Top quark decay with dimension-six operators at NLO in QCD

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Motivation

In the SM, $t \to b + W$ has been calculated at NNLO in QCD.

A. Czarnecki et al.
1005.2625

In Effective Field Theory,

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{SM}} + \left(\frac{1}{\Lambda^2} C_i O_i + \text{h.c.} \right)$$

The dimension-six operators

- Modify W helicity fractions in $t \to b + W$.
- Give rise to flavor changing decay $t \to c + V$ and $t \to c + h$.

A model-independent calculation of 2-body top decay, at $\mathcal{O}(\alpha_s \frac{1}{\Lambda^2})$.

Motivation

- Look for NP in FCNC decay channels $t \to c + V$ and $t \to c + h$, and W helicity fractions in $t \to b + W$.

- Understand NLO QCD calculation in Effective Field Theory. (Possible technical issues such as operator running and mixing, higher rank loop integrals, etc.)

Main decay channel

Main decay channel $t \to bW$

- $t \to bW$ helicity fraction with a general Wtb coupling. $\surd$

J.Drobnak et al.
1010.2402

$$\mathcal{L}_{\mathrm{tbW}} = \frac{g_W}{\sqrt{2}}\bar{t}\left[a_L\gamma^\mu P_L - b_{LR}\frac{2i\sigma^{\mu\nu}}{m_t}q_\nu P_R + (L \leftrightarrow R)\right] bW_\mu$$

$a_{L,R}$ and $b_{LR,RL}$ coming from

$$O_{\varphi q}^{(3)} = i\frac{1}{2}y_t^2\left(\varphi^\dagger \overleftrightarrow{D}^I_\mu\varphi\right)(\bar{Q}\gamma^\mu\tau^I Q)\ , \quad O_{\varphi\varphi} = iy_t^2(\tilde{\varphi}^+ D_\mu\varphi)(\bar{t}\gamma^\mu b)$$
$$O_{tW} = y_t g_W(\bar{Q}\sigma^{\mu\nu}\tau^I t)\tilde{\varphi}W^I_{\mu\nu}\ , \quad O_{bW} = y_t g_W(\bar{Q}\sigma^{\mu\nu}\tau^I b)\varphi W^I_{\mu\nu}$$

- $t \to bW$ helicity fraction with a CMDM operator. $\times$

$$O_{tG} = g_s(\bar{Q}_L\sigma^{\mu\nu}T^A t_R)\tilde{\varphi}G^A_{\mu\nu}, \qquad Q_L = (t_L, b_L)\ , \quad \varphi = \text{higgs}$$

Flavor changing decay channel

FCNC decay channel

J. Drobnak et al.
1007.2552

J. J. Zhang et al.
1004.0898

- $t \to cV$, with dimension-four and five operators. $\surd$

$$\mathcal{L}_{\text{eff}} = \frac{v^2}{\Lambda^2} a_L^Z \left[g_Z Z_\mu (\bar{q}_L \gamma^\mu t_L) \right] + \frac{v}{\Lambda^2} b_{LR}^Z \left[g_Z Z_{\mu\nu} (\bar{q}_L \sigma^{\mu\nu} t_R) \right]$$
$$+ \frac{v}{\Lambda^2} b_{LR}^\gamma \left[e F_{\mu\nu} (\bar{q}_L \sigma^{\mu\nu} t_R) \right] + \frac{v}{\Lambda^2} b_{LR}^g \left[g_s G_{\mu\nu}^A (\bar{q}_L \sigma^{\mu\nu} T_A t_R) \right]$$
$$+ (L \leftrightarrow R) + \text{h.c.}$$

Note in EFT we add φ to restore the full SM symmetry, i.e.

$$\left[g_s G_{\mu\nu}^A (\bar{q}_L \sigma^{\mu\nu} T_A t_R) \right] \to g_s (\bar{q}_L \sigma^{\mu\nu} T^A t_R) \tilde{\varphi} G_{\mu\nu}^A \equiv O_{uG}^{(13)}$$

- $t \to ch$, through dimension-six operators. $\times$

LO: $O_{u\varphi}^{(23)} = -y_t^3 (\varphi^\dagger \varphi)(\bar{q}_L t_R)\tilde{\varphi}$, $O_{u\varphi}^{(32)} = -y_t^3 (\varphi^\dagger \varphi)(\bar{Q}_L c_R)\tilde{\varphi}$

