The NRQED Lagrangian at order $1/M^4$

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Gabriel Lee

University of Chicago
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Outline

Motivation

- Proton charge radius puzzle. Pohl et al. (2010)
- Two-photon radiative corrections to $ep, \mu p$ scattering. e.g., Arrington et al. (2011)
- Lattice calculations. Christensen et al. (2004), Detmold et al. (2006, 2010)

A Primer on Heavy Quark Effective Theory (HQET)

- The momenta of heavy fermions with velocity v can be written as $p = mv + k$, where k is a residual momentum from interactions.
- At tree-level, decompose the fermion appearing in QED/QCD Lagrangian $\mathcal{L} = \overline{Q}(i\not{D} - m)Q$ into two velocity-dependent fields:

$$Q(x) = e^{-imv\cdot x}(Q_v(x) + \mathcal{Q}_v(x)),$$
$$Q_v(x) = e^{imv\cdot x}\frac{1+\not{v}}{2}Q(x), \qquad \mathcal{Q}_v(x) = e^{imv\cdot x}\frac{1-\not{v}}{2}Q(x),$$
$$\not{v}Q_v = Q_v, \qquad \not{v}\mathcal{Q}_v = -\mathcal{Q}_v$$

- Integrating out the field $\mathcal{Q}_v$ gives the point-particle HQET Lagrangian

$$\mathcal{L} = \overline{Q}_v\Big(iv\cdot D - i\not{D}_\perp \frac{1}{iv\cdot D + 2m} i\not{D}_\perp\Big)Q_v.$$

where $D_\perp^\mu = D^\mu - v^\mu v\cdot D$.

- However, for composite particles, there is no analogy of a "full Lagrangian". How do we construct the effective field theory (EFT)?

Operator Basis: One-Fermion Sector

- For a general heavy fermion in a background electromagnetic field, we construct all PT-conserving, rotationally invariant, Hermitian operators with $\boldsymbol{\sigma}, \boldsymbol{E}, \boldsymbol{B}, \boldsymbol{D}, D_t$, i.e. with $v = (1,0,0,0)$.
- Up to order $1/M^2$, the NRQED Lagrangian is

$$\mathcal{L} = \psi^\dagger \left\{ iD_t + c_2 \frac{\boldsymbol{D}^2}{2M} + c_F g \frac{\boldsymbol{\sigma}\cdot\boldsymbol{B}}{2M} + c_D g \frac{[\boldsymbol{\partial}\cdot\boldsymbol{E}]}{8M^2} + ic_S g \frac{\boldsymbol{\sigma}\cdot(\boldsymbol{D}\times\boldsymbol{E} - \boldsymbol{E}\times\boldsymbol{D})}{8M^2} + \dots \right\}\psi.$$

- Terms up to $\mathcal{O}(1/M^3)$ are written in Manohar (1997).
- We find 12 new operators at $1/M^4$:

Spin	1 field strength	2 field strengths
SI	3	1
SD	2	6

Lorentz Invariance

- Lorentz invariance is not broken in the effective theory; we can use this to constrain Wilson coefficients.
- There are two methods:
  - Variational method: enforce invariance under infinitesimal boosts.
  - Invariant operators: construct Lagrangian from quantities that transform with a phase under a reparametrization invariance. Luke & Manohar (1992)
- Implementation of the latter method detailed in the literature yields discrepancies at order $1/M^4$. Heinonen et al. (2012)
- Enforcing Lorentz invariance constrains the number of new (SI, SD) coefficients at each order in $1/M$ in the expansion:

$$1/M : (0,1); \qquad 1/M^2 : (1,0); \qquad 1/M^3 : (1,2); \qquad 1/M^4 : (1,6).$$

$$\{\boldsymbol{D}^i, [\boldsymbol{E}^i \times \boldsymbol{B}^i]\}, \quad \boldsymbol{\sigma} \cdot \boldsymbol{B}[\boldsymbol{\partial} \cdot \boldsymbol{E}] + (2 \text{ perm.}),$$

$$[E^i \boldsymbol{\sigma} \cdot \boldsymbol{\partial} B^i] + (1 \text{ perm.}), \quad \boldsymbol{\sigma} \cdot \boldsymbol{E} \times [\partial_t \boldsymbol{E} - \boldsymbol{\partial} \times \boldsymbol{B}].$$

Operator Basis: Two-Fermion Sector

- We can also write operators for a heavy fermion coupled to another heavy fermion or a relativistic fermion.
- For the heavy-heavy case:

