quark mass ratios!

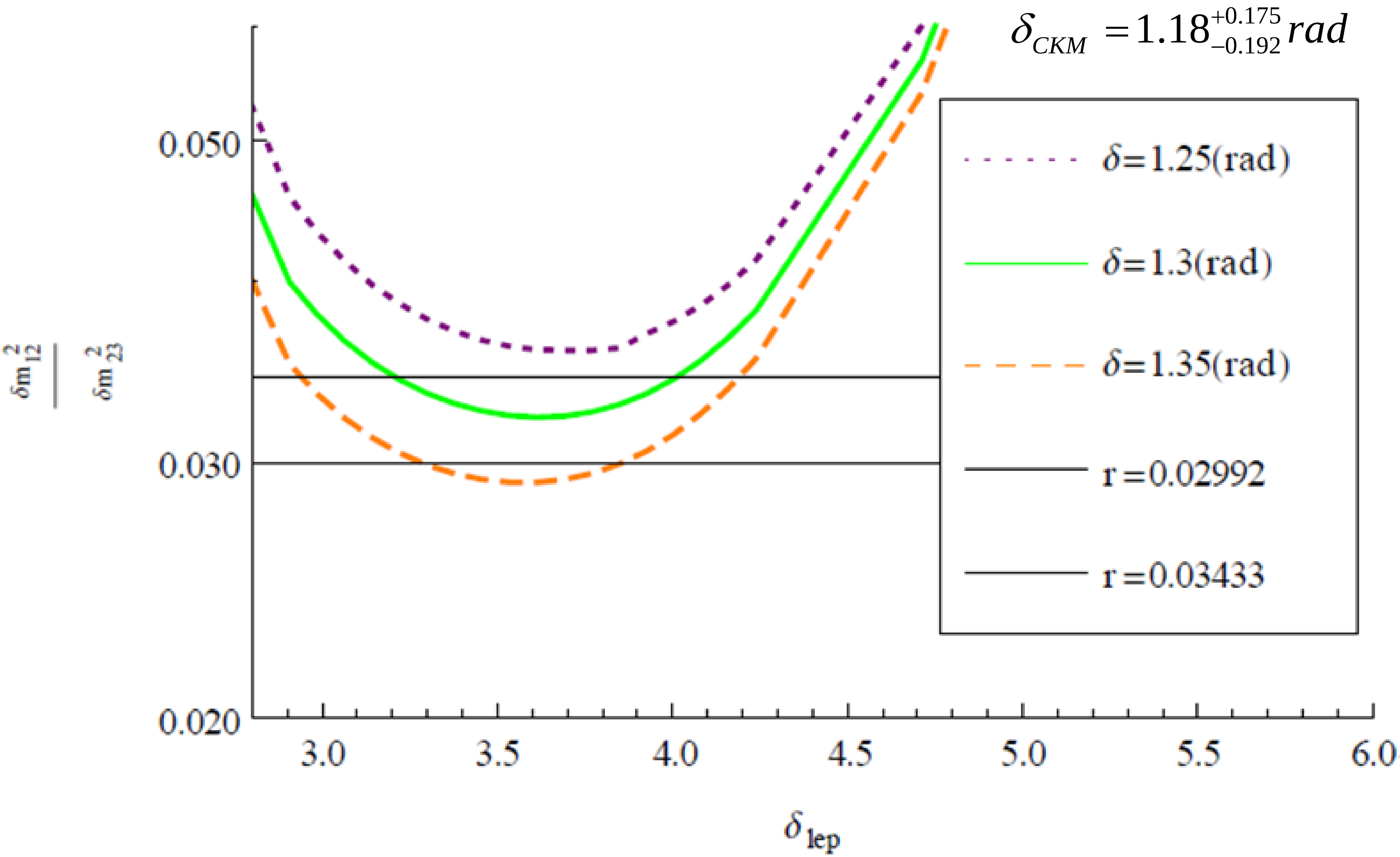
predictions of model

Quantity	Values in fit	Experiment
μ_ν	0.1428 eV	—
$pe^{i\alpha}$	$0.1525e^{-2.734i}$	—
$qe^{i\beta}$	$0.01405e^{-0.352i}$	—
m_b/m_s	52.9	52.9 ± 2.6
m_s/m_d	19	17 to 22
$\lvert V_{us}\rvert$	0.2252	0.2252 ± 0.0009
$\lvert V_{cb}\rvert$	0.0409	0.0409 ± 0.0011
$\lvert V_{ub}\rvert$	0.00415	0.00415 ± 0.00049
δ	1.30 rad	$1.187^{+0.175}_{-0.192}$ rad
θ_{sol}	34.1°	$33.89^\circ\ {}^{+0.976^\circ}_{-0.971^\circ}$
θ_{atm}	40°	$45^\circ \pm 6.5^\circ$
θ_{13}	9.12°	$9.122^\circ\ {}^{+0.609^\circ}_{-0.647^\circ}$
δm^2_{23}	2.32×10^{-3} eV^2	$2.32^{+0.12}_{-0.08} \times 10^{-3}$ eV^2
δm^2_{12}	7.603×10^{-5} eV^2	$(7.5 \pm 0.2) \times 10^{-5}$ eV^2
δ_{lep}	1.15π rad	$1.1\pi^{+0.3\pi}_{-0.4\pi}$ rad
$(M_\nu)_{ee}$	0.0020 eV	



$\theta_{atm} = 45^\circ \pm 7.1^\circ$
0.050
0.030
0.020
$\frac{\delta m^2_{12}}{\delta m^2_{23}}$
$\theta_{atm} = 39°$
$\theta_{atm} = 40°$
$\theta_{atm} = 41°$
$\theta_{atm} = 42°$
$\theta_{atm} = 43°$
r = 0.02992
r = 0.03433
3.0
3.5
4.0
4.5
5.0
5.5
δ_{lep}


$\theta_{sol} = 34.06^{\circ +1.16}_{\ -0.84}$
$\theta_{sol}=34°$
$\theta_{sol}=34.4°$
$\theta_{sol}=35°$
r = 0.02992
r = 0.03433
$\delta m^2_{12} / \delta m^2_{23}$
δ_{lep}
0.050
0.030
0.020
3.0
3.5
4.0
4.5
5.0
5.5
6.0


$\delta_{CKM} = 1.18^{+0.175}_{-0.192}\, rad$
δ=1.25(rad)
δ=1.3(rad)
δ=1.35(rad)
r=0.02992
r=0.03433
0.050
0.030
0.020
3.0
3.5
4.0
4.5
5.0
5.5
6.0
$\frac{\delta m^2_{12}}{\delta m^2_{23}}$
δ_{lep}

Conclusion:

This model is quite simple in concept, based on two ideas:

SU(5) invariance and
that all interfamily mixing is due to mixing of "usual" SM families
with extra vectorlike fermions in $\mathbf{5} + \overline{\mathbf{5}}$ pairs.

One ends up with a quite predictive scheme that incorporates the "lopsided" mass matrix idea that explains why the MNS angles are large and the CKM angles are small (and related to quark mass ratios).

It also predicts several neutrino parameters, and constrains certain already (but somewhat poorly) known quantities, such as the solar and atmospheric angles, the strange quark mass, and the quark CP phase.