$(q_L = (c_L, s_L), \quad Q_L = (t_L, b_L))$

NLO: $O_{uG}^{(23)} = y_t g_s (\bar{q}_L \sigma^{\mu\nu} T^A t_R) \tilde{\varphi} G_{\mu\nu}^A$, $O_{uG}^{(32)} = y_t g_s (\bar{Q}_L \sigma^{\mu\nu} T^A c_R) \tilde{\varphi} G_{\mu\nu}^A$

Results

1. Top color dipole contribution to W helicity fractions in main decay channel.

2. FC decay $t \to c + h$.

$t \to b + W$, zero helicity

Define W helicity fractions, $\Gamma = \Gamma_+ + \Gamma_0 + \Gamma_-$, $F_i = \Gamma_i/\Gamma$

NP effects from:

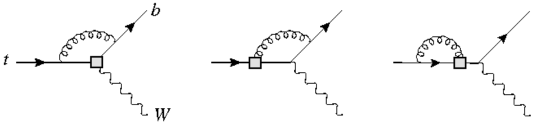
1. Anomalous couplings of Wtb:

$$O_{\varphi q}^{(3)} = i\frac{1}{2}y_t^2\left(\varphi^\dagger \overleftrightarrow{D}^I_\mu \varphi\right)(\bar{Q}\gamma^\mu\tau^I Q)\ , \quad O_{\varphi\varphi} = iy_t^2(\tilde{\varphi}^+ D_\mu\varphi)(\bar{t}\gamma^\mu b)$$
$$O_{tW} = y_t g_W(\bar{Q}\sigma^{\mu\nu}\tau^I t)\tilde{\varphi}W^I_{\mu\nu}\ , \quad O_{bW} = y_t g_W(\bar{Q}\sigma^{\mu\nu}\tau^I b)\varphi W^I_{\mu\nu}$$

   - $O_{\varphi\varphi}$, O_{bW} constrained from $b \to s\gamma$
   - $O_{\varphi q}^{(3)}$ does not change helicities.
   - Focus on O_{tW}

B. Grzadkowski et al.
0802.1413

2. Top CMDM operator: $O_{tG} = y_t g_s(\bar{Q}_L\sigma^{\mu\nu}T^A t_R)\tilde{\varphi}G^A_{\mu\nu}$



$t \to b + W$, zero helicity

Result on zero helicity fraction

$$F_0 = 0.689 - 0.040 C_{tW} \frac{1\mathrm{TeV}^2}{\Lambda^2} + \alpha_s \left(0.005 C_{tW} \frac{1\mathrm{TeV}^2}{\Lambda^2} + 0.007 C_{tG} \frac{1\mathrm{TeV}^2}{\Lambda^2} \right)$$

$t \to b + W$, zero helicity

Measurements from ttbar events

CMS PAS TOP-12-025
ATLAS-CONF-2013-033

$$F_0 = 0.626 \pm 0.034(\mathrm{stat.}) \pm 0.048(\mathrm{syst.})$$
$$F_- = 0.359 \pm 0.021(\mathrm{stat.}) \pm 0.028(\mathrm{syst.})$$

Constraints:
LO

$$\frac{C_{tW}(\mathrm{TeV})^2}{\Lambda^2} = 1.32 \pm 0.86$$

NLO

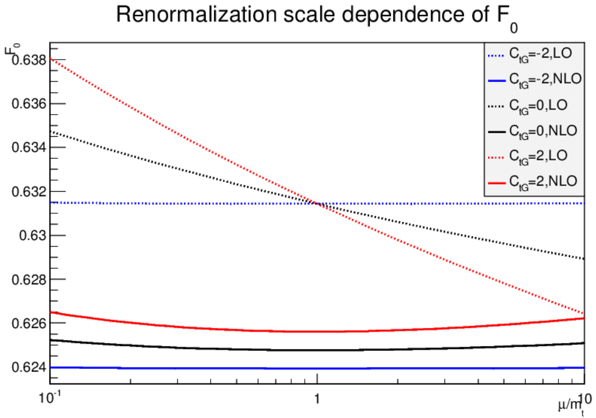
$$\frac{C_{tW}(\mathrm{TeV})^2}{\Lambda^2} = 1.168 + 0.015\frac{C_{tG}(\mathrm{TeV})^2}{\Lambda^2} \pm 0.86$$

