EFFECTIVE THEORY OF SELF INTERACTING DARK MATTER

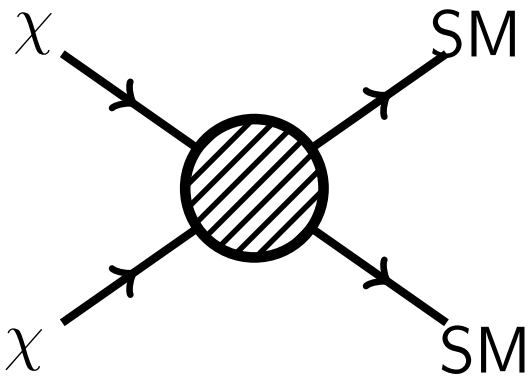
Mathieu Cliche

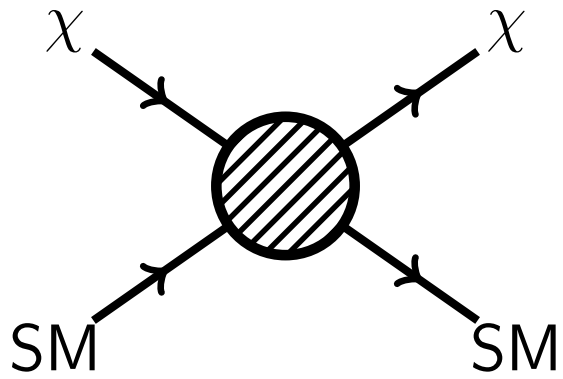
In collaboration with Brando Bellazzini and Flip Tanedo

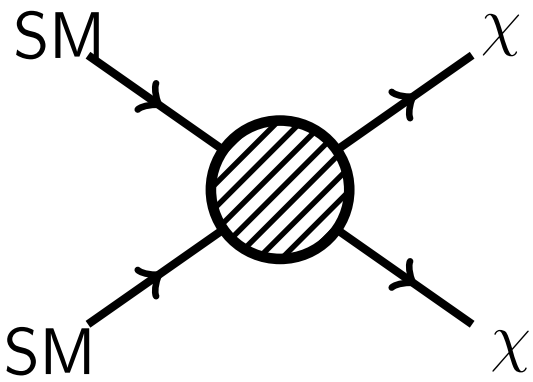
May 7, 2013

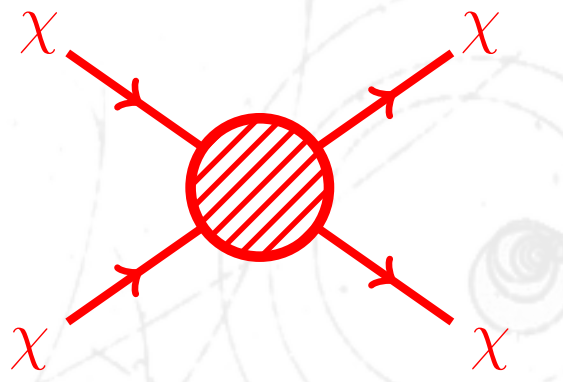
Introduction

Dark matter interactions:

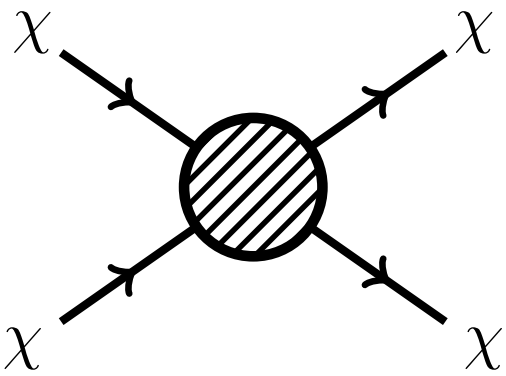




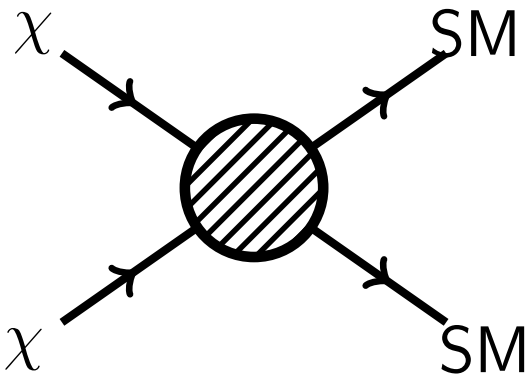




σ_{elast}



σ_{ID}



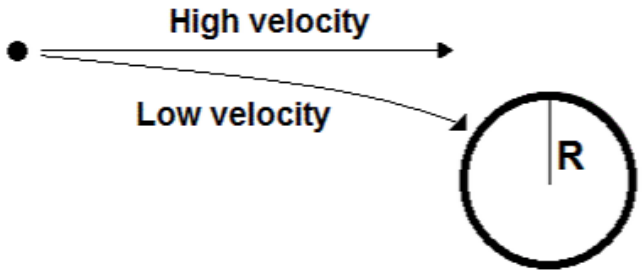
Indirect detection and elastic scattering can feel the dark matter self interaction at low velocity.

Possible consequences:

- Positron excess and γ signals
- Core vs cusp problem

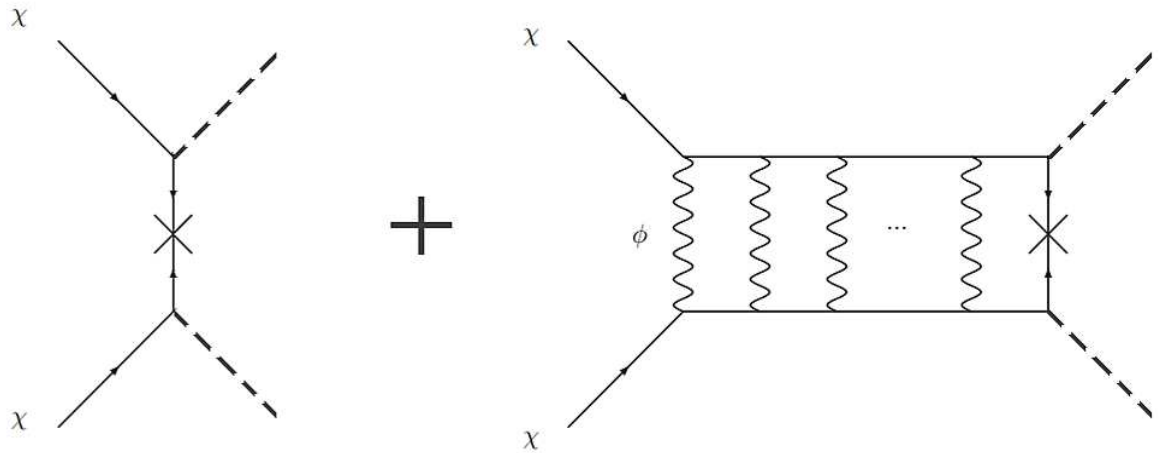
Sommerfeld enhancement: Intuition

Classical intuition:



$$\sigma = \pi R^2 \rightarrow \pi R^2 \left(1 + \frac{v_{esc}^2}{v^2}\right)$$

QFT intuition:

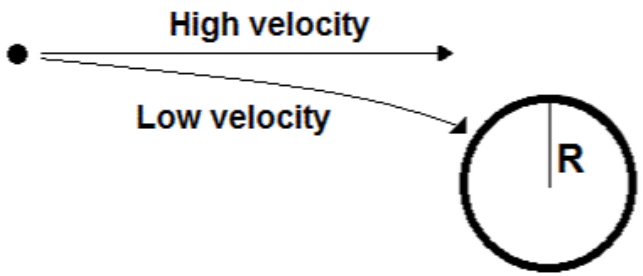


Sommerfeld enhancement: Derivation

- Annihilation interaction is short range: $H_{ann} = U_{ann}\delta^3(\vec{r})$.
- Solve the Schrodinger equation with the long range potential $V(r)$:

$$-\frac{1}{2M}\nabla^2\psi_k(r) + V(r)\psi_k(r) = \frac{k^2}{2M}\psi_k$$

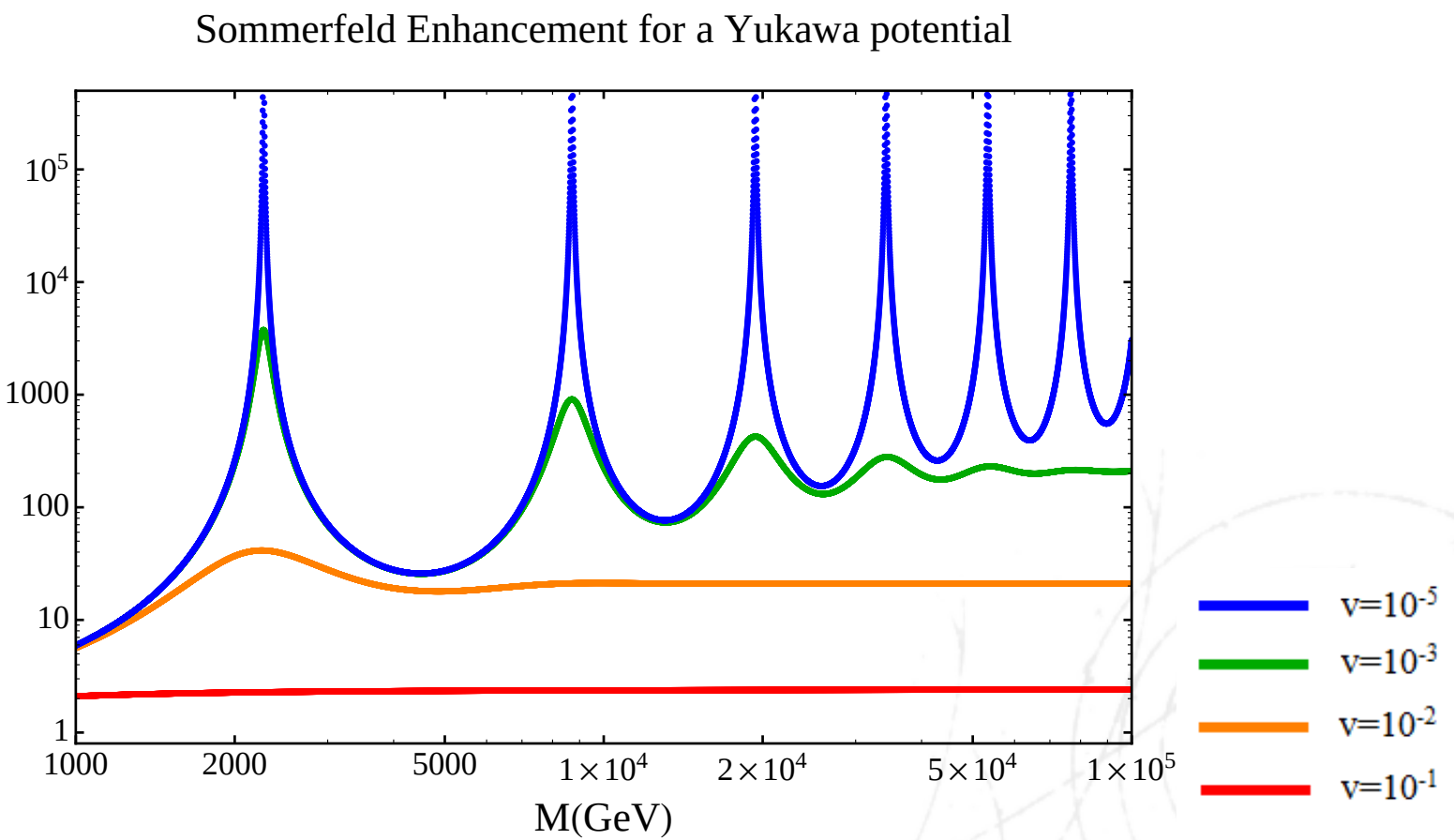
- Extract Sommerfeld enhancement S: $\sigma = |\psi_k(0)|^2\sigma_0$.