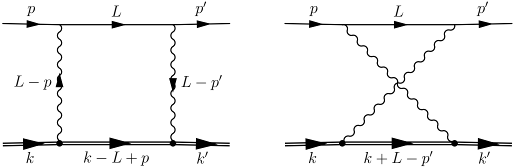
$$\mathcal{L}_{\psi\chi} = \frac{d_1}{M^2}\psi^\dagger\sigma^i\psi\ \chi^\dagger\sigma^i\chi + \frac{d_2}{M^2}\psi^\dagger\psi\ \chi^\dagger\chi + \dots$$

- 19 operators at $1/M^4$, 10 independent from Lorentz invariance.
- In the particle-antiparticle case, our results reproduce Brambilla et al. (2009).
- For the heavy-relativistic case:

$$\mathcal{L}_{\psi\ell} = \frac{b_1}{M^2}\psi^\dagger\psi\ \bar{\ell}\gamma^0\ell + \frac{b_2}{M^2}\psi^\dagger\sigma^i\psi\ \bar{\ell}\gamma^i\gamma_5\ell + \dots$$

- 7 operators at $1/M^3$, 5 independent from Lorentz invariance.
- For a dynamical photon field, we can also include the pure gauge sector, the Euler-Heisenberg Lagrangian.

Radiative Corrections to Lepton-Nucleon Scattering



- Extraction of hadronic quantities from scattering observables requires a computation of radiative corrections.
- At low energies, NRQED provides a systematic and model-independent approach to computing corrections and to relating observables at different energies.
- Leading $\mathcal{O}(\alpha)$ correction to $ep \to ep$, $m_e \ll E \ll M$:

$$\mathcal{M} = \frac{4\pi\alpha}{Q^2}\bar{u}(p)\gamma^0 u(p)\left\{1 + \alpha\left[\frac{\pi}{2}\frac{Q}{2E+Q} + i\left(-2\log\frac{\lambda}{Q} + \frac{Q^2}{(2E)^2 - Q^2}\log\frac{Q}{2E}\right)\right]\right.$$

- $1/M : c_F$, $1/M^2 : c_D, b_1, b_2$.
- Can also perform the analysis $E \ll m_\mu \sim M$ using the heavy-heavy case.

Spin polarizabilities and static properties of nucleons

- NRQED can be used to relate parameters in scattering observables with those in static nucleon properties.
- For a constant external EM field, shifts in energy of a heavy neutral particle can be written as:

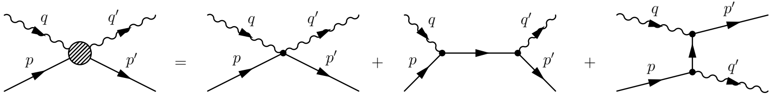
$$\delta M(\boldsymbol{E},\boldsymbol{B}) = \lim_{\boldsymbol{p}\to 0} \Sigma(\boldsymbol{p}) = c_F g \frac{\boldsymbol{\sigma}\cdot\boldsymbol{B}}{2M} + c_{A1} g^2 \frac{\boldsymbol{B}^2 - \boldsymbol{E}^2}{8M^3} - c_{A2} g^2 \frac{\boldsymbol{E}^2}{16M^3} + \dots,$$

- Scalar electric and magnetic polarizabilities as defined in Compton scattering are not proportional to c_{A1}, c_{A2}, but

$$\frac{4M^3}{\alpha}\alpha_E = -c_{A1} - \frac{1}{2}c_{A2} + Z^2 + 2Zc_M + c_F c_S - c_F^2\,,$$

$$\frac{4M^3}{\alpha}\beta_M = c_{A1} - Z^2\,.$$

Matching to Virtual Compton Scattering



- Match the tree-level virtual Compton scattering amplitude in the full theory and NRQED by a conventional separation,

$$\mathcal{M}^{\mu\nu} \equiv \mathcal{M}^{\mu\nu}_{\text{Born}} + \mathcal{M}^{\mu\nu}_{\text{non-Born}} \,,$$

 using "Sticking in Form Factors" to compute the Born term.
- The non-Born term can be expressed in terms of a basis of 12 functions. Drechsel et al. (1998)
- Real Compton scattering determines 4 of the 6 SD Wilson coefficients at $1/M^4$; these can be expressed in terms of spin polarizabilities of Ragusa (1993).

Future Work

- NRQCD.
  - Generalization to non-Abelian case.
- Spin-one heavy vector.
  - Higher-order correction for deuterium bound states.
- Parity-violating ep scattering.
  - Better understand SM prediction before considering BSM contributions.
- Higher-order $1/M$, α radiative corrections to ep, μp scattering.
  - Corrections beyond Rosenbluth separation at low Q^2.
  - Backgrounds to dark boson searches, e.g. APEX, DarkLight, HPS at JLAB.
- Applications to effective theories of dark matter.
  - Most DM EFT's implement Galilean invariance. e.g. Fitzpatrick et al. (2012)

Conclusion

- We have derived the complete NRQED Lagrangian for a heavy fermion to order $1/M^4$.
- We have corrected the existing formalism in the literature for implementing Lorentz invariance in heavy particle EFT's.
- We demonstrated that NRQED continues to be a useful theory for describing interactions of heavy particles with electromagnetic probes, and that it provides a systematic approach to relating static nucleon properties and scattering observables.
- More applications to explore.