$t \to b + W$, zero helicity, scale dependence

The $\mathcal{O}(\alpha_s)$ mixing between O_{tG} and O_{tW} given by

$$C_{tG}(\mu) = C_{tG}(m_t)\left(\frac{\alpha_s(\mu)}{\alpha_s(m_t)}\right)^{-\frac{2}{3\beta_0}}, \quad \beta_0 = 11 - \frac{2}{3}N_f$$

$$C_{tW}(\mu) = C_{tW}(m_t)\left(\frac{\alpha_s(\mu)}{\alpha_s(m_t)}\right)^{-\frac{4}{3\beta_0}} - 2C_{tG}(m_t)\left[\left(\frac{\alpha_s(\mu)}{\alpha_s(m_t)}\right)^{-\frac{2}{3\beta_0}} - \left(\frac{\alpha_s(\mu)}{\alpha_s(m_t)}\right)^{-\frac{4}{3\beta_0}}\right]$$



($\Lambda = 1$ TeV, $C_{tw} = 1$)

$t \to c + h$

The *tch* coupling comes from

$$O_{u\varphi}^{(23)} = -y_t^3(\varphi^\dagger\varphi)(\bar{q}_L t_R)\tilde{\varphi}\,, \quad O_{u\varphi}^{(32)} = -y_t^3(\varphi^\dagger\varphi)(\bar{Q}_L u_R)\tilde{\varphi}$$

- Typical value for BR($t \to ch$) in SM and NP models

J. A. Saavedraa et al.
hep-ph/0409342

SM	Quark singlet	2HDM	2HDM (flavor conserving)	MSSM
3×10^{-15}	4×10^{-5}	1.5×10^{-3}	10^{-5}	10^{-5}

- Indirect bounds from $Z \to c\bar{c}$:
  BR($t \to ch$)$< 3 \times 10^{-3}$

F. Larios et al.
hep-ph/0412222

- 3σ discovery limits $(100\mathrm{fb}^{-1})$:

  BR($t \to ch$)$\approx 5.8 \times 10^{-5}$

  corresponds to $\frac{C_{u\varphi}}{\Lambda^2} \approx \frac{0.3}{1\mathrm{TeV}^2}$

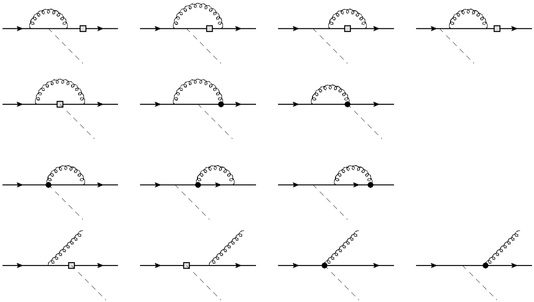
J. A. Saavedraa et al.
hep-ph/0409342

$t \to c + h$, diagrams

Operators:

1. $O_{u\varphi}^{(23)} = -y_t^3(\varphi^\dagger\varphi)(\bar{q}_L t_R)\tilde{\varphi}$
   $O_{u\varphi}^{(32)} = -y_t^3(\varphi^\dagger\varphi)(\bar{Q}_L c_R)\tilde{\varphi}$
2. $O_{uG}^{(23)} = y_t g_s(\bar{q}_L \sigma^{\mu\nu} T^A t_R)\tilde{\varphi} G_{\mu\nu}^A$
   $O_{uG}^{(32)} = y_t g_s(\bar{Q}_L \sigma^{\mu\nu} T^A c_R)\tilde{\varphi} G_{\mu\nu}^A$

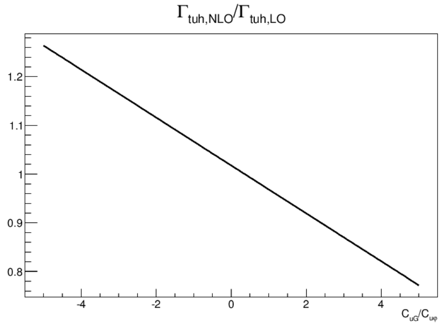
NLO diagrams



$t \to c + h$, result

Operators:

1. $$O_{u\varphi}^{(23)} = -y_t^3(\varphi^\dagger\varphi)(\bar{q}_L t_R)\tilde{\varphi}$$
   $$O_{u\varphi}^{(32)} = -y_t^3(\varphi^\dagger\varphi)(\bar{Q}_L c_R)\tilde{\varphi}$$
2. $$O_{uG}^{(23)} = y_t g_s(\bar{q}_L \sigma^{\mu\nu} T^A t_R)\tilde{\varphi} G_{\mu\nu}^A$$
   $$O_{uG}^{(32)} = y_t g_s(\bar{Q}_L \sigma^{\mu\nu} T^A c_R)\tilde{\varphi} G_{\mu\nu}^A$$



- Result

$$BR(t \to ch) = \left[0.545|C_{u\varphi}|^2 + \alpha_s\left(0.09|C_{u\varphi}|^2 - 0.25\mathrm{Re}C_{u\varphi}C_{uG}^*\right)\right] \times 10^{-3}\left(\frac{1\mathrm{TeV}^4}{\Lambda^4}\right)$$

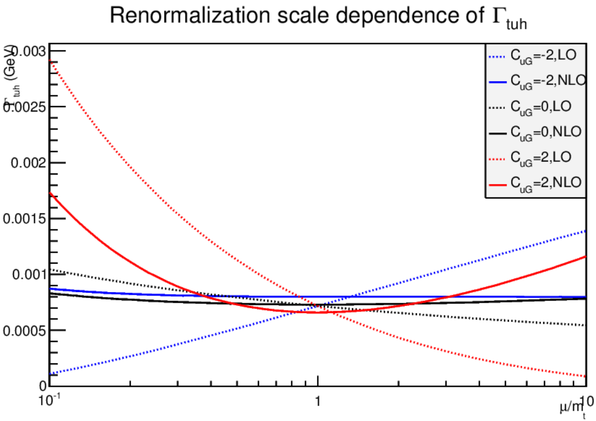
- NLO correction can vary at 10% level if $C_{uG} \sim C_{u\varphi}$

$t \to c + h$ scale dependence

The $\mathcal{O}(\alpha_s)$ mixing between O_{uG} and $O_{u\varphi}$

$$C_{uG}(\mu) = C_{uG}(m_t)\left(\frac{\alpha_s(\mu)}{\alpha_s(m_t)}\right)^{-\frac{2}{3\beta_0}}, \quad \beta_0 = 11 - \frac{2}{3}N_f$$

$$C_{u\varphi}(\mu) = C_{u\varphi}(m_t)\left(\frac{\alpha_s(\mu)}{\alpha_s(m_t)}\right)^{\frac{4}{\beta_0}} + \frac{12}{7}C_{uG}(m_t)\left[\left(\frac{\alpha_s(\mu)}{\alpha_s(m_t)}\right)^{\frac{4}{\beta_0}} - \left(\frac{\alpha_s(\mu)}{\alpha_s(m_t)}\right)^{-\frac{2}{3\beta_0}}\right]$$



$(\Lambda = 1$ TeV, $C_{u\varphi} = 1)$

Summary

We calculate $t \to b + W$, $t \to c + V$ and $t \to c + h$ in EFT at order $\mathcal{O}\left(\frac{\alpha_s}{\Lambda^2}\right)$.

Backup slides

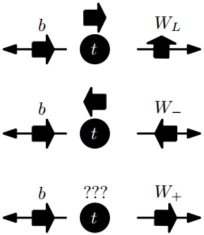
$t \to b + W$, positive helicity

- $F_+ = \Gamma_+/\Gamma$, fraction of positively polarized W.
- In SM, $F_+ = 0$ at tree-level if $m_b = 0$.
- $F_+ = 0.0017$ at NNLO in QCD including $m_b \neq 0$ effects.

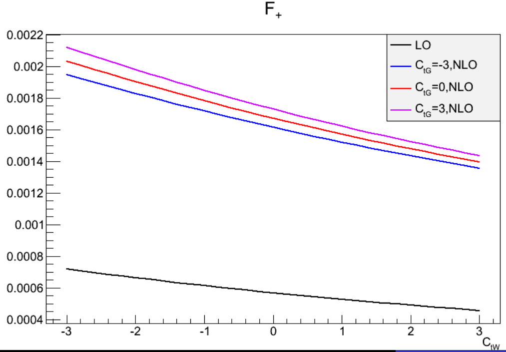
A. Czarnecki et al.
1005.2625

- Sensitivity for LHC ($L = 10\mathrm{fb}^{-1}$): $F_+ \sim \pm 0.002$.