Sommerfeld enhancement: Previous results

Coulomb$\rightarrow \mathcal{L} = -e\bar{\psi}\gamma^{\mu}\psi A_{\mu} \rightarrow V(r) = -\alpha/r$: $S \approx 2\pi\left(\frac{\alpha}{v}\right)$.

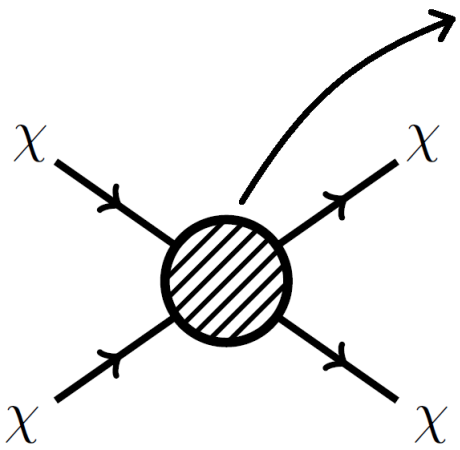
Yukawa $\rightarrow \mathcal{L} = -g\bar{\psi}\phi\psi \rightarrow V(r) = -\alpha e^{-m_{\phi}r}/r$:



Arkani-Hamed 0810.0713, Lattanzi 0812.0360

Effective self-interactions I

We want to be **agnostic** about the possible self interactions.



$$\int_0^\infty d\mu^2 \frac{\rho(\mu^2)}{|\vec{q}|^2 + \mu^2}$$

$$\mathcal{M} = \int_0^\infty d\mu^2 \frac{\rho(\mu^2)}{|\vec{q}|^2 + \mu^2} \sum_i g_i \mathcal{O}_i \left(\frac{\vec{q}}{\Lambda}, \vec{s}_{1,2} \right)$$

Effective self-interactions II

List of Parity invariant QM operators

$$\begin{aligned}\mathcal{O}_1 =& 1\,,\\ \mathcal{O}_2 =& \vec{s}_1\cdot\vec{s}_2\,,\\ \mathcal{O}_3 =& \frac{1}{\Lambda^2}(\vec{s}_1\cdot\vec{q})(\vec{s}_2\cdot\vec{q})\end{aligned}$$

These terms give rise to an **effective potential**

$$V_{\rm eff}^P = \left[g_1 + g_2(\vec{s}_1\cdot\vec{s}_2) + \frac{g_3}{\Lambda^2 r^2}\left(3\vec{s}_1\cdot\hat{r}\;\;\vec{s}_2\cdot\hat{r} - \vec{s}_1\cdot\vec{s}_2\right)\right] V_0(r)$$

where $V_0(r)$ is

$$V_0(r) = \int d\mu^2 \rho(\mu^2)\frac{e^{-\mu r}}{r}\,.$$

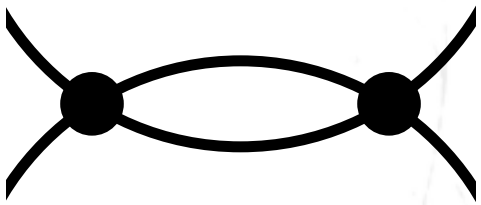
Effective self interactions III

Example of exotic potentials:

- Tree level gives $V_0(r) \propto \frac{1}{r}$.
- Strong dynamics can generate $\rho(\mu^2)$.
- Weak coupling, but loop level $V_0(r) \sim \{\frac{1}{r^3}, \frac{1}{r^5}, \frac{1}{r^7}\}$.