Full NRQED Lagrangian to Order $1/M^4$

$$
\begin{aligned}
\mathcal{L} = \psi^\dagger \Big\{ & iD_t + c_2 \frac{\boldsymbol{D}^2}{2M} + c_4 \frac{\boldsymbol{D}^4}{8M^3} + c_F g \frac{\boldsymbol{\sigma}\cdot\boldsymbol{B}}{2M} + c_D g \frac{[\boldsymbol{\partial}\cdot\boldsymbol{E}]}{8M^2} + ic_S g \frac{\boldsymbol{\sigma}\cdot(\boldsymbol{D}\times\boldsymbol{E} - \boldsymbol{E}\times\boldsymbol{D})}{8M^2} \\
& + c_{W1} g \frac{\{\boldsymbol{D}^2, \boldsymbol{\sigma}\cdot\boldsymbol{B}\}}{8M^3} - c_{W2} g \frac{D^i \boldsymbol{\sigma}\cdot\boldsymbol{B} D^i}{4M^3} + c_{p'p} g \frac{\boldsymbol{\sigma}\cdot\boldsymbol{D}\boldsymbol{B}\cdot\boldsymbol{D} + \boldsymbol{D}\cdot\boldsymbol{B}\boldsymbol{\sigma}\cdot\boldsymbol{D}}{8M^3} \\
& + ic_M g \frac{\{\boldsymbol{D}^i, [\boldsymbol{\partial}\times\boldsymbol{B}]^i\}}{8M^3} + c_{A1} g^2 \frac{\boldsymbol{B}^2 - \boldsymbol{E}^2}{8M^3} - c_{A2} g^2 \frac{\boldsymbol{E}^2}{16M^3} \\
& + c_{X1} g \frac{[\boldsymbol{D}^2, \boldsymbol{D}\cdot\boldsymbol{E} + \boldsymbol{E}\cdot\boldsymbol{D}]}{M^4} + c_{X2} g \frac{\{\boldsymbol{D}^2, [\boldsymbol{\partial}\cdot\boldsymbol{E}]\}}{M^4} + c_{X3} g \frac{[\boldsymbol{\partial}^2 \boldsymbol{\partial}\cdot\boldsymbol{E}]}{M^4} \\
& + ic_{X4} g^2 \frac{\{\boldsymbol{D}^i, [\boldsymbol{E}\times\boldsymbol{B}]^i\}}{M^4} \\
& + ic_{X5} g \frac{D^i \boldsymbol{\sigma}\cdot(\boldsymbol{D}\times\boldsymbol{E} - \boldsymbol{E}\times\boldsymbol{D}) D^i}{M^4} + ic_{X6} g \frac{\epsilon^{ijk}\sigma^i D^j [\boldsymbol{\partial}\cdot\boldsymbol{E}] D^k}{M^4} \\
& + c_{X7} g^2 \frac{\boldsymbol{\sigma}\cdot\boldsymbol{B}[\boldsymbol{\partial}\cdot\boldsymbol{E}]}{M^4} + c_{X8} g^2 \frac{[\boldsymbol{E}\cdot\boldsymbol{\partial}\boldsymbol{\sigma}\cdot\boldsymbol{B}]}{M^4} + c_{X9} g^2 \frac{[\boldsymbol{B}\cdot\boldsymbol{\partial}\boldsymbol{\sigma}\cdot\boldsymbol{E}]}{M^4} \\
& + c_{X10} g^2 \frac{[E^i \boldsymbol{\sigma}\cdot\boldsymbol{\partial} B^i]}{M^4} + c_{X11} g^2 \frac{[B^i \boldsymbol{\sigma}\cdot\boldsymbol{\partial} E^i]}{M^4} + c_{X12} g^2 \frac{\boldsymbol{\sigma}\cdot\boldsymbol{E}\times[\partial_t \boldsymbol{E} - \boldsymbol{\partial}\times\boldsymbol{B}]}{M^4} \Big\} \psi \,.
\end{aligned}
$$