J. A. Saavedraa et al.
0705.3041



$$F_+ = \left[1.67 - 0.043 C_{tW} \frac{1\mathrm{TeV}^2}{\Lambda^2} + (-0.52 C_{tW} + 0.13 C_{tG}) \frac{\alpha_s \times 1\mathrm{TeV}^2}{\Lambda^2}\right] \times 10^{-3}$$



Existing bounds

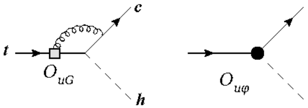
- O_{tW}, from F_0 measurement $C_{tW} \sim (0.3, 2.0)$
- O_{tG}, from ttbar cross section $C_{tG} \sim (-0.7, 2.2)$

α_s contribution?

In EFT, the order of operator mixing can depend on convention. . .

- Consider mixing between

$$O_{uG}^{(23)} = (\bar{q}_L \sigma^{\mu\nu} T^A t_R)\tilde{\varphi} G^A_{\mu\nu} \quad \text{and} \quad O_{u\varphi}^{(23)} = -(\varphi^\dagger \varphi)(\bar{q}_L t_R)\tilde{\varphi}$$



$$\delta Z \sim g_s y_t^2 \quad \text{(but expect } g_s^2\text{)}$$

- Redefine

$$O_{uG}^{(23)} = y_t g_s (\bar{q}_L \sigma^{\mu\nu} T^A t_R)\tilde{\varphi} G^A_{\mu\nu} \quad \text{and} \quad O_{u\varphi}^{(23)} = -y_t^3 (\varphi^\dagger \varphi)(\bar{q}_L t_R)\tilde{\varphi}$$

$$\Rightarrow \delta Z \sim \alpha_s$$

- In general, start with 2 fermions, add top Yukawa y_t for each additional Higgs field, and gauge coupling for each additional gauge field.

$\mathcal{O}(\alpha_s)$ mixing, flavor diagonal

$$O_{tG} = y_t g_s (\bar{Q}\sigma^{\mu\nu} T^A t)\tilde{\varphi} G^A_{\mu\nu}$$
$$O_{tW} = y_t g_W (\bar{Q}\sigma^{\mu\nu} \tau^I t)\tilde{\varphi} W^I_{\mu\nu}$$
$$O_{tB} = y_t g_Y (\bar{Q}\sigma^{\mu\nu} t)\tilde{\varphi} B_{\mu\nu}$$
$$O_{t\varphi} = -y_t^3 (\varphi^\dagger \varphi)(\bar{Q}t)\tilde{\varphi}$$
$$O_{\varphi G} = \frac{1}{2} y_t^2 (\varphi^\dagger \varphi) G^A_{\mu\nu} G^{\mu\nu A}$$
$$O_{\varphi \tilde{G}} = \frac{1}{2} y_t^2 (\varphi^\dagger \varphi) \tilde{G}^A_{\mu\nu} G^{\mu\nu A}$$
$$O_G = \frac{1}{2} g_s f^{ABC} G^{A\nu}_{\mu} G^{B\rho}_{\nu} G^{C\mu}_{\rho}$$

CP-even: $\mathrm{Re}C_{tG}$, $\mathrm{Re}C_{tW}$, $\mathrm{Re}C_{tB}$, $\mathrm{Re}C_{t\varphi}$, $C_{\varphi G}$, C_G:

$$\gamma = \frac{2\alpha_s}{\pi} \begin{pmatrix} \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{9}{8} \\ \frac{1}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 \\ \frac{5}{9} & 0 & \frac{1}{3} & 0 & 0 & 0 \\ -4 & 0 & 0 & -1 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 & \frac{N_f}{6} - \frac{11}{4} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{N_f}{6} + \frac{7}{4} \end{pmatrix}$$

CP-odd: $\mathrm{Im}C_{tG}$, $\mathrm{Im}C_{tW}$, $\mathrm{Im}C_{tB}$, $\mathrm{Im}C_{t\varphi}$, $C_{\varphi \tilde{G}}$:

$$\gamma = \frac{2\alpha_s}{\pi} \begin{pmatrix} \frac{1}{6} & 0 & 0 & 0 & 0 \\ \frac{1}{3} & \frac{1}{3} & 0 & 0 & 0 \\ \frac{5}{9} & 0 & \frac{1}{3} & 0 & 0 \\ -4 & 0 & 0 & -1 & 0 \\ -\frac{1}{4} & 0 & 0 & 0 & \frac{N_f}{6} - \frac{11}{4} \end{pmatrix}$$