  Example:

$$\mathcal{L} = \bar{f}\gamma^\mu(1-\gamma^5)f\bar{\chi}\gamma_\mu(a-b\gamma^5)\chi$$

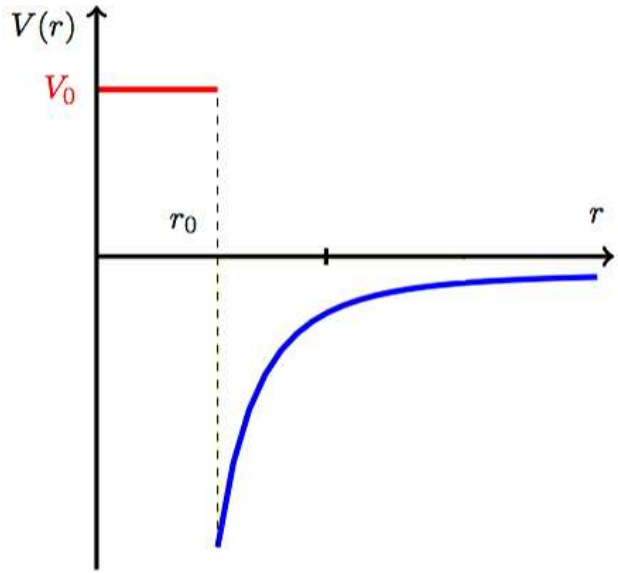


$$\rightarrow V(r) \propto \frac{1}{r^5}\left[a^2 - b^2\left(\frac{3}{2}s_1\cdot s_2 - \frac{5}{2}(\hat{r}\cdot s_1)(\hat{r}\cdot s_2)\right)\right]$$

Singular potentials I

To deal with singular potentials we apply the following recipe:

1. **Regularize** the potential.