Constraints from Lorentz Invariance: One-Fermion

$$c_2 = 1\,, \quad c_S = 2c_F - Z\,, \quad c_4 = 1\,,$$
$$2c_M = c_D - c_F\,, \quad c_{W2} = c_{W1} - Z\,, \quad c_{p'p} = c_F - Z\,.$$

$$\begin{aligned}
32c_{X1} &= \frac{5Z}{4} - c_F + c_D\,,\\
32c_{X2} &= -\frac{Z}{2} + 2c_F\,,\\
32c_{X4} &= -Z^2 - 4c_F(Z - c_F) + 4Zc_D - 2c_{A2}\,,\\
32c_{X5} &= -Z + 4c_F\,,\\
32c_{X6} &= 4(Z - c_F) + c_D - 4c_{W1}\,,
\end{aligned}$$

Invariant Operators

$$\Psi_v = \Gamma \psi_v, \quad \Psi_v \to e^{iq\cdot x}\Psi_v,$$

$$\Gamma = 1 + \frac{i\not{D}_\perp}{2M} + \frac{1}{M^2}\left\{-\frac{1}{8}(iD_\perp)^2 - \frac{1}{2} i\not{D}_\perp iv\cdot D\right\} + \frac{1}{M^3}\left\{\frac{1}{4}(iD_\perp)^2 iv\cdot D \right.$$
$$\left. + \frac{i\not{D}_\perp}{2}\left[-\frac{3}{8}(iD_\perp)^2 + (iv\cdot D)^2\right] + \frac{gZ}{8}F_{\mu\nu}v^\mu D_\perp^\nu + \frac{gZ}{16}\sigma_\perp^{\mu\nu}F_{\mu\nu} i\not{D}_\perp\right\} + \ldots,$$

$$\begin{aligned}\mathcal{L} &= \mathcal{L}(\Psi_v, v^\mu + iD^\mu/M) \\ &= \overline{\Psi}_v\left\{M(/\!\!\!\mathcal{V} - 1) - a_F g\frac{\sigma^{\mu\nu}F_{\mu\nu}}{4M} + ia_D g\frac{\{\mathcal{V}_\mu, [M\mathcal{V}_\nu, F^{\mu\nu}]\}}{16M^2}\right. \\ &\quad \left. - a_{W1}g\frac{[M\mathcal{V}^\alpha, [M\mathcal{V}_\alpha, \sigma^{\mu\nu}F_{\mu\nu}]]}{16M^3} + a_{A1}g^2\frac{F_{\mu\nu}F^{\mu\nu}}{16M^3} + a_{A2}g^2\frac{\mathcal{V}_\alpha F^{\mu\alpha}F_{\mu\beta}\mathcal{V}^\beta}{16M^3}\right\}\Psi_v \\ &\quad + a_{X3}\mathcal{B}_{X3} + \sum_{i=7}^{12} a_{Xi}\mathcal{B}_{Xi}.\end{aligned}$$

Constraints from Lorentz Invariance: Two-Fermion

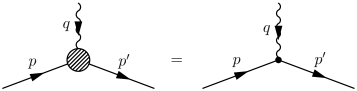
- Heavy-heavy:

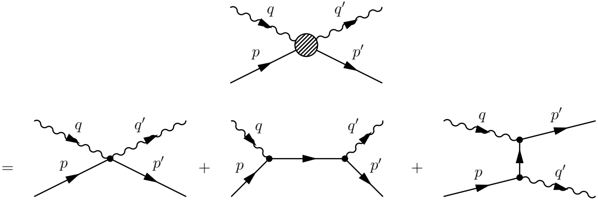
$$
\begin{aligned}
rd_4 + d_5 &= \frac{d_2}{4}\,, \qquad d_5 = r^2 d_6\,,\\
8r(d_8 + rd_9) &= -rd_2 + d_1\,, \qquad 8r(rd_{11} + d_{12}) = -d_2 + rd_1\,,\\
rd_{14} + d_{18} &= \frac{d_1}{4}\,, \qquad d_{18} = r^2 d_{20}\,,\\
2rd_{16} + d_{19} &= \frac{d_1}{4}\,,\\
r(d_{16} + d_{17}) + d_{19} &= 0\,, \qquad d_{19} = r^2 d_{21}\,.
\end{aligned}
$$

- Heavy-relativistic:

$$
b_4 = -\frac{1}{2} b_1\,, \quad b_9 = -\frac{1}{2} b_2\,.
$$

Diagrams for Matching





$$\mathcal{M}^{\mu\nu}_{\text{Born}} \equiv -g^2 \bar{u}(p') \left\{ \Gamma^\nu(-q') \frac{1}{\not{p} + \not{q} - M} \Gamma^\mu(q) + \Gamma^\mu(q) \frac{1}{\not{p} - \not{q}' - M} \Gamma^\nu(-q') \right\} u(p) \,.$$