$\mathcal{O}(\alpha_s)$ mixing, flavor diagonal

$$
\begin{aligned}
O_{bG} &= y_t g_s (\bar{Q}\sigma^{\mu\nu} T^A b)\varphi G^A_{\mu\nu} \\
O_{bW} &= y_t g_W (\bar{Q}\sigma^{\mu\nu} \tau^I b)\varphi W^I_{\mu\nu} \\
O_{bB} &= y_t g_Y (\bar{Q}\sigma^{\mu\nu} b)\varphi B_{\mu\nu} \\
O_{b\varphi} &= -y_t^3 (\varphi^\dagger \varphi)(\bar{Q}b)\varphi
\end{aligned}
$$

C_{tG}, C_{tW}, C_{tB}, $C_{t\varphi}$:

$$
\gamma = \frac{2\alpha_s}{\pi} \begin{pmatrix} \frac{1}{6} & 0 & 0 & 0 \\ \frac{1}{3} & \frac{1}{3} & 0 & 0 \\ -\frac{1}{9} & 0 & \frac{1}{3} & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}
$$

and $\gamma = 0$ for

$$
O^{(3)}_{\varphi Q} = i\frac{1}{2}y_t^2 \left(\varphi^\dagger \overleftrightarrow{D}^I_\mu \varphi\right)(\bar{Q}\gamma^\mu \tau^I Q), \qquad O^{(1)}_{\varphi Q} = i\frac{1}{2}y_t^2 \left(\varphi^\dagger \overleftrightarrow{D}_\mu \varphi\right)(\bar{Q}\gamma^\mu Q)
$$

$$
O_{\varphi t} = i\frac{1}{2}y_t^2 \left(\varphi^\dagger \overleftrightarrow{D}_\mu \varphi\right)(\bar{t}\gamma^\mu t), \qquad O_{\varphi b} = i\frac{1}{2}y_t^2 \left(\varphi^\dagger \overleftrightarrow{D}_\mu \varphi\right)(\bar{b}\gamma^\mu b), \quad O_{\varphi\varphi} = iy_t^2(\tilde{\varphi}^+ D_\mu \varphi)(\bar{t}\gamma^\mu b)
$$

$\mathcal{O}(\alpha_s)$ mixing, flavor changing

$$\begin{aligned}
O_{uG}^{(13)} &= y_t g_s (\bar{q}\sigma^{\mu\nu} T^A t)\tilde{\varphi} G^A_{\mu\nu} \\
O_{uW}^{(13)} &= y_t g_W (\bar{q}\sigma^{\mu\nu} \tau^I t)\tilde{\varphi} W^I_{\mu\nu} \\
O_{uB}^{(13)} &= y_t g_Y (\bar{q}\sigma^{\mu\nu} t)\tilde{\varphi} B_{\mu\nu} \\
O_{u\varphi}^{(13)} &= -y_t^3 (\varphi^\dagger \varphi)(\bar{q}t)\tilde{\varphi}
\end{aligned}$$

$$\gamma = \frac{2\alpha_s}{\pi} \begin{pmatrix} \frac{1}{6} & 0 & 0 & 0 \\ \frac{1}{3} & \frac{1}{3} & 0 & 0 \\ \frac{5}{9} & 0 & \frac{1}{3} & 0 \\ -2 & 0 & 0 & -1 \end{pmatrix}$$

$$\begin{aligned}
O_{dG}^{(13)} &= y_t g_s (\bar{q}\sigma^{\mu\nu} T^A b)\varphi G^A_{\mu\nu} \\
O_{dW}^{(13)} &= y_t g_W (\bar{q}\sigma^{\mu\nu} \tau^I b)\varphi W^I_{\mu\nu} \\
O_{dB}^{(13)} &= y_t g_Y (\bar{q}\sigma^{\mu\nu} b)\varphi B_{\mu\nu} \\
O_{d\varphi}^{(13)} &= -y_t^3 (\varphi^\dagger \varphi)(\bar{q}b)\varphi
\end{aligned}$$