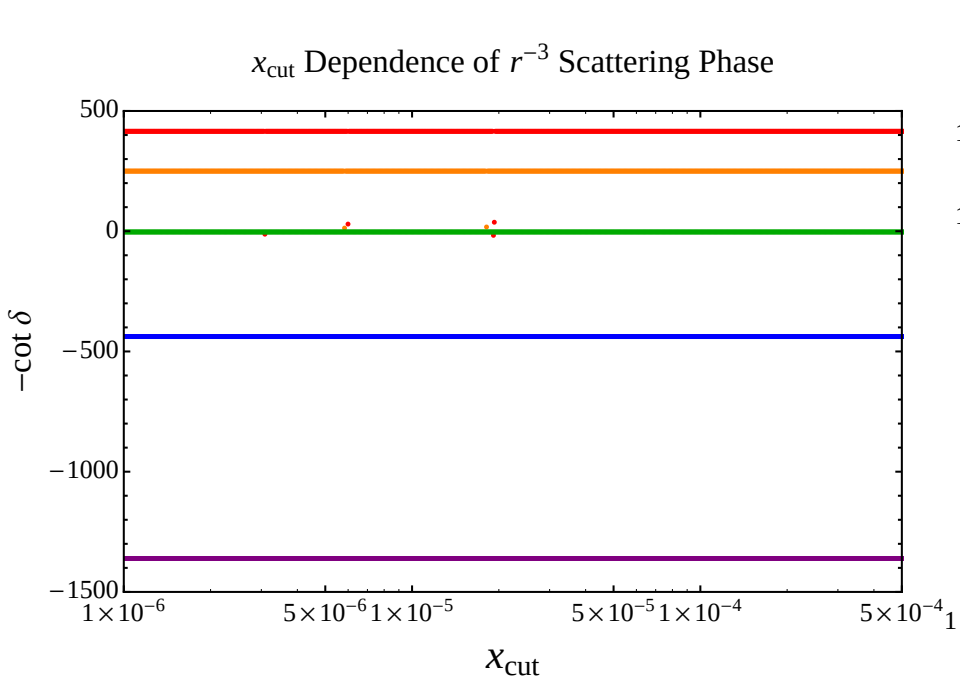


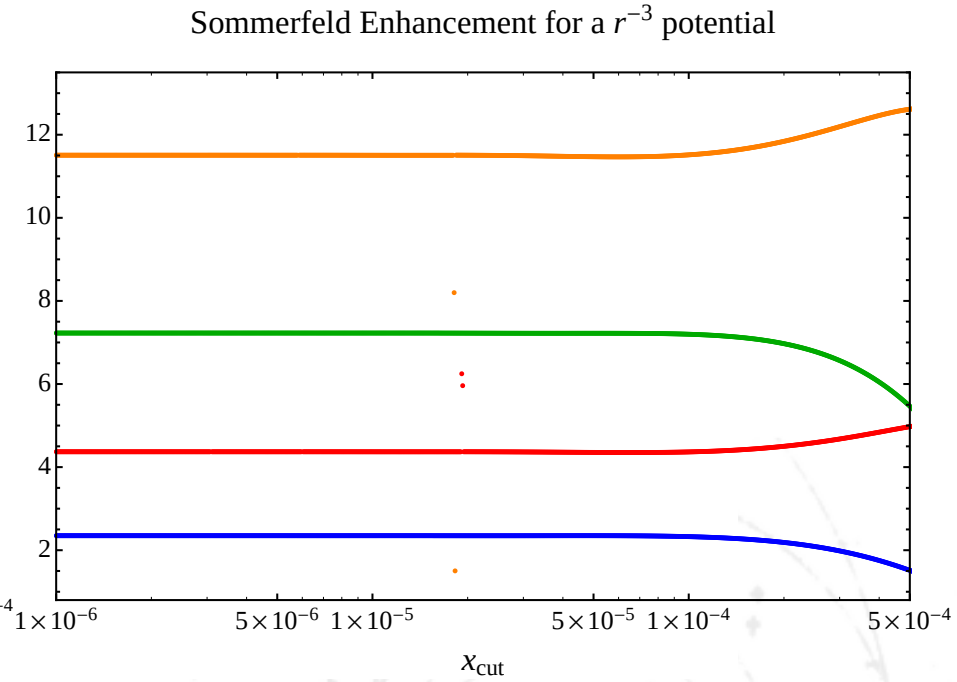
$$V(r) \longrightarrow V(r)\theta(r - r_0) + V_0\theta(r_0 - r)$$

2. **Renormalize** by matching with IR observables.

Singular potentials II

Cutoff independence with $V(r) = -\frac{\alpha}{f^2 r^3}$.