$$\gamma = \frac{2\alpha_s}{\pi} \begin{pmatrix} \frac{1}{6} & 0 & 0 & 0 \\ \frac{1}{3} & \frac{1}{3} & 0 & 0 \\ -\frac{1}{9} & 0 & \frac{1}{3} & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

and $\gamma = 0$ for

$$\begin{aligned}
O_{\varphi q}^{(3,1+3)} &= i\frac{1}{2} y_t^2 \left(\varphi^\dagger \overleftrightarrow{D}^I_\mu \varphi\right)(\bar{q}\gamma^\mu \tau^I Q) \\
O_{\varphi q}^{(1,1+3)} &= i\frac{1}{2} y_t^2 \left(\varphi^\dagger \overleftrightarrow{D}_\mu \varphi\right)(\bar{q}\gamma^\mu Q)
\end{aligned}$$

$\mathcal{O}(\alpha_s)$ mixing, flavor changing

$$
\begin{aligned}
O_{uG}^{(31)} &= y_t g_s (\bar{Q}\sigma^{\mu\nu} T^A u)\tilde{\varphi} G_{\mu\nu}^A \\
O_{uW}^{(31)} &= y_t g_W (\bar{Q}\sigma^{\mu\nu} \tau^I u)\tilde{\varphi} W_{\mu\nu}^I \\
O_{uB}^{(31)} &= y_t g_Y (\bar{Q}\sigma^{\mu\nu} u)\tilde{\varphi} B_{\mu\nu} \\
O_{u\varphi}^{(31)} &= -y_t^3 (\varphi^\dagger \varphi)(\bar{Q}u)\tilde{\varphi}
\end{aligned}
$$

$$
\gamma = \frac{2\alpha_s}{\pi} \begin{pmatrix} \frac{1}{6} & 0 & 0 & 0 \\ \frac{1}{3} & \frac{1}{3} & 0 & 0 \\ \frac{5}{9} & 0 & \frac{1}{3} & 0 \\ -2 & 0 & 0 & -1 \end{pmatrix}
$$

$$
\begin{aligned}
O_{dG}^{(31)} &= y_t g_s (\bar{Q}\sigma^{\mu\nu} T^A d)\varphi G_{\mu\nu}^A \\
O_{dW}^{(31)} &= y_t g_W (\bar{Q}\sigma^{\mu\nu} \tau^I d)\varphi W_{\mu\nu}^I \\
O_{dB}^{(31)} &= y_t g_Y (\bar{Q}\sigma^{\mu\nu} d)\varphi B_{\mu\nu} \\
O_{d\varphi}^{(31)} &= -y_t^3 (\varphi^\dagger \varphi)(\bar{Q}d)\varphi
\end{aligned}
$$

$$
\gamma = \frac{2\alpha_s}{\pi} \begin{pmatrix} \frac{1}{6} & 0 & 0 & 0 \\ \frac{1}{3} & \frac{1}{3} & 0 & 0 \\ -\frac{1}{9} & 0 & \frac{1}{3} & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}
$$

and $\gamma = 0$ for

$$
\begin{aligned}
O_{\varphi u}^{(1+3)} &= i\frac{1}{2}y_t^2 \left(\varphi^\dagger \overleftrightarrow{D}_\mu \varphi\right)(\bar{t}\gamma^\mu u) \\
O_{\varphi d}^{(1+3)} &= i\frac{1}{2}y_t^2 \left(\varphi^\dagger \overleftrightarrow{D}_\mu \varphi\right)(\bar{b}\gamma^\mu d) \\
O_{\varphi\varphi}^{(13)} &= i y_t^2 (\tilde{\varphi}^+ D_\mu \varphi)(\bar{t}\gamma^\mu d) \\
O_{\varphi\varphi}^{(31)} &= i y_t^2 (\varphi^+ D_\mu \tilde{\varphi})(\bar{b}\gamma^\mu u)
\end{aligned}
$$