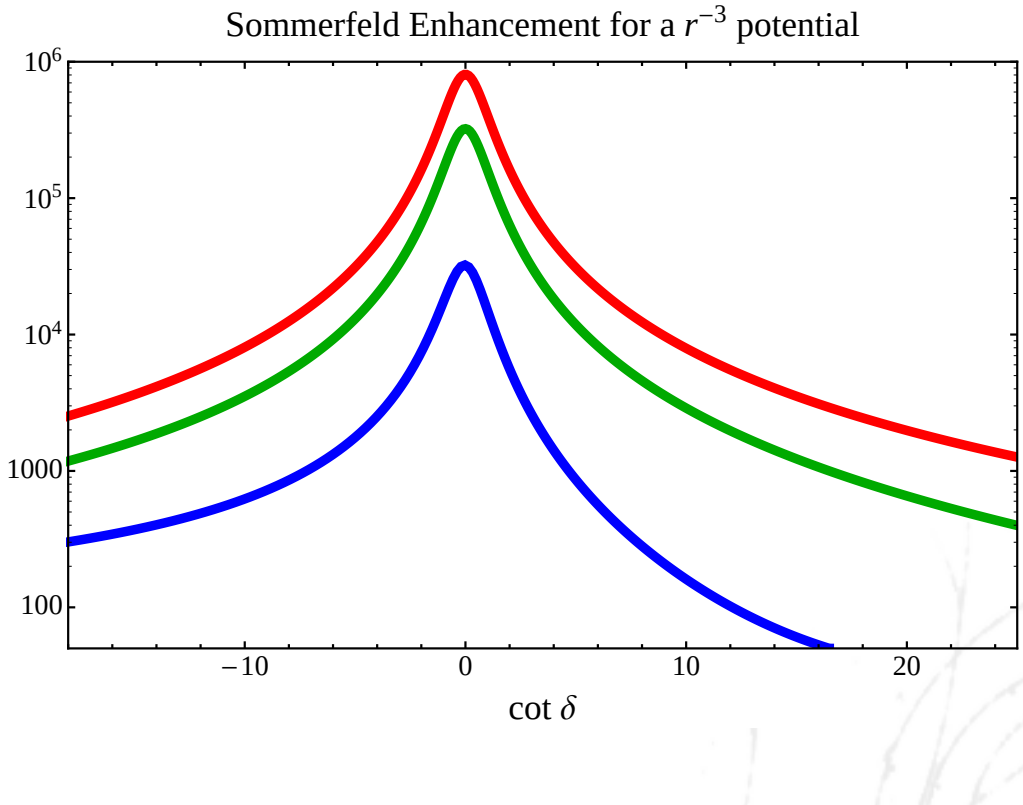




Bedaque 0907.0235

Numerical results

We can use this recipe to plot Sommerfeld enhancement for $V(r) = \frac{-\alpha}{f^2 r^3}$ as a function of $\cot(\delta)$:



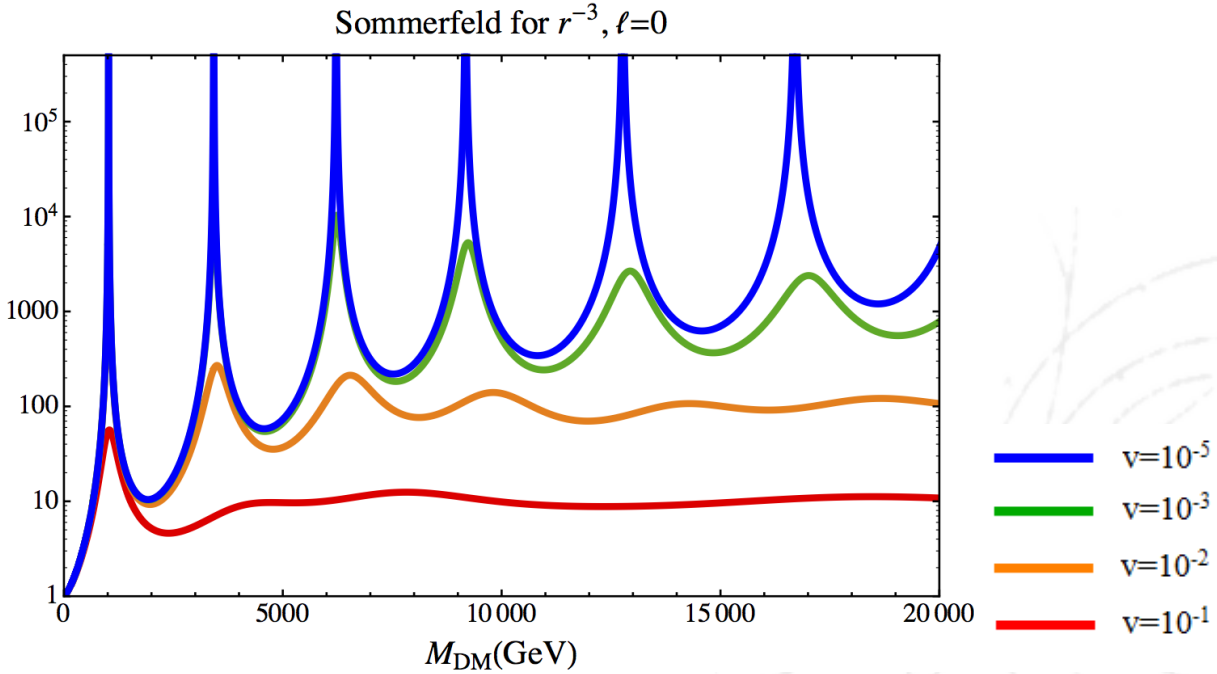
Recall $\sigma \approx \frac{4\pi}{k^2 \cot^2(\delta)}$, and .

NDA approximations

Problem: $\cot(\delta)$ depends on the UV.

Solution: A NDA type of analysis.

1. Physical cutoff: Relativistic $V(r_0) = M$
2. Fix V_0 using $V_0 = V(r_0)$.



Conclusion

To sum up:

- We provided the parametrization of self interacting dark matter in the non-relativistic regime.
- We showed how to renormalize singular QM potentials.

Possible future directions:

- Further astro phenomenology.