Analytical result, $t \to b + W$

$x = m_W/m_t$

$$\Gamma^{(+)} = \frac{\mathrm{Re}C_{tG}}{\Lambda^2}\frac{\alpha\alpha_s m_t^3}{6\pi s_W^2 x^2}\left\{x\log(x)\left[(1+x)^3\log(1+x)-(1-x)^3\log(1-x)\right]-2x^2(1-x^2)\log(1+x)\right.$$
$$+x^2\left[(x-1)(x+9)/2+(2+x^2)\pi^2/3-(2+3x^2)\log x\right]$$
$$\left.+x\left[(x^3-3x^2+3x-1)\mathrm{Li}_2(x)+(3x^3+3x^2+5x+1)\mathrm{Li}_2(-x)\right]\right\}$$

$$\Gamma^{(0)} = \frac{\mathrm{Re}C_{tG}}{\Lambda^2}\frac{\alpha\alpha_s m_t^3}{6\pi s_W^2 x^2}\left\{x^2(1-x^2)^2\log\frac{mt^2}{\mu^2}-(1-x^2)^3\log(1-x^2)\right.$$
$$-2x\log x\left[(1+x)^3\log(1+x)-(1-x)^3\log(1-x)-\frac{x}{2}(9x^2+4)\right]$$
$$\left.-(26x^6+33x^4-66x^2+7)/12-\pi^2x^2(3+x^2)/3+2x\left[(1-x)^3\mathrm{Li}_2(x)-(1+x)^3\mathrm{Li}_2(-x)\right]\right\}$$

$$\Gamma^{(-)} = \frac{\mathrm{Re}C_{tG}}{\Lambda^2}\frac{\alpha\alpha_s m_t^3}{6\pi s_W^2 x^2}\left\{2x^2(1-x^2)^2\log\frac{mt^2}{\mu^2}+x\log x\left[(1+x)^3\log(1+x)-(1-x)^3\log(1-x)\right]\right.$$
$$-(1-x^2)\left[(1-4x^2+x^4)\log(1+x)+(1-x^2)^2\log(1-x)\right]-x^2(2+3x^2)\log x$$
$$\left.-\frac{x^2}{2}(11x^4-31x^2+8x+12)+\frac{\pi^2x^2}{3}-x\left[(1-3x+3x^2-x^3)\mathrm{Li}_2(x)-(1+x+3x^2-x^3)\mathrm{Li}_2(-x)\right]\right\}$$

Analytical result, $t \to c + h$

$$\Gamma^{(0)} = \frac{|C_{u\varphi}|^2}{\Lambda^4} \frac{\alpha m_t^7}{8 m_W^2 s_W^2} \left(1 - \frac{m_H^2}{m_t^2}\right)^2$$

$x = m_H / m_t$

$$\begin{aligned}
\frac{\Gamma^{(1)}}{\Gamma^{(0)}} =& \frac{\alpha_s}{36\pi(1-x^2)^2} \frac{|C_{uG}|^2}{|C_{u\varphi}|^2} \Big[x^8 - 8x^6 - 342x^4 + 620x^2 - 271 \\
&+ 6x\sqrt{4-x^2}(26-5x^2)\left(\pi - 6\sin^{-1}\frac{x}{2}\right) + 12(9x^4 + 76x^2 - 8)\log x \Big] \\
&- \frac{2\alpha_s}{9\pi(1-x^2)^2} \frac{\mathrm{Re}(C_{uG}C_{u\varphi}^*)}{|C_{u\varphi}|^2} \Bigg\{ 6\Bigg[6(1-x^2)^2 \log\frac{m_t}{\mu} + (5x^4 + 2x^2 + 4\log(1-x^2) - 2\log x)\log x \\
&\quad + \left(\sqrt{\frac{4}{x^2} - 1}(x^4 - 6x^2 + 8) + 2\pi\right)\sin^{-1}\frac{x}{2} + 6\left(\sin^{-1}\frac{x}{2}\right)^2 \Bigg] \\
&+ 12\left[\mathrm{Li}_2(x^2) - 2\mathrm{ReLi}_2\left(\left(x - \frac{1}{x}\right)\left(\frac{x}{2} - i\sqrt{1 - \frac{x^2}{4}}\right)\right)\right] \\
&+ \left[3\pi\sqrt{4-x^2}(x^2-2)x - 3(x^4 + 8x^2 - 9)x^2 - 5\pi^2 \right] \Bigg\} \\
&- \frac{\alpha_s}{9\pi}\Bigg[\left(36\log\frac{m_t}{\mu} + 4\pi^2 - 51\right) + 24\mathrm{Li}_2 x^2 + 24\log x \log(1-x^2) \\
&+ 24\frac{x^2}{1-x^2}\log x + 6\left(5 - \frac{2}{x^2}\right)\log(1-x^2) \Bigg]
\end{aligned